A Comparative Evaluation of Damage Influence on Aircraft Structure Safety

Stanisław Danilecki
Warsaw University of Technology
Institute of Aeronautics and Applied Mechanics
Warsaw – Poland

Abstract

The analysis concerns a comparison of Lundberg and Eggwertz's concept of the fatigue life of aircraft structure with a new approach to the problem based on some probabilistic premises. The obtained theoretical and experimental results confirm that the new approach can be more effective and useful when analysing the safety of aircraft structure. Procedure for the changes of safety evaluation is worked out and presented, especially when the changes of safety are induced by cracks occurring in the structure. The proposed Admissible Damage Coefficient (ADC) can be applied as a measure in the analysis. A number of practical examples are presented.

1. Introduction

Lundberg and Eggwertz's method of calculating the fatigue life of aircraft structure, introduced in the 1960's and applied so far, is based on the hypothesis of the linear fatigue wear cumulation. This approach makes use of a generally accepted concept of the fatigue wear, defined earlier by Miner. The representative spectrum course of external loadings, changing in time, shall be known to determine the life by this method. Combining these two pieces of information allows for determination of the fatigue life, frequently called the operation life, by the well known formula:

$$T=\frac{L_0}{DV}, \qquad (1)$$

where: L_o - number of kilometres being the measurement basis for the accepted spectrum of loads [5],

V - average operational speed of the aircraft,

D - the fatigue wear, calculated from the formula:

$$D=\frac{H_0}{\alpha}h^{-\beta}e^{-h\delta_z}\Gamma(\beta+1), \qquad (2)$$

where: H_0, h - parameters of the external load spectrum,

α, β, σ_z - parameters of the fatigue curve.

Introducing the parameters of the fatigue curve for the required safety level into the formulae, the operation life can be easily determined.

The author proposes another approach to the evaluation of the aircraft structure safety. It is slightly differently based on probabilistic premises. This approach enables to determine the probability of that that the internal force, inducing a defined state of stresses σ and generated in the considered element of the structure by the external loading Q, will exceed the load-carrying ability N of the structure element made of the material of R_m strength, i.e.:

$$(Q)\leqslant N(R_m) \qquad (3)$$

and constitutes the condition of structure safety in the new approach.

2. Probabilistic premises of not obtaining aircraft structure failure

The condition for probability of not obtaining failure can be defined as follows:

$$R=P(N>\sigma)=P(N-\sigma)>0, \qquad (4)$$

where: N - load-carrying ability,

σ - stress in the structure.

Distributions of probability density for both the load-carrying ability $f_N(N)$ and stresses $f_\sigma(\sigma)$ can be combined and commonly introduced in a flat reference system (Fig. 1). Then it can be noticed that the probability of stresses σ occurs in a small range of width $d\sigma$. The value of the probability is equal to the area of the $d\sigma$ element enclosed under the curve of the density distribution:

$$P\left(\sigma_0-\frac{d\sigma}{2}\leqslant\sigma\leqslant\sigma_0+\frac{d\sigma}{2}\right)=f_\sigma(\sigma_0)d\sigma. \qquad (5)$$

Whereas the probability, that the load-carrying ability N exceeds the values of stresses σ_0, is given as:

$$P(N>\sigma_0)=\int_{\sigma_0}^{\infty}f_N(N)dN. \qquad (6)$$

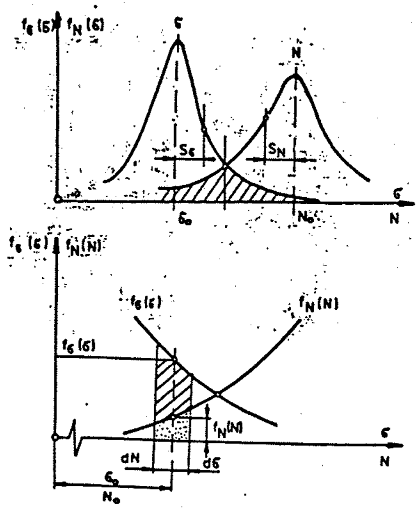

Fig. 1. Connections between loadability and stresses reliability densities

Thus, the probability that for the value of stresses, enclosed in the small range $d\sigma$, the load-carrying ability N exceeds the stress σ (also in the same range), assuming that both random values of σ and N are independent one of another, is as follows:

$$f_\sigma(\sigma_0)d\sigma \int_{\sigma_0}^{\infty} f_N(N)dN. \tag{7}$$

The probability of not obtaining failure is the probability of that that the load-carrying ability N exceeds the stress σ, and then the condition (4) can be written down as

$$R=\int_{-\infty}^{+\infty} f_\sigma(\sigma)\left[\int_{\sigma}^{\infty} f_N(N)dN\right]d\sigma \tag{8}$$

The probability of failure occurrence is then defined as:

$$\bar{R}=1-R=P(N\leqslant\sigma). \tag{9}$$

Substituting probability in the formula (9) by the one determined in the formula (8), the following is obtained:

$$\bar{R}=P(N\leqslant\sigma)=1-\int_{-\infty}^{+\infty} f_\sigma(\sigma)\left[\int_{\sigma}^{\infty} f_N(N)\,dN\right]d\sigma \tag{10}$$

When the difference between the load-carrying ability N and the stress σ is made and marked as below:

$$y=N-\sigma, \tag{11}$$

then the probability of not obtaining failure, depending on the difference, can be formulated as:

$$R=P(y>0). \tag{12}$$

Now an assumption is made, that both the load-carrying ability N and the stress σ are independent of each other, thus they are not bonded by any random incidental values. Then the distribution density of the random value y has a form shown in Fig. 2.

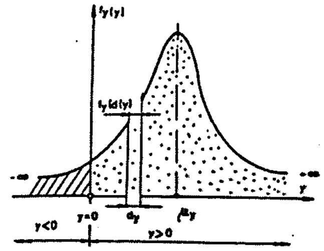

Fig. 2. Reliability density for function y

$$f_y(y)=\int_{\sigma} f_N(y+\sigma)f_\sigma(\sigma)\,d\sigma. \tag{13}$$

When $y\geq 0$, it can be written down that the function of distribution density y has the following form:

$$f_y(y)=\int_{0}^{+\infty} f_N(y+\sigma)f_\sigma(\sigma)\,d\sigma, \tag{14}$$

and when:

$$y\leq 0,$$

the function has the form:

$$f_y(y)=\int_{-\infty}^{0} f_N(y+\sigma)f_\sigma(\sigma)\,d\sigma. \tag{15}$$

Finally, the probability of obtaining failure $\bar{R}$ can be determined from the following formula:

$$\bar{R}=\int_{-\infty}^{0} f_y(y)\,dy=\int_{-\infty}^{0}\int_{-y}^{\infty} f_N(y+\sigma)f_\sigma(\sigma)\,d\sigma\,dy \tag{16}$$

or, more frequently used in further considerations, the probability of not obtaining failure R will be equal:

$$R=\int_{0}^{+\infty} f_y(y)\,dy=\int_{0}^{+\infty}\int_{0}^{+\infty} f_N(y+\sigma)f_\sigma(\sigma)\,d\sigma\,dy \tag{17}$$

Both formula (16) and (17) compose a certain state, static in time t, thus they are valid for the functions of load-carrying ability N and the stress σ, known in the given moment of time t. The formulae compose a certain static model.

For requirements of a safety comparative evaluation, formulae which enable tracing changes of probability R as the time t goes on, are needed. To be able to determine values of the probability from the formulae (16) and (17), types of the density distributions of both the stresses $f_\sigma(\sigma)$ and the load-carrying ability $f_N(N)$ shall be known. While determining the probability R for aircraft structures, either a simpler case can be used, when both mentioned above distributions are normal distributions, or the case which slightly better describes an actual state, i.e. when the load-carrying ability distribution is the Weibull distribution and the stress distribution is the normal one. As the first case is simpler for calculations, it will be taken under further consideration. Then the distribution density of the load-carrying ability has the following form:

$$f_N(N)=\frac{1}{S_N\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{N-\mu_N}{S_N}\right)^2\right], \tag{18}$$

and the distribution density of the stresses has a similar form:

$$f_\sigma(\sigma)=\frac{1}{S_\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\sigma-\mu_\sigma}{S_\sigma}\right)^2\right], \quad (19)$$

and denotations in the formulae are as follows:

μ_N - average value of load-carrying ability N,
S_N - standard deviation of load-carrying ability N,
μ_σ - average value of stresses,
S_σ - standard deviation of stresses.

If the distribution is normalised by introducing the variable $z=(y-\mu_y)/S_y$, then $\mu_y dz=dy$. But let us consider the case when y is equal at the utmost to zero, i.e.:

$$Z=\frac{0-\mu_y}{S_y}=-\frac{(\mu_N-\mu_\sigma)}{\sqrt{S_N^2+S_\sigma^2}} \quad (20)$$

and then for y approaching infinity, the upper limit of integration will be $+\infty$, thus it will be obtained that R can be written down with the variable as follows:

$$R=\frac{1}{\sqrt{2\pi}}\int_{-z_0}^{+\infty} e^{-\frac{z^2}{2}}dz. \quad (21)$$

Since z is the normalised function, the same type of normal distribution is obligatory for it. Thus, to determine the probability R, the tables of normal distribution functions can be used. Denoting by z_0 the lower limit of integration in the formula (21)

$$z_0=\frac{\mu_N-\mu_\sigma}{\sqrt{S_N^2+S_\sigma^2}}, \quad (22)$$

the relation between z_0 and the probability R of not obtaining failure.

The value z determined from the formula (20) can now be considered as the conventional index of safety. It will undergo a change as the time t goes on. Thus, if the index would be a measure of safety changing with time, the function of changes with time for both, the stresses $\mu_\sigma=F(t)$ and the load-carrying ability $\mu_N=\Psi(t)$, together with analogical changes for both types of standard deviations, shall be given. When such courses are known, they can be used for the safety analysis at any moment of time t.

3. Course of stress μ_s changes with time

For the purpose of the analysis, the most suitable form of recording load spectrum is such that gives an average number of events in one hour of flight. The events consist in reaching or exceeding a given level of stresses μ_σ, usually related to the initial load-carrying ability μ_{No} of a material or structure. The spectra set up in Fig. 3 are typical in such case.

4. Course of load-carrying ability μ_N changes with time

The estimation of load-carrying ability change depending on the time of use is carried out in several stages:

a) in the first stage, the course of crack length change with time is determined,

b) in the second stage, the crack length and the drop of load-carrying ability is related,

c) in the final stage, the previous stages are so related that the change of load-carrying ability with time can be directly determined.

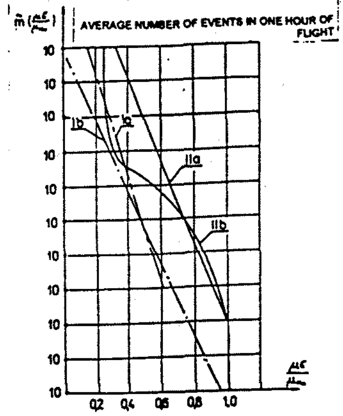

Fig.3 Set of external loading spectrum acc. reports [1]

Application of the probabilistic method and its comparison with Lundberg and Eggwertz's method can be observed in the numerical example given below.

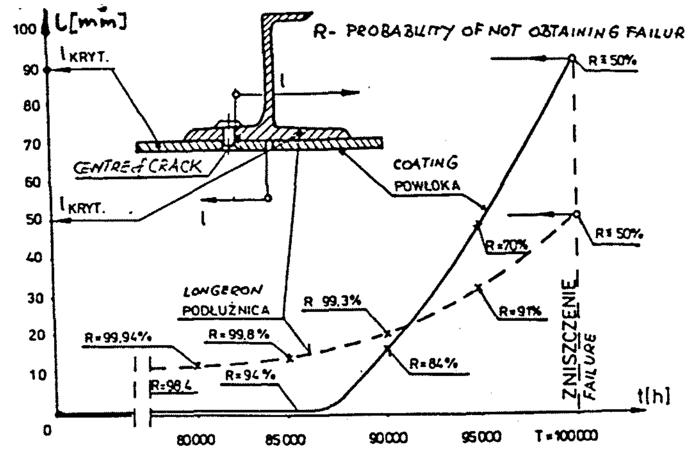

Fig. 4 The course of changes in crack length in relation to using period acc.

5. Example of calculation of construction safety probability

One of the elements determining the fatigue life of a transport aircraft wing structure is joining outer coatings with a longeron, the joint type frequently applied in such structure. Type of such joint is shown in Fig. 4.

In this type of joint fatigue cracks were found. Observation of the cracks allowed to establish the way, in which the cracks extended in both the longeron and the coating. The real course of load-carrying ability in the longeron and the coating, until their contemporary failure, which occurred after the time equal to $T=10^5$ hours [4] (Fig. 4) was established by investigations.

according to [1], that the course of changes is as follows:

for the time interval, when $0\leq t/T\leq 0.6$, the load-carrying ability did not change (stage I),

for the time interval, when $0.6\leq t/T\leq 0.97$, $\mu_N=(1.3-0.5t/T)\mu_{No}$ (stage II), for the time interval, when $0.97 \leq t/T \leq 1$, $\mu_N = (11.0 - 10.5t/T)\mu_{No}$ (stage III).

Fig. 5 shows results of calculations for the probability R, where in the linear-Gauss system, the probability of not obtaining failure, resulting from Lundberg and Eggwertz's method, was calculated for the standard deviation of life equal $S_{logt}=0.17$, assumed on the ground of many life investigations, also made by the author [5]. Using tables of tStudent's distribution for three levels of safety, i.e. 10^{-2}, 10^{-4} and 10^{-6}, and for the assumed degree of freedom $k=100$, results of calculations were obtained and plotted in the same diagram in Fig. 5

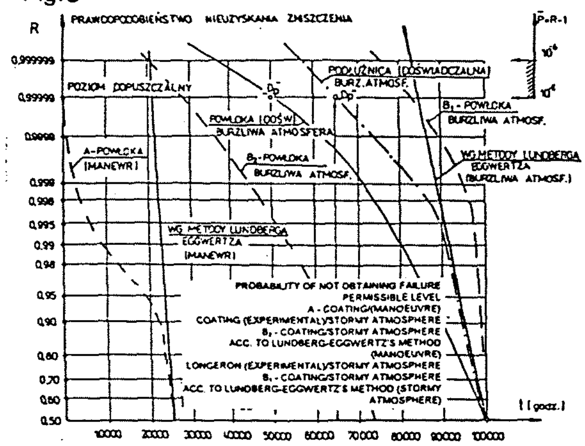

Fig. 5 . Comparison of safety resulting of Lundberg – Eggwertz method and reliability method (in linear-gauss system)

6. Summary

Several observations can be made from the presented results of calculations:

- the reliability method appeared more rigorous than Lundberg and Eggwertz's one. A certain exception is the B_1 case, however, it strays from a real state, but a difference concerns only the probability above R=99.995%, thus out of the range of present requirements;

- in a certain small range of probability both methods gave practically identical results for both, the transport and the fighter aircraft cases;

- the analytical method of reliability includes the risk of failure caused by a decrease of initial load-carrying ability, while Lundberg and Eggwertz's method excludes the risk;

- the reliability method can consider failures resulting from the exceeding of permissible loads, which can occur in any moment. It is significant for both, the methods of carrying fatigue life tests and the observations of failures during the time of use;

- the reliability method allows to carry any superposition of stress spectra;

- the reliability method also allows to determine the time in which an inspection of structure is required. For example, for the requirement that the probability is equal to $R=99.999\%$ ($p=10^{-5}$), the first inspection of a longeron should be carried out not later than after 65,000 hours of use (Fig. 5), whereas basing on Lundberg and Eggwertz's method the time would be extended by 19,000 hours;

- finding a crack of defined length, probability of further work without failure, corresponding to the crack, can be directly attributed to it. It leads to the author's suggestion to apply another expression of a diagram, in which a course of crack length extension is given. Owing to an application of the reliability method, a risk of failure can be determined in any point of such diagram. It allows to plot $l=f(t)$ diagram, completed with very important information. Such information is given in Fig. 4. It is worth emphasising that a certain case can occur when a crack of a threshold length is found, as in the longeron equal to $l_p=12.5$ mm, what means that the part of structure is unfit for use from the point of view of a safety criterion, based on the reliability method. A repair is then required, because at the moment when the crack of the threshold length $l_p=12.5$ mm is found, the probability R is equal only to 99.95%, thus much less than the present requirements;

- the reliability method can be effectively applied for dimensioning of aircraft structures. A selection of stress levels can be directly made when a limitation for the value of safety index z_0 is imposed;

- Lundberg and Eggwertz's method is applied to determine the safe period of structure use of the so called "safe life", in which the time of crack extension is very small, as compared to the total life of the crack. Usually it amounts to several percent of the safe period of use. The reliability method does not introduce any limitations and can be applied wider, also for structures of "safe failure".

Bibliography

(1) Hooke, F.H.: A Comparison of Reliability and Conventional Estimation of Safe Fatigue Life and Safe Inspections Intervals. Advanced Approaches to Fatigue Evaluation - Sixth ICAF Symposium held at Miami Beach, Florida, May 13-14, 1971.

(2) Diamond, P., Payne, O.A.: Reliability Analysis Applied to Structural Tests. Advanced Approaches to Fatigue Evaluation - Sixth ICAF Symposium held at Miami Beach, Florida, May 13-14, 1971.

(3) Hooke, F.H.: A New Look at Structural Reliability and Risk Theory. A Collection of Technical Papers - 19th AIAA/ASME Symposium, Bethesda, April 3-5, 1978.

(4) Craig, I.E., Goranson, U.G.: Airworthiness Assessment of Boing Jet Transport Structure. International Committee on Aeronautical Fatigue, X Symposium, Brussels, 16th May, 1979.

(5) Danielecki, S.: An Application of Fatigue Life Analysis for Evaluation of Effects of Local Failures and Results of Repairing Aircraft Wing Coatings. Technika Lotnicza, No. 4, 5, 6, 1982.