STABILITY AND ELEMENTS POSTBUCKLING BEHAVIOR OF COMPLEX COMPOSITE STRUCTURES

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Abstract

A method of buckling analysis for complex structures comprising a cylindrical system of interacted bars, plates and shells made of composites is developed. Curvature, load and thickness variation of the system along its contour as well as stacking sequence, anysotropy of elements, discrete stiffening and possibility of general and local buckling modes are taken into account. The method is based on the formulation of the respective boundary problems, the separation of variables, the "exact" numerical solution of differential equations using the discrete orthogonalization method as well as on the finite element method algorithms to unite the elements into a system. An approximate analytic solution is obtained, which describes deflection and stress-strain state (SSS) of orthotropic composite plates after local buckling under combined loading. A method of stiffness characteristics reducing for such plates is developed using substantiative hypotheses and classical concept of reduction coefficients (by von Karman, K. Marquerre). Basing upon this approach, a method to analyze global nonlinear SSS and buckling behavior of complex thin-walled structures with due account of postbuckling behavior of some plate elements is proposed.

Introduction

There exist two kinds of methods to analyze the static stability of metallic/composite structures. In classical methods the stability differential equations are solved in closed form or numerically for relatively simple elements. The advantages of this way are high accuracy and time savings, but it cannot be used in the buckling analysis of more complex structures. The second kind is presented by the finite element methods for the computer modelling of structures with complex geometry, but the accuracy and reliability of this approach to the stability analysis is often doubtful, the solution durations being too long. In this paper we want to draw attention to a new, unified method for a numerical analysis of stability of complex aerospace structures. It simultaneously offers sufficient possibilities to describe the structural geometry, achieve a high precision and attain time saving. The structure comprises cylindrical and slightly conic systems of interacting bars, plates and shells made of both the metallic and composite materials under combined thermo-mechanical loading.

Typical examples of such systems are shown in Figure 1.

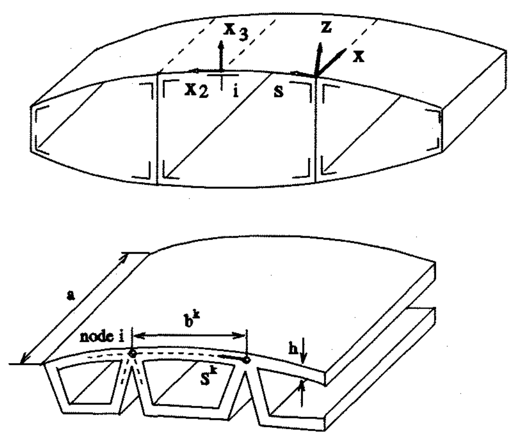

FIGURE 1 - Typical Structures Under Consideration

The curvature, loading and thickness variation, initial out-of-plane deflection and subcritical deformation of the system along its contour as well as stacking sequence, anisotropy of components, discrete stiffening and the possibility of general and local buckling modes are taken into account.

The method is based on the formulation of the respective homogeneous boundary problems, the separation of variables and the "exact" numerical solution of stability equations for structural members using the discrete orthogonalization method[1]. Finite element method algorithms are used to unite all elements into a system.

It is known, that the separate elements of stiffened thin-walled structures may buckle long before the general buckling or overall structural failure. In this case, prebuckling stress-strain state determination problem arises as well as the general instability problem for complex structure with due regard for nonlinear postbuckling behavior of mentioned elements.

In engineering statement, this problem is solved by method of reduction coefficients taking into account the reduction of stiffness characteristics of elements after buckling. The application of corresponding solutions and reduction coefficients to metal structures was considered by von Karman, K.Marquerre[2]. The general approach was reported by the authors[3].

In this work, the reduction procedure for rectangular composite plates is developed which consists in the replacement of real buckled plate by the unbuckled plate made of nonlinear elastic orthotropic material which exhibits the same stiffness. The approximate analytical solution is obtained both for deflections/SSS of simply supported plate and for its reduced stiffness characteristics under combined loads. On this base, the numerical method developed before[3] for the prediction of stress-strain state of metal thin-walled structures with buckled skin is applied to the structures with composite elements. Correspondingly the method of buckling analysis for complex composite structures stated below becomes applicable to the analysis of general instability of structures with previously buckled skin.

Stability

Single element

Let us consider an arbitrary (not circular) anisotropic cylindrical shell which is loaded in subcritical stress-strain state by $N^0_x(s,t)$, $N^0_s(s,t)$, $N^0_{xs}(s,t)$; $t \geq 0$ is the load growth factor. The complete set of stability equations describing the buckling effects for this shell can be written as three groups of relations[4].

Neutral equilibrium equations

$$
\begin{aligned}
&\frac{\partial N_x}{\partial x}+\frac{\partial S}{\partial s}=-\frac{\partial}{\partial x}(N_x^0\frac{\partial u}{\partial x}+N_{xs}^0\frac{\partial u}{\partial s})-\\
&-\frac{\partial}{\partial s}(N_{sx}^0\frac{\partial u}{\partial x}+N_x^0\frac{\partial u}{\partial s})-p\frac{\partial w}{\partial x},\\
&\frac{\partial N_s}{\partial s}+\frac{\partial S}{\partial x}+\frac{1}{R}\frac{\partial M_{sx}}{\partial x}+\frac{Q_s}{R}=-\frac{\partial}{\partial x}[N_x^0\frac{\partial v}{\partial x}+N_{xs}^0(\frac{\partial v}{\partial s}+\frac{w}{R})-\\
&-\frac{\partial}{\partial s}[N_{sx}^0\frac{\partial v}{\partial x}+N_s^0(\frac{\partial v}{\partial s}+\frac{w}{R})]-p(\frac{\partial w}{\partial s}-\frac{v}{R}),\\
&\frac{N_s}{R}-\frac{\partial Q_x}{\partial x}-\frac{\partial Q_s}{\partial s}=-\frac{N_{sx}^0}{R}\frac{\partial v}{\partial x}-\frac{N_s^0}{R}(\frac{\partial v}{\partial s}+\frac{w}{R})-p(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial s}+\frac{w}{R}),\\
&Q_x=\frac{\partial M_x}{\partial x}+\frac{\partial M_{sx}}{\partial s}-N_x^0\theta_x-N_{xs}^0\theta_s-(\theta_s^0+\theta^0)S,\\
&Q_s=\frac{\partial M_s}{\partial s}+\frac{\partial M_{xs}}{\partial x}-N_s^0\theta_s-N_{sx}^0\theta_x-(\theta_s^0+\theta^0)N_s,\\
&(S=N_{xs}-\frac{M_{sx}}{R}=N_{sx},\quad H=\frac{1}{2}(M_{xs}+M_{sx})).
\end{aligned}
\tag{1}
$$

Elasticity relations

$$
\begin{aligned}
&\begin{Bmatrix}\{N\}\\\{M\}\end{Bmatrix}=\begin{bmatrix}[B] & [C]\\ [C] & [D]\end{bmatrix}\begin{Bmatrix}\{\varepsilon\}\\\{\kappa\}\end{Bmatrix},\\
&\{N\}=[N_x\ N_s\ S]^T,\quad \{M\}=[M_x\ M_s\ H]^T,.\\
&\{\varepsilon\}=[\varepsilon_x\ \varepsilon_s\ \gamma_{xs}]^T,\quad \{\kappa\}=[\kappa_x\ \kappa_s\ 2\kappa_{xs}]^T,\\
&[B]=[B_{ij}],\quad [C]=[C_{ij}],\quad [D]=[D_{ij}]
\end{aligned}
\tag{2}
$$

Geometrical relations

$$
\begin{aligned}
&\varepsilon_x=\frac{\partial u}{\partial x},\quad \varepsilon_s=\frac{\partial v}{\partial s}+\frac{w}{R}+(\theta_s^0+\theta_0)\theta_s,\\
&\gamma_{xs}=\frac{\partial u}{\partial s}+\frac{\partial v}{\partial x}+(\theta_s^0+\theta_0)\theta_x\\
&\kappa_x=\frac{\partial\theta_x}{\partial x},\quad \kappa_s=\frac{\partial\theta_s}{\partial s},\quad \kappa_{xs}=\frac{\partial\theta_s}{\partial x}=\frac{\partial\theta_x}{\partial s}+\frac{1}{R}\frac{\partial v}{\partial x},\\
&\theta_x=-\frac{\partial w}{\partial x},\quad \theta_s=-\frac{\partial w}{\partial s}+\frac{v}{R},\\
&(\theta_0=-\frac{dw_0}{dx},\quad \theta^0=-\frac{dw^0}{dx}).
\end{aligned}
\tag{3}
$$

Here u, v, w, θ_x, θ_s are the additional displacements and rotation angles of the shell; ε_x, ε_s, γ_{xs}, κ_x, κ_s, κ_{xs} are the strains and curvature variation components; $w_0(s)$, $w^0(s)$ are the initial and accumulated subcritical transverse displacements, respectively; B_{ij}, C_{ij}, and D_{ij} $(i, j=1, 2, 3)$ are stiffness characteristics of the shell.

The boundary conditions at the edges $s = 0$ and $s=b$ of the single shell are given by

$$w|_{s=0}(1-\gamma_0)+\overline{Q}_s|_{s=0}\gamma_0=0,\quad w|_{s=b}(1-\gamma_1)+\overline{Q}_s|_{s=b}\gamma_1=0,$$
$$\theta_s|_{s=0}(1-\delta_0)+M_s|_{s=0}\delta_0=0,\quad \theta_s|_{s=b}(1-\delta_1)+M_s|_{s=b}\delta_1=0,$$
$$u|_{s=0}(1-\varphi_0)+\overline{S}|_{s=0}\varphi_0=0,\quad u|_{s=b}(1-\varphi_1)+\overline{S}|_{s=b}\varphi_1=0, \tag{4}$$
$$v|_{s=0}(1-\psi_0)+\overline{N}_s|_{s=0}\psi_0=0,\quad v|_{s=b}(1-\psi_1)+\overline{N}_s|_{s=b}\psi_1=0,$$

where the constants γ_0, δ_0, φ_0, Ψ_0, γ_1, δ_1, φ_1, Ψ_1, are equal to 0 or 1 for kinematic or static conditions and

$$\overline{S}=S+N^0_{sx}\frac{\partial u}{\partial x}+N^0_s\frac{\partial u}{\partial s},\quad \overline{Q}_s=Q_s+\frac{\partial M_{sx}}{\partial x}$$
$$\overline{N}_s=N_s+N^0_{sx}\frac{\partial v}{\partial x}+N^0_s(\frac{\partial v}{\partial s}+\frac{w}{R}).$$

The critical value $t=t_*$ of the load growth factor should be found as the minimum value of t for which a nonzero solution of the homogeneous boundary problem (1) - (4) (with additional simple-support boundary conditions at the edges $x=0$ and $x=a$) exists. These equations provide the possibility to analyze the general instability of structures, as well as local and thermal buckling.

Cylindrical system

Similarly, the stability equations can be written for the complex stiffened cylindrical structure composed of interacting cylindrical shells, plates and bars; the bars may be considered as distributed or discrete members. For instance, equations for the ith discrete rib (see Figure 1) are written as follows:

$$\frac{dN_i}{dx}+\overline{S}^{\,i+1}-\overline{S}^{\,i}=\frac{d}{dx}N_i^0\frac{du_i}{dx},$$
$$\frac{dQ_{2i}}{dx}+N_2^{i+1}-N_2^i=0,$$
$$\frac{dQ_{3i}}{dx}+Q_3^{i+1}-Q_3^i=0,$$
$$\frac{dH_i}{dx}+M_s^{i+1}-M_s^i-\Delta_2^{i+1}Q_3^{i+1}-\Delta_2^iQ_3^i-\Delta_3^{i+1}N_2^{i+1}-\Delta_3^iN_2^i=0, \tag{1$'$}$$
$$\frac{dM_{2i}}{dx}-N_i^0\varphi_{2i}-Q_{2i}+\Delta_2^{i+1}\overline{S}^{\,i+1}+\Delta_2^i\overline{S}^{\,i}=0,$$
$$\frac{dM_{3i}}{dx}-N_i^0\varphi_{3i}-Q_{3i}-\Delta_3^{i+1}\overline{S}^{\,i+1}-\Delta_3^i\overline{S}^{\,i}=0,$$
$$(N_2=\overline{N}_S\sin\alpha-\overline{Q}_S\cos\alpha,\ Q_3=\overline{N}_S\cos\alpha+\overline{Q}_S\sin\alpha);$$

$$N_i=EF_i\varepsilon_i,\qquad M_{2i}=EI_{2i}\kappa_{2i}+EI_{23i}\kappa_{3i},$$
$$H_i=GI_i\kappa_i,\qquad M_{3i}=EI_{3i}\kappa_{3i}+EI_{23i}\kappa_{2i}; \tag{2$'$}$$

$$\varepsilon_i=\frac{du_i}{dx},\ \kappa_{2i}=\frac{d\varphi_{2i}}{dx},\ \kappa_{3i}=\frac{d\varphi_{3i}}{dx},\ \kappa_i=\frac{d\varphi_i}{dx},$$
$$\varphi_{2i}=-\frac{dv}{dx},\ \varphi_{3i}=-\frac{dw}{dx}, \tag{3$'$}$$

where u_i, v_i, w_i, φ_i, φ_{2i}, φ_{3i}, are the generalized displacements of the bar; N_i, Q_{2i}, Q_{3i}, H_i, M_{2i}, M_{3i} are the stress resultants;

Δ_2, Δ_3 are the eccentricities of the bar; EF_i, EI_{2i}, EI_{3i}, GI_i are the stiffness; the superscripts i, $(i+1)$ correspond to the left (right) edge of the ith ((i+1)th) shell. Note that the local coordinate system x, x_2, x_3 of the bar has been used here; α is the angle between axes x_3 and s on the rib. The displacement compatibility conditions for the shells under consideration and the rib must also be written in the form

$$v_i=u_2^i-\Delta_3^i\varphi_i=u_2^{i+1}+\Delta_3^{i+1}\varphi_i,$$
$$w_i=u_3^i-\Delta_2^i\varphi_i=u_3^{i+1}+\Delta_2^{i+1}\varphi_i,$$
$$u_i=u^i+\Delta_2^i\varphi_{2i}-\Delta_3^i\varphi_{3i}=u^{i+1}-\Delta_2^{i+1}\varphi_{2i}+\Delta_3^{i+1}\varphi_{3i}, \tag{4$'$}$$
$$\varphi_i=\theta_s^i=\theta_s^{i+1};$$
$$(u_2=v\sin\alpha-w\cos\alpha,\ u_3=v\cos\alpha+w\sin\alpha).$$

Equations (1) - (3) are written for every shell and plate ($R\to\infty$). Equations similar to $(1)'$- $(3)'$ are valid for each bar. Certain conditions must be added to them at all junctures of shells and plates; usually, those include the equilibrium equations and the requirements for equal generalized displacements at the junctures. All these relations are evident and not presented here.

Solution method

Let us first consider the particular case of an orthotropic system without subcritical shear: $N^0_{sx}=0$, $B_{13}=B_{23}=C_{13}=C_{23}=D_{13}=D_{23}=0$. The solution to be found for every shell may be presented as follows:

$$\{P\}=\{P_m\}\cos\frac{m\pi x}{a},\qquad \{F\}=\{F_m\}\sin\frac{m\pi x}{a}, \tag{5}$$

where:

$$\{P\}=[u\theta_x\gamma_{xs}\kappa_{xs}SHQ]^T,\ \{P_m\}=[u_m\theta_{xm}\gamma_m\kappa_{xsm}S_mH_mQ_{xm}]^T,$$
$$\{F\}=[vw\theta_S\varepsilon_x\varepsilon_S\kappa_x\kappa_sN_xN_SM_xM_SQ_S]^T,$$
$$\{F_m\}=[v_mw_m\theta_m\varepsilon_{xm}\varepsilon_m\kappa_{xm}\kappa_mN_{xm}N_mM_{xm}M_mQ_m]^T,$$

and m is the number of the buckling half-waves on the shell in its longitudinal direction; the functions with the subscript m depend on coordinate s only. The stability equations (1)-(3) can in this case be transformed into the system of 8 ordinary differential equations (for any m and t):

$$\frac{d\{u\}}{ds}=[A]\{u\} \tag{6}$$

where

$$\{u\}=[w_m\theta_mM_m\overline{Q_m}u_mv_m\overline{N_m}\overline{S_m}]^T$$

is the vector to be found,

$$\overline{S_m}=S_m+N_s^0\frac{du_m}{ds},\ \overline{N_m}=N_m+N_s^0(\frac{w}{R}+\frac{dv_m}{ds}),\ \overline{Q_m}=Q_m-\frac{m\pi}{a}H_m$$

[A] is a square matrix of coefficients which are certain functions of s.

The boundary conditions (4) for the single element can be written in the form

$$[\Gamma_0]\{u(0)\}=0,\quad [\Gamma_1]\{u(b)\}=0 \tag{7}$$

where $[\Gamma_0]$, $[\Gamma_1]$ are constant (4 x 8) matrices.

The problem (6), (7) can be effectively solved by the numerically stable discrete orthogonalization method (by means of solving some Cauchy problems). The solution at the point $s=b$ is given by

$$\{u(b)\}=[Z(b)]\{c(b)\}$$

and the condition for obtaining the critical value $t=t_*$ is written as follows:

$$\det[D(t)]=0, \tag{8}$$

where $[D]=[\Gamma_1][Z(b)]$, $[Z(b)]$ is certain (8 x 4) solution matrix.

Equations (1$'$) - (4$'$) for the discrete ribs can be transformed similarly and some relations are obtained for the solutions $\{u\}^{l+1}$, $\{u\}^l$ on the left and right edges of ribs.

For complex cylindrical systems the following solution method is proposed. The solutions like (5) are written for all elements.

Solutions of equation (6) for every shell and plate are obtained under the "unit-displacement" kinematic boundary conditions, for example, under the following conditions at the points $s = 0$ and $s = b$:

$$w_m^k|_{s^k=0}=1,\ Q_m^k=u_m^k=v_m^k|_{s^k=0}=0,\ w_m^k=Q_m^k=u_m^k=v_m^k|_{s^k=b^k}=0,$$

where the superscript k is the element number.

Thus the generalized stiffness matrices may be constructed for every element ($[K]^k$) and for structure as a whole ([K]) by means of usual assumed displacement finite element method algorithms. The resulting structural neutral equilibrium equations are written in the form of linear algebraic equations (for arbitrary m and t)

$$[K]\{y\}=0 \tag{9}$$

where {y} is the generalized displacement vector for all nodes in a cross section of the system. The vector {y} is composed of the vectors of nodal displacements

$$\{y_i\}=[w_{mi}\varphi_{mi}u_{mi}v_{mi}]^T,\ i=1,2...,N,$$

taking into account the corresponding coordinate transformations in nodes.

The equation for obtaining the t_* is given by

$$\det[K(t)]=0 \tag{10}$$

At the first step we must use (10) to find t_m for any m, then t_* is obtained as $min_m\ t_m$ at $m=m_*$. Finally, equation (9) is used to find the critical value $\{y_*\}$; thereafter the structure buckling shape is found.

<u>Accounting for shear load and anisotropy of structures</u>

In the general case of a non-zero subcritical shear $N^0_{sx} \neq 0$ and/or general elasticity relations (2) for at least one element the solution to the problem on structural instability may be written in the form

$$\begin{aligned}\{P\}&=\{P_m\}\cos\frac{\pi x}{l}+\{\tilde{P}_m\}\sin\frac{\pi x}{l},\\ \{F\}&=\{F_m\}\sin\frac{\pi x}{l}+\{\tilde{F}_m\}\cos\frac{\pi x}{l},\end{aligned} \tag{5$'$}$$

where the variables of the first group, $\{P_m\}$ and $\{F_m\}$, are the same as in (5), and the second group is comprised of the variables

$$\{\tilde{P}_m\}=[\tilde{u}_m\tilde{\theta}_{xm}\tilde{\gamma}_m\tilde{\kappa}_{xsm}\tilde{S}_m\tilde{H}_m\tilde{Q}_{xm}]^T,$$
$$\{\tilde{F}_m\}=[\tilde{v}_m\tilde{w}_m\tilde{\theta}_m\tilde{\varepsilon}_{xm}\tilde{\varepsilon}_m\tilde{\kappa}_{xm}\tilde{\kappa}_m\tilde{N}_{xm}\tilde{N}_m\tilde{M}_{xm}\tilde{M}_m\tilde{Q}_m]^T.$$

In this case, the structure is assumed to be prolong ($a>>b$) and l is the longitudinal half-wave length (in practical analyses, one may take roughly $l=l_m = a/m$, $m = 1, 2, 3, ...$).

Substituting $(5)'$ into shell buckling equations (1) - (3) makes it possible to separate variables to obtain 16 simultaneous ordinary differential equations of the form (6) with a variable matrix **[A]** not written here for brevity. Analogous transformation should be applied to equations $(1)'$-$(3)'$ for stiffeners, to the boundary conditions of the type (4), and to the compatibility conditions for the longitudinal edges of the components. The subsequent matrix formulation and the solution method are just the same as those for the particular case above; the dimensions of all matrices and vectors are duplicated due to the addition of nodal displacements of the second group

$$\{y_i\}=[w_{mi}\,\varphi_{mi}\,u_{mi}\,v_{mi}\,\tilde{w}_{mi}\,\tilde{\varphi}_{mi}\,\tilde{u}_{mi}\,\tilde{v}_{mi}]^T,\ i=1,2...,N$$

Let us note that the integral stiffness B_{ij}, C_{ij}, D_{ij} of multilayer composite structures are to be determined by integrating through the thickness(4),(5) on the basis of elastic characteristics of the unidirectional layers the structures are comprised of. By applying the reduction factor concept, the regularly stiffened panels may be considered as structurally-orthotropic ones with the skin simulated by orthotropic layer with reduced characteristics(4). Only in the particular case of a symmetrical laminate layup and the stiffeners symmetrical with respect to the skin midsurface the usual assumption of **[C]** = 0 is valid.

Many classical problems exactly and/or approximately solved, as well as new problems on composite component stability which occur in design practice can be solved with the use of the technique proposed and the computer program developed.

Postbuckling behavior of composite plate

For analysis of postbuckling composite skin deformation, let us consider the behavior of rectangular plate stiffened by mutually perpendicular ribs, Fig.2.

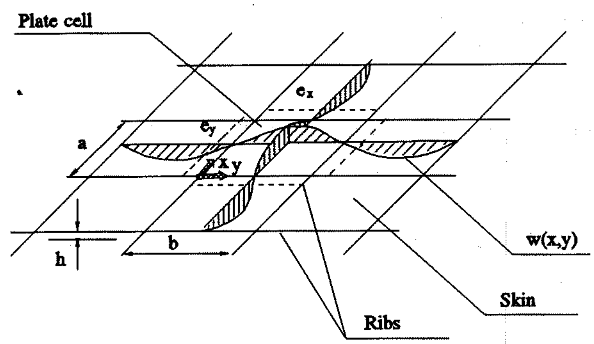

FIGURE 2 - Siffened Composite Panel

Each typical part of plate with adjoining parts of ribs in biaxial compression and shear behave similar to the others retaining the rectilinearity of ribs and edges $x=0$, $x=a$, $y=0$, $y=b$ of plate cell. Prior to buckling, the uniform flat stress state is realized in the plate with average stresses $p_x=T_x/h$, $p_y=T_y/h$, $\tau=S/h$. After buckling, the plate get the deflection $w(x,y)$; the distribution of membrane forces balancing external loads T_x, T_y, S becomes nonuniform. Let us consider that the ribs are simply supported; the plate is orthotropic with symmetrical layup arrangement, so that the elastic relationships (2) with **[C]** = 0, $B_{13} = B_{23} = D_{13} = D_{23} = 0$ are valid. Postbuckling plate behavior is described by nonlinear differential equations of von Karman type

$$L_1(\Phi)+\frac{1}{2}L_3(w,w)=0,\quad L_2(w)-L_3(\Phi,w)=0, \tag{11}$$

where Φ is force function,

$$L_1=A_{22}\frac{\partial^4}{\partial x^4}+2A_3\frac{\partial^4}{\partial x^2\partial y^2}+A_{11}\frac{\partial^4}{\partial y^4},\quad A_3=A_{12}+A_{33}/2$$
$$L_2=D_{11}\frac{\partial^4}{\partial x^4}+2D_3\frac{\partial^4}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4}{\partial y^4},\quad D_3=D_{12}+2D_{33} \tag{12}$$
$$L_3=\frac{\partial^2}{\partial x^2}\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial y^2}\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial x\partial y}\frac{\partial^2}{\partial x\partial y},\quad [A]=[A_{ij}]=[B]^{-1}$$
$$\frac{\partial^2\Phi}{\partial y^2}=N_x,\quad \frac{\partial^2\Phi}{\partial x^2}=N_y,\quad -\frac{\partial^2\Phi}{\partial x\partial y}=N_{xy},\quad \{\varepsilon\}=[A]\{N\}.$$

Plate midsurface strains are coupled with its displacements $u(x,y)$, $v(x,y)$, and deflection $w(x,y)$ by relationships

$$\varepsilon_x=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2,\ \varepsilon_y=\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2,$$
$$\gamma_{xy}=\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}, \tag{13}$$

so that for relative mutual displacements of plate cell edges the following is valid:

$$e_x=-\frac{1}{a}\int_0^a\frac{\partial u}{\partial x}dx=$$
$$=-\frac{1}{a}\int_0^a\left[A_{11}\frac{\partial^2\Phi}{\partial y^2}+A_{12}\frac{\partial^2\Phi}{\partial x^2}-\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right]dx=Const,$$
$$e_x=-\frac{1}{b}\int_0^b\frac{\partial v}{\partial y}dy= \tag{14}$$
$$=-\frac{1}{b}\int_0^b\left[A_{12}\frac{\partial^2\Phi}{\partial y^2}+A_{22}\frac{\partial^2\Phi}{\partial x^2}-\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right]dy=Const.$$

We will give the approximate solution of the problem by using the assumption that the postbuckling deflection pattern is close to that for buckling pattern and by obtaining the latter from the equations (12) using Bubnov-Galerkin's method.

Biaxial compression

In this case we shall follow the solution scheme proposed by K. Marquerre[(2)] in application to metal plates. According to (14), we consider the relative mutual displacements e_x, e_y as given ones (by defining the absolute displacements of edges). The plate deflection we represent as

$$w(x,y)=f_0\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b},\quad m,n=1,2,3,..., \tag{15}$$

that corresponds to all possible buckling modes of simply supported plate

$$w|_{x=0,a}=\frac{\partial^2 w}{\partial x^2}|_{x=0,a}=0,\ w|_{y=0,b}=\frac{\partial^2 w}{\partial y^2}|_{x=0,b}=0. \tag{16}$$

Boundary conditions (16) are also satisfied in postbuckling phase of deforming at any amplitude of deflection f_0. Substituting equation (15) into first equation (11), the solution of the latter can be expressed as

$$\Phi(x,y)=-\frac{T_x y^2}{2}-\frac{T_y x^2}{2}+$$
$$+\frac{f_0^2}{32}\left(\frac{a^2n^2}{b^2m^2A_{22}}Cos\frac{2m\pi x}{a}+\frac{b^2m^2}{a^2n^2A_{11}}Cos\frac{2n\pi y}{b}\right). \tag{17}$$

Hence the loads in plate are expressed as

$$N_x(x,y)=-T_x-\frac{\pi^2m^2}{8a^2A_{11}}f_0^2Cos\frac{2n\pi y}{b},$$
$$N_y(x,y)=-T_y-\frac{\pi^2m^2}{8b^2A_{22}}f_0^2Cos\frac{2m\pi y}{a},\quad N_{xy}(x,y)=0. \tag{18}$$

Here T_x, T_y are average compressive plate loads in x and y directions. They should satisfy the conditions

$$T_x=-\frac{1}{b}\int_0^b N_x(x,y)\,dy=-\frac{1}{b}\int_0^b\frac{\partial^2\Phi}{\partial y^2}dy=Const,$$
$$T_y=-\frac{1}{a}\int_0^a N_y(x,y)\,dx=-\frac{1}{a}\int_0^a\frac{\partial^2\Phi}{\partial x^2}dx=Const. \tag{19}$$

It is obvious that T_x, T_y are independent of coordinates x,y. Substituting equations (15), (17) into conditions (14) together with simultaneous using the average laminate stresses P_x, P_y result in relationships

$$e_x=\frac{p_x}{E_x}-\mu_{xy}\frac{p_y}{E_y}+\frac{\pi^2m^2}{8a^2}f_0^2,$$
$$e_y=\frac{p_y}{E_y}-\mu_{yx}\frac{p_x}{E_x}+\frac{\pi^2n^2}{8b^2}f_0^2, \tag{20}$$

where

$$E_x=\frac{1}{A_{11}h},E_y=\frac{1}{A_{22}h},\mu_{yx}=-\frac{A_{12}}{A_{11}}=\frac{B_{12}}{B_{22}},$$
$$\mu_{xy}=-\frac{A_{12}}{A_{22}}=\mu_{yx}\frac{E_y}{E_x} \tag{21}$$

are laminate elastic moduli and Poisson ratios (average of lamina moduli and ratios[(5)]). According to (12)

$$A_{11}=\frac{B_{22}}{B},\ A_{22}=\frac{B_{11}}{B},\ A_{12}=-\frac{B_{12}}{B},\ A_{33}=\frac{1}{B_{33}},\ B=B_{11}B_{22}-B_{12}^2$$

For f_o determination we use the approximate solution of second equation (11) by Bubnov-Galerkin's method. According to it

$$\int_0^a\int_0^b[L_2(w)-L_3(\Phi,w)]\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}dxdy=0.$$

Substituting equations (15), (17) into this equation, for $f_0 \neq 0$, the following relation can be obtained after some transformations

$$D(m,n)-p_x\frac{m^2}{4a^2}-p_y\frac{n^2}{4b^2}+E(m,n)f_0^2=0, \qquad (22)$$

where

$$D(m,n)=\frac{\pi^2}{4h}\left(D_{11}\frac{m^4}{a^4}+2D_3\frac{m^2n^2}{a^2b^2}+D_{22}\frac{n^4}{b^4}\right),$$
$$E(m,n)=\frac{\pi^2}{64}\left(E_x\frac{m^4}{a^4}+E_y\frac{n^4}{b^4}\right).$$

Critical state of the plate is determined from the condition of nontrivial solution $f_0 \neq 0$ of equation (22). For proportional loading $p_y = \psi p_x$, we can find

$$p_x^* = \min_{m,n} p_x(m,n) = p_x(m_0,n_0),\ p_y^* = \psi p_x^*$$
$$p_x(m,n)=\frac{D(m,n)}{\dfrac{m^2}{4a^2}+\psi\dfrac{n^2}{4b^2}}, \qquad (23)$$

where m_0, n_0 = numbers of half-waves in buckle pattern of plate cell. Fixing these numbers for postbuckling plate deformation phase at $p_x > p_x^*$, we can obtain from (22) the following dependency of f_o^2 on p_x, p_y

$$f_0^2=\frac{D_0}{E_0}\left(\frac{p_x}{p_x^*}-1\right)=\frac{D_0}{E_0}(\bar{p}-1), \qquad (24)$$

where

$$\bar{p}=\bar{p}_x+\bar{p}_y,\ \bar{p}_x=\frac{p_x}{p_{x0}},\ \bar{p}_y=\frac{p_y}{p_{y0}},$$
$$D_0=D(m_0,n_0),\ E_0=E(m_0,n_0), \qquad (25)$$
$$p_{x0}=\frac{4a^2}{m_0^2}D_0,\ p_{y0}=\frac{4b^2}{n_0^2}D_0.$$

Substituting equation (24) into equation (20) at $m=m_0$, $n=n_0$, $\bar{p} \geq 1$ results in two relations between e_x, e_y and p_x, p_y similar to that for prebuckling deformation phase $\bar{p} \leq 1$, when at $f_0=0$, $e_x = \epsilon_x$, $e_y = \epsilon_y$, the usual relations (12) of prebuckling plane stress state of plate are valid

$$\varepsilon_x=\frac{p_x}{E_x}-\mu_{xy}\frac{p_y}{E_y},\ \varepsilon_y=\frac{p_y}{E_y}-\mu_{yx}\frac{p_x}{E_x} \qquad (26)$$

So the postbuckling state of representative plate cell is fully determined including its deflections and bending stress-strain state; the boundary conditions could be determined by mutual relative displacements e_x, e_y, by external forces $T_x=p_xh$, $T_y=p_yh$ or by their combinations e_x, T_y and e_y, T_x.

The characteristic features of obtained solution are that the shear forces are equal to zero including the forces on the edges of plate cell as well as the nonlinear variation of contour tangent displacements along the edge. This result in inaccurate satisfaction of compatibility conditions for displacements of the plate and ribs in the direction along rib, when these conditions are satisfied integrally (the so-called edge slipping). Marquerre[(2)] has shown that this has no appreciable effect on the SSS of plate and ribs and all the more on reduction factors.

Reduction factors

As far as stiffness characteristics of buckled composite plate are concerned, let us consider the most interesting case of uniaxial longitudinal compression of elongated plate with $a>>b$. We will use usual nondimensional parameters of the theory of orthotropic plates

$$\alpha=\frac{b^2}{a^2}\sqrt{\frac{D_{11}}{D_{22}}},\ \beta=\frac{D_3}{\sqrt{D_{11}D_{22}}},\ \kappa_x^*=\frac{b^2hp_x}{\pi^2\sqrt{D_{11}D_{22}}}. \qquad (27)$$

The solution of instability problem (23) for $\psi=0$, $1/\sqrt{\alpha} \geq 3$ can be expressed by known relations

$$n_0=1,\ m_0=\frac{1}{\sqrt{\alpha}},\ p_x^*=\frac{\pi^2\sqrt{D_{11}D_{22}}}{b^2h}\kappa_x^*,\ \kappa_x^*=2(1+\beta). \qquad (28)$$

For the amplitude of postbuckling deflection f_0 from (24) we can obtain

$$\frac{f_0}{\sqrt{A_{11}D_{11}}}=f_0\sqrt{\frac{E_xh}{D_{11}}}=4\sqrt{2}\sqrt{\frac{(1+\beta)}{(1+\gamma)}(\frac{p_x}{p_x^*}-1)}. \qquad (29)$$

Here an additional nondimensional stiffness parameter is introduced

$$\gamma=\frac{A_{11}D_{11}}{A_{22}D_{22}}=\frac{B_{22}D_{11}}{B_{11}D_{22}}=\frac{E_yD_{11}}{E_xD_{22}}, \qquad (30)$$

influencing the postbuckling behavior of plate.
Substitution of equation (29) at $p_y=0$ into relations (20) results in the following relationships

$$e_x=\frac{p_x\left(3+\gamma-2\frac{p_x^*}{p_x}\right)}{E_x(1+\gamma)}=\frac{p_x}{E_x^s},\ p_x=\frac{E_xe_x(1+\gamma)}{(3+\gamma)}\left[1+\frac{2e_x^*/e_x}{(1+\gamma)}\right]=E_x^se_x$$

$$e_y=-\frac{p_x}{E_x}\left[\mu_{yx}-\frac{2\sqrt{D_{11}/D_{22}}}{(1+\gamma)}\left(1-\frac{p_x^*}{p_x}\right)\right]=-\mu_{yx}^s\frac{p_x}{E_x^s},\ (e_x^*=p_x^*/E_x) \qquad (31)$$

As we can see from the comparison of relations (31) and (26), the buckled skin behave like non-buckled plate made of nonlinear elastic material exhibiting the reduced averaged elastic modulus $E^s_x=1/A^s_{11}h=\varphi^s_x E_x$ and corresponding reduced stiffness characteristic $A^s_{11}=A_{11}/\varphi^s_x$. From equations (31) changing e_x by ϵ_x we can obtain two equivalent expressions for secant reduction factor $\varphi^s_x=E^s_x/E_x \leq 1$

$$\varphi_x^s=\frac{(1+\gamma)}{\left(3+\gamma-2\frac{p_x^*}{p_x}\right)}=\frac{(1+\gamma)}{(3+\gamma)}\left[1+\frac{2\varepsilon_x^*/\varepsilon_x}{(1+\gamma)}\right], \qquad (32)$$

where $\varphi_x^s=1$ at $p_x=p_x^*$, $\epsilon_x=\epsilon_x^*$ and prior to buckling. In particular case of isotropic plate when $\gamma=1$, this expression can be transformed into the following one

$$\varphi_x^s=\frac{1}{2-\frac{p_x^*}{p_x}}=\frac{1}{2}\left(1+\frac{\varepsilon_x^*}{\varepsilon_x}\right), \qquad (33)$$

which corresponds to the solution[2]. Thus the unique parameter determining the distinctions of reduction factors for orthotropic and isotropic plate is the nondimesional parameter γ. In accordance with equation (30) for homogeneous and quasi-homogeneous plates, the parameter γ is also equal to unit because the following is valid

$$D_{11}=\frac{E_xh^3}{12(1-\mu_{xy}\mu_{yx})},\ D_{22}=\frac{E_yh^3}{12(1-\mu_{xy}\mu_{yx})}.$$

Thus we obtain an important result, that such plates behave like the isotropic metal plate with respect to the longitudinal stiffness.

In addition, we obtain from the second relation (31), that the Poisson ratio is also reduced according to relationship

$$\mu_{yx}^s=\varphi_x^s\left[\mu_{yx}-\frac{2d}{(1+\gamma)}(1-p_x^*/p_x)\right]=\varphi_x^s\mu_{yx}-(1-\varphi_x^s)d, \qquad (34)$$

where the additional parameter $d=\sqrt{D_{11}/D_{22}}$ has appeared. In particular case of homogeneous material, this parameter may be replaced by $d=\sqrt{E_x/E_y}$. For isotropic skin we can obtain

$$\mu_{yx}^s=\varphi_x^s(\mu-1+p_x^*/p_x)=\varphi_x^s(1+\mu)-1.$$

It is obvious that the value of reduced Poisson ratio can be expressed through the reduction factor φ_x^s, which in its turn depends upon the extent of critical state exceedance p_x/p_x^* or ϵ_x/ϵ_x^*.

Figure 3 shows the typical generalized dependence of p_x/p_x^* upon ϵ_x/ϵ_x^* obtained from equation (31). These functions do not depend on parameter β; they are piecewize linear. The obtained solution gives the constant slopes of these curves in postbuckling region or p_x/p_x^* is independent upon ϵ_x/ϵ_x^*. The tangent modulus $E_x^t=1/A_{11}^th=\varphi_x^tE_x$ and reduction factor

$$\varphi_x^t=\frac{E_x^t}{E_x}=\frac{1}{E_x}\frac{dp_x}{d\varepsilon_x}=\frac{1+\gamma}{3+\gamma},\ (\varphi_x^t|_{\gamma=1}=0.5). \qquad (35)$$

These results qualitatively correspond to the results of numerical solution of the problem under consideration given by Stein[6]; they coincide for the initial phase of postbuckling deforming. The dotted curve in Figure 3 shows the particular case of isotropic plate, which correspond to the relationship (36) reliably verified by tests[2]

$$\varphi_x^s=\sqrt{\frac{p_x^*}{p_x}}=\sqrt[3]{\frac{\varepsilon_x^*}{\varepsilon_x}},\ \varphi_x^t=\frac{2}{3}\varphi_x^s. \qquad (36)$$

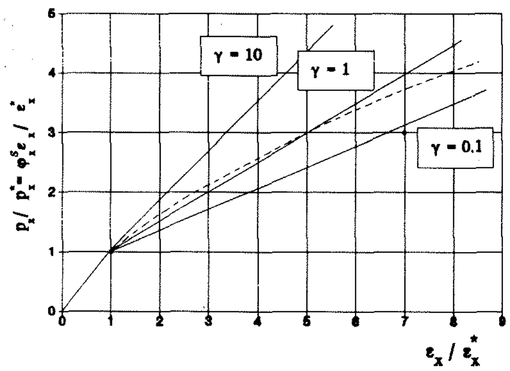

FIGURE 3 - Stess-Strain Curves for Uniaxial Compression

The obtained solution is rather accurate in the region of "moderate postbuckling" when the conditions $\epsilon_x/\epsilon_x^* \leq 7$, $p_x/p_x^* \leq 4$ are satisfied. If buckling state is exceeded more than in these conditions, the value of φ_x^s appreciably differs from the numerical result[(6)] and results of calculation by equations (36). In accordance with equation (32) this value approaches $(1+\gamma)/(3+\gamma)=\varphi_x^t$ as $p_x/p_x^* \to \infty$, while the tests and equation (36) give considerably smaller values $\varphi_x^s \to 0$. The reason of this discrepancy is certainly the constant representation of plate deflection mode (15), while the real mode modifies during the plate loading up to the possible full change of wave pattern[(5)]. For quasi-homogeneous plates in this case, it is possible to replace the relationships (32), (35) by the relationships (36).It should be noted that for uniaxial loading, the introduction of elastic modulus reduction factor φ_x^s with respect to the longitudinal stiffness of skin is the same as the introduction of effective skin width by von Karman $b_x=\varphi_x^s b$ at the same E_x, because $E_x^s b = E_x b_x = \varphi_x^s E_x b$.

In case of biaxial loading, after substituting equation (24) into equation (20) and taking into account that $e_x = \epsilon_x$, $e_y = \epsilon_y$ we can go over the usual relations (26) with reduced stiffness characteristics

$$\varepsilon_x=\frac{p_x}{E_x^s}-\mu_{xy}^s\frac{p_y}{E_y^s},\quad \varepsilon_y=\frac{p_y}{E_y^s}-\mu_{yx}^s\frac{p_x}{E_x^s},\tag{37}$$

where

$$E_x^s=\varphi_x^s E_x=\frac{1}{A_{11}^s h},\quad E_y^s=\varphi_y^s E_y=\frac{1}{A_{22}^s h},\quad \mu_{yx}^s=-\frac{A_{12}^s}{A_{11}^s},$$
$$\mu_{xy}^s=-\frac{A_{12}^s}{A_{22}^s}=\mu_{yx}^s\frac{A_{11}^s}{A_{22}^s}=\mu_{ys}^s\frac{E_y^s}{E_x^s},\tag{38}$$

and the secant reduction factors φ_x^s, φ_y^s, Poisson ratios μ_{xy}^s, μ_{yx}^s can be expressed by the following way

$$\varphi_x^s=\frac{1+\bar\gamma}{3+\bar\gamma-2/\bar p},\quad \varphi_y^s=\frac{1+\bar\gamma}{1+3\bar\gamma-2\bar\gamma/\bar p},$$
$$\mu_{xy}^s=\varphi_y^s\left[\mu_{xy}-\frac{2\bar\gamma}{\bar d+(1+\bar\gamma)}\left(1-\frac{1}{\bar p}\right)\right]=\varphi_y^s\mu_{xy}-\frac{1-\varphi_y^s}{\bar d},\tag{39}$$
$$\mu_{yx}^s=\varphi_x^s\left[\mu_{yx}-\frac{2\bar d}{(1+\bar\gamma)}\left(1-\frac{1}{\bar p}\right)\right]=\varphi_x^s\mu_{yx}-(1-\varphi_x^s)\bar d.$$

The following notation are introduced in (39)

$$\bar\gamma=\bar d^2\frac{E_y}{E_x}=\frac{l_x^4 E_y}{l_y^4 E_x},\quad \bar d=\frac{l_x^2}{l_y^2}=\sqrt{\bar\gamma\frac{E_x}{E_y}},\quad l_x=\frac{a}{m_0},\quad l_y=\frac{b}{n_0}.$$

The latter two depend on the direction of loading (ψ). Thus, as in case of uniaxial loading, the buckled plate can be replaced by the stiffness-equivalent nonbuckled plate made of nonlinear elastic material with characteristics (38),(39). The extent of critical state exceedance under biaxial compression is characterized by the load index $\bar p$ becoming equal to unit in critical state. In particular case of uniaxially loaded elongated plate, when $\bar\gamma=\gamma$, $\bar d=d$, $p_{x0}=p_x^*$, the relations (39) result in the above mentioned relations (32),(34). The important inverse statement is also true, that the general expressions (39) can be simply obtained from (32), (34) by substituting $\bar p$, $\bar\gamma$, $\bar d$ for p_x/p_x^*, γ, d respectively. After that φ_y^s, μ_{xy}^s can be obtained from φ_x^s, μ_{yx}^s by the simple change of variables $x\to y$, $y\to x$, $\bar\gamma\to 1/\bar\gamma$, $\bar d\to 1/\bar d$. . Stain-stress relations (37) in the obtained solution are piecewize linear with constant secant elastic characteristics

$$E_x^t=\varphi_x^t E_x=\frac{1}{\dfrac{\partial\varepsilon_x}{\partial p_x}}=\frac{1+\bar\gamma}{3+\bar\gamma}E_x,\quad A_{11}^t=E_x^t h,$$

$$E_y^t=\varphi_y^t E_y=\frac{1}{\dfrac{\partial \varepsilon_y}{\partial p_y}}=\frac{1+\bar{\gamma}}{1+3\bar{\gamma}}E_y, \quad A_{22}^t=E_y^t h,$$

$$\mu_{xy}^t=-E_y^t\frac{\partial \varepsilon_x}{\partial p_y}=\varphi_y^t\left[\mu_{xy}-\frac{2\bar{\gamma}}{\bar{d}(1+\bar{\gamma})}\right]=\varphi_y^t\mu_{xy}-\frac{1-\varphi_y^t}{\bar{d}}, \tag{40}$$

$$\mu_{yx}^t=-E_x^t\frac{\partial \varepsilon_y}{\partial p_x}=\varphi_x^t\left[\mu_{yx}-\frac{2\bar{d}}{1+\bar{\gamma}}\right]=\varphi_x^t\mu_{yx}-(1-\varphi_x^t)\bar{d},$$

$$A_{12}^t=-\mu_{yx}^t A_{11}^t=-\mu_{xy}^t A_{22}^t, \quad \mu_{xy}^t=\mu_{yx}^t\frac{E_y^t}{E_x^t}.$$

It should be noted that all stiffness matrixes of the buckled skin $[A_{ij}^s]$, $[A_{ij}^t]$, $[B_{ij}^s]=[A_{ij}^s]^{-1}$, $[B_{ij}^t]=[A_{ij}^t]^{-1}$ can be uniquely determined by the introduced secant and tangent reduction factors and other elastic characteristics of material (39), (40).

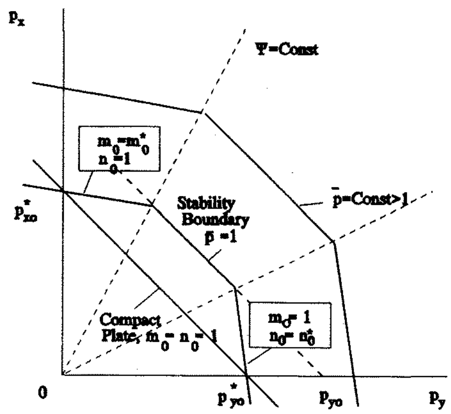

FIGURE 4 - Example of Stability Boundary and Postbuckling Plate Behavior

If numbers m_0, n_0 are fixed, all the reduced stiffness characteristics depend on the loads identically. The character of these functions is determined by the index $\bar{p}$; it is independent on the load path. Since in accordance with equation (23) in general case m_0, n_0 depend on nondimensional plate parameters α, β, a/b and they discretely change as ψ changes, the relations (39), (40) behave similarly. The boundary of stability $\bar{p}=1$ in p_x, p_y-frame of reference is the piecewize linear function; these pieces are created by the straight lines corresponding to m_0, n_0 = Const. For the constant values of $\bar{p}>1$, the similar piecewize boundaries with discrete change of m_0, n_0 and relations (39), (40) at corresponding values of ψ (see Fig.4).

In each rectilinear piece of this boundary, the reduced plate stiffness characteristics are constant and independent on the load direction. For the plate which form is close to the square when m_0, n_0 are independent on ψ and equal to unit, we obtain unique relations of reduction factors and loading index $\bar{p}$ for all possible load spectra. These relations (30), (40) are the same for all load paths including the case of uniaxial compression in x or y directions.

Shear consideration in compact plates

The plate is considered to be "compact" if α parameter satisfies the inequality $0.5 \le \alpha \le 2$. In application to such plate, similar approximate solution for postbuckling skin behavior were obtained for the combination of biaxial compression and shear $S \neq 0$. The same relationships (11)-(14), (16) and binomial approximation for deflection were used[7]

$$w(x,y)=f_{11}\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}+f_{22}\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{b}. \tag{41}$$

The expressions were obtained for reduced stiffenes characteristics introduced above as well as for the shear reduction factor $\varphi_\gamma^s=G_{xy}^s/G_{xy}$ $(\gamma_{xy}=\tau/G_{xy}^s)$ as a function of plate parameters and load level $\bar{p}$, $\bar{\tau}=\tau/\tau_0$, respectively. That solution is not shown here for the sake of paper brevity.

Analysis of stress state and stability of composite structures with the local skin buckling

It is known that the skin of thin-walled aerospace structures often buckle locally under load which is much less than the general failure load. The nonlinear problem arises to determine the general stress state and load-carrying capability of such structures with buckled skin[3]. For example, the static bending failure of wing box shown in Figure 1 is usually caused by the general instability of upper stringer-stiffened panel. The static strength in this case can be analyzed by the method described in the first part of present paper. After skin buckling, the panel is considered as the structurally orthotropic panel with reduced skin elastic characteristics which depend on the load level. These characteristics enter into the known expressions for stiffness **[B]**, **[D]** of structurally-orthotropic panel. Thus, two different problems should be solved. First, the prebuckling stress state in the structure with buckled skin is determined for the increased load (t parameter) by using some method, e.g. engineering "beam" method or finite element method. The stress resultants (membrane forces) are determined which nonlinearly depend on the external load ($N_x^0(s,t)$ in case of wing box bending). These resultants are necessary to solve the general instability problem for each t value. For cylindrical compound structures (i.e. wing, fuselage), it is advisable to solve this second problem by the above developed numerical method.

Let us consider in a general way both mentioned problems taking into account specific features of composite structures.

The numerical method for thin-walled structural stress state determination in application to the metal structures was reported by the authors on the Second Word Congress on the Computational Mechanics in Stuttgart[3]. With the help of special methods of reduction and special finite elements, the problem is reduced in essence to classical iteration methods of variable elasticity parameters and the others which are applied for the stress analysis of the structures made of nonlinear elastic materials[8]. The specific character of composite skin is only in the necessity of calculation of its secant reduced elastic characteristics in postbuckling phase by using the relatively simple relationships obtained above. Thus, the method of analysis proposed in paper[3] can be applied to the structures comprising composite elements.

If the restrictions stipulated in the beginning of this paper are valid, the second problem of general instability of both separate structurally-orthotropic panel and cylindrical structure as the whole can be solved for each load level by the described method. However in contrast to the SSS problem, the tangent stiffness should be used in stability equations, which characterize the relations between the increments of generalized forces and the deformations of elements. This result in the necessity of calculation of above mentioned tangent elastic characteristics for buckled composite skin, in particular according to relationships (40). Naturally, further theoretical and experimental investigations are necessary to make both secant and tangent reduction factors for locally buckled composite elements more accurate. This specially concerns with nonplanar, nonrectangular elements at complicated boundary conditions.

The authors clearly realize the approximate and restricted character of the proposed approach to due regard for postbuckling deformations of composite skin when analyzing the general stress-strain state and stability of complex structures. But the real alternative is only finite-element solution of this problem in general geometrically nonlinear statement for very dense meshes, and with revealing all the local and general buckling modes and bifurcation points up to the structural failure loads. This method of analysis is theoretically possible, if modern FEM computer systems are in use, but the reliability of the obtained results will always be doubtful.

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