VIBRATIONS AND STABILITY OF A BISELF-EXCITED SURFACE STRUCTURE IN SUPERSONIC FLOW SUBJECTED TO A FOLLOWER FORCE

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Abstract

A surface structure in supersonic flow having a deformable support and subjected to a follower compressing force, which preserves its direction tangential to the deformed surface, is a biself-excited system enclosing two independent physical factors being the reason of self-excited vibrations.

In the paper a study of vibrations and stability of such a structure is presented by way of example of rectangular plate in one-side supersonic flow subjected to a follower force. The plate is considered under assumption that the conditions of rigid support are satisfied at the plate edges parallel to the unperturbed flow direction. One of the remaining edges is clamped, while the second one has a deformable support.

A number of numerical calculations have been performed. The analysis indicates a variety of phenomena resulting from simultaneous action of the two independent factors decisive for self-excitation of the structure under consideration.

Nomenclature

$W=W(x,y,t)$ - transversal displacement of the plate
x,y - rectangular coordinates of the plate
t - time

$$D=\frac{Eh^3}{12(1-\nu^2)}$$ - flexural rigidity of the plate

h - plate thickness
E -Young's moduls
ν - Poisson's ratio
ρ_p - density of plate material
l - plate length
b - plate width
$\beta=b/l$
θ_0 - damping of the plate material
N_x, N_y - forces acting in the plane of the plate
k, c - coefficients of elasticity and damping of the deformable support
a_0, ρ_0 - sound velocity and gas density in unper turbed flow
$M=U_0/a_0 > 1$ - Mach number of unperturbed flow

$$\mu=\sqrt{M^2-1}$$

$$\sigma_x=S_x/\pi^2, \qquad \sigma_y=S_y/\pi^2$$

1.Introduction

Vibrations and stability of plates and shells in supersonic flow have been considered in a number of papers (cf. for instance [1],[2],[5]). The effect of a follower force, which preserves its direction tangential to the deformed surface of a structure has been investigated in [3],[6],[7]

Plates of finite length in plane supersonic flow subjected to a follower compressing force have been studied in [4], [8], [9].

In this paper vibrations and stability of a rectangular plate in supersonic flow,having a deformable support and subjected to a compressing follower force, are considered. It is assumed that the conditions of rigid support are satisfied at the plate edges parallel to the unperturbed flow direction. One of the remaining edges is clamped, while the second one has a deformable support. External and internal damping and forces acting in the plane of the plate are taken into account. The solution representing vibrations of the plate in supersonic flow is obtained in a finite form making use of the Laplace transformation.

Numerical analysis of the solution enables us to determine the boundaries of stable and unstable vibrations depending on the parameters of the problem.

2. Statement of the Problem

Let us consider autonomous vibrations of a rectangular plate of length l and width b which is exposed to one-side flow of a supersonic stream of gas whose unperturbed velocity is $U_o > a_o$ (Fig.1). Two directions of the flow velocity U_o are assumed, as it is shown in Fig.1 - that is the case *(1)* or the case *(2)*.

The conditions of rigid support are satisfied at the plate edges parallel to the unperturbed flow directions for $y=0$ and $y=b$. The edge for $x=0$ is assumed to be clamped,while the second one, for $x=l$, has a deformable support (Fig.1).

The plate is subjected to the pressure difference $\Delta p=\Delta p[W(x,y,t)]$ produced by the plate motion in the flow and the plate edges are acted on by constant forces N_x, N_y in the plane of the plate.We assume that the force N_x is a follower one which preserves its direction tangential to the deformed surface of the plate. Internal damping of the plate material is taken into account by means of the Voigt model, similarly to the previous papers [1]- [5].

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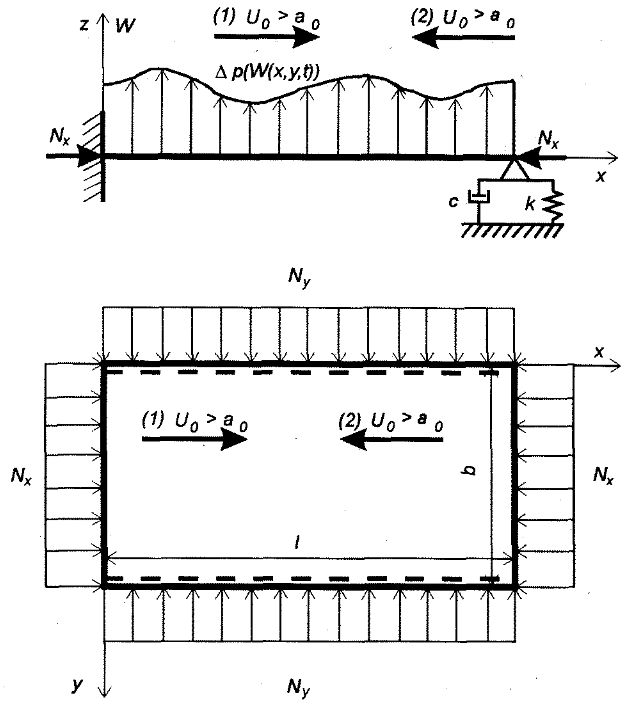

Fig. 1

We obtain the equation of motion in the following form

$$D\left(1+\theta_0\frac{\partial}{\partial t}\right)\nabla^2\nabla^2 W + N_x\frac{\partial^2 W}{\partial x^2} + N_y\frac{\partial^2 W}{\partial y^2} + \\ +\rho_p h\frac{\partial^2 W}{\partial t^2} = \Delta p[W(x,y,t)] \tag{2.1}$$

The pressure difference $\Delta p[W(x,y,t)]$ is determined on the basis of the potential theory of linearized supersonic flow after neglecting the secondary part of the expression for Δp (cf.[1],[2]). We can adopt the following approximate formula

$$\Delta p[W(x,y,t)] = \Delta p_{1,2} = p_0 - p_{1,2} = \\ = \frac{-\rho_0 U_0}{\mu}\left[\left(1-\frac{1}{\mu^2}\right)\frac{\partial W}{\partial t} \pm U_0\frac{\partial W}{\partial x}\right] \tag{2.2}$$

which is valid for both directions of the unperturbed flow (Fig.1). The boundary conditions of the plate are assumed :

for $y=0$ and $y=b$: $$W = \frac{\partial^2 W}{\partial y^2} = 0 \tag{2.3}$$

for $x=0$: $$W = \frac{\partial W}{\partial x} = 0 \tag{2.4}$$

and for $x=l$ $$\frac{\partial^2 W}{\partial x^2} + \nu\frac{\partial^2 W}{\partial y^2} = 0 \tag{2.5}$$

$$D\left(1+\theta_0\frac{\partial}{\partial t}\right)\left[\frac{\partial^3 W}{\partial x^3} + (2-\nu)\frac{\partial^3 W}{\partial x \partial y^2}\right] = \\ = kW + c\frac{\partial W}{\partial t} \tag{2.6}$$

The equation of motion (2.1), (2.2) will be written in a dimensionless form assuming that the coordinates x,y and the normal displacement of the plate W are referred to the plate length l, and time t-to the quotient $1/\omega_o$, where

$$\omega_0 = \frac{\pi^2}{l^2}\sqrt{\frac{D}{h\rho_p}} \tag{2.7}$$

We then obtain the equation

$$D\left(1+\theta\frac{\partial}{\partial t}\right)\nabla^2\nabla^2 W + S_x\frac{\partial^2 W}{\partial x^2} + S_y\frac{\partial^2 W}{\partial y^2} + \\ +\gamma_1\frac{\partial W}{\partial x} + \pi^4\frac{\partial^2 W}{\partial t^2} + \gamma_2\frac{\partial W}{\partial t} = 0 \tag{2.8}$$

where

$$\theta = \theta_0\omega_0, \quad S_x = \frac{N_x l^2}{D}, \quad S_y = \frac{N_y l^2}{D} \tag{2.9}$$

$$\gamma_1 = \pm\frac{\rho_0 U_0^2 l^3}{\mu D} \tag{2.10}$$

$$\gamma_2 = |\gamma_1|\delta, \quad \delta = \frac{\omega_0 l}{a_0}\frac{M^2-2}{M\left(M^2-1\right)} \tag{2.11}$$

It should be noted that the coefficient γ_1 takes positive values for the unperturbed flow direction (1) (Fig.1) and negative values for the direction(2).

The boundary conditions (2.3), (2.4) and (2.5) remain unchanged, while the condition (2.6) becomes for x=1

$$\left(1+\theta\frac{\partial}{\partial t}\right)\left[\frac{\partial^3 W}{\partial x^3} + (2-\nu)\frac{\partial^3 W}{\partial x \partial y^2}\right] = \\ = \kappa_p W + \gamma_p\frac{\partial W}{\partial t} \tag{2.12}$$

where

$$\kappa_p = \frac{kl^3}{D}, \quad \gamma_p = \frac{cl^3\omega_0}{D} \tag{2.13}$$

Equation (2.8) involves two independent dimensionless parameters S_x and γ_1 which can cause the self-excitation of the system under consideration. For this reason it is called a biself- excited system.

3. Salution of the Problem

The solution of equation (2.8) will be sought in the form

$$W(x,y,t) = V_n(x)\sin\left(\frac{n\pi}{\beta}y\right)e^{ipt} \tag{3.1}$$

where the function $V_n(x)$ satisfies the equation

$$V_n''''(x) + b_2V_n''(x) + b_1V_n'(x) + b_0V_n(x) = 0 \tag{3.2}$$

and the coefficients b_k are defined by the formulae

$$\begin{aligned} b_0 &= \left(\frac{n\pi}{\beta}\right)^4 - \left[S_y\left(\frac{n\pi}{\beta}\right)^2 + \pi^4p^2 - i\gamma_2 p\right](1+ip\theta)^{-1} \\ b_1 &= \frac{\gamma_1}{1+ip\theta} \\ b_2 &= \frac{S_x}{1+ip\theta} - 2\left(\frac{n\pi}{\beta}\right)^2 \end{aligned} \tag{3.3}$$

On applying the Laplace transformation to equation (3.2), we obtain the salution (cf. [1]-[4])

$$V_n(x) = v_2K_n'(x) + v_3K_n(x) \tag{3.4}$$

where v_2, v_3 are initial values of the derivatives of the function $V_n(x)$

$$v_2 = V_n''(0), \qquad v_3 = V_n'''(0) \tag{3.5}$$

It is already assumed in (3.4) that

$$V_n(0) = V_n'(0) = 0 \tag{3.6}$$

The function $K_n(x)$ is determined by the formula

$$K_n(x) = \sum_{j=1}^{4} c_j \exp(s_jx) \tag{3.7}$$

where the coefficients s_j are roots of the characteristic equation

$$d(s) = s^4 + b_2s^2 + b_1s + b_0 = 0 \tag{3.8}$$

and the coefficients c_j are given by

$$c_j = \frac{1}{d'(s_j)} = \left(4s_j^3 + 2b_2s_j + b_1\right)^{-1}; \; j = 1,2,3,4 \tag{3.9}$$

In order to determine critical parameters at the limits of stable and unstable vibrations, we should take into account the boundary conditions (2.5) and (2,12) for x=1.

Making use of (3.1), (3.4) and the mentioned boundary conditions we obtain the frequency equation in the form:

$$\begin{aligned} \Delta &= \left[K_n'''(1) - \nu\left(\frac{n\pi}{\beta}\right)^2 K_n'(1)\right]\Big\{(1+ip\theta)\Big[K_n'''(1) + \\ &-(2-\nu)\left(\frac{n\pi}{\beta}\right)^2 K_n'(1)\Big] - (\kappa_p + ip\gamma_p)K_n(1)\Big\} + \\ &-\left[K_n''(1) - \nu\left(\frac{n\pi}{\beta}\right)^2 K_n(1)\right]\Big\{(1+ip\theta)\Big[K_n''''(1) + \\ &-(2-\nu)\left(\frac{n\pi}{\beta}\right)^2 K_n''(1)\Big] - (\kappa_p + ip\gamma_p)K_n'(1)\Big\} = 0 \end{aligned} \tag{3.10}$$

and the function (3.4) can be written as

$$V_n(x) = v_2\left[K_n'(x) + \alpha K_n(x)\right] \tag{3.11}$$

where

$$\alpha = \frac{v_3}{v_2} = \frac{K_n'''(1) - \nu\left(\frac{n\pi}{\beta}\right)^2 K_n'(1)}{\nu\left(\frac{n\pi}{\beta}\right)^2 K_n(1) - K_n''(1)} \tag{3.12}$$

The solution representing the autonomous vibration of the structure under study can be written finally in the form:

$$\begin{aligned} W(x,y,t) &= \sin\left(\frac{n\pi}{\beta}y\right)\mathrm{Real}\left[V_n(x)e^{ipt}\right] = \\ &= \sin\left(\frac{n\pi}{\beta}y\right)a_n(x)\cos\left(pt + \varphi_n(x)\right) \end{aligned} \tag{3.13}$$

where

$$\begin{aligned} a_n(x) &= \sqrt{\mathrm{Re}V_n^2(x) + \mathrm{Im}V_n^2(x)} \\ tg\varphi_n(x) &= \frac{\mathrm{Im}V_n(x)}{\mathrm{Re}V_n(x)} \end{aligned} \tag{3.14}$$

On substituting

$$p = q - i\varepsilon \tag{3.15}$$

into (3.10) we can find, for given values of S_x, S_y, γ_1, δ, θ, κ_p, γ_p, natural frequencies $q = q_m$, $m = 1,2,3,,$ and their decrements of decrease ($\varepsilon = \varepsilon_m < 0$) for damped vibrations and those of increase ($\varepsilon = \varepsilon_m > 0$) in the case of unstable self-excited vibrations. From equation (3.10) we can also determine the critical parameters of self-excited vibrations S_{xcr}, q_{cr} *or* γ_{1cr}, q_{cr} for given values of other parameters and $\varepsilon = 0$. From this equation we can also find the critical parameters of divergence S_{xcr} *or* γ_{1cr} for $q = \varepsilon = 0$.

4. Numerical Analysis of the Frequency Equation

In order to determine the critical parameters of the biself-excited system under consideration, we shall examine solutions of the frequency Eq. (3.10) in the vicinity of the first and second natural vibrations of the structure.

On substituting (3.15) into (3.10) and splitting Δ into real and imaginary parts we obtain the equations

$$Re\Delta = 0\,, \quad Im\Delta = 0 \tag{4.1}$$

Values of the function $V_n(x)$ and its derivatives for $x = 1$ which occur in (3.10) can be found in the course of calculations and then real roots q can be determined separately from the first and the second equation (4.1) in terms of

$$\sigma_x = S_x / \pi^2\,, \quad \sigma_y = S_y / \pi^2 \tag{4.2}$$

and γ_1, δ, θ, κ_p, γ_p, β, n in the case of harmonic ($\varepsilon = 0$) and nonstationary ($\varepsilon \neq 0$) vibrations. From Eqs. (4.1) we can obtain on the γ_1, q - plane and on the σ_x, q - plane the lines of roots of $Re\Delta = 0$ and $Im\Delta = 0$ which are presented by way of examples in Figs.2 and 3.

Diagrams in Figs.2 and 3 have been determined for $\varepsilon = \sigma_y = \gamma_p = \theta = 0$, $\beta = n = 1$, $\delta = 0.1$ and a number of values of σ_x, γ_1, κ_p. The intersection points of the lines (4.1) determine the critical parameters of self-excited vibrations γ_{1cr}, q_{cr} or σ_{xcr}, q_{cr} and the critical parameters of divergence γ_{1cr} or σ_{xcr} for $q = 0$.

The lines $Re\Delta = 0$ in Figs.2 and 3 show the course of changes of the first and the second natural frequencies in terms of the parameters γ_1, σ_x, κ_p which are decisive for self-excitation of the structure. The values of critical parameters obtained in Fig.2 are given in Table 1 and those from Fig.3 - in Table 2.

Table 1

$\sigma_y=0,\ \gamma_p=0,\ \beta=1,\ n=1,\ \Theta=0,\ \delta=0.1$				
No	σ_x	κ_p	γ_{1cr}	q_{cr}
1	0	10^8	670.218	5.229
1'	0	10^8	-670.218	5.229
2	0	10^3	615.602	5.109
2'	0	10^3	-568.195	4.953
3	0	10^2	453.293	4.455
3'	0	10^2	-211.690	2.746
4	6	10^8	191.244	2.166
4'	6	10^8	-191.244	2.166
5	6	10^8	120.267	0
5'	6	10^8	-120.267	0
6	6	10^3	153.759	2.206
6'	6	10^3	-175.660	1.968
7	6	10^3	76.904	0
7'	6	10^3	-120.150	0
8	6	10^2	62.320	1.951
8'	6	10^2	-135.450	1.173
9	6	10^2	-8.095	0
9'	6	10^2	-119.400	0

Table 2

$\sigma_y=0,\ \gamma_p=0,\ \beta=1,\ n=1,\ \Theta=0,\ \delta=0.1$				
No	γ_1	κ_p	σ_{xcr}	q_{cr}
1'	0	10^8	4.847	0
1"	0	10^8	8.331	0
2'	±100	10^8	5.519	0
2"	±100	10^8	7.746	0
3	±200	10^8	5.857	2.273
4	±300	10^8	4.370	3.199
5'	0	10^2	6.351	0
5"	0	10^2	7.369	0
6	100	10^2	5.235	2.361
7'	-100	10^2	5.506	0
7"	-100	10^2	7.779	0
8	200	10^2	3.475	3.140
9	-200	10^2	0.838	2.615

From Fig.2 it is seen that for the rigid support ($\kappa_p=10^8$) the course of natural frequencies and critical parameters are the same for both flow directions in Fig.1 and there appears asymmetry for a deformable support. The diagrams in Fig. 3 show that for deformable support the force N_x can act as a follower one and it is a reason of self-excitation of the structure.

Making use of the results of analysis on the γ_1, q and σ_x, q planes we can establish the diagrams of stability and instability limits of the structure under study on the γ_1, σ_x plane for various parameters of the problem. They are shown in subsequent Figs.4 to 7.

In Fig.4 the diagrams of stability limits are presented for the rigid support ($\kappa_p = 10^8$) of a square plate ($n = \beta = 1$) and a plate of finite length and infinite width ($n = 0$). In this figure we can see the flutter limits and the divergence limits for the first and second mode of vibration and for both directions of flow (Fig.1).

Flutter and divergence limits for plates with a deformable support ($\kappa_p = 10^2$) are shown in Fig.5. The square plate ($n= 1$) and the plate of infinite width ($n = 0$) are considered.

In Fig.4 we can see the symmetric course of diagrams for $\gamma_1 > 0$ and $\gamma_1 < 0$, while in Fig.5 the diagrams lose their symmetry and the regions of divergence are shifted to the interval $\gamma_1 < 0$. The diagrams of stability limits for the square plate with rigid support ($\kappa_p = 10^8$) and deformable one ($\kappa_p = 10^2$) are given in Fig.6.

The effect of material damping $\theta = 0.05$ on the course of diagrams of stability limits is considered in Fig.7 for the square plate.

In the next Figs.8 to 11 modes of vibration of a square plate are presented in the case of rigid support (Figs.8 and 9) and deformable ones (Figs.10 and 11) for both flow directions considered. Asymmetry of the modes and moving nodes can be seen.

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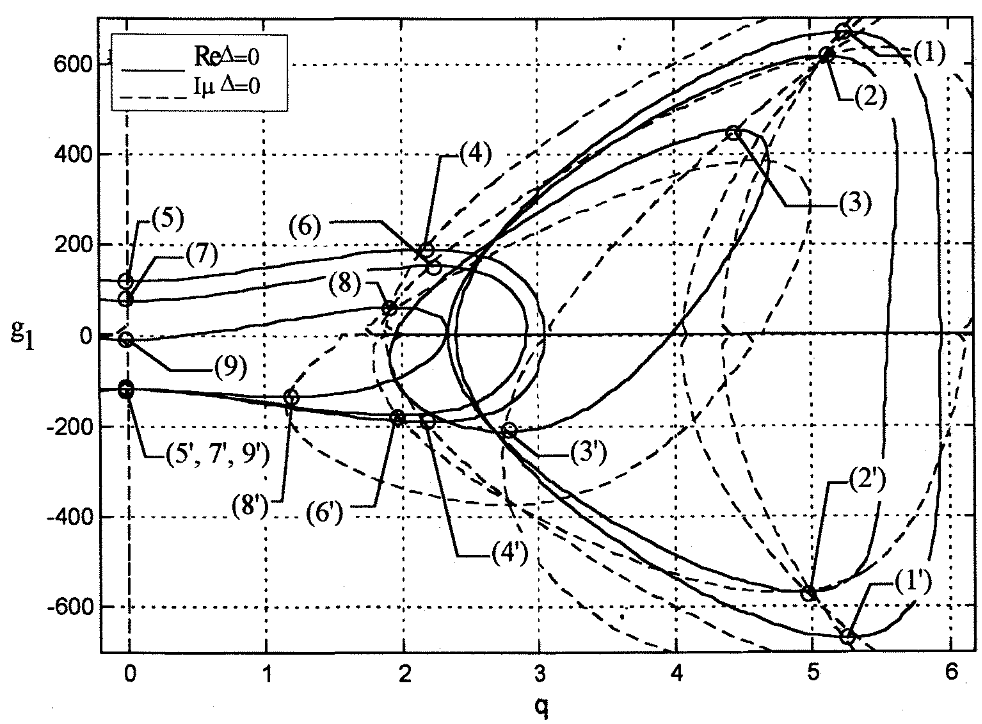

Fig.2. The lines ReΔ=0 and ImΔ=0 on the γ_1, q plane

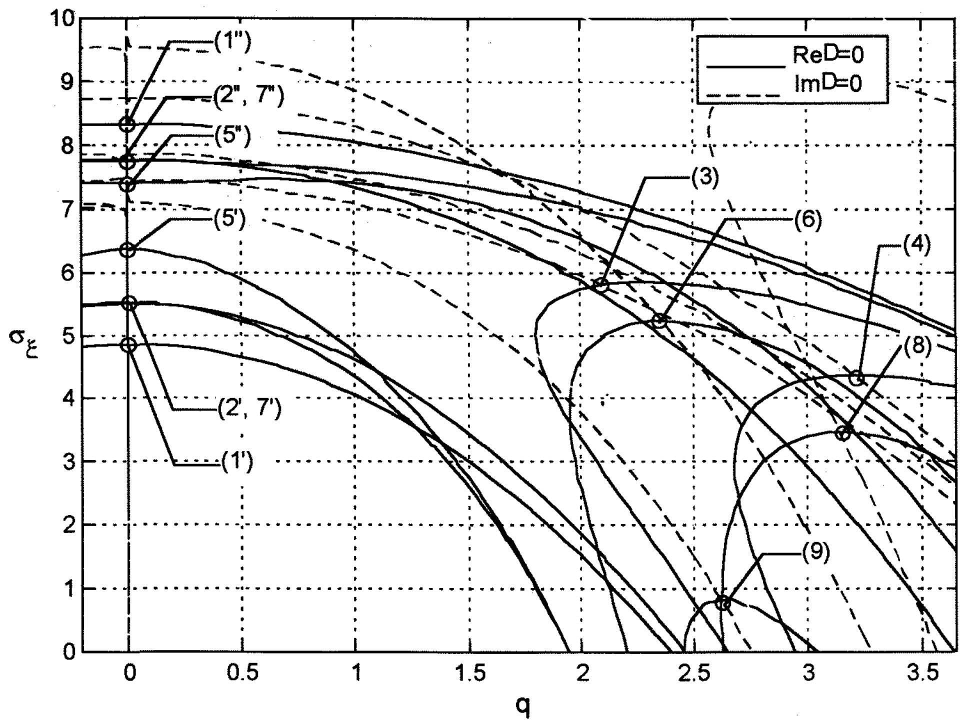

Fig. 3 . The lines ReΔ=0 and ImΔ=0 on the σ_x, q plane

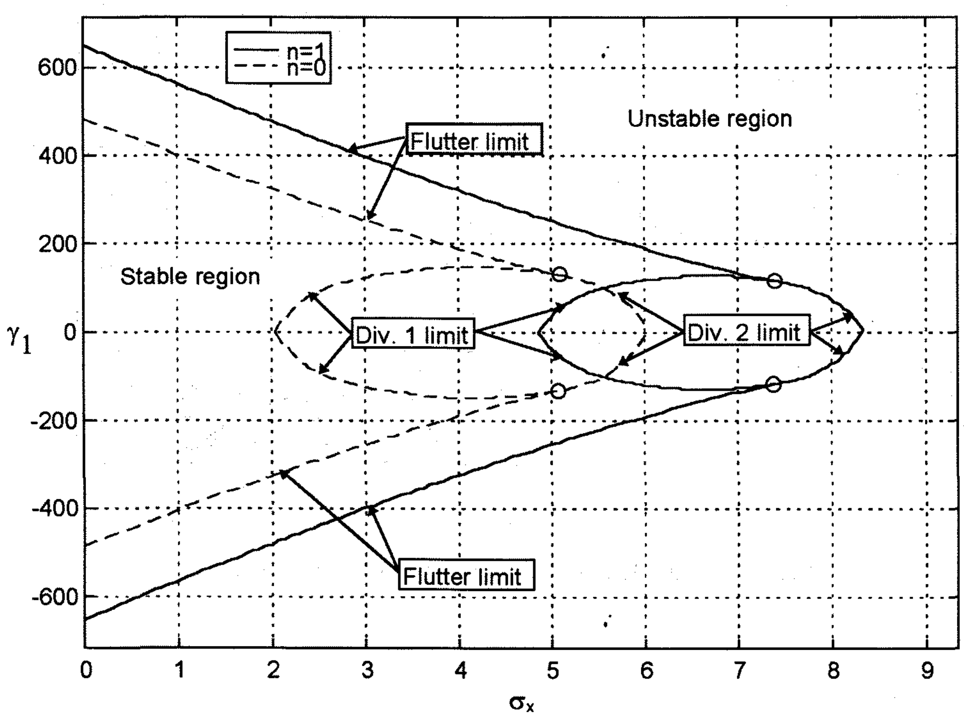

Fig.4. Stability limits on the γ_1, σ_x plane
($\Theta=0$, $\gamma_2=|\gamma_1|\delta$, $\delta=0.1$, $\kappa_p=10^8$, $\gamma_p=0$, $\sigma_y=0$, $\beta=1$)

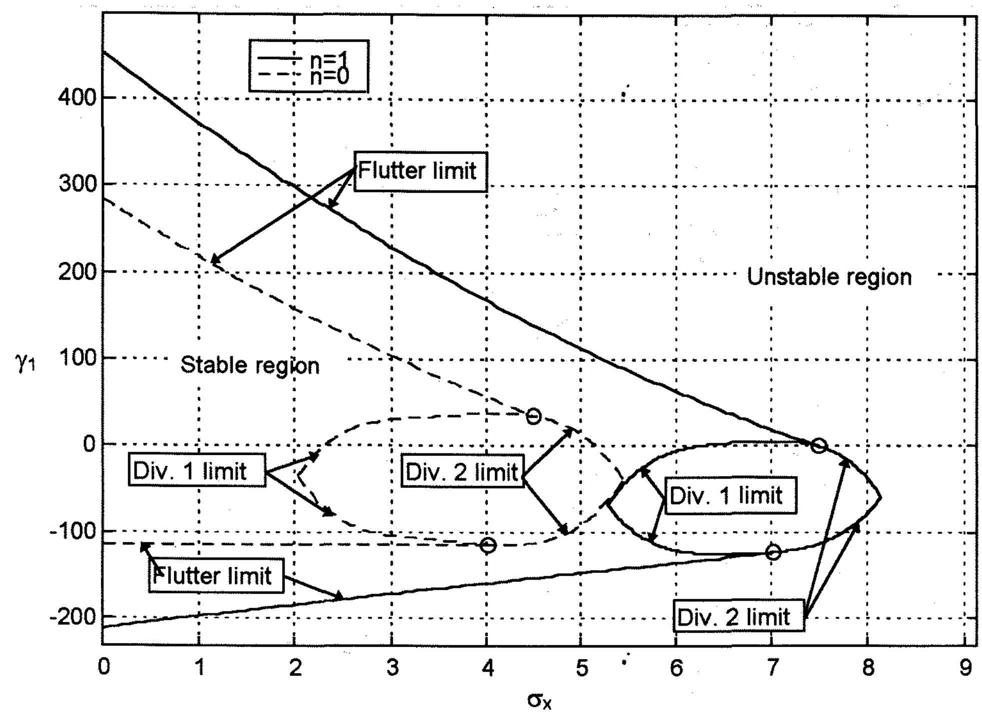

Fig.5. Stability limits on the γ_1, σ_x plane
(Θ=0, γ_2=$|\gamma_1|\delta$, δ=0.1, κ_p=10^2, γ_p=0, σ_y=0, β=1)

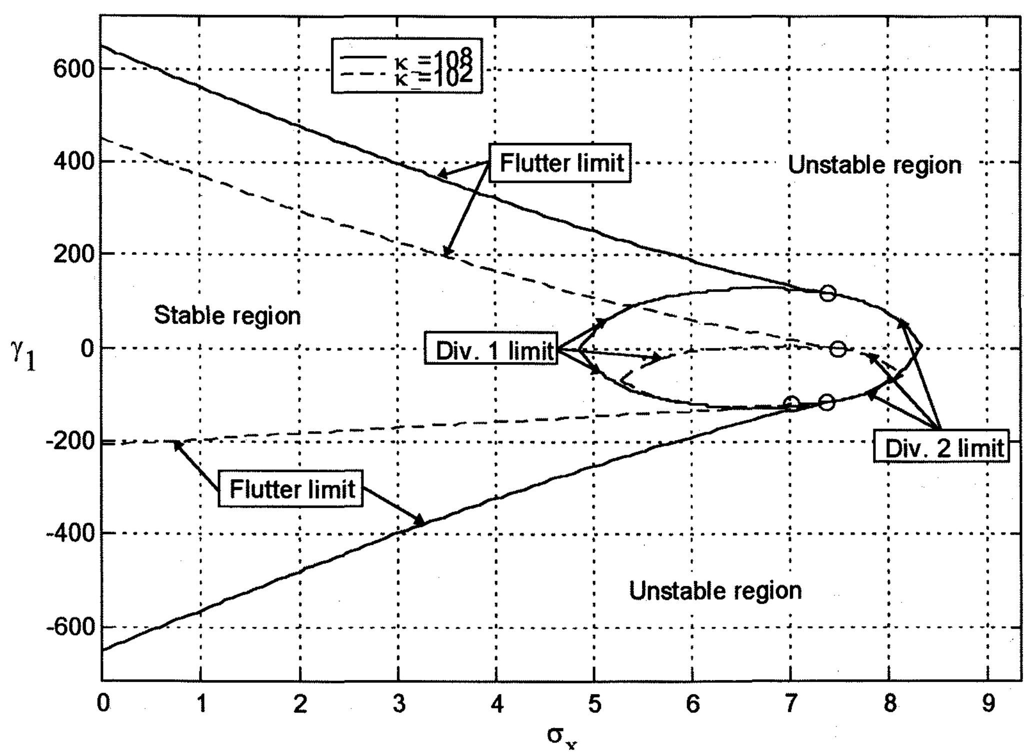

Fig.6. Stability limits on the γ_1, σ_x plane
(γ_2=$|\gamma_1|\delta$, δ=0.1, Θ=0, γ_p=0, σ_y=0, β=1, n=1)

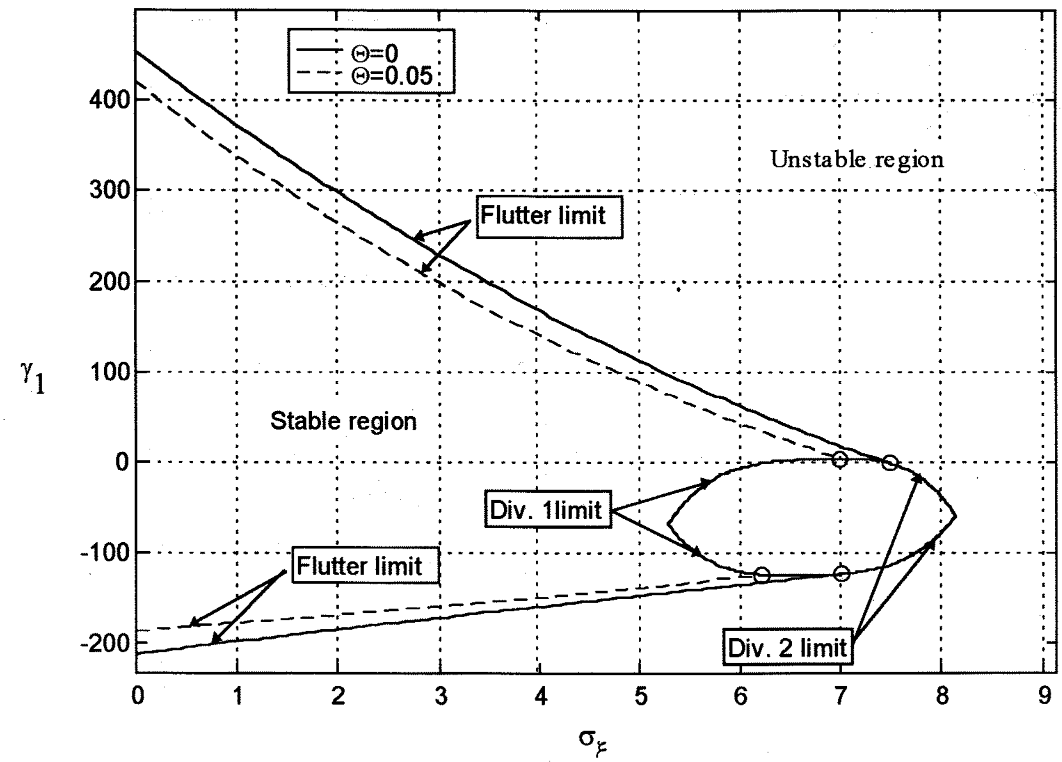

Fig.7. Stability limits on the γ_1, σ_x plane
($\gamma_2=|\gamma_1|\delta$, $\delta=0.1$, $\kappa_p=10^2$, $\gamma_p=0$, $\sigma_y=0$, $\beta=1$, n=1)

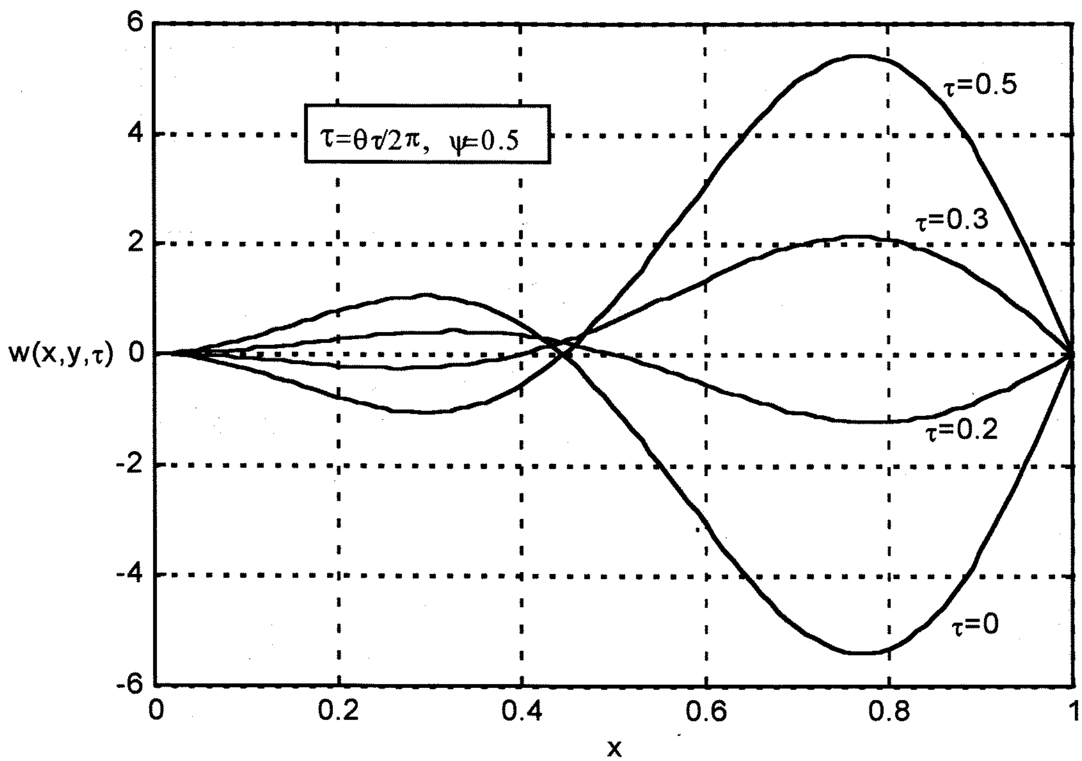

Fig.8. The mode of vibrations for rigid supports and the flow direction(1)
(q=2.962, $\gamma_1=290.45$, $\gamma_2=\gamma_1\,\delta$, $\delta=0.1$, $\kappa_p=10^8$, $\gamma_p=0$, $\sigma_x=2.5$, $\sigma_y=0$, $\beta=1$, n=1)

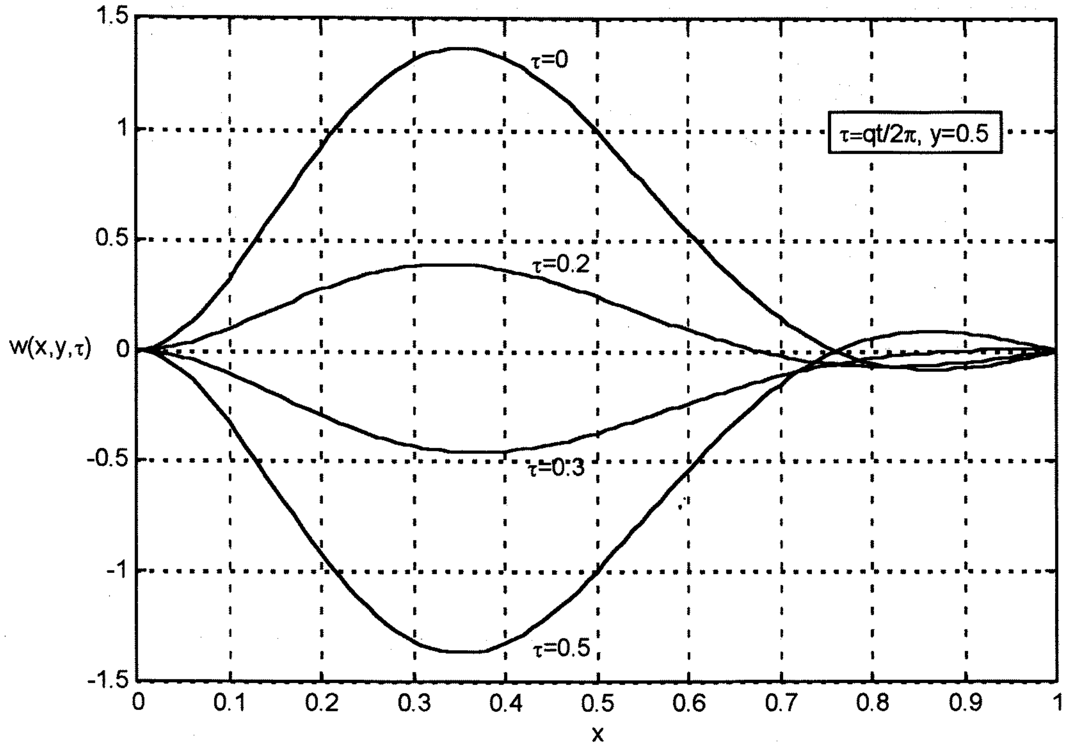

Fig.9. The mode of vibrations for rigid supports and the flow direction (2)
($q=2.962$, $\gamma_1=-290.45$, $\gamma_2=|\gamma_1|\,\delta$, $\delta=0.1$, $\kappa_p=10^8$, $\gamma_p=0$, $\sigma_x=2.5$, $\sigma_y=0$, $\beta=1$, $n=1$)

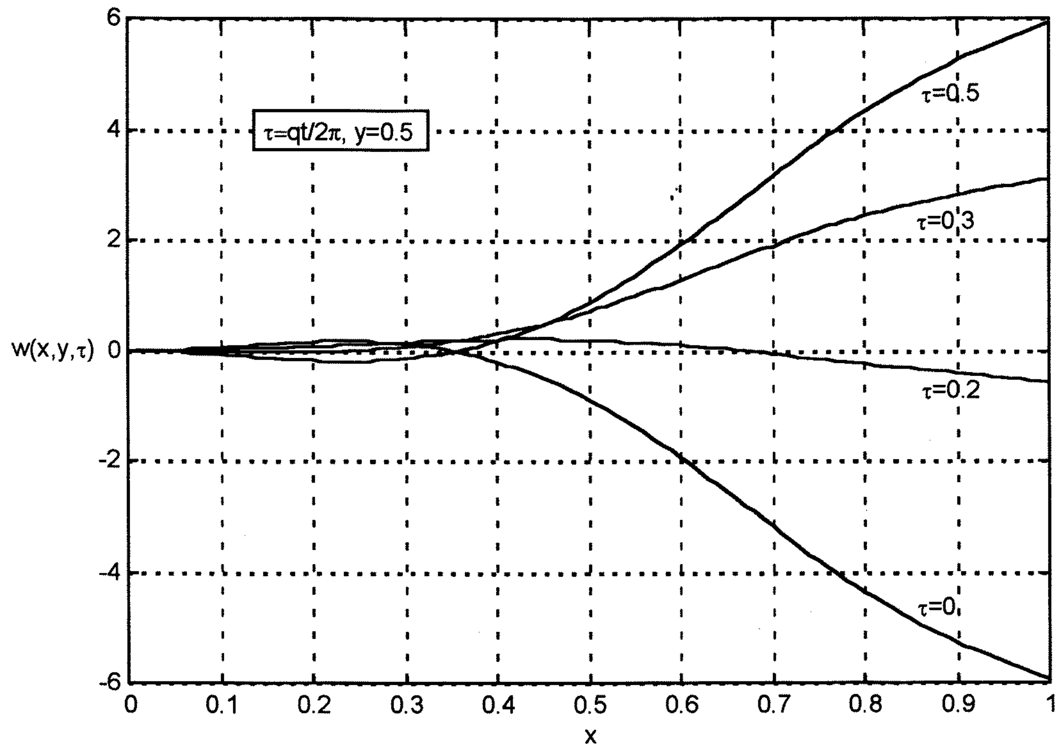

Fig.10. The mode of vibrations for a deformable support and the flow direction(1)
($q=3.716$, $\gamma_1=298.37$, $\gamma_2=\gamma_1\,\delta$, $\delta=0.1$, $\kappa_p=100$, $\gamma_p=0$, $\sigma_x=2$, $\sigma_y=0$, $\beta=1$, $n=1$)

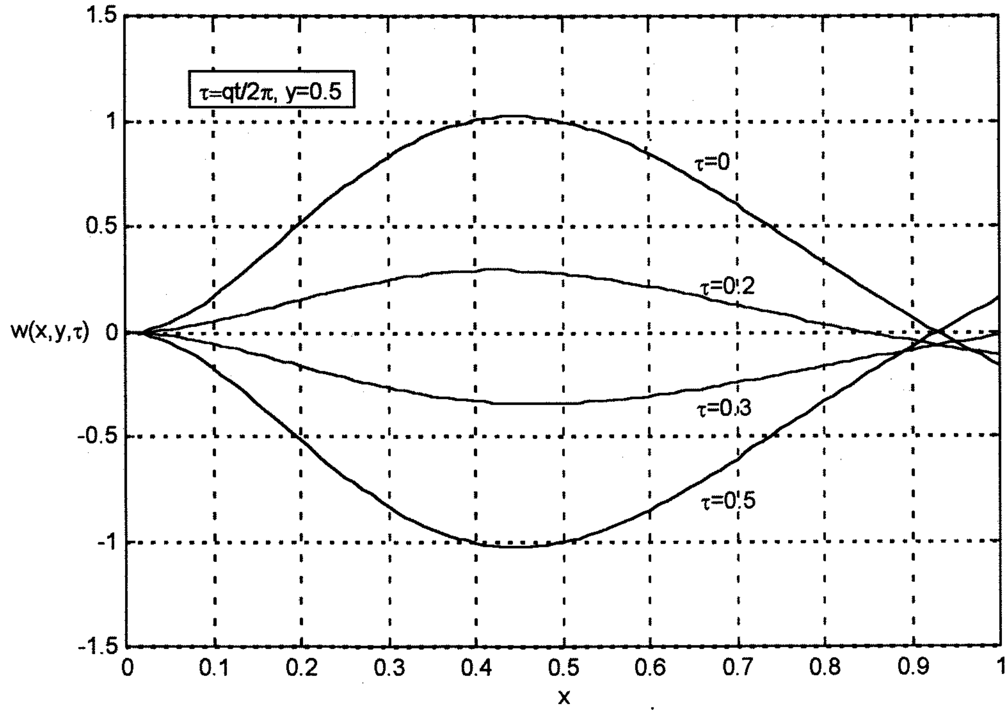

Fig.11.The mode of vibrations for a deformable support and the flow direction(2) ($q=2.409$, $\gamma_1=-184.49$, $\gamma_2=|\gamma_1|\delta$, $\delta=0.1$, $\kappa_p=100$, $\gamma_p=0$, $\sigma_x=2$, $\sigma_y=0$, $\beta=1$, $n=1$)