ANALYTICAL-COMPUTATIONAL METHOD FOR MATCHING OPTIMAL CONTROL FORMULATIONS

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Abstract

An analytical-computational method for matching two optimal control formulations is presented. Both formulations rely on the classical time domain, constant matrix representation of linear control systems. The first formulation, referred to as Direct System, consists in directly enforcing a prescribed time behavior of output quantities and solving for the control activity. The second formulation, referred to as Quadratic System, is associated to the use of linear quadratic regulator theory. Matching the two formulations is accomplished by enforcing the time behavior identity of control, state, output vectors using an appropriate form of the regulator equations, and verifying the conditions for the mathematical existence of the Quadratic System: a routine is presented which computes sets of weight matrices associated to a quadratic performance index that exactly generate the matched feedback control feedback matrix as summarized here above.

A numerical application to ascertain theoretical results is run onto aircraft maneuver load alleviation, but the generality of the method makes it also suitable for studying other problems in aeroservoelasticity and advanced guidance and control of missiles, for instance analyzing optimal guidance versus other criteria such as proportional navigation.

Nomenclature

A/C	:	aircraft
[A], [Å], [A#]	:	dynamic matrices
[B], [B#]	:	control-state matrices
BM20, BM40	:	wing bending moment at 1/3, 2/3 span station
[C], [Ç], [C^], [C#]	:	state-output matrices
C(I,J)	:	generic element of [C]
CG	:	aircraft center of gravity
[D], [D^], [D#]	:	control-output matrices
D(I,J)	:	generic element of [D]
DS	:	direct system
{e}	:	error on state vector
J	:	quadratic performance index
[K]	:	regulator control matrix
LQR	:	linear quadratic regulator
[N]	:	weight matrix of J
NZ	:	vertical acceleration at CG
P	:	matrix order
[Q]	:	weight matrix of J
{Q°}	:	vector form of [Q]
q	:	aircraft pitch rate
QS	:	quadratic system
[R]	:	weight matrix of J
{R°}	:	vector form of [R]
[S]	:	solution of the Riccati equation
{S°}	:	vector form of [S]
TQ20, TQ40	:	wing torque at 1/3, 2/3 span station
u	:	aircraft forward velocity
{v}	:	control vector
$\{v_e\}$	:	error on control vector
$\{v_K\}$	:	regulator control vector
w	:	aircraft vertical velocity
{X}	:	state vector
{Y}	:	output vector
θ	:	aircraft pitch angle
λ	:	level of alleviation
η	:	aileron deflection
ξ	:	elevator deflection
$[\Pi 1]$, $[\Pi 2]$	:	solver matrices for {S°}
$\{\phi\}$	:	input vector to DS
$(^*)$, $(_d)$, $(_q)$, $(_Q)$, $(_R)$	:	quantity refers to unalleviated system, DS, QS, [Q], [R]
$()^T$	:	() transposed

1. Introduction

The application of LQR in problems of minimization relies on the use of a quadratic performance index associated to the well-known Riccati equation. An intrinsic complication therein is represented by the fact that the quality of the solution is strongly affected by the form of the performance index integrand (presence of cross-coupling terms) and the numerical values of the weight matrices. Minimization of the quadratic index is rarely the true design objective.[1,2] In the open literature, it is sometimes found that authors do not use the cross-coupling terms and rely on the use of unit matrices.[3,4] A better way of handling the problem is to use the so-called Bryson rule[2,5,6,7,8] for setting reasonable initial values of the weight matrices. An alternative method consists in using energy,[4,9] pole placement and actuator saturation[1,5] considerations. Once initial values are found, the further step consists in trial-and-error looping the elements of the weight matrices which ends when design requirements are met, and therefore the so-called optimal solution is identified.

Referring to the common applications of LQR to aeroservoelasticity and flight dynamics, the control law is usually formulated to stabilize unstable short period modes while maintaining or augmenting the stability of the aeroelastic modes and to achieve gust load alleviation and ride improvement: in all of these cases, the control system is required to fulfill a pole-placement assignment and/or destroy the effects of a gust disturbance input.

If more demanding constraints are required, as for instance time-matching a prescribed behavior of selected dynamic quantities, the afore-mentioned criteria for selecting weight matrices combined with a poor formulation of the performance index integrand may fail. Nevertheless, LQR-based solutions can be very precise: a technique for generating a LQR-controlled dynamic system which time-matches a non-LQR optimal system is assessed thoroughly in the following sections.

2. Summary of Equations

2.1 Basic System

One refers to a steady matrix formulation for linear systems represented by the following set of equations:

$$\begin{cases} \{dX/dt\} = [A]\{X\}+[B]\{v\} & (2.1.1) \\ \{Y\} = [C]\{X\}+[D]\{v\} & (2.1.2) \end{cases}$$

In the following, one will refer to an "unalleviated" system when dealing with a dynamic system which does not fulfill prescribed sets of constraints and levels of optimization of output quantities. The relative arrays will be indicated with a (*) superscript. In this way, the unalleviated system looks like:

$$\begin{cases} \{dX^*/dt\} = [A]\{X^*\}+[B]\{v^*\} & (2.1.3) \\ \{Y^*\} = [C]\{X^*\}+[D]\{v^*\} & (2.1.4) \end{cases}$$

2.2 Direct System

This section deals with a dynamic system in which the control action is derived from directly prescribing a set of constraints and optimization levels of output quantities. This system will be referred to as "Direct System" (DS) and the relative arrays will be indicated with a (d) subscript. The full derivation of the equations is reported in Section 4.2, which deals with an application to aircraft maneuver load alleviation (incidentally, the DS represents the standard routine for solving this kind of aeronautical problem). The DS represents the non-LQR method which the LQR system will be matched to. Anyway the final form of the equations is of the same type as in standard feedback systems and looks like:

$$\begin{cases} \{dX_d/dt\} = [A_d]\{X_d\}+[B]\{v_d\} & (2.2.1) \\ \{Y_d\} = [C_d]\{X_d\}+[D]\{v_d\} & (2.2.2) \end{cases}$$

$$[A_d]=[A]+[\mathring{A}_d] \qquad (2.2.3)$$

$$[\mathring{A}_d]=[B][D\hat{\ }]^{-1}[C\hat{\ }] \qquad (2.2.4)$$

$$[C_d]=[C]+[\c{C}_d] \qquad (2.2.5)$$

$$[\c{C}_d]=[D][D\hat{\ }]^{-1}[C\hat{\ }] \qquad (2.2.6)$$

$$\{v_d\}=[D\hat{\ }]^{-1}\{\phi\} \qquad (2.2.7)$$

2.3 Quadratic System

One refers here to a dynamic system (Quadratic System - QS) which is controlled by a LQR. The QS can be written as:

$$\begin{cases} \{dX_q/dt\} = [A_q]\{X_q\}+[B]\{v_q\} & (2.3.1) \\ \{Y_q\} = [C_q]\{X_q\}+[D]\{v_q\} & (2.3.2) \end{cases}$$

$$[A_q]=[A]-[B][K] \qquad (2.3.3)$$

$$[C_q]=[C]-[D][K] \qquad (2.3.4)$$

$$[\mathring{A}_q]= -[B][K] \qquad (2.3.5)$$

Matching QS and DS formulations is accomplished in two steps:

a. Mathematical identification of QS and DS formulas, leading to [K] of QS

b. Solving for the weight matrices of a quadratic performance index which guarantee the mathematical existence of [K].

3. Matching QS and DS Formulations

3.1 Enforcing the Mathematical Identity of the two Formulations

One introduces an error on the state vector {e} defined as:

$$\{e\}=\{X_q - X_d\} \qquad (3.1.1)$$

and an error on the control vector $\{v_e\}$ defined as:

$$\{v_e\}=\{v_q-v_d\} \qquad (3.1.2)$$

Subtracting Eq.(2.2.1) from Eq.(2.3.1) one gets the following differential equation:

$$\{de/dt\}= [A]\{e\}+[B]\{v_e\}+([\AA_q]\{X_q\}-[\AA_d]\{X_d\}) \quad (3.1.3)$$

Eq.(3.1.3) can be reduced to a regulator equation provided that:

$$\{v_e\}=\{0\} \quad (3.1.4)$$

$$[\AA_q]=[\AA_d] \quad (3.1.5)$$

One gets by identification of Eq.(3.1.5):

$$[K]=-[D^\wedge]^{-1}[C^\wedge] \quad (3.1.6)$$

Thus QS is exactly represented by the same set of equations relative to DS. Eq.(3.1.3) yields:

$$\{de/dt\}= ([A]+[\AA_d])\{e\}=([A]-[B][K])\{e\} \quad (3.1.7)$$

Defining:

$$\{v_K\}= -[K]\{e\} \quad (3.1.8)$$

QS relies on the minimization of a quadratic performance index (which provides [K]) referred to {e} and $\{v_K\}$:

$$J= \int_0^\infty (\{e\}^T[Q]\{e\}+\{v_K\}^T[R]\{v_K\}+2\{e\}^T[N]\{v_K\})dt \quad (3.1.9)$$

3.2 Assigning Weight Matrices for a Prescribed Feedback Control Matrix [K]

The starting point is represented by the equation of Riccati[1,5] related to the performance index introduced in Eq.(3.1.9):

$$[dS/dt]=[0]=[S][A]+[A]^T[S]-([S][B]+[N])[R]^{-1}([N]^T+[B]^T[S])+[Q] \quad (3.2.1)$$

Here, the unknown matrix is [S], and the equation becomes meaningless when [R] is singular. The equation which defines [K] versus [S] is:

$$[K]=[R]^{-1}([N]^T+[B]^T[S]) \quad (3.2.2)$$

Combining Eq.(3.2.2) and Eq.(3.2.1) yields:

$$[S]([A]-[B][K])+[A]^T[S]+[S]^T[B][K]-[K]^T[R]^T[K]+[Q]=[0] \quad (3.2.3)$$

One introduces "standard"[1] hypotheses for matrices [Q], [R], [S]: [Q] symmetric, positive semidefinite, [R] and [S] symmetric, positive definite. In this way, Eq.(3.2.3) can be simplified as:

$$([S][A])^T+[S][A]-[K]^T[R][K]+[Q]=[0] \quad (3.2.4)$$

One can verify that $[K]^T[R][K]$ is symmetric. In this way, one can transform matrix equation (3.2.4) into a vector equation by rearranging into vector arrays the independent elements of [S], [Q] and [R] (this operation can be rapidly performed with symbolic algebra software codes). The number of components of each vector is P(P+1)/2, where P is the order of the matrix considered. Let be {S°} the vector form of [S], {R°} the vector form of [R], {Q°} the vector form of [Q]. One gets:

$$[\Pi 1]\{S°\}+[\Pi 2]\{R°\}+\{Q°\}=\{0\} \quad (3.2.5)$$

Eq.(3.2.4) is a Lyapunov equation. The Lyapunov theorem[5] states that the existence of a unique, positive definite [S] is guaranteed by the stability of [A] and the condition that $([Q]-[K]^T[R][K])$ must be positive semidefinite. If all the conditions are met, [Π1] is symmetric and certainly not singular. Eq.(3.2.5) yields:

$$\{S°\}=-[\Pi 1]^{-1}([\Pi 2]\{R°\}+\{Q°\}) \quad (3.2.6)$$

The generic vector {S°} belongs to a space of $(P_Q(P_Q+1)+P_R(P_R+1))/2$ dimensions: any choice of {R°} and {Q°} leads to a different [S] but to the same [K]. The procedure consists in selecting the components of {R°} and {Q°} in order to find {S°}. Here, the only constraint (which is a very soft one) is represented by the fact that {R°} and {Q°} have to fulfill the afore-mentioned hypotheses. [N] can be computed through back-substitution of [S] and [R] into Eq.(3.2.2).

4. Application to Aircraft Maneuver Load Alleviation

The demonstration of the principles depicted in the upper sections is performed on an example of aircraft maneuver load alleviation. It is worth repeating that this problem is normally framed using DS which rely on the direct assignment of prescribed alleviation levels and load factors, and not using LQR routines, which are normally used within the frame of flutter suppression and gust load alleviation problems.[10,11,12] The proven identity of QS and DS is such to highlight the precision, and thus the good performance of QS even in those problems where more suitable methods are normally used.

It is worth mentioning that maneuver loads primarily affect fighter A/C,[10,13] whereas gust loads are the dominant loads on transport A/C[13-19], even though fighters may also experience severe gust loads during "terrain following missions".[10] Because of the restrictions associated to the publication of data on military aircraft, a numerical application is run here on the longitudinal model of a transport aircraft with the help of the MATLAB[20] simulation package.

4.1 Description of the Aircraft Dynamic Model

The dynamic characteristics of the A/C, represented in Fig.1, are investigated for a pull-up maneuver. This is defined as a pitch demand resulting from a step of the elevator control surface reaching a maximum, steady-state value of 5 degrees in 0.8 seconds. Because of this, loads arise and consequently wing bending moments increase from a steady, horizontal-flight, zero-reference condition. All of the dynamic quantities will be regarded here as an unalleviated case and indicated with a (*) superscript. An A/C rigid model is derived from Kaynes.[13] Since the period of short oscillations is T_S=2.86s, the validity of the time evolution of the physical quantities is restrained to a bounded interval, say $t \in [0,5s]$. For longer time scales thrust changes would be made, which is beyond the scope of this document. Define {X} and {Y} as perturbation vectors:

$$\{X\}=\begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} = \begin{bmatrix} \text{forward velocity} \\ \text{vertical velocity} \\ \text{pitch rate} \\ \text{pitch angle} \end{bmatrix}$$

$$\{Y\}=\begin{bmatrix} BM20 \\ BM40 \\ TQ20 \\ TQ40 \\ NZ \end{bmatrix} \begin{bmatrix} \text{bending moment at 1/3 span} \\ \text{bending moment at 2/3 span} \\ \text{torque at 1/3 span} \\ \text{torque at 2/3 span} \\ \text{vertical acceleration at CG} \end{bmatrix}$$

$$\{v\}=\begin{bmatrix} \xi \\ \eta \end{bmatrix} = \begin{bmatrix} \text{elevator deflection} \\ \text{aileron deflection} \end{bmatrix}$$

The equations of motion are Eqs.(2.1.1)and (2.1.2) with [A], [B], [C] and [D] from Kaynes.[13] The dynamic quantities relative to the unalleviated case are obtained using ($\eta^*=0$) and (ξ^*= step of 5°). They are represented in Fig.2-a to Fig.2-k.

4.2 Direct Method

Since in the A/C model presented in the previous section one can rely only on the two control surfaces ξ and η, it is quite evident these cannot fulfill more than two conditions, e.g. alleviate BM20, BM40 and constrain NZ at the same time. Constraining NZ is essential to preserve the feasibility of the maneuver: this represents an equality one cannot afford loosing. Moreover, pitch motion is primarily dictated by ξ, which is also very much responsible for the value of the unconstrained BM20. Thus under these circumstances the alleviation can be effectively performed solely on BM40. Of course BM20 is indirectly subject to some (minor) changes with respect to the (*) condition. These changes physically can only represent positive variations, which may turn the whole design into a worthless task: one will thus assume the design is acceptable within a maximum 5% increase in BM20 (5% uncertainty can be considered as an upper noise level margin).

The first step of the method consists in expanding Eq.(2.1.2) and focusing only on the BM40 and NZ rows:

$$\begin{cases} D(2,1)\xi+D(2,2)\eta=BM40-C(2,1)u-C(2,2)w-C(2,3)q-C(2,4)\theta \\ D(5,1)\xi+D(5,2)\eta=NZ\quad -C(5,1)u-C(5,2)w-C(5,3)q-C(5,4)\theta \end{cases} \quad (4.2.1)$$

Let be λ, the prescribed level of alleviation on BM40, defined on the whole time interval [0, 5s]:

$$\lambda = 1 - (BM40 / BM40^*) \quad (4.2.2)$$

Let be {ϕ} such that:

$$\{\phi\}^T=\{(1-\lambda)BM40^*,NZ^*\} \quad (4.2.3)$$

Eq.(4.2.1) can be transformed into the compact form:

$$\{v\}=[D^\wedge]^{-1}(\{\phi\}+[C^\wedge]\{X\}) \quad (4.2.4)$$

which leads, after replacement into Eqs.(2.1.1) and (2.1.2), to the final set of equations (2.2.1) and (2.2.2) already encountered in Section 2.2. These equations represent a dynamic system where NZ and BM40 are prescribed.

The implementation of the DS requires a full-state feedback, the selection of the maneuver schedule NZ* (pilot input), the generation of the BM40* signal through NZ*. Taking the second Eq.(4.2.1) and enforcing NZ(t)=NZ*(t) with the assumption that in the unalleviated case there is no aileron activity ($\eta^*=0$), one gets in compact form:

$$\xi=D(5,1)^{-1}(NZ^*-[Ç]\{X\}) \quad (4.2.5)$$

Introducing Eq.(4.2.5) and ($\eta^*=0$) into Eqs.(2.1.3) and (2.1.4) leads to a system driven by NZ*:

$$\begin{cases} \{dX^*/dt\}=[A\#]\{X^*\}+[B\#]NZ^* & (4.2.6) \\ \{Y^*\} \quad =[C\#]\{X^*\}+[D\#]NZ^* & (4.2.7) \end{cases}$$

Matrices [Ç], [A#], [B#], [C#], [D#] can be identified straightforward. The control loop of DS is shown in Fig.3 and Fig.2-a to Fig.2-k refer to λ=0.25.

5. Results

5.1 Results related to QS and DS Formulation Matching

The first output is the identification of [K], given by Eq.(3.1.6):

$$[K]=\begin{bmatrix} 0.0127 & 0.1400 & 0.4584 & 0 \\ 0.0023 & 0.0224 & 0.0871 & 0 \end{bmatrix}$$

In the present case, [S] is 4x4, [Q] is 4x4, [R] is 2x2, [N] is 4x2. A first set of matrices which generate [K] is:

$$[R]=\begin{bmatrix} 1.0 & 0.2 \\ 0.2 & 1.0 \end{bmatrix} \quad [Q]=\begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

$$[N]=10^5\begin{bmatrix} -0.0010 & -0.0005 \\ 0.0187 & 0.0073 \\ -1.2340 & -0.4869 \\ -5.7344 & -2.2437 \end{bmatrix}$$

A second set is:

$$[R] = \begin{bmatrix} 1.20 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} \quad [Q] = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

$$[N] = 10^5 \begin{bmatrix} -0.0010 & -0.0005 \\ 0.0186 & 0.0072 \\ -1.2450 & -0.4920 \\ -5.7350 & -2.2439 \end{bmatrix}$$

Matrices [Q] and [R] fulfill all hypotheses of Section 3.2. A back-check through the Riccati equation proof (Eq.3.2.1) perfectly confirms in both cases the numerical value of [K] with round off errors smaller than 10^{-4}. The corresponding variations on {Y} are smaller than $4x10^{-4}$.

5.2 Results related to Aircraft Motion

This section does not represent an A/C technical assessment; It only helps visualize the accuracy of the use of QS.

Table1 contains peak values of BM20, BM40 and NZ in the cases considered (unalleviated, DS=QS) and their relative variations. Design constraint on (+5%) peak increase on BM20 (see Section 4.2) and the identity of NZ are fulfilled, together with (-25%) on BM40. BM20 and BM40 curves are shown in Fig.2-c and Fig.2-d.

Table 1: Comparison of Bending Moments and CG Accelerations

case	NZ	BM20		BM40	
		BM20	ΔBM/ BM(1)	BM40	ΔBM/ BM(1)
(1): unall.	0.8542	1.5268	0.0	0.7102	0.0
(2): DS=QS	0.8542	1.5954	0.045	0.5326	-0.250

The eigenvalues of the A/C longitudinal model are presented in Table2.

Table2: Eigenvalues

case	eigenvalues	natural frequency	natural damping
unalleviated	-0.9900±1.9601i	2.1959rad/s	0.4508
	-0.0059±0.0488i	0.0049rad/s	0.1200
DS=QS	-2.0089±8.1819i	8.4249rad/s	0.2384
	-0.0081±0.0791i	0.0080rad/s	0.1019

The stability conditions are ensured. One can check that the alleviated curves of u, w, q, θ (Fig.2-h to Fig.2-k) perfectly match the unalleviated behavior, although this result does not follow from a direct requirement. The quality of the eigenvalue results may seem immaterial to flight mechanics specialists. One can definitely add some measures of additional compensation to render the A/C "friendlier". However, a flight mechanical analysis of the feedback matrix gains is beyond the scope of the document. One only wants to illustrate the mathematical technique depicted in the previous sections.

The characteristics of control surface activity are shown in Table3.

Table3: Maximum Control Surface Deflection and Rate

case	ξ (°)	η (°)	dξ/dt (°/s)	dη/dt (°/s)
unalleviated	5.00	0.0	31.6	0.00
direct	4.93	0.28	28.9	0.86

The time behavior of ξ and η is shown in Fig.2-a and Fig.2-b. The extremely low value of η is probably hard to implement in the real world. As already stated, one is interested here only in describing a mathematical technique.

8. Conclusions

A LQR-based solution intrinsically owns a very high level of accuracy if one correctly formulates the performance index and drives the weight matrices as described in the paper: the solution proposed is easy to implement and requires relaxed conditions on the matrices.

The associated matching method is general, has the advantage of requiring standard feedback systems and can be used to analyze aeroservoelasticity problems and advanced missile guidance and control laws.

9. References

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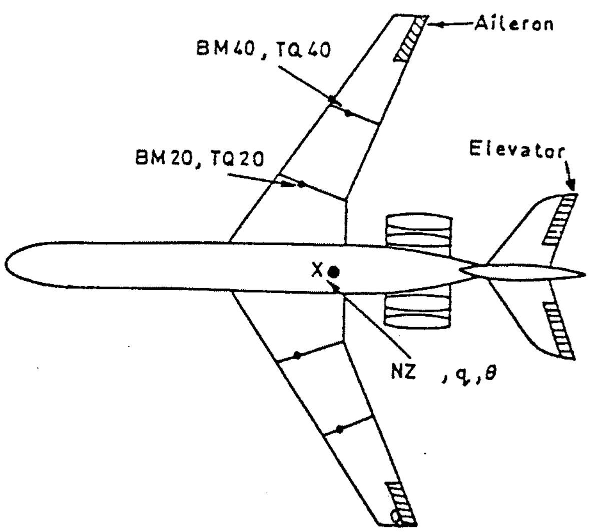

Fig.1: Aircraft Layout

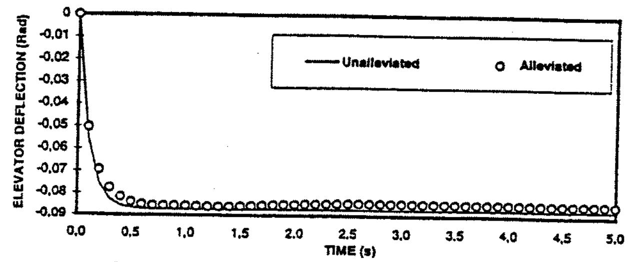

Fig. 2-a : ELEVATOR DEFLECTION

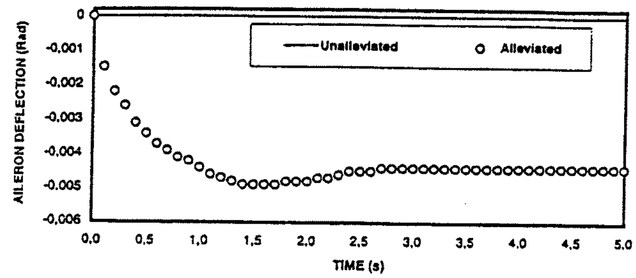

Fig. 2-b : AILERON DEFLECTION

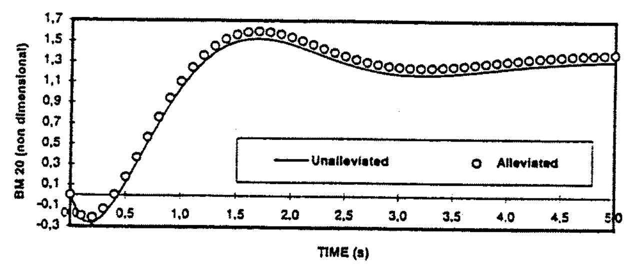

Fig. 2-c : BENDING MOMENT AT 1/3 SPAN STATION

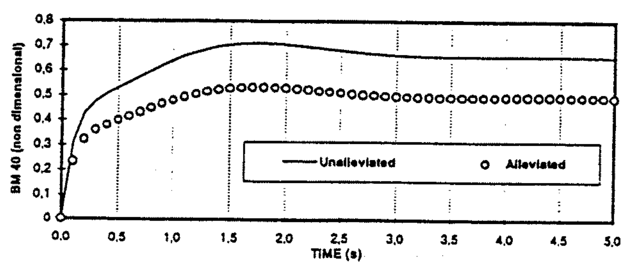

Fig. 2-d : BENDING MOMENT AT 2/3 SPAN STATION

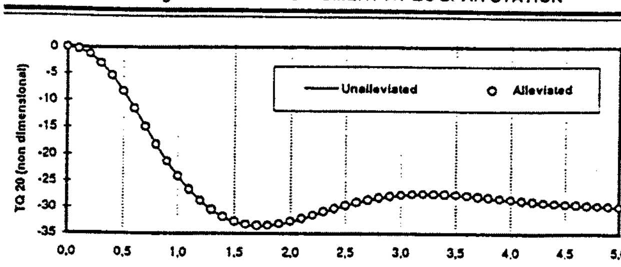

Fig. 2-e : TORQUE AT 1/3 SPAN STATION

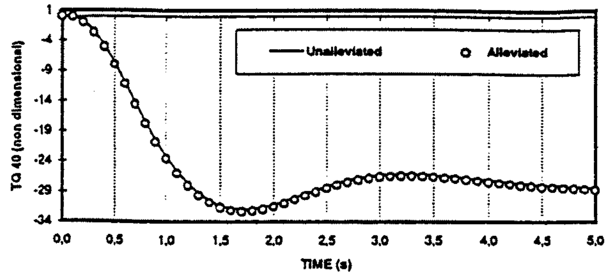

Fig. 2-f : TORQUE AT 2/3 SPAN STATION

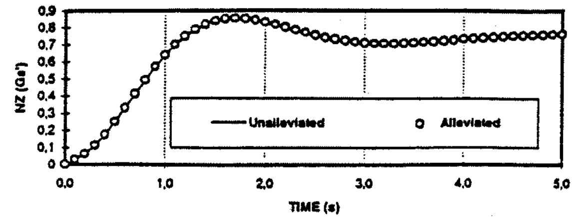

Fig. 2g : VERTICAL ACCELERATION

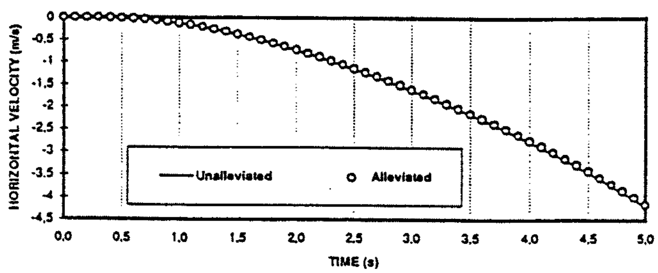

Fig. 2-h : HORIZONTAL VELOCITY

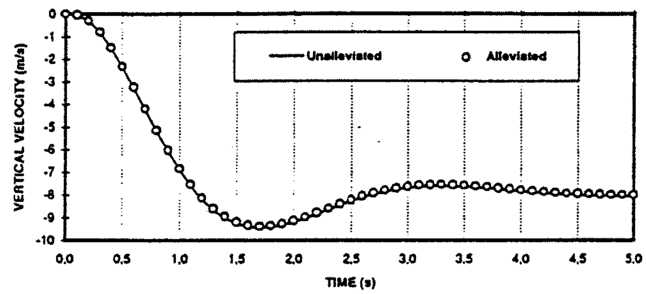

Fig. 2-i : VERTICAL VELOCITY

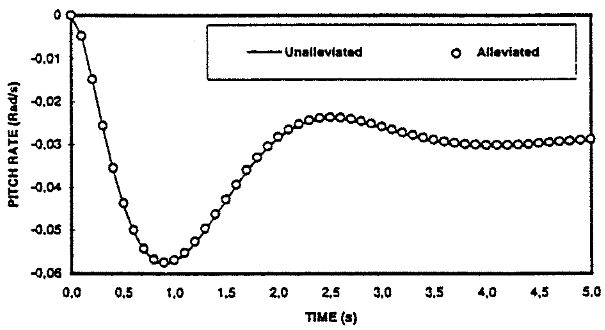

Fig. 2-j : PITCH RATE

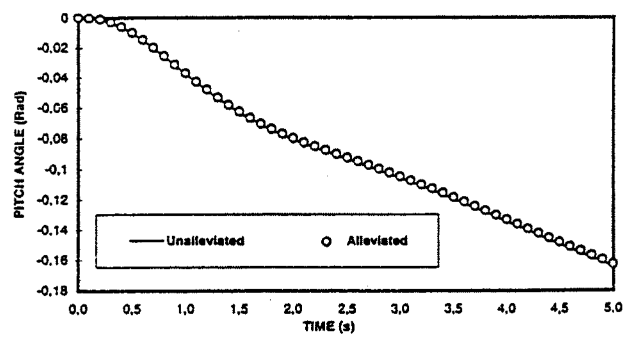

Fig. 2-k : PITCH ANGLE

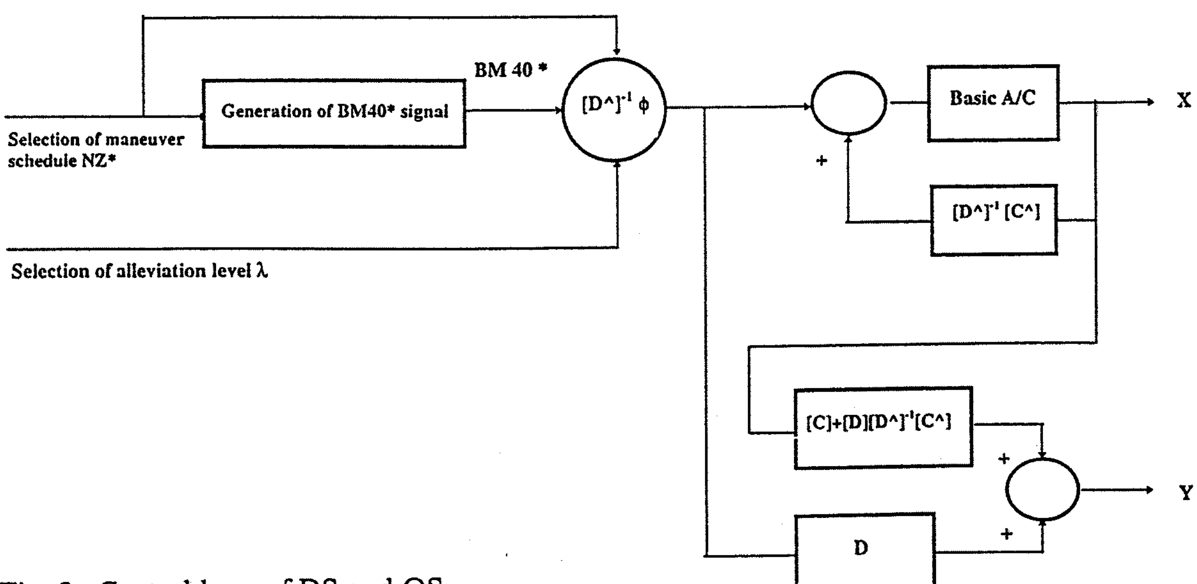

Fig. 3 : Control loop of DS and QS