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RATE LIMITERS WITH PHASE COMPENSATION

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Abstract

In a modern fighter, like JAS 39 Gripen, rate limiting of control surfaces is an important issue. In addition to the naturally limited rates of the hydraulic control surface servos there are software rate limiters in the flight control system (FCS). When a rate limiter is saturated, the phase shift drastically reduces the stability margins of the closed loop and increases the risk for pilot-induced oscillation (PIO).

This paper describes a novel method for compensating for the phase lag of a rate limiter. In contrast to earlier phase compensation methods, this method uses feedback instead of logic or feedforward. Open loop and closed loop properties of the method are discussed. The method gives a drastic improvement on stability margins and reduces PIO tendencies. The enhanced stability margins are demonstrated on a F-16 example and the reduced PIO tendencies have been demonstrated during in-flight simulation. Phase compensated rate limiters are now used in JAS 39 Gripen FCS production software.

Keywords: aircraft control, flight control system, rate saturation, describing function, pilot-induced oscillation.

1. Introduction

In the past few years rate limiting has become an important issue in flight control system design for modern fighter aircraft. Typically these aircraft are aerodynamically unstable and have very high demands on agility, precision, and flying qualities. They usually have hydraulically powered control surfaces.

The hydraulic control surface servos have limited rates, but they are not the only rate limitations in the aircraft. Software rate limiters (SRL's) are also used in the flight control system (FCS). SRL's are placed, e.g., immediately before the servo command outputs in order not to rate saturate the servos. These rate limiters are usually dominating compared to the inherent servo rate limits.

When a rate limiter is saturated using sinusoidal input, the output has a smaller amplitude and essentially a phase shift compared to the input signal. Especially the phase shift is critical since it drastically reduces the stability margin of the closed loop. For an aerodynamically unstable aircraft, like JAS 39 Gripen, the FCS may typically not provide more than 45° phase margin, and a rate limiter easily gives such phase shifts. A corresponding phase shift also affects the pilot (outer) loop and creates an effect corresponding to a time delay. This extra delay explains why a pilot-induced oscillation (PIO) is more likely to occur once the pilot has started to command with large and rapid stick inputs. PIO caused by rate limiting has been observed on many modern military and civil aircraft, and some of these PIO incidents have led to crashes.

In Section 2 different rate limiters will be discussed. Earlier algorithms for phase compensation of rate limiters typically involve logical expressions, if-then-else constructions, where states and outputs of the algorithm are determined by conditions on the sign and magnitude of, e.g., the input rate(1)(2)(3). Other algorithms use feedforward(4). These methods will be further discussed in Section 2.1.

The novel phase compensation methods described in this paper rely solely on feedback. The methods are inspired by anti-windup methods(5), which feed back the difference between the input and the output of a nonlinearity. The methods, which have been filed for patent rights, will be described in detail in Section 2.2. In Section 3 the rate limiters will be analysed.

It was found that stability results for rate limiters could always be explained by the describing function (DF) method(6)(7)(8). This will be further discussed in Section 4. This section will also demonstrate the stability properties of conventional and compensated rate limiting on an aircraft, in this case a F-16 model(9).

Section 5 and Section 6 discuss rate limiting and phase compensation in JAS 39 Gripen. Topics covered include the rate limiter structure, hydraulic system issues, and different tests, e.g., in-flight simulation, used during the development of control laws with phase compensation. Phase compensation is now used in the production FCS software. Conclusions are given in Section 7.

2. Rate limiters

This section will discuss different rate limiter algorithms. Unless otherwise stated the rate limit $r = 1$ (unit/s). A simple continuous time model of a conventional rate limiter is given in Figure 1 a. This model illustrates that the rate limiter output y always tries to be equal to the input u. The gain K must be chosen sufficiently high compared to the frequencies in u. For the sinusoidal input $u = a\sin(\omega t)$, rate limiting occurs if

$$\rho = \frac{r}{a\omega} < 1 , \qquad (1)$$

where ρ is a nondimensional parameter, and r the rate limit. For $\rho \geq 1$ there is no rate limiting. In the sequel rate limiter elements are denoted by the symbol in Figure 1 b.

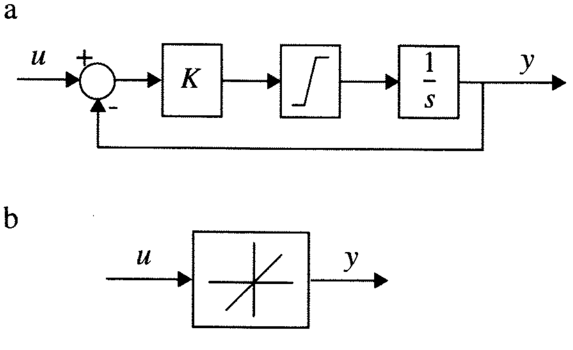

Figure 1. a) A simple model of a rate limiter element. b) Rate limiter symbol.

2.1. Earlier phase compensation methods

The idea of avoiding or compensating for rate limiters is not new(1)(2)(3)(4). In Reference 4 two algorithms, which both reduce a gain, are evaluated. One of the algorithms reduce a gain as a function of input frequency, the other as a function of input rate. By reducing a gain, rate limiting is avoided. These algorithms are intended for use in the pilot command path of the control law and were developed at NASA after a PIO incident during the first Space Shuttle runway landing.

In Reference 1 - 3 phase compensation is obtained essentially by a three step procedure, differentiate - limit - integrate, see Figure 2. Thus we will always have

$$\operatorname{sgn}\left(\frac{dy}{dt}\right) = \operatorname{sgn}\left(\frac{du}{dt}\right) , \qquad (2)$$

i.e., the output y will always have the same direction as the input u, in other words perfect phase compensation. However, after rate limiting we will usually have $y \neq u$ since the limiter has discarded information. In order to make $y = u$, logical conditions on, e.g., the input rate du/dt, are used to select other inputs to the integrator. This could be denoted "input recovery", and the method will then resemble Figure 1 a. It is often possible to construct inputs u which give undesirable output y due to "unforeseen" effects of the logic. The methods are also noise sensitive, since they may rapidly shift between phase compensation and input recovery(1)(3).

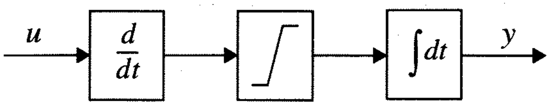

Figure 2. A simple phase compensation method.

2.2. Rate limiters with feedback

The inspiration to the novel methods came from conventional anti-windup methods(5), which among other things have the property of increasing or advancing the phase of a transfer function around the nonlinearity. In anti-windup an error signal, the difference between the input and output of a nonlinearity, is fed back in order to adjust, or stabilize, some of the controller states. The first attempt was to feed back error signals to an integrator. A drawback with this approach was that the integrator had usually built up an unacceptable bias when rate limiting ceased.

Instead a stable low pass filter was used, see Figure 3. When the rate of the input signal u is greater than the rate limit r, the feedback signal becomes negative and reduces the input signal to the rate limiter. If the input reverses direction the output will almost immediately reverse direction too, i.e., less phase lag is obtained. When the input u has a rate smaller than r, the feedback signal decays to zero. The low pass filter gives less phase compensation compared to an integrator, but on the other hand it does not give any bias either.

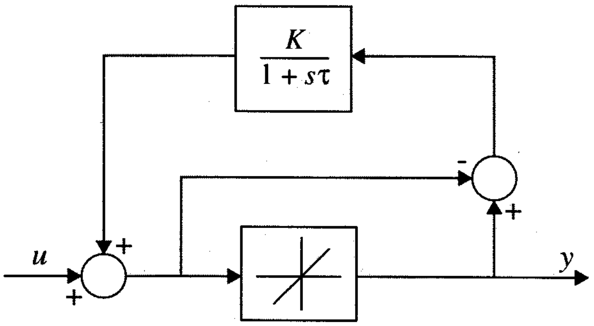

Figure 3. Rate limiter with feedback.

It was found that the circuit in Figure 3 is not a good solution in the presence of high frequency signals. The reason is that high frequency components of the input signal u almost blocks the circuit for low frequencies. For high frequencies the circuit does not provide phase compensation and it is not necessary either. Thus the problem would be solved if only the low frequency components of u were phase compensated. A number of different rate limiter circuits were tested, and the circuit in Figure 4 was finally selected. Since the low frequency components of u are limited by the first rate limiter (with phase compensation), only the high frequency components of u are limited in the second (uncompensated) rate limiter. Note that both rate limiters have equal limits.

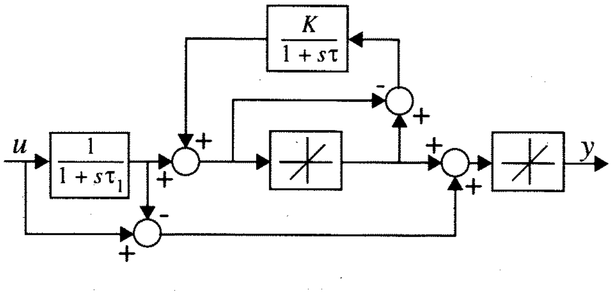

Figure 4. Rate limiter with feedback and bypass.

3. Analysis and simulation of rate limiters

The properties of the different rate limiter circuits were studied by means of analysing the magnitude and phase properties of the describing function (DF)(7)(8), and by carrying out simulations for different input signals.

Since rate limiters are dynamic nonlinearities, the describing functions Y_N depend on both amplitude and frequency of the input signal. The describing function for a nonlinearity N with input $u = C\sin(\omega t)$ and output $y(t)$ is given by

$$Y_N(C,\omega) = \frac{b_1 + ia_1}{C} = \frac{c_1 e^{i\phi}}{C} \tag{3}$$

where the Fourier coefficients a_1 and b_1 are given by

$$\begin{aligned} a_1 &= \frac{\omega}{\pi}\int_0^{\frac{2p}{\omega}} y(t)\cos(\omega t)\,dt \\ b_1 &= \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} y(t)\sin(\omega t)\,dt \end{aligned} \tag{4}$$

and where $c_1 = \sqrt{a_1^2 + b_1^2}$ and $\phi = \operatorname{atan}(a_1/b_1)$ are the magnitude and phase respectively of the DF. The Fourier coefficients of the DF's were calculated using numerical integration.

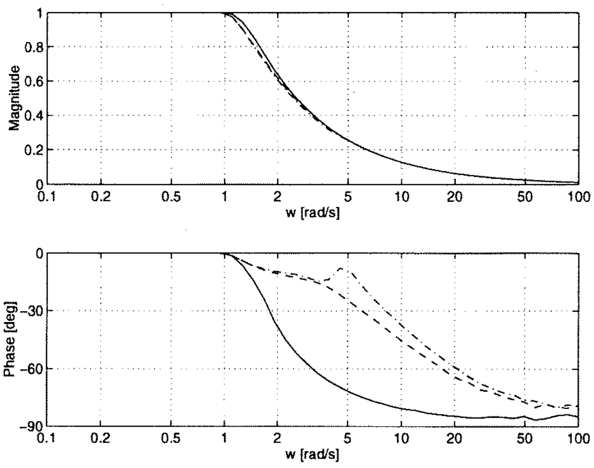

Figure 5. Magnitude and phase of the describing function $Y_N(1,\omega)$ for a conventional rate limiter (solid), rate limiter with feedback (dashed) and rate limiter with feedback and bypass (dash-dotted).

In Figure 5 the magnitude and phase of different rate limiter DF's are shown as function of frequency for the input signal amplitude 1. Filter parameters are $K = 8$, $\tau = 1$ and $\tau_1 = 0.1$ In the figure it is shown that the feedback has almost no effect on the amplitude, while the phase is improved for compensated rate limiters, especially around 5 rad/s. The gain $K \approx 10$ is a good choice, while τ and τ_1 must be chosen with respect to, e.g., the bandwidth of the application, but at least $\tau_1 < \tau$ should hold. In Figure 5 it is noted that the circuit with bypass, see Figure 4, has a better phase compensation compared to the first circuit, see Figure 3. The explanation is that there is phase advance in both the high frequency and low frequency paths, and this phase advance is preserved when the two signals are added.

In Figure 6 the responses to $u = \sin(5t)$ for the different circuits are shown. The amplitude is about the same for all circuits, while the phase lag is less for the rate limiters with feedback.

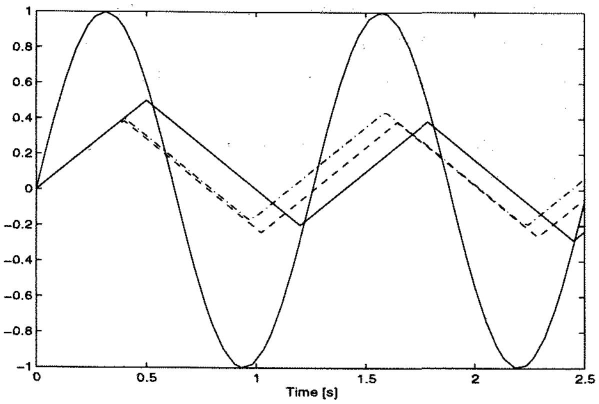

Figure 6. Responses to $u = \sin(5t)$ for a conventional rate limiter (solid), rate limiter with feedback (dashed) and rate limiter with feedback and bypass (dash-dotted).

The responses of the rate limiters to a step input are shown in the first part of Figure 7. The convergence rate of the responses from rate limiters with feedback depends on the time constant τ. It may seem as if it is better to stay as long as possible in the rate limit. But if the step input is followed by another large command in the opposite direction it may be an advantage that the response is a bit slow. This is found when studying the response to a square wave, i.e., the rest of Figure 7. This type of input may be pilot commands, e.g., roll commands. Since the pilot typically commands pitch and roll rate, the integral of the rate limiter outputs are pitch and roll attitude angles. These integrals have smaller amplitudes for rate limiters with feedback, due to the exponential decay of the feedback filter. Thus these rate limiters rather are equivalent to a smaller gain for step inputs. The phase is less important as long as the output reverses direction when the input does.

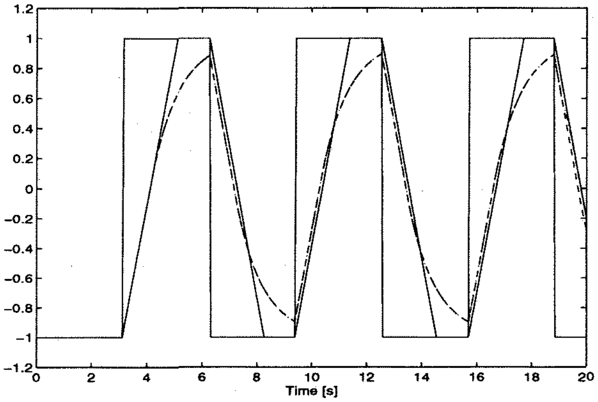

Figure 7. Response to square wave for a conventional rate limiter (solid), rate limiter with feedback (dashed) and rate limiter with feedback and bypass (dash-dotted).

4. Rate limiter properties in closed loop

In order to demonstrate the closed loop properties of the proposed rate limiter circuits, the pitch dynamics of a textbook F-16 model is considered[9]. A state feedback controller is designed for the linearized dynamics at 200 km/h and altitude 1000 m. A first order servo model is added to the dynamics, and the control signal u is rate limited before being fed to the elevator servo, see Figure 8. The linear open loop dynamics, i.e., servo, aircraft and feedback, has transfer function G.

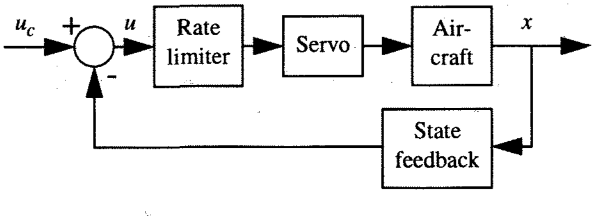

Figure 8. Block diagram of F-16 pitch control loop.

Normally $G(i\omega)$ and $-1/Y_N$, the negative inverse of the DF, are plotted in, e.g., a Nichols diagram, and they must not intersect if stability is to be predicted by the DF method. Since Y_N depends on both amplitude and frequency, it was instead chosen to plot the loop transfer $Y_N(C, \omega) \cdot G(i\omega)$, which then must encircle -1 according to the Nyquist criterion in order to predict stability of the closed loop.

The loop transfer $Y_N G$, using a conventional rate limiter and rate limiter with feedback and bypass respectively, were calculated in closed loop. A sinusoidal input signal with fixed amplitude was applied at the command input, u_c, and the sensitivity functions, $S = (1 + Y_N G)^{-1}$, using rate limiters, were computed by numerical integration for frequencies in the interval 0.1 to 10 radians/s. The frequency interval was restricted to 10 radians/s to make the assumption of first harmonic dominance valid.

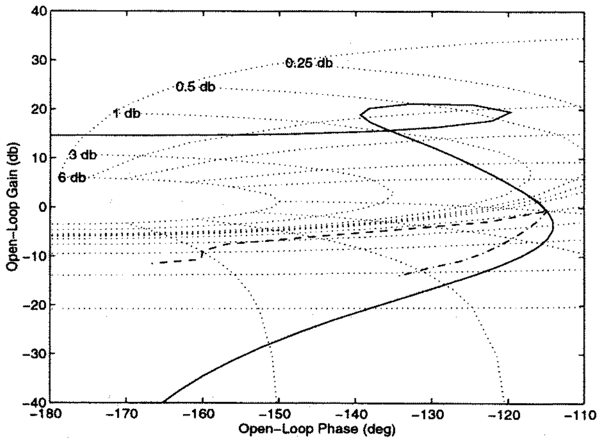

Figure 9. Nichols diagram for the linear transfer function G (solid), and the nonlinear loop transfers $Y_N G$ using a conventional rate limiter (dashed) and rate limiter with feedback and bypass (dash-dotted).

The open loop transfer functions $Y_N G$ were then calculated from the sensitivity functions and are shown in Figure 9, together with the linear open loop transfer function G. From the figure it is found that the stability properties are improved when a compensated rate limiter is used, compared to when a conventional rate limiter is used.

Phase compensation methods essentially try to obtain phase advance in order to compensate for the phase shift of the rate limiter. Such algorithms pay more attention to the input rate than to the input itself. Thus high-frequency disturbances may almost decouple the input and the output of a compensated rate limiter. It has been demonstrated$^{(6)}$ that a high-frequency disturbance may destabilize a closed loop involving a compensated rate limiter for a smaller amplitude compared to when a conventional rate limiter was used. However, it is also shown that the DF method explains the phenomenon. Without disturbances the compensated rate limiters were better than a conventional rate limiter.

Note, however, that if an unstable process is controlled by a controller with a limited and/or rate limited control signal, an input signal of sufficiently large amplitude will destabilize the closed loop. The only difference between compensated and conventional rate limiters is the amplitude required for destabilization.

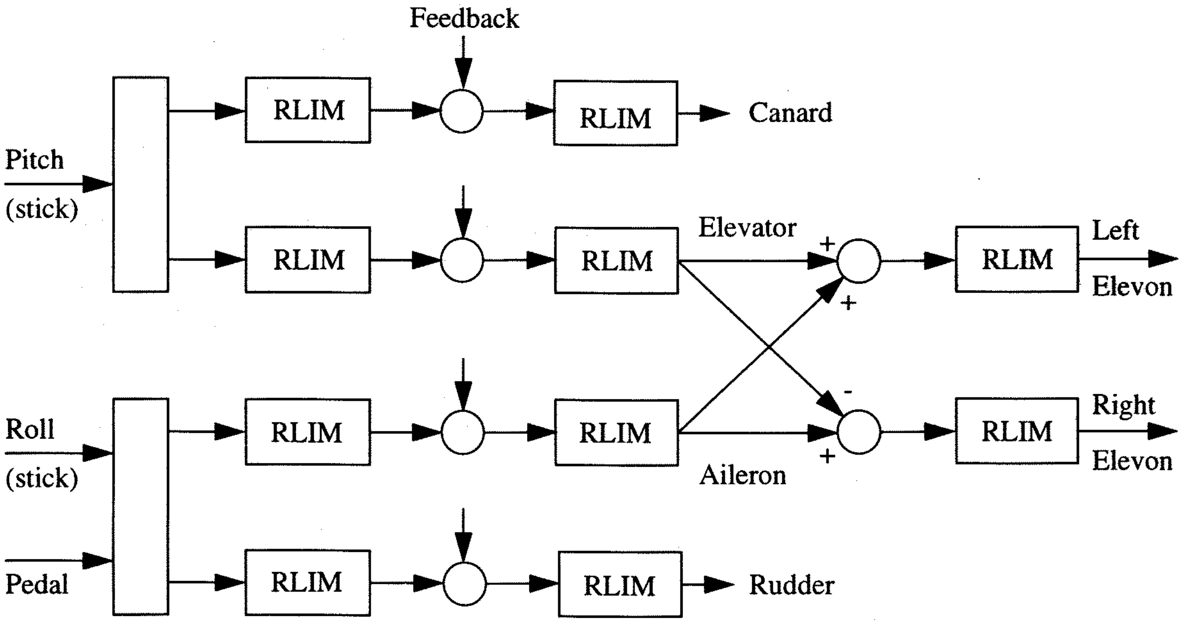

Figure 10. Rate limiter structure in JAS 39 Gripen. The structure of feedforward and feedback is suppressed, and some of the rate limiters are omitted. Pilot inputs enter from the left and are rate limited, then the feedback signals are added and the sum is rate limited. Then control surface commands are computed and rate limited.

5. Rate limiting in JAS 39 Gripen

In contrast to the F-16 model above, the JAS 39 Gripen is multi-variable in both longitudinal and lateral-directional axes. Pilot commands, total aileron, elevator, canard, and rudder commands are rate limited. Then control surface commands are computed and rate limited. This is illustrated in Figure 10, which shows the main rate limiter structure in JAS 39 Gripen. Some rate limiters are omitted in the figure, e.g., the elevons are split into inner and outer elevons, each being separately rate limited. All in all there are 14 rate limiters (with phase compensation) in the pitch, roll, and yaw control loops. Thus it is not as easy as above, see Section 4, to demonstrate enhanced stability margins.

5.1. Hydraulic system issues

The control surfaces are powered by hydraulics, and the load on the hydraulic system depends heavily on how the servo commands are rate limited. In addition to protecting the servos from being commanded at a too high rate, the purpose of rate limiting is also to avoid overloading the hydraulic system. Thus the rate limiter structure, see Figure 10, deserves a few comments regarding the effects on the hydraulic system.

The rate limits on the pilot commands (the 4 leftmost limiters in Figure 10) depend on the hydraulic pressure such that these rate limits are automatically reduced at low hydraulic pressure. The rate limiters after the feedback signals have been added (the next 4 limiters in Figure 10) have larger rate limits than the pilot command limiters. This is necessary for an unstable aircraft in order to ensure that the pilot cannot entirely suppress the stabilization of the aircraft.

The compensated rate limiters on the individual control surface commands reduce the load on the hydraulic system and yields significantly less phase lag in the closed loop. The reason is that during high-gain tasks the surface commands are relatively close to sinusoidals and they do not exceed the rate limit too much, i.e., ρ is not too small, (see (1)). A compensated rate limiter preserves the soft signal shape of a sinusoidal, unless ρ is too small, while uncompensated rate limiters give a triangular wave as output. The triangular wave has constant rate, except when it changes direction, while the output of the compensated rate limiter changes direction in a smooth way, see Figure 11. The lower rate during these intervals reduces the load on the hydraulic system.

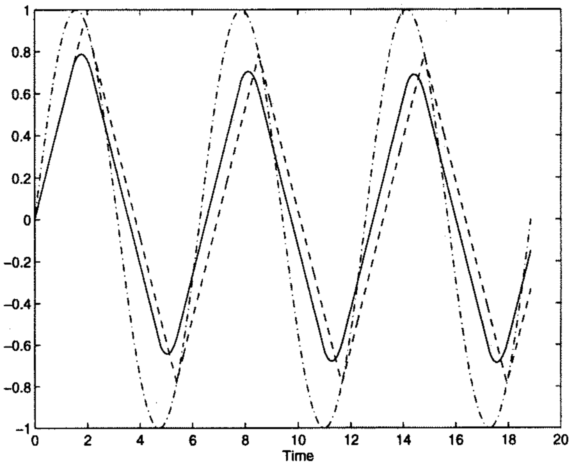

Figure 11. Comparison between the outputs from a compensated rate limiter (solid) and conventional rate limiter (dashed) when the input is sinusoidal with $\rho = 0.5$.

5.2. A non-linear lag-lead filter

A non-linear lag-lead filter, see Figure 12, is primarily used for reducing the pitch rate overshoot. Parameters of the filter are chosen such that $0 < K < 1$ and $\tau = 0.8\,\tau_{\theta 2}$. A typical response of the non-linear lead-lag filter is shown in Figure 13. A secondary effect of the nonlinear lag-lead filter is that the filter, when properly tuned, significantly reduces the unwanted washout effect from the lag filter of the compensated rate limiter, see Figure 7.

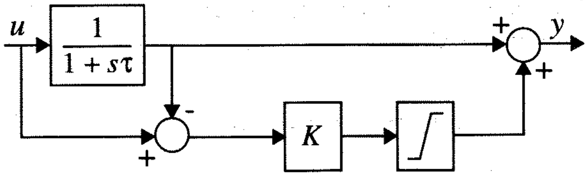

Figure 12. Nonlinear lag-lead filter on pilot pitch stick command.

5.3. Summary of rate limiting in JAS 39 Gripen

Thus rate limiters with phase compensation, the circuit in Figure 4, and a careful tuning of the pilot command path gains and filters are the measures used in order to get a safe high authority FCS edition for the JAS 39 Gripen. As a result it is very rare that the sums of pilot control surface commands and stabilizing feedback commands are rate limited. There is a trade-off between stability, authority and flying qualities when choosing parameters τ and τ_1 for the phase compensation. The roll axis of an aircraft has faster dynamics than the pitch axis and thus needs a smaller time constant τ in the phase compensation.

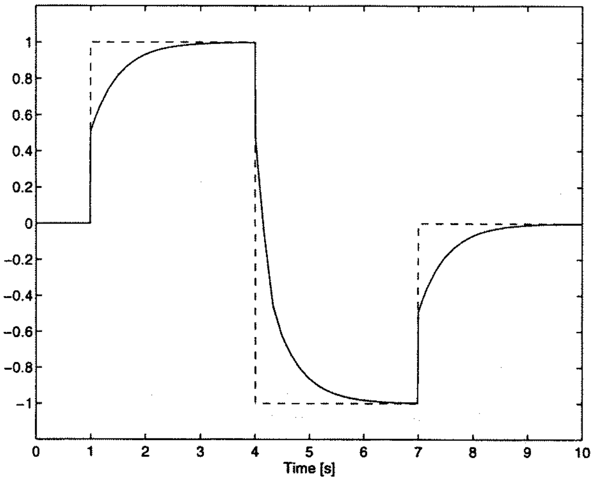

Figure 13. Response of nonlinear lag-lead filter for pitch stick command.

6. Evaluation of compensated rate limiters

Phase compensated rate limiters have been tested in many ways before they finally were cleared for use in flight in the JAS 39 Gripen. A special pilot model, essentially a type of relay controller, is used in off-line batch simulations in order to determine if the FCS edition is safe or if it is possible to depart the aircraft into high angles of attack. The sensitivity for high-frequency disturbances has also been investigated. Other real-time tests will be discussed below.

6.1. In-flight simulation

Before using compensated rate limiters in the JAS 39 Gripen FCS it was decided to first test them in in-flight simulation (IFS). The reason was to expose the limiters for real signals, obtained in flight, and to gain confidence. Using the Calspan Variable Stability Learjet, it was possible to flight test the limiters during safe conditions, since the FCS in the Learjet can be disconnected. This turns the Learjet back into a conventional aircraft with mechanical controls, which can be safely flown and landed$^{(10)}$. One of the in-flight simulations was an up-and-away bank angle tracking task. One such sequence of experiments are shown in Figure 14 - Figure 16.

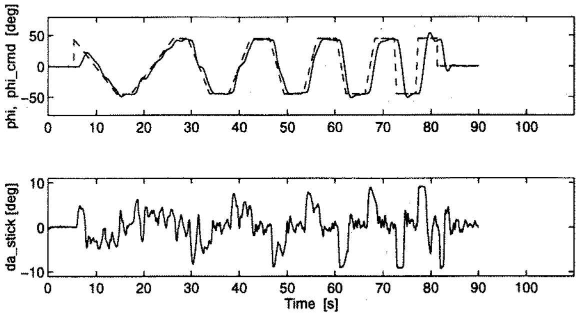

Figure 14. Bank angle tracking task without rate limiting. Desired and obtained bank angle (upper), and pilot input (lower) are shown.

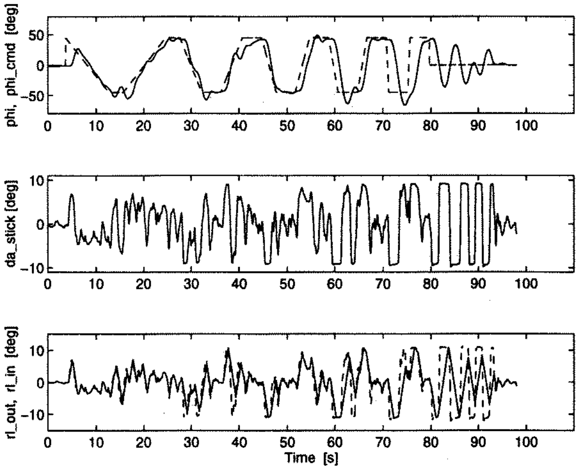

Figure 15. Bank angle tracking task with uncompensated rate limiting. Desired and obtained bank angle (upper), pilot input (middle), and rate limiter input and output (lower) are shown. Note the PIO during the final part.

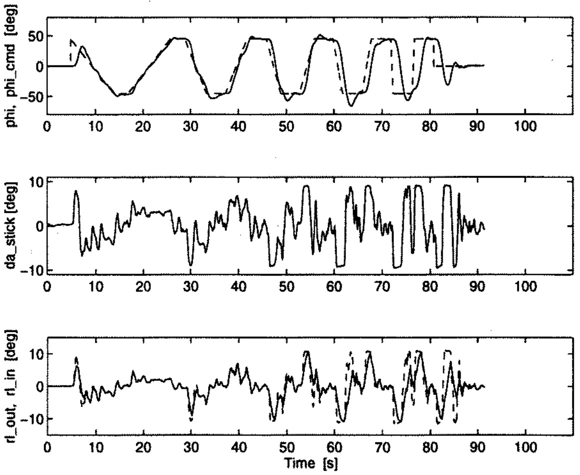

Figure 16. Bank angle tracking task with phase compensated rate limiting (Figure 4). Desired and obtained bank angle (upper), pilot input (middle), and rate limiter input and output (lower) are shown.

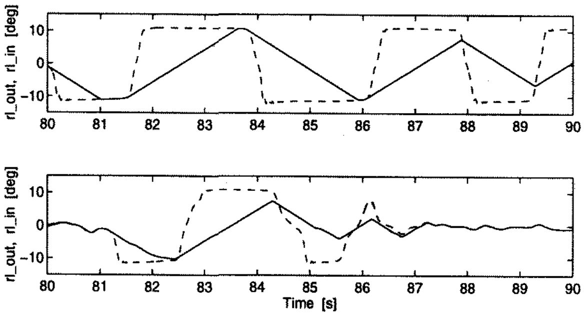

Figure 17. Detail of rate limiter input (dashed) and output (solid) from Figure 15 (upper) and Figure 16 (lower).

In the experiment the desired bank angle is swept between ±45°, using successively steeper slopes. The pilot task is to track the bank angle as good as possible. In Figure 14 the experiment is performed without rate limiting, i.e., with a very high rate limit. This experiment serves as a basis for comparisons. The pilots were instructed to use high gain control in the experiment. As can be seen the pilot is able to track the desired bank angle with a reasonably small error.

In Figure 15 the pilot roll command is rate limited corresponding to an aileron rate of 10°/s, without phase compensation. In addition to pilot command and bank angles, the input and output of the rate limiter are also shown. Note the final PIO when the pilot is trying to capture bank angle 0°.

In Figure 16 the pilot roll command is rate limited corresponding to an aileron rate of 10°/s, but this time with phase compensation according to Figure 4. The figure shows the same signals as in the uncompensated case, Figure 15. When comparing these two figures it is clear that the pilot has better control and performance when phase compensation is used. Both workload (stick input) and bank angle error are smaller, almost back at the level of no rate limiting at all, see Figure 14.

Due to the time scale in the figures the use of phase compensation in Figure 16 is not visible. In Figure 17 the rate limiter inputs and outputs using uncompensated (Figure 15) and compensated (Figure 16) rate limiting respectively are shown for a shorter time interval. The bank angle references are not synchronized in the two figures, but this is not important. It is, however, clearly visible that phase compensation is not used in Figure 15, since the output reverses direction only when the difference between input and output changes sign. It is also clearly visible at a few instances during the time interval 84 - 87 s, that phase compensation is used in Figure 16. At these instants the output changes direction when the input does.

Phase compensation was also tested during offset landings, which is another high-gain task. With conventional rate limiters the aircraft was almost not controllable during offset landings. Several landings were interrupted by the safety pilot due to loss of control. When phase compensation was used the aircraft remained controllable and the landings could be fulfilled. Thus phase compensation preserved controllability of the aircraft despite severe rate limiting.

The conclusions of the in-flight simulations were that phase compensation performed as expected, and that no unexpected deficiencies or side effects were obtained. It was also clearly demonstrated that compensated rate limiters reduced pilot workload and gave better performance and control compared to conventional rate limiters.

6.2. Real-time simulation

Phase compensation has also been extensively tested in real-time ground simulators, e.g., during the validation process. Many pilots have tried to provoke the aircraft to depart into high angles of attack, but they have not succeeded. The main tool for checking that the aircraft is safe is, however, the off-line batch simulations with a special pilot model.

The main purpose of real-time simulation has been control law design. Gains, time constants, limits, etc., have been adjusted in order to give the aircraft desirable flying qualities.

6.3. Flight tests

Finally, phase compensation has been flight tested in the JAS 39 Gripen. These tests have included normal high-gain tasks such as aiming, bank angle capture, segment rolls, etc., but also fast open-loop stick inputs of the same type used by the pilot model in off-line batch simulations. In all cases the compensated rate limiters performed as expected, and good flying qualities were obtained in the high-gain tasks. Thus the compensated rate limiters are now qualified for use in the production FCS software. More results on the flight tests and the design issues of the JAS 39 Gripen FCS are presented in Reference 11.

7. Conclusions

This paper has described feedback-based phase compensation of rate limiters. The methods have been analysed both in open and closed loop and they give a significant reduction of the phase lag. The describing function method is a reliable method for stability analysis of these problems. If high-frequency disturbances are present, the compensated rate limiters are, however, not always better than a conventional rate limiter.

Phase compensation of rate limiters has been one of the keys in obtaining good performance and safe handling of JAS 39 Gripen. Extensive analysis, simulations and flight tests have been carried out to ensure that rate limiting is no threat to good handling qualities of the JAS 39 Gripen. The pilot comments are favourable. The FCS edition yields high authority, accuracy, and good predictability for both small and large stick inputs and is now qualified for production use.

It should be pointed out that compensated rate limiters can not improve a system which has a too low rate limit for a given task. In such cases the only remedy is to increase the available rate. For tasks which could require rates close to or above the rate limit, the compensated rate limiters may improve performance and safety by reduction of the phase lag. Thus the compensated rate limiters may make it possible to, with preserved safety and stability, fully utilize the available rates of the servos. Without compensated rate limiters it may instead be necessary to reduce the authority of the FCS to a sufficiently low level, such that rate limiting never occurs.

8. Acknowledgements

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