GENERALIZATION OF DATA REDUCTION ROUTINES FOR MODEL ATTITUDE, FORCE AND MOMENT MEASUREMENTS IN THE WIND TUNNELS OF THE AERONAUTICAL INSTITUTE ŽARKOVO

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Abstract

With the introduction of data reduction computers to several wind tunnel laboratories of the Aeronautical Institute it became evident that the application software should implement generalized algorithms permitting the processing, with the same code, of data taken with various types of model supports and wind tunnel balances. Therefore, extensive modifications were made to the originally used data reduction software. The new code can be used with almost any model support internal or external wind tunnel balance by simply entering the appropriate setup into the database. The focus of this paper are the fundamentals of the new algorithms, based on the definitions of "generic" model support and force balance. Generalized procedure is outlined for calculations of model attitude in the test section in terms of Euler and aerodynamic angles and Cartesian coordinates, and reduction of aerodynamic forces and moments to body axes, extensively using matrix transformations. Correction of tare loads, and also inertial forces during sweep tests, is performed using the generalized "buoyant offset" concept and least squares approximations by trigonometric functions. Output is possible in any of the generally used axis systems. Provisions are made for compensation of wall interference and test section calibration.

Introduction

In the seventies, digital computers were introduced as data reduction machines in two wind tunnel laboratories of the Aeronautical Institute. The software then used to determine model aerodynamic angles, force and moment coefficients, treated each type of model support and each type of balance used in these wind tunnels as a particular case, and different software routines were used to process data for each case.

When the T-38 1.5×1.5m trisonic wind tunnel was designed, it was felt that for this facility a more general approach was needed. Foreseeing the modernization of other wind tunnels in the Institute, an integrated, 'wind tunnel independent' software package for data reduction was created.

However, mainly due to inertia, some of the peculiarities of different types of model supports and wind tunnel balances were again hard coded into routines used only for certain kinds of measuring devices. It soon became evident that this resulted in very unwieldy software and difficult setup of data reduction parameters. Therefore, extensive modifications were made to the software, with the intent of making it simpler and more general.

Generalizations

The generic model support

A wind tunnel model is almost always held in position in the test section by some sort of a mechanism, called the model support, by which its orientation and position relative to the flow direction and test section walls can be set, maintained and accurately measured. While many types of model supports are used in wind tunnels, they can all be reduced to a combination of simple, one degree of freedom mechanisms. These elementary mechanisms are linked in a train, the 'first' one being fixed to the wind tunnel structure, and the 'last' one holding a model, possibly via a force balance. A local frame of reference (coordinate axes system) is associated to each elementary mechanism m, with the origin at the point where it is joined to the 'preceeding' elementary mechanism. A set of left-handed frames of reference is used throughout the calculations.

An elementary support mechanism can perform either a translation along one of the coordinate axes or a rotation around it. This displacement δ, relative to the preceeding mechanism, is assumed to be measured by an appropriate transducer.

A set of three angles, $\Delta\psi_m$, $\Delta\theta_m$ and $\Delta\phi_m$ describe the misalignment of axes of mechanism m to mechanism $m-1$ ($m=0$ refers to the wind tunnel structure). Each mechanism comprises an arm of length l, with the connection of the next mechanism $m+1$ being located on its endpoint, at coordinates l_x, l_y and l_z in the frame of reference of the mechanism m.

A 'translating' mechanism can be envisaged as sliding along one of the axes (x, y, z). The mechanism is defined by the relevant translation axis, e.g. **x**, **y** or **z**.

A 'rotating' mechanism is modelled as an arm rotating around one of the axes at the end where it is connected to the 'preceeding' mechanism. The mechanism is identified by the axis around which the rotation is performed, e.g. **l**, **m** or **n** (for axes x, y and z). The **l** type is a "roll" mechanism. The **m** and **n** types are "pitch" and "yaw" mechanisms, respectively.

The elementary transformation matrices used to compute components of a vector in a frame of reference rotated through an angle δ around axes x, y or z are, respectively:

$$[\phi] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\delta & \sin\delta \\ 0 & -\sin\delta & \cos\delta \end{bmatrix} \quad (1)$$

$$[\theta] = \begin{bmatrix} \cos\delta & 0 & -\sin\delta \\ 0 & 1 & 0 \\ \sin\delta & 0 & \cos\delta \end{bmatrix} \quad (2)$$

$$[\psi] = \begin{bmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Transformation matrix $[M]$ for a series of n rotations is a product of elementary rotation transformation matrices:

$$[M] = [M_n]\,[M_{n-1}]\ldots[M_1] \quad (4)$$

where each $[M_i]$ is one of the elementary matrices 1, 2 or 3, depending on the axis of rotation. Any rotation transformation matrix $[M]$ is orthogonal, therefore, for the inverse transformation, $[M]^{-1} = [M]^T$.

There are cases where the two subsequent elementary mechanisms do not move independently, but are connected by some sort of mechanical linkage, the reason for this usually being to ensure that the rotated model remains at test section centerline, as in Fig. 3. A connection code is associated with each elementary mechanism to describe the dependence (or otherwise) of its movement relative to the preceeding mechanism.

A particular model support system can, therefore, be defined by a sequence of elementary mechanism types (see Fig.1 to Fig.3), accompanied by the data on connection codes and arm lengths. These definitions are entered into the wind tunnel's database only once for each model support. When reducing the data from wind tunnel tests only a reference to a certain model support 'name' is needed in the data setup.

Often, the elements comprising the mechanisms can not be considered rigid, but deformable under aerodynamic loads and model weight. These deformations can be taken into account by putting strain gauges on model support elements and calibrating deflections under load. The element with strain gauges is then to be included in the model support as an additional elementary mechanism, as explained above. This arrangement can be simplified if only the last element of the model support mechanism (i.e. the sting or the strut) is deformable, and if a force balance is included in the test. Loads measured by the balance can then be used to compute changes in model position and attitude, and additional deformation measuring elements are not needed. This case is dealt with in the section where calculation of balance loads is presented.

The generic balance

A wind tunnel force balance consists of 1 to 6 load sensing elements, most often of the strain gauge type. Each element measures a single load "component" (a force or a moment). The correspondence between the balance output $\{e\}$ (usually electrical signals), zero offsets $\{e_0\}$ and loads $\{F\}$ applied to these elements is contained in the "calibration matrix" $[C]$:

$$\{e\} - \{e_0\} = \{\Delta e\} = [C]\,\{F^{**}\} \quad (5)$$

where $\{F^{**}\}$ is a vector $\{F_1 \ldots F_n, F_1{}^2 \ldots, F_1F_2 \ldots\}$ of loads and load products up to the desired order (usually second or third). For convenience, the calibration matrix $[C]$ is normalized, as suggested in [(1)], by the inverse of its main diagonal elements $[D]$, and the normalized matrix $[X]$ is divided into linear and nonlinear parts $[X_1]$ and $[X_2]$ so that:

$$\{\Delta e\} = [D]^{-1}\,[X_1]\,\{F\} + [D]^{-1}\,[X_2]\,\{F^*\} \quad (6)$$

with $\{F^*\}$ being the nonlinear part of $\{F^{**}\}$. The inverse relation can be determined only iteratively, as:

$$\{F\}_i = [X_1]^{-1}\,[D]\{\Delta e\} - [X_1]^{-1}\,[X_2]\,\{F^*\}_{i-1} \quad (7)$$

Various types of wind tunnel balances exist, with arrangements of components governed by constraints of manufacture and application. The experimenter is usually interested in reducing component loads $\{F\}$ to three components of force (e.g. R_x, R_y, R_z) and three components of moment (e.g. M_x, M_y, M_z), acting at a defined balance centre and usually aligned with geometric axes of the 'live' end of the balance. As shown in the equations 5 and 6, it is customary to include the nonlinear elements of this transformation, (mostly caused by balance deformation under load) in the calibration matrix. It follows that the relationship between k arbitrary balance components, three components of force vector and three components of moment vector is linear and given by a k×6 matrix $[S]$:

$$\left\{ \begin{array}{c} \{R\} \\ \{M\} \end{array} \right\} = \left\{ \begin{array}{c} R_x \\ R_y \\ R_z \\ M_x \\ M_y \\ M_z \end{array} \right\} = [S]\,\{F\} \quad (8)$$

As the matrix $[S]$ is not a square matrix, the inverse relation is given as:

$$\{F\} = \left[\,[S]^T[S]\,\right]^{-1}[S]^T \left\{ \begin{array}{c} \{R\} \\ \{M\} \end{array} \right\} \quad (9)$$

For precise calculations of model attitude, it is usually necessary to define the change of orientation of the 'live' end of the loaded balance, relative to the supported end. These deflections, assuming they are small, can be expressed, in terms of three Euler angles and three Cartesian coodrinates, as functions of balance component loads $\{F\}$:

$$\left\{ \begin{array}{c} \Delta\psi \\ \Delta\theta \\ \Delta\phi \\ \Delta x \\ \Delta y \\ \Delta z \end{array} \right\} = [A]\,\{F\} \tag{10}$$

In case of an external balance, the 'live' end of the balance is considered to be at the point where the model is connected to the balance - usually coincident with a hinge of the attitude changing mechanism.

The above elasticity matrix should be determined for the balance integrated with its sting or strut. The rest of the model support system is considered to be rigid.

Matrix $[S]$ is dependent only on balance geometry. The elasticity matrix $[A]$ should be determined by a calibration under load. These two matrices should always accompany the balance in use, together with the calibration matrix $[C]$. They are sufficient to compute the principal load components and attitude changes (all in the balance axes system) from the output of an arbitrary balance.

When reducing the vectors $\{R\}$ and $\{M\}$ to model body axes it must be noted that there exist essentially two types of wind tunnel balances, the first one conditionally named the "internal" and the second "external", though this may not reflect the actual position of the balance. The live end of the internal balance is immovably fixed to the model. The external balance contains a sort of mechanism which changes the orientation of the model relative to the balance, while the balance itself usually retains a constant orientation relative to the test section of the wind tunnel, except in the yaw plane. Additional transformations are necessary to reduce the data from an external balance to model body axes.

Data segmentation

The algorithm supports both the step-by-step motion and the continuous motion of the model during wind tunnel measurements. In the first case, each 'step' represents a data integrating segment. In the second case, all acquired data must be segmented at intervals centered on user defined model attitudes/positions, and the information on the attitude is contained in the data itself. The software must search through a sequence of samples from model position transducers and determine the limits of each integration segment. As the data from position transducers may contain noise, segmentation may become uncertain, so it is necessary to apply some kind of low-pass filtering prior to segmentation. The problem becomes slightly complicated when different types of transducers are used in the test: e.g. a digital output position transducer in the model support, and analog outputs from a force balance. Hardware low-pass filtering, customary for analog signals, introduces a time lag in data relative to the unfiltered digital signals, so balance and position data become unsynchronized. The solution accepted here is to restrict hardware low-pass filtering to a higher cut-off frequency, adequate to prevent anti-aliasing during D/A conversion. Software low-pass filtering to a desired cut-off frequency is performed on all all digitalized data, from analog and digital sources. A simple two-stage non-causal digital filter computing the moving average was found to be satisfactory. The position data filtered in this manner is easily segmented. Suitable cut-off frequency is a characteristic of each support system and it forms a part of that support system's description in the database.

Further filtering of the data from attitude and position measuring transducers is performed, within the bounds of the integration interval, by approximating the output from each transducer by a second order polynomial:

$$\delta(t) = c_0 + c_1 t + c_2 t^2 \tag{11}$$

Deviations of data samples from this curve are checked to ensure that the approximation is adequate.

By diferentiation and integration, average values of mechanism's measured displacement δ from the initial position, velocity $d\delta/dt$ and accelerations $d^2\delta/dt^2$ are obtained for the integration interval. For the 'translating' elementary mechanism, these values represent distance, linear velocity and acceleration. For the 'rotating' mechanism they represent angle, rate of turn and angular acceleration.

It was found that the repeatability of the measurements improves if the length of integration intervals is adjusted so as to contain an integer number of periods of the model support dominant frequency of oscillation, which is obtained by frequency analysis of some representative signal, e.g. a balance output.

Computing Model Attitude

The objective of the calculation of model attitude is to obtain position and orientation of the model relative to the wind tunnel test section. A convenient way to express the attitude is in terms of Euler angles. From these, model aerodynamic angles can be computed. For the purpose of correcting the support and wall interference, it is often necessary to determine model position, in terms of Cartesian coordinates, relative to the test section walls. When a wind tunnel test with rapid continuous motion (sweep) is performed, calculation of model velocities, accelerations and rates of turn sometimes become necessary in order to compensate the inertial forces and induced velocities [5].

For the "generic" mode support, attitude and position calculations are executed step by step, performing the same process for each elementary mechanism, and accumulating the transformation matrix as shown in equation 4.

The relative position $\{l\}$ of the arm of a translating type of mechanism (i.e. the position in the mechanism's frame of reference) is obtained by adding that mechanism's movement δ from the initial position to the relevant component of $\{l\}$. Appropriate velocity components $\{v\}$ and acceleration components $\{a\}$ are obtained directly from the diferentiation of equation 11. Angular velocity $\{\omega\}$ and angular acceleration $\{\varepsilon\}$ are zero.

The relative position of the arm end in the rotating type of mechanism that is moved through the angle δ from the initial position is computed as

$$\{l_r\} = [M_\delta]\,\{l\} \tag{12}$$

$[M_\delta]$ being one of the matrices 1 to 3, depending on the mechanism's axis of rotation. The components $\{\omega\}$ and $\{\varepsilon\}$ of the vectors of angular velocity and acceleration are obtained directly from equation 11, depending on the rotation axis. Linear velocity and acceleration are obtained as:

$$\{v\} = \{\omega\} \times \{l_r\} \tag{13}$$

$$\{a\} = \{\varepsilon\} \times \{l_r\} + \{\omega\} \times \{v\} \tag{14}$$

The notation "×" will be used for brevity, although formally not quite correct, between two vectors displayed in matrix form, to indicate a vector product with the result being resolved to components, e.g. in the equation 13 the product $\vec{\omega} \times \vec{l_r}$:

$$\{\omega\} \times \{l_r\} = \left\{ \begin{array}{c} \omega_y l_{rz} - \omega_z l_{ry} \\ \omega_z l_{rx} - \omega_x l_{rz} \\ \omega_x l_{ry} - \omega_y l_{rx} \end{array} \right\} \tag{15}$$

A transformation matrix $[\Delta_m]$ for rotations through the mechanism's misalignment angles relative to the preceeding mechanism is obtained from three elementary rotation matrices:

$$[\Delta_m] = [\Delta\phi_m]\,[\Delta\theta_m]\,[\Delta\psi_m] \tag{16}$$

Transformation matrices $[M_{m0}]$ and $[M_m]$ for rotation from the wind tunnel frame of reference to the origin and rotated frame of reference of an elementary mechanism in the train are obtained from its misalignment angles transformation matrix and the previous mechanism's transformation matrix $[M_{m-1}]$.

$$[M_{m0}] = [\Delta_m]\,[M_{m-1}] \tag{17}$$

$$[M_m] = [M_\delta]\,[M_{m0}] \tag{18}$$

The position, velocity and accleration of the arm are resolved to the wind tunnel frame of reference:

$$\{l_t\} = [M_{m0}]^T \{l_r\} \tag{19}$$

$$\{v_t\} = [M_{m0}]^T \{v\} \tag{20}$$

$$\{a_t\} = [M_{m0}]^T \{a\} \tag{21}$$

$$\{\omega_t\} = [M_{m0}]^T \{\omega\} \tag{22}$$

$$\{\varepsilon_t\} = [M_{m0}]^T \{\varepsilon\} \tag{23}$$

The absolute position, velocity and acceleration of the mechanism's endpoint, referring to the wind tunnel frame of reference, are obtained by adding the relative and transfer movements.

$$\{r_m\} = \{r_{m-1}\} + \{l_t\} \tag{24}$$

$$\{v_m\} = \{v_{m-1}\} + \{\omega_{m-1}\} \times \{l_t\} \tag{25}$$

$$\begin{aligned}\{a_m\} = {} & \{a_{m-1}\} + \{\varepsilon_{m-1}\} \times \{l_t\} + \\ & +\{\omega_{m-1}\} \times (\{\omega_{m-1}\} \times \{l_t\}) + \\ & +2\{\omega_{m-1}\} \times \{v_t\}\end{aligned} \tag{26}$$

$$\{\omega_m\} = \{\omega_{m-1}\} + \{\omega_t\} \tag{27}$$

$$\{\varepsilon_m\} = \{\varepsilon_{m-1}\} + \{\varepsilon_t\} \tag{28}$$

To obtain the final transformation matrix $[M]$ for rotation from the wind tunnel to the model frame of reference, the length and bending angles of a string or strut can be added as the parameters of the last mechanism in the train. The velocities and accelerations of the model fixed to the last mechanism in the train are then computed as shown above.

If a transformation matrix for rotation involving all three Euler angles is equated with the matrix $[M]$ created for a particular set of rotations, it can be seen that Euler angles are easily determined from the appropriate elements of $[M]$:

$$\theta = -\arcsin M_{13} \tag{29}$$

$$\psi = \arctan \frac{M_{12}}{M_{11}} \tag{30}$$

$$\phi = \arctan \frac{M_{23}}{M_{33}} \tag{31}$$

or, when $\theta = \pm\pi/2$:

$$\psi = \arctan \frac{-M_{21}}{M_{22}}\ ;\ \phi = 0 \tag{32}$$

In a similar manner, aerodynamic and aeroballistic angles can be computed:

$$\alpha = \arctan \frac{M_{31}}{M_{11}} \tag{33}$$

$$\beta = \arcsin M_{21} \tag{34}$$

$$\alpha' = \arccos M_{11} \tag{35}$$

$$\phi' = \arctan \frac{M_{21}}{M_{31}} \tag{36}$$

Computing Balance Loads

In order to compute accurately balance component loads using equation 7, zero offsets $\{e_0\}$ must be known. As they vary slowly with time and temperature, they must be determined immediately prior to measurement. In order to calculate balance loads due to aerodynamic forces, tare loads $\{F_g\}$, comprising model and balance weight and other inertial forces must also be known. Galway has developed a convenient way [1] to determine the 'true' zero offsets (called the "buoyant offsets") and tare loads of an arbitrary balance from several 'tare' measurements taken at various model attitudes prior to the wind tunnel run. That concept is here generalized and improved and some of its restrictions amended.

If the balance used in the test is nonlinear, and the pattern of attitude changes in the run is such that loads on some balance components remain constant, the algorithm requires that data on approximate model weight and centre of gravity position be supplied. In that case, model weight is resolved, for each of the model positions in the wind tunnel run, including tare measurements and wind-on data, to vectors of force and moment $\{R_w\}$ and $\{M_w\}$ in the balance axes system, using the inverse transformation matrix $[M]^T$. Corresponding approximate balance component loads $\{F_{gw}\}$ are determined using equation 9. All the values of $\{F_{gw}\}$ are then checked to determine on which balance components the loads remain constant, within the limits of balance accuracy, throughout the run. A vector $\{F_c\}$ of these "constant" loads is formed, containing zeros in places for variably loaded components.

It should be noted that it is not necessary to supply model weight and centre of gravity position with great accuracy, as these data only serve to account for higher order effects of balance calibration.

Tare loads $\{F_g\}$ are approximated by a 'gravity matrix' $[G]$ and a vector $\{T\}$ of trigonometric functions as:

$$\{F_g\} = [G]\,\{T\} \tag{37}$$

The software analyses the amount of change of the model Euler angles in the wind tunnel run, and determines $\{T\}$ as the appropriate one of the following:

When $\psi,\ \theta,\ \phi = const$:

$$\{T\} = \{1\} \tag{38}$$

When $\theta,\ \phi = const;,\ \psi = var$:

$$\{T\} = \left\{ \begin{array}{c} 1 \\ \sin\psi \\ \cos\psi \end{array} \right\} \tag{39}$$

When $\psi,\ \phi = const;\ \theta = var$:

$$\{T\} = \left\{ \begin{array}{c} 1 \\ \sin\theta \\ \cos\theta \end{array} \right\} \tag{40}$$

When $\psi, \theta = const;\ \phi = var$:

$$\{T\} = \left\{ \begin{array}{c} 1 \\ \sin\phi \\ \cos\phi \end{array} \right\} \tag{41}$$

When $\psi = const;\ \phi,\ \theta, \phi = var$;

$$\{T\} = \left\{ \begin{array}{c} 1 \\ \sin\theta \\ \cos\theta\sin\phi \\ \cos\theta\cos\phi \end{array} \right\} \tag{42}$$

When $\phi = const;\ \theta, \psi = var$:

$$\{T\} = \left\{ \begin{array}{c} 1 \\ \sin\theta \\ \cos\theta\sin\psi \\ \cos\theta\cos\psi \end{array} \right\} \tag{43}$$

For the 'constant' angles, a certain amount of change is allowed that does not significantly alter tare data.

There are provisions for expanding the choice of functions if necessary, but the above set was found to be sufficient for all model supports used in the Aeronautical Institute and all reasonable patterns of model attitude changes.

Functions $\{T\}$ are calculated for all tare measurements, forming columns in a matrix $[T]$.

Initial values od buoyant offsets $\{e_0\}$ are set to 0. The following procedure is then performed iteratively:

For a set of tare measurements, balance loads $\{F_g\}$ are calculated using the equation 7, forming columns in a matrix $[F_g]$.

Optimum approximation of inertial loads, in a least-squares sense, is obtained by determining the gravity matrix $[G]$:

$$[G] = [F_g]\,[T]^T\left[\,[T]\,[T]^T\right]^{-1} \tag{44}$$

The first column of matrix $[G]$ is extracted to form a vector $\{G_1\}$. From this, the vector of constant loads $\{F_c\}$ is subtracted to form a vector $\{G_0\}$ of apparent loads due to uncompensated-for buoyant offsets. In this way, the nonlinearity of balance components subjected to constant loads is taken into account:

$$\{G_0\} = \{G_1\} - \{F_c\} \tag{45}$$

An increment $\{\Delta e_0\}$ of buoyant offset is calculated using equation 6:

$$\{\Delta e_0\} = [D]^{-1}\,[X_1]\,\{G_0\} + [D]^{-1}\,[X_2]\,\{G_0{}^*\} \tag{46}$$

And a new approximation of buoyant offset is obtained by adding that increment to the buoyant offset from the previous iteration:

$$\{e_0\}_i = \{e_0\}_{i-1} + \{\Delta e_0\} \tag{47}$$

The whole procedure is then repeated, until $\{\Delta e_0\}$ becomes acceptably small relative to the balance accuracy and full scale output.

For the algorithm to function properly, the number of tare measurements must be at least equal to the number of rows in $[T]$ which, in turn, depends on the type of $\{T\}$ chosen for the pattern of model attitude changes in a particular wind tunnel run. It is desirable, though, to use a greater than minimum number of tare measurements. The redundant data give the opportunity to check the quality of the least-squares fitting by calculating individual deviations from the approximation, and to signal unacceptably large errors that may be caused by instrumentation malfunction or some other undesired influence.

It is obvious that tare positions must be chosen so that the matrix $\left[[T]\,[T]^T \right]$ is non-singular, and that the angles variable in the wind-on phase of the test should also be variable in the tare measurements (the variable angle ψ being relevant only for external balances).

When the buoyant offset is known, balance component loads $\{F\}$ can be calculated using the equation 7. Corrections of model attitude and position due to deflections under loads can then be made using equation 10. A corrected wind-tunnel-to-model rotation transformation matrix $[M_c]$ is calculated as:

$$[M_c] = [\Delta\phi]\,[\Delta\theta]\,[\Delta\psi]\,[\phi]\,[\theta]\,[\psi] \qquad (48)$$

Model attitude is determined from $[M_c]$ using equations 29 through 32.

In order to compensate tare weights and inertial forces, for each position of the model in the wind tunnel run the acceleration $\{a_{cg}\}$ of the model's centre of gravity is obtained by going one step further with the calculations performed to compute model support acceleration. The value of g is added to this acceleration as acting vertically upwards to obtain the apparent total acceleration $\{a_g\}$ of the model's centre of gravity:

$$\{a_g\} = \left\{ \begin{array}{c} a_{cgx} \\ a_{cgy} \\ a_{cgz} + g \end{array} \right\} \qquad (49)$$

From a_{gx}, a_{gy} and a_{gz} the direction of the apparent (to the hypothetical observer in the model) local vertical is determined in terms of corections $\Delta\phi_g$ and $\Delta\theta_g$. A 'corrected vertical' wind-tunnel-to-model rotation transformation matrix $[M_g]$ is calculated as:

$$[M_g] = [M_c]\,[\phi_g]\,[\theta_g] \qquad (50)$$

Euler angles ψ_g, θ_g and ϕ_g, relevant to determination of tare weights are extracted from $[M_g]$ using equations 29 to 32. A check is performed to ensure that the chosen type of tare approximation functions $\{T\}$ is still valid for the corrected attitudes. Assuming that total acceleration in the tare phase is equal to g, tare loads are calculated using the gravity matrix multiplied by the ratio of the current total acceleration a_g and g:

$$\{F_g\} = \frac{a_g}{g}\,[G]\,\{T\} \qquad (51)$$

In this way, the effect of acceleration acting differently on different balance components is taken into account in the same way as different weights sensed by the components.

The whole procedure of determining buoyant offsets and model loads is repeated, thus taking into account the inertial forces and load-induced changes of attitude in the tare measurements. Iterations are performed until an acceptable convergence is achieved. Apparent Euler angles ψ_g, θ_g and ϕ_g are used instead of real angles in equations 38 to 43.

Balance loads due to aerodynamic forces are obtained by subtracting the computed tare loads:

$$\{F_a\} = \{F\} - \{F_g\} \qquad (52)$$

If an external balance is used this relation is only approximate, as the tare loads $\{F_g\}$ may be different in the wind-on measurement because of the shift of model's centre of gravity caused by sting/strut deflections under aerodynamic loads (the deflections caused by weight alone have already been taken into account). In this case an iterative computation of tare loads is performed. Approximate deflections Δx_a, Δy_a and Δz_a due to aerodynamic loads are calculated using $\{F_a\}$ and equation 10. Tare forces $\{R_g\}$ are computed from $\{F_g\}$ using equation 9. A change of tare moments $\{\Delta M_g\}$ is computed as:

$$\{\Delta M_g\} = \{R_g\} \times \left\{ \begin{array}{c} -\Delta x_a \\ -\Delta y_a \\ -\Delta z_a \end{array} \right\} \qquad (53)$$

and corresponding change $\{\Delta F_g\}$ of balance component loads is computed using equation 8. $\{\Delta F_g\}$ is added to $\{F_g\}$ and a new approximation of aerodynamic loads is made using equation 52. This procedure is repeated until $\{\Delta F_g\}$ becomes acceptably small.

When aerodynamic loads of balance components are determined they are resolved, using the equation 8, to three components of force $\{R_a\}$ and three components of moment $\{M_a\}$ acting at balance centre. If an external balance is used this procedure involves computing the "unrotated" forces and moments $\{R_b\}$ and $\{M_b\}$ using equation 8, and then translating and rotating the axes system to account for the motion of the model relative to the balance:

$$\{R_a\} = [M_B]\,\{R_b\} \qquad (54)$$

$$\{M_a\} = [M_B]\,\{\{M_b\} + \{R_b\} \times \{r\}\} \qquad (55)$$

$[M_B]$ being the transformation matrix for the change of model attitude relative to the balance, and $\{r\}$ being the position of the model relative to it. At present, the algorithm is restricted to those external balances in which model motion relative to the balance involves only angles θ and ϕ, the assumption being made that the whole balance acts as a turntable rotating in the yaw plane through an angle ψ.

A reduction of moments to a user-defined reference point, located at position $\{r_r\}$ relative to the balance centre, is then performed:

$$\{R_r\} = [M_r]\{R_a\} \quad (56)$$
$$\{M_r\} = [M_r]\{\{M_a\} + \{R_a\} \times \{r_r\}\} \quad (57)$$

where $[M_r]$ is a rotation transformation matrix involving misalignment angles (if there are any) of model and balance axes.

If there is more than one balance utilised in the test, a similar procedure for computing aerodynamic loads is performed for each additional balance. It is assumed that the additional balances are placed at known positions and attitudes on the model The final, corrected attitude of the model is taken as the basis for determining the attitude of additional balances. The buoyant offsets, total, tare and aerodynamic loads, and attitude corrections of the balance relative to the model are determined.

A set of nondimensional aerodynamic coefficients that are resolved to model (or model element) body axes is obtained in the usual way from the obtained aerodynamic forces and moments that are reduced to the chosen reference point.

Provision for Wind Tunnel Corrections

Basic reduction of measurements of forces and moments ends with non-dimensional coefficients reduced to model body axes and model attitude expressed in Euler angles. At this point a provision is made for wall interference and test section callibration corrections. The non-dimensional coefficients can easily be resolved to wind tunnel axes by using the transposed transformation matrix $[M]^T$. These coefficients can be used in calculations of wall interference, e.g. from boundary pressure measurements. Corrections $\Delta\psi_c$ and $\Delta\theta_c$ of the position of the model support relative to flow direction in the horizontal and vertical plane can thus be determined. A modified transformation matrix $[M_a]$ is formed using the elementary rotation matrices for $[\Delta\psi_c]$ and $[\Delta\theta_c]$:

$$[M_a] = [M]\,[\Delta\theta_c]\,[\Delta\psi_c] \quad (58)$$

and the corrected Euler angles (i.e. relative to flow direction) are calculated from it, using equations 29 to 32. Non-dimensional coefficients are recomputed to account for the Mach number correction. The corrected aerodynamic or aeroballistic angles are calculated from $[M_a]$ using the equations 33 and 34 or 35 and 36. In the similar manner, the corrected aerodynamic angles of model elements (e.g. the tail) can be calculated, using the residual flow curvature obtained during the calculation of wall interference corrections.

The aerodynamic coefficients are reduced to the desired axes system using the appropriate rotation transformation matrix; e.g. for the reduction of body-axes force coefficients C_A, C_Y, C_N to aerodynamic axes:

$$\begin{Bmatrix} C_x \\ C_y \\ C_z \end{Bmatrix} = [\beta]\,[-\alpha] \begin{Bmatrix} C_A \\ C_Y \\ C_N \end{Bmatrix} \quad (59)$$

assuming that left-handed, downstream-oriented axes systems are always used; changes of sign are performed if necessary.

An equivalent rotation transformation matrix to aeroballistic axes system is $[-\phi']$, and to unrotated aerodynamic axes system is $[-\alpha']\,[-\phi']$.

Implementation

The routines implementing the algorithms for the calculation of model position and orientation and for the reduction of data from force balances form a part of the software package used in the Aeronautical Institute for wind tunnel data reduction. The package is being used on VAX computers running under VMS operating system. The new programs have been coded in VAX FORTRAN and integrated with programs and databases used to process other wind tunnel measurements. They have been founded on a set of basic routines for performing matrix operations, calculations of Euler and aerodynamic angles, vector additions and products, conversions of balance outputs to component loads and vice versa. In order to ensure the necessary accuracy, the matrix calculations and trigonometric functions for determining model aerodynamic angles have been implemented in double precision. Single precision calculations have proved to be adequate for the reduction of balance data. At present, a single model support is permitted in a wind tunnel run. Up to eight force balances can be used simultaneously to measure forces and moments on the model or on particular elements of the model. To reduce the amount of data entry, the misalignment angles of elementary mechanisms are assumed to be zero, and mechanism arm orientation is restricted to cases where only l_x or l_z exists. Connection code of non-independent rotating mechanisms is restricted to the case where $l_{m-1}\sin\delta_{m-1} = l_m\sin(\delta_{m-1} - \delta_m)$ as in Fig. 3. The balance calibration model defined by equation 5 is implemented as in [1], with second order calibration and separate normalized matrices for 'positive' and 'negative' loads, the matrices $[X_1]$ and $[X_2]$ being synthetized for each load case.

Performance Evaluation

The code implementing the presented generalized procedures has been used for some time in the Institute. The performance has been satisfactory, with numerous advantages when compared to the previously used software:

- The same code is used for reduction of data from different wind tunnels, balances and model supports. This makes software maintenance and upgrade much easier;
- The algorithm is applicable to most model support devices;
- It is applicable to any wind tunnel balance used in the Aeronautical Institute;
- It supports both the step-by-step and continuous motion (sweep) measurements;
- It can be used both with linear and non-linear balance calibration;
- Tare loads are calculated in the optimum way, by a least-squares approximation, using the "buoyant offset" concept, for any practical pattern of model attitude changes;
- Application of digital filtering and adjustment of integration times have improved the repeatability of measurements;
- Constant, non-zero loads acting on balance components are included in tare calculations, for precise measurements with non-linear balances;
- Several balances can be used simultaneously in a test;
- Any wind tunnel corrections scheme that is reducable to corrections of the Mach number and freestream direction is easily implemented, without changes to the basic algorithm and existing code;
- Output of data is possible in any of the widely used aerodynamic axes systems and sets of angles;
- Data setup for unfamiliar test configurations is greatly simplified.

During the exploitation of the software some of the defficiencies have also come into view. They partly stem from the fact that the implementation of the algorithm had to be embedded into the existing application software package with a rigid and limited database structure and user interface. Some of the observed shortcomings, pointing to possible improvements in the future, are:

- Provisions for model supports with non-orthogonal axes have not been implemented in the software;
- Application to vertical wind tunnels is not explicitely supported;
- Balance loads due to moments of inertia and angular accelerations are not accounted for;
- The generalization of a force balance is not complete; an external balance is still, in some aspects, being treated as a particular case;
- The assumption of a rigid model support structure (except the sting or the strut) is not entirely justified;
- It seems, though the evidence is not conclusive, that the second-order balance calibration model with separate matrices for positive and negative loads is not the best choice for high-precision single-piece balances [12]; a change to a third order model may follow;
- A set of exclusively left-handed axes systems that is used in the Aeronautical Institute and implemented in the algorithm does not appear to be coincident, in terms of axes orientation, with any of the axes systems widely used today. Changes of signs are almost always performed in data supplied to outside users. The simplest, and probably the best, amendment would be a complete change to ISO axes systems [4] as they are all right-handed and with orientation of all axes opposite to the one presently used. In fact, provisions for this change have been made in necessary places in the software, awaiting the hoped-for adoption of ISO axes systems;
- Porting of the software to another computer operating systems is not possible without major changes.

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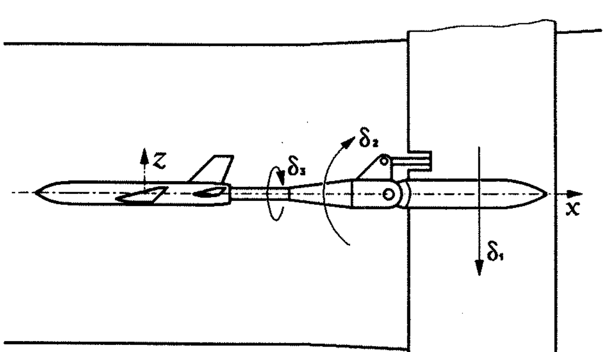

Fig.1: A "**zml**" (elevate-pitch-roll) model support

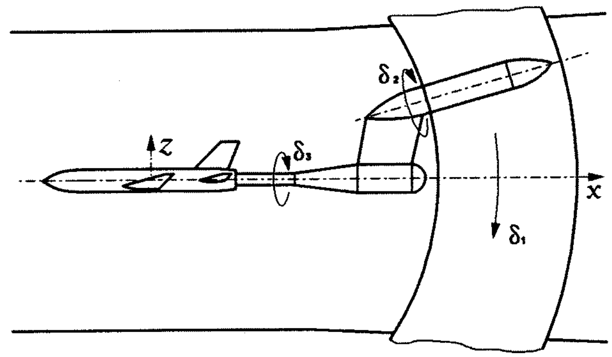

Fig.2: A "**mll**" (pitch-roll-roll) model support

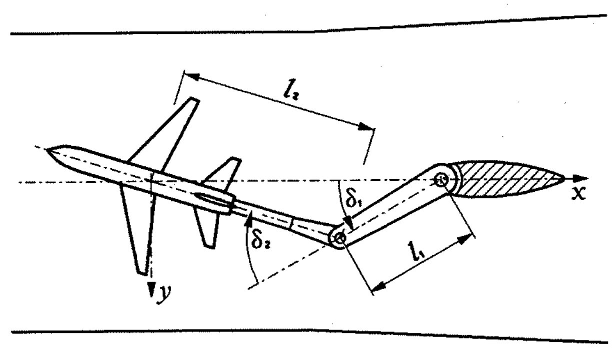

Fig.3: A "**nn**" (yaw-yaw) model support