UNSTEADY VORTEX FLOW PHENOMENA ON DELTA WINGS AT HIGH ANGLES OF INCIDENCE

by
D G Mabey
Visting Professor, Department of Aeronautics, Imperial College, London UK

Abstract

Vortex flow phenomena at high angles of incidence are of great interest to the designers of advanced combat aircraft. The steady phenomena (such as steady lift and pitching moments) are understood fairly well, whereas the unsteady phenomena are still uncertain. This paper addresses two important unsteady phenomena on delta wings. With regard to the frequency parameter of the quasi-periodic excitation caused by vortex bursting, a new correlation is established covering a range of sweep back from 60° to 75°.

With regard to the much lower frequency parameter of limit-cycle rigid-body wing-rock, a new experiment shows conclusively that although the motion is non-linear, the frequency parameter can be predicted by quasi-steady theory. As a consequence, for a given sweep angle, the frequency parameter is inversely proportional to the square root of the inertia in roll. This is an important observation when attempting to extrapolate from model tests to wind tunnels to predict the wing-rock characteristics of aircraft.

Introduction

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With regard to the much lower frequency parameter of limit-cycle rigid-body wing rock, a new experiment shows conclusively that although the motion is non-linear, the frequency parameter can be predicted by quasi-steady theory. As a consequence, for a given sweep angle, the frequency parameter of the wing-rock on a given configuration depends upon the inertia in roll parameter, $\sqrt{\rho S b \bar{c}^2 / I}$ (sometimes known as the apparent mass parameter). This is an important observation when attempting to extrapolate from model tests to wind tunnels to predict the wing rock characteristics of aircraft.

Review of vortex shedding frequencies

The tremendous cost of replacing the fins of some combat aircraft after only a few hours flying has created great interest in the prediction of the buffet excitation frequency on the fins of combat aircraft at moderate/high angles of incidence, α, (say $20° < \alpha < 50°$). The present author noted that for every wing/fin configuration tested within many RAE/DRA experiments, the frequency parameter, $n = f\bar{c}/U$, of the peak in the buffet excitation parameter, $\sqrt{nG(n)}$ (fully defined according to the AGARD notation in Ref 1) varied with the angle of incidence according to the relation

$$(f\bar{\bar{c}}/U) \sin \alpha = constant , \qquad (2\text{-}1)$$

where the constant varied from about 0.2 to 0.8 with the different configurations (eg widely different wing shapes, single and twin fins). The universal form of equation (2-1) suggests that large scale separations have global characteristics independent of their particular origin. This inference is confirmed by a review of the experiments on aerofoils, flat plates and delta wings[2].

Experimental data

Aerofoils and flat plates

Fage and Johansen observed that the large scale vortex structures shed from aerofoils and flat plates of infinite aspect ratio are nearly the same[3]. For both aerofoils (with flows separated completely from the leading edge to the trailing edge) and for flat plates with sharp edges they suggested that the length scale, L, determining the wake shedding frequency was the approximate wake width

$$L = c \sin \alpha \ . \qquad (2\text{-}2)$$

Using this length scale they found a relationship for the frequency parameter, Fig 1.

For aerofoils and flat plates for $30° < \alpha < 90°$

$$(fc/U) \sin \alpha = 0.15 \ . \qquad (2\text{-}3)$$

Ref 4 shows that even for a flat plate of aspect ratio 1.6, equation (2-3) applies. For the square flat plate[4] with a highly three-dimensional flow there is a wider variation (see Fig 1) over the incidence range from $\alpha = 38°$ to $\alpha = 90°$.

Delta wings

In a study of the principal helical mode vortex instability on delta wings, Gursul showed[5] how the frequency of the quasi-periodic pressure fluctuations measured on the wing decreased with the streamwise distance from the apex, x. (cf Ref 5, Fig 10.) The curves (referenced to the root chord c) varied both with the angle of incidence, α, and the sweep angle, Λ, of the four delta wings considered (Λ = 60°, 65°, 70° and 75°). The predominant pressure fluctuation frequency on the 60° delta wing at x/c = 0.89 was close to that observed by a hot wire in the wake (Ref 5, Fig 7). Thus the wake frequency from the measurements of Ref 5 is given by

$$x = c \ . \qquad (2\text{-}4)$$

Fig 2 shows that for Λ = 60°, 65° and 75° there is a characteristic 'plateau' region where

$$(fc/U) \sin \alpha = constant \ . \qquad (2\text{-}5)$$

and where the constant varies from about 0.4 for Λ = 60° to about 0.9 for Λ = 75°; this range of constant is comparable with that of equation (2-1). For the lowest angles of incidence equation (2-5) is not satisfied for Λ = 60°, 65° and 75°, presumably because here the vortex breakdown position is just downstream of the trailing edge, so that the similarity rules appropriate to the vortex burst phenomenon are inapplicable. Again, for these three sweep angles equation (2-5) is not satisfied for the highest angles of incidence, presumably because now the vortex burst is close to the apex of every wing. Equation (2-5) is less satisfactory for Λ = 70°, a minor anomaly caused by some feature in the vortex development.

The systematic variation in reduced frequency for Λ = 60°, 65° and 75° suggests that a unique frequency parameter might be found if a reference length L appropriate to the phenomenon could be identified. These disparate measurements can be reduced to a unique wake frequency parameter which determines the excitation on fins, giving a useful design rule for delta wings related with the semi-span, s. Equation (2.2) is replaced with the reference length

$$L = s \sin \alpha = c \cot \Lambda \sin \alpha \ . \qquad (2\text{-}6)$$

Fig 3 shows that equation (2-6) is appropriate for Λ = 60° 65° and 75°, the constant being

$$(fs/U) \sin \alpha = (fc \cot \Lambda/U) \sin \alpha = 0.25 \pm 0.02 \ . \qquad (2\text{-}7)$$

With regard to the minor anomaly for Λ = 70° the plateau is not well defined. However, it is significant that now all the values given are intermediate between those for Λ = 65° and 75°.

Semi-empirical theory for wings

Currently there is no theory to predict the vortex burst excitation frequency observed on delta wings or on wings with widely varying planforms and camber distributions. However, the universal form of equation (2-1) suggests an heuristic framework (Fig 4) which explains the character of all the experiments and implies an interesting expression for an appropriate reference length, d′. It is important to appreciate that the cross-flow velocity, U sin α, is common to all three flow regimes to be described within Fig 4. For simplicity and brevity, the flow about delta wings is described, but similar remarks would apply to other wings, which might have combinations of vortices and swept bubbles.

Low angles of incidence

For low angles of incidence (Fig 4a, typically $0 \langle \alpha \langle 20°$) there are small, unburst vortices situated close to the wing. Although there is local excitation (buffet) on the wing, the separations in the cross-flow plane are closed, so that the width, d′, of the cross-flow shear layer is given by the relation

$$d' \equiv 0 \ . \qquad (2\text{-}8)$$

Thus for low angles of incidence, although there may be local excitation on the wing or on a horizontal tailplane in the wake of the wing, there will be little excitation at the top of a fin. Hence there will be little fin buffeting in this flow regime.

Moderate/high angles of incidence

For these angles of incidence (Fig 4b, typically 20° $\langle$ α $\langle$ say 60°) there are large, burst vortices situated some distance away from the wing. Now the separation in the cross-flow plane must be open so that vorticity can be shed. Thus there will be a high level of excitation at the top of a fin and hence severe fin buffeting, in addition to heavy wing buffeting. Equation (2-7) suggests an interesting, semi-empirical expression for d′. A three-dimensional relationship analogous with the two-dimensional equation (2-3) would be

$$S^* = (fd'/U \sin \alpha) = \textit{universal constant} \ . \quad (2\text{-}9)$$

because the cross-flow velocity is always U sin α. By inspection, equations (2-9) and (2-7) can be compatible only if

$$d' = \textit{constant} \ s \sin^2 \alpha \ . \quad (2\text{-}10)$$

It is interesting that such a simple expression could explain such a diverse collection of measurements. Of course, at present we cannot be sure that there is a universal relation making equation (2-9) valid, but it is a reasonable working hypothesis. It is plausible to hope also that the constant is close to that given by equation (2-3) for two-dimensional shear layers, although we have no direct measurements of d′ to confirm this. Making this assumption in equation (2-9),

$$(fd'/U \sin \alpha) = 0.15 \ , \quad (2\text{-}11)$$

we find from equations (2-11) and (2-7) that for the intermediate range of incidence

$$d' = 0.60 \ s \sin^2 \alpha \ . \quad (2\text{-}12)$$

High angles of incidence

For high angles of incidence (Fig 4c, typically 60° $\langle$ α $\langle$ 90°) the wake from a wing is the same in character as that from a bluff body. Thus close to the wing there will be low level, random excitation covering a wide range of frequencies. This excitation gives light/moderate levels of wing buffeting and some buffeting on horizontal tailplanes. However, there could be some excitation at a higher level (but now covering a broad range of frequencies) at the edges of the shear layer: on particular configurations this might also excite light/moderate levels of fin buffeting.

Review of wing-rock

Physical phenomena

The control of many aircraft at high angles of incidence (with separated flows) is limited by the phenomenon of wing-rock. Wing-rock is a Limit Cycle Oscillation (LCO) in the rigid body roll mode discussed by many authors, eg Nguyen and Ross[6]. Prediction methods for the onset and frequency of wing rock, considered as a non-linear problem, have been derived and applied successfully to several different aircraft[7].

The present author's interest in the phenomenon stems from the frequency of the oscillation, which has received little comment. Suppose a model (a delta wing is considered here) is mounted in a wind tunnel (Fig 5) with a single, rigid-body freedom in roll (generally provided by a roller bearing but ideally by a frictionless air bearing). Then, as the angle of incidence increases slowly at constant speed, two alternative sequences of events may occur. The model motion in roll may be random buffeting which increases steadily in rms amplitude, reaches a maximum and then decreases. This behaviour is typical of delta wings with low to moderate sweep (Fig 5a). This motion is similar to the variation of the buffet excitation on an aerofoil with separated flow[8]. An alternative sequence is that the model motion may initially be random buffeting, but at a higher range of incidence it may develop into a large amplitude LCO in roll. For an even higher angle of incidence the roll motion reverts to low amplitude buffeting: this behaviour is typical of highly swept delta wings (Fig 5b). The interesting feature of these radically different sequences is that the frequency, and hence the frequency parameter, of these motions is generally much the same throughout the whole range of incidence. This suggests the hypothesis that although the equations governing the motion are non-linear[7], there must be a constant, quasi-linear stiffness, giving the nearly constant frequency parameter. In contrast, complex variations in the aerodynamic damping will determine the character of the motions and, in particular, the roll LCO amplitude.

Implications of quasi-steady theory

Suppose that a quasi-steady aerodynamic stiffness does exist. Then the roll stiffness, k, will be given by

$$k = (C_{l\phi})qSb \ . \qquad (3\text{-}1)$$

For quasi-steady motions the aerodynamic and friction terms (giving damping) may be neglected. Hence the equation of quasi-steady motion in roll becomes

$$I\ddot{\phi} + k\phi = 0 \ . \qquad (3\text{-}2)$$

It is easy to show that the frequency parameter of the roll-motion is given by

$$n = \frac{fc}{U} = \frac{1}{2\pi}\sqrt{(C_{l\phi})\frac{\rho Sbc^2}{2I}} \ . \qquad (3\text{-}3)$$

Thus if a quasi-steady aerodynamic stiffness exists for a given model geometry the frequency parameter of the motion will be independent of the velocity but inversely proportional to the square root of the inertia in roll. In addition, passive devices, (eg fins or leading-edge fences) which reduce wing-rock leave the frequency parameter of the motion almost unaltered. Hence these devices cannot be altering the aerodynamic stiffness in roll (represented in equation (3.3) by $C_{l\phi}$). Thus the quasi-steady equation (3.3) is crucial in studying wing-rock.

The choice of the most appropriate value of $C_{l\phi}$ from a typical low frequency, non-linear LCO (Fig 6) raises interesting questions. The region within the cycle where the direction of the motion is clockwise is unstable and energy flows into the system. This region could be approximated by a straight line through the image points A1, A2 and the origin, with a high slope

$$C_{l\phi} = A \ . \qquad (3\text{-}4)$$

Use of equation (3-4) would, of course, ignore the region within the cycle where the direction of the motion is anticlockwise and stable. Here energy is dissipated. Both regions might be included, and perhaps the stiffness forces might be better approximated, by a straight line through the extreme image points (defining the largest roll amplitude) B1, B2 and the origin, with low slope

$$C_{l\phi} = B \ . \qquad (3\text{-}5)$$

Equations (3-4) and (3-5) are only linear approximations to a difficult non-linear phenomenon. However, as an engineering approximation for a quasi-steady theory equation (3-5) has the apparent advantage of including both regions within the LCO. Despite this we shall see later that equation (3-4) gives better agreement than equation (3-5) with a diverse collection of experiments (Fig 9). For the limit cycle oscillation of an 80° swept delta wing at $\alpha = 30°$ (close to the condition for maximum amplitude in roll), Fig 4.22 of Ref 9 shows that the higher slope is A = 0.20/radian and the lower slope is B = 0.10/radian. Fig 8 includes straight lines for these values.

Results

The experiments were made as student research projects in the 5ft x 4ft Low Speed Wind Tunnel in the Department of Aeronautics at Imperial College: a consistent and repeatable wing-rock was established with a leading-edge swept 80°. This motion was studied in tests when the inertia in roll was varied[10] (Fig 7).

The results are described fully elsewhere[11,12]. Fig 8 shows the variation of the frequency parameter, n, of the Limit Cycle Oscillations with the angle of incidence, where the amplitude, ϕ, typically varies from about 30° to 45°. Apart from some scatter, for every tip pod the frequency parameter is independent of the velocity, consistent with equation (3-3). With regard to the variation with the angle of incidence, for every tip pod n increases steadily from about $\alpha = 25°$ to 40°. For the brass and aluminium tip pods (Figs 8a and 8b respectively) the oscillation is completely suppressed at $\alpha = 45°$ (n = 0!), due to the combined effects of bearing friction and high inertia[11,12]. In marked contrast, Fig 8c shows that for the balsa pod n actually reached a maximum at $\alpha = 45°$, even though the amplitude there is only about $\phi = 14°$. Subsequently n falls steadily from $\alpha = 45°$ to 55°.

Comparison of the measurements of frequency parameter, n, at $\alpha = 30°$ with the inertia parameter, $\sqrt{\rho Sbc^2/I}$ from the present tests[10] and widely disparate sources provides a severe test of the validity of equation (3-3). [The value selected for A is 0.20.] Fig 9 shows that the measurements on the three small models (with c about 0.4m and I about 1×10^{-3}kg m^2) fit equation (3-3) quite well. This is not surprising, because A was evaluated from tests on one of these models. The measurement from the single very large model (with c about 1.8m and I about 92×10^{-3}kg m^2)

also compares reasonably well with equation (3-3). Hitherto this high frequency parameter (n nearly 0.2) has been regarded as anomalous.

Conclusions

This paper suggests four main conclusions which will be justified more fully elsewhere[12]. With regard to the vortex development on fixed delta wings the conclusions are:

1 A new correlation of the quasi-periodic shedding frequency associated with vortex bursting on delta wings has been suggested (equation 2-7).

2 A tentative, semi-empirical expression for the cross-flow wake width at intermediate angles of incidence has been developed (equation 2.12). This expression would be consistent with the quasi-periodic excitation frequencies referred to above.

With regard to Limit Cycle Oscillations (LCO) in roll the conclusions are:

3 Although the LCO is a non-linear phenomenon the experimental measurements are consistent with an approximate, quasi-steady linear theory.

4 The measurements of frequency parameter have been correlated by the aerodynamic inertia parameter $\sqrt{\rho S c b^2 / I}$, and are consistent with a wide range of measurements on similar configurations (Fig 9).

List of symbols

A, B	particular values of $C_{l\phi}$ (Fig 6)
b	wingspan
c, $\bar{\bar{c}}$	root or aerodynamic mean chord of wing
$C_{l\phi}$	quasi-steady rolling moment coefficient/radian (eqn 3-1)
d'	width of shear layer
f	excitation frequency
I	inertia in roll
k	aerodynamic roll stiffness (eqn 3-1)
L	characteristic length
$\sqrt{nG(n)}$	buffet excitation parameter (defined in Ref 1)
S	wing area
s	semi-span of delta wing
α	angle of incidence (°)
Λ	sweep angle (°)
ϕ	roll amplitude (radians or °)

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Acknowledgement

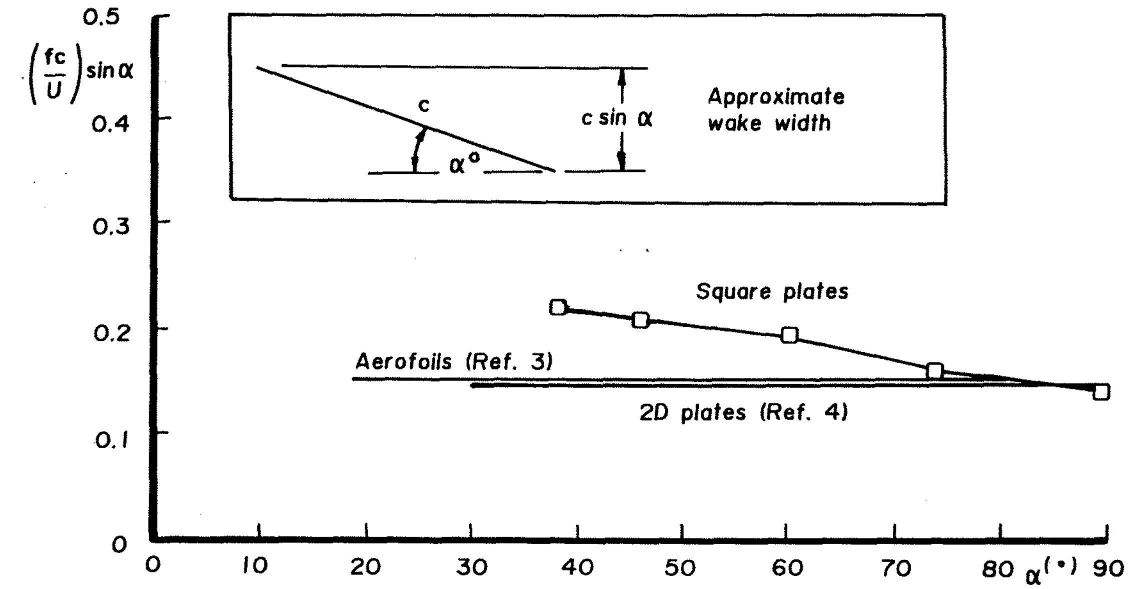

Fig 1 Correlation of shedding frequencies for aerofoils and 2D-plates

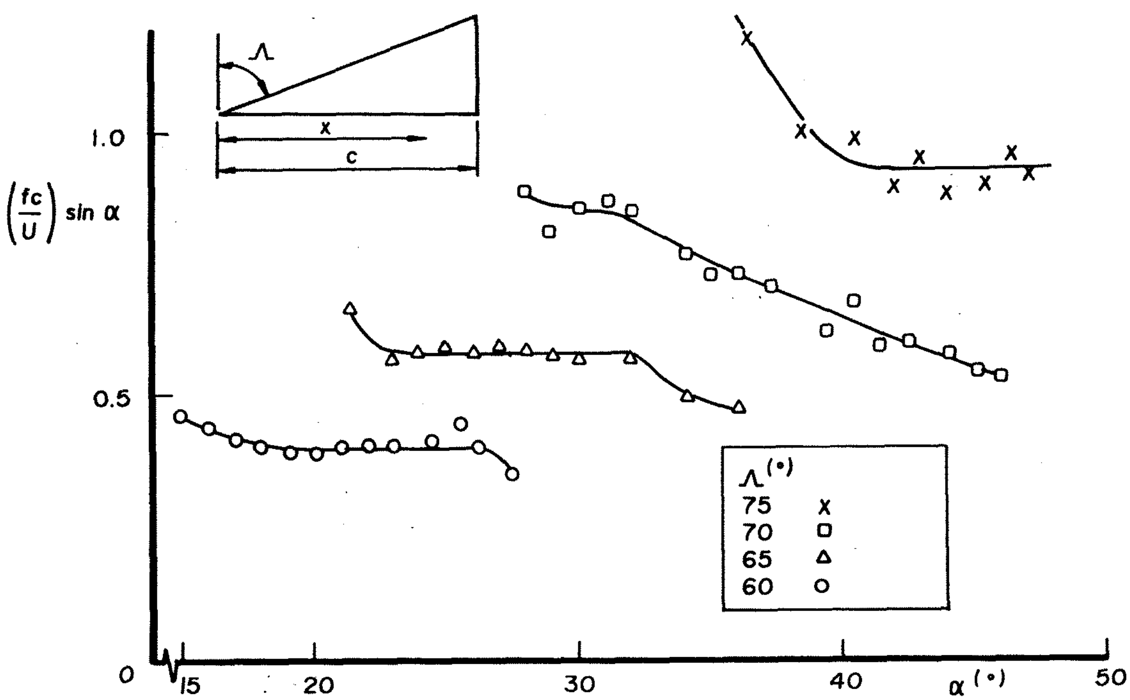

Fig 2 Variation of dimensionless frequency, (fc/U) sin α, as a function of angle of incidence for delta wings of varying sweep

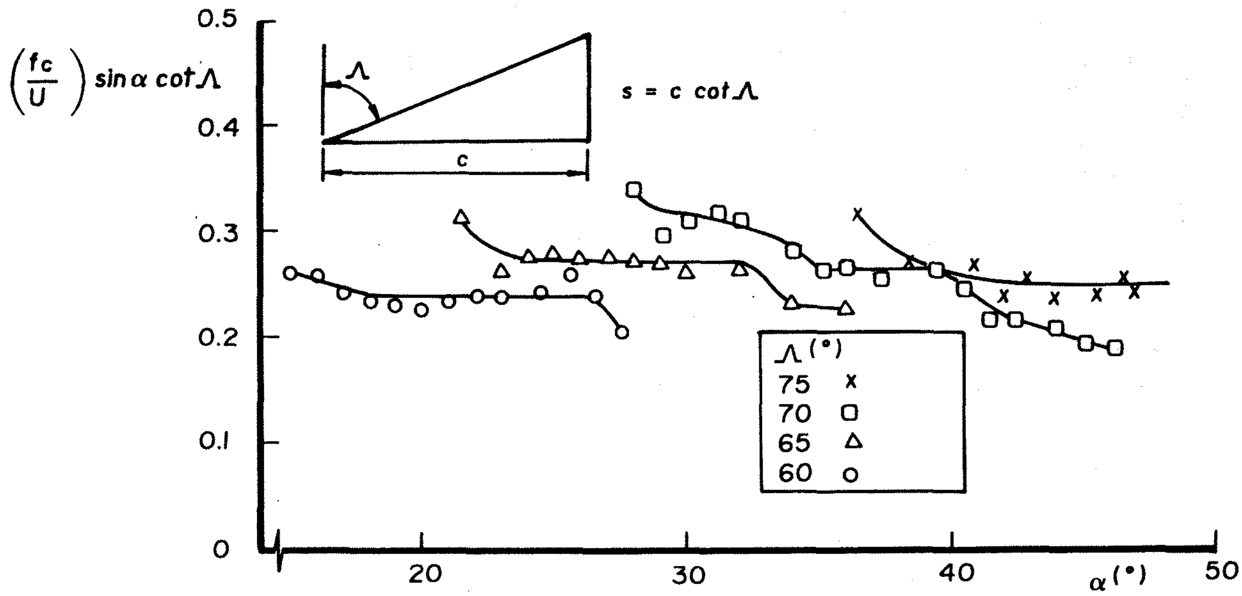

Fig 3 Variation of dimensionless frequency based on semi-span, (fc/U) sin α cot Λ, as a function of angle of incidence

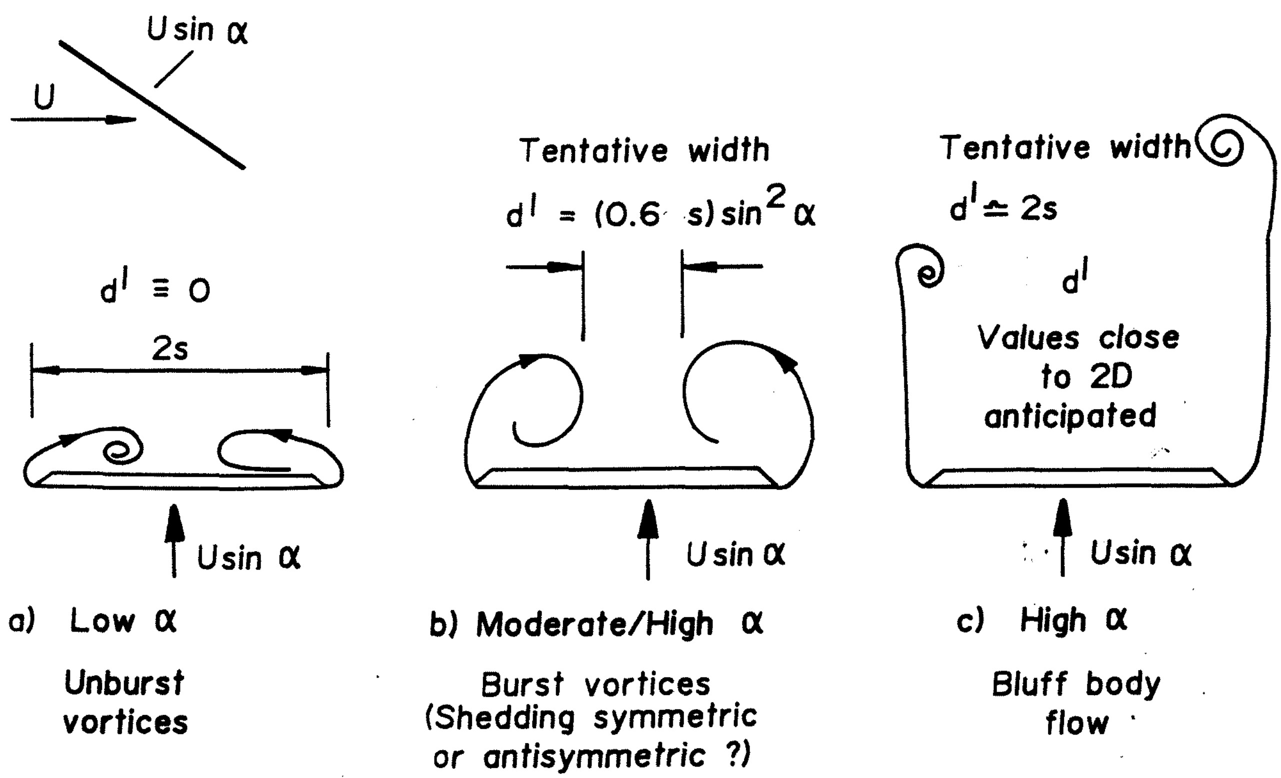

Fig 4 Three-dimensional wings at trailing edge

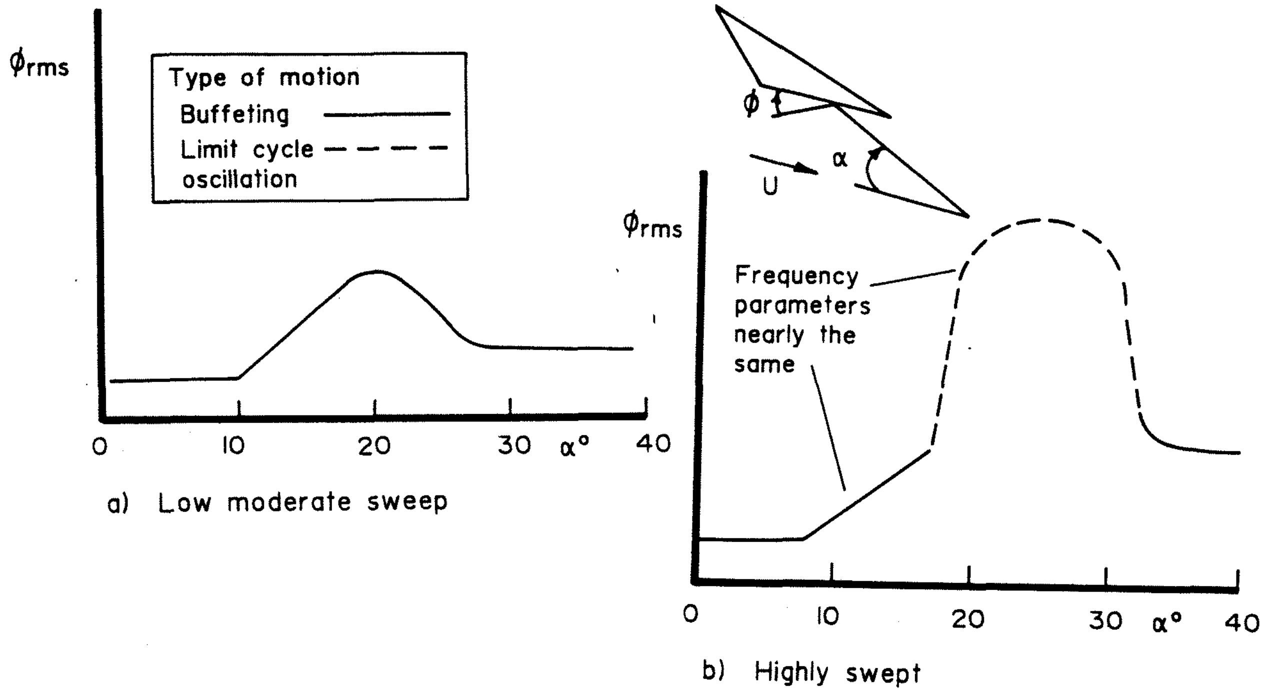

Fig 5 Delta wings with rigid body freedom in roll

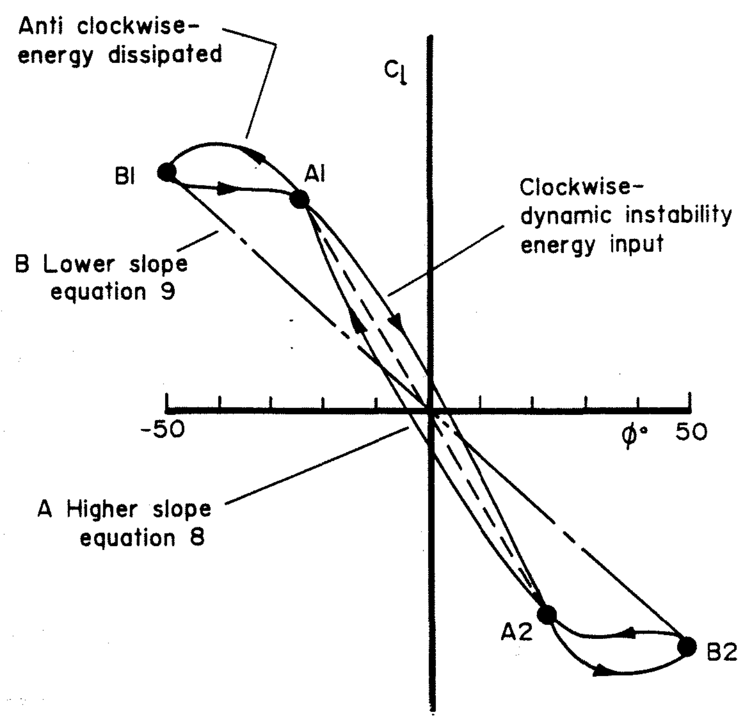

Fig 6 Choice of quasi-steady rolling moment slope for typical LCO

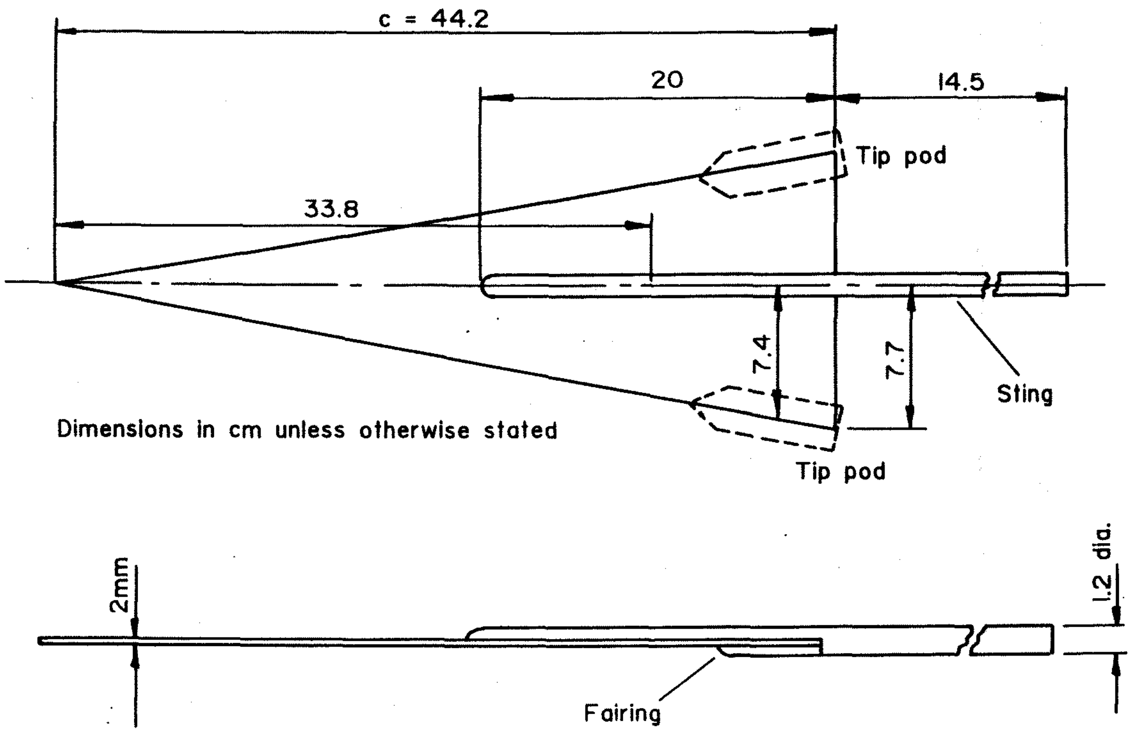

Fig 7 80° Delta Wing Model

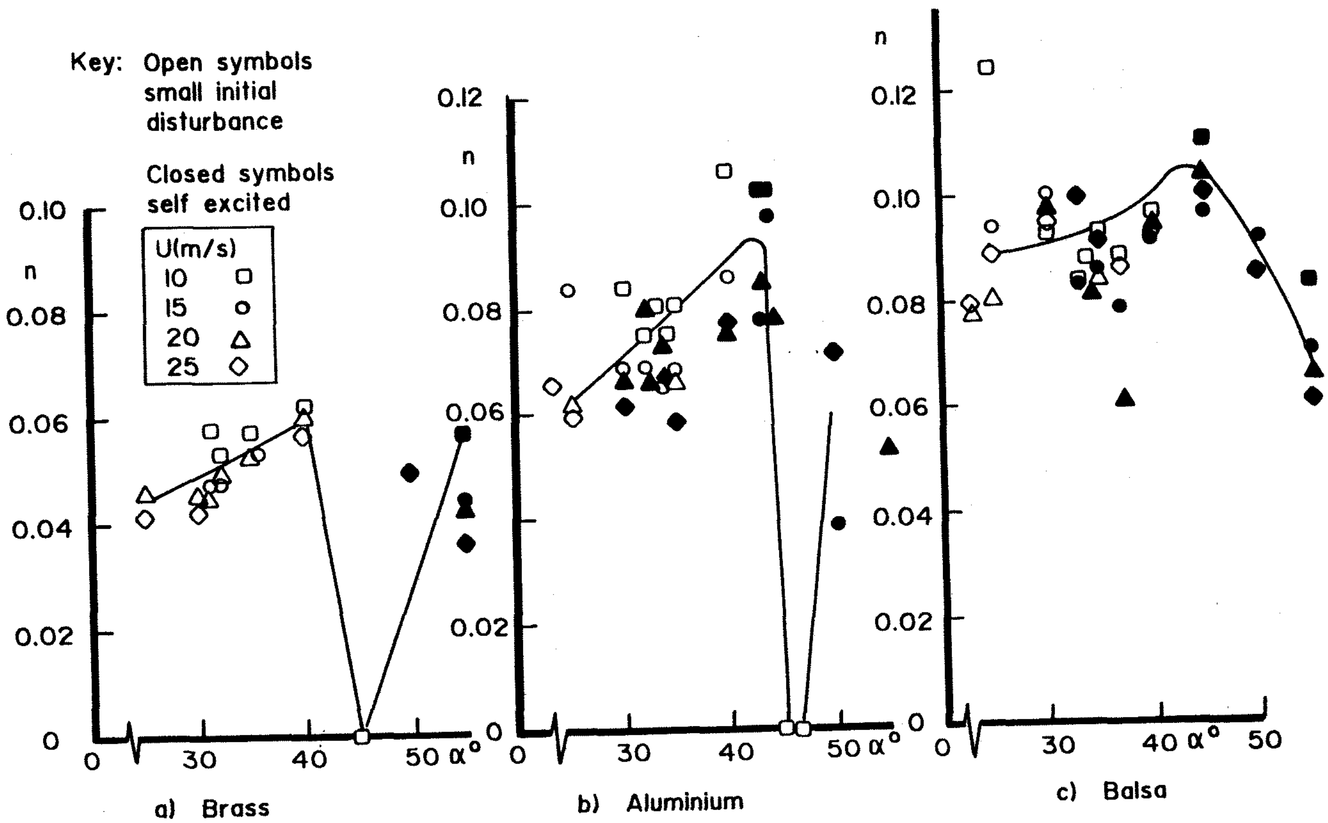

Fig 8 Variation of wing-rock frequency parameter (n) with angle of incidence (α) for three tip tanks of varying inertia

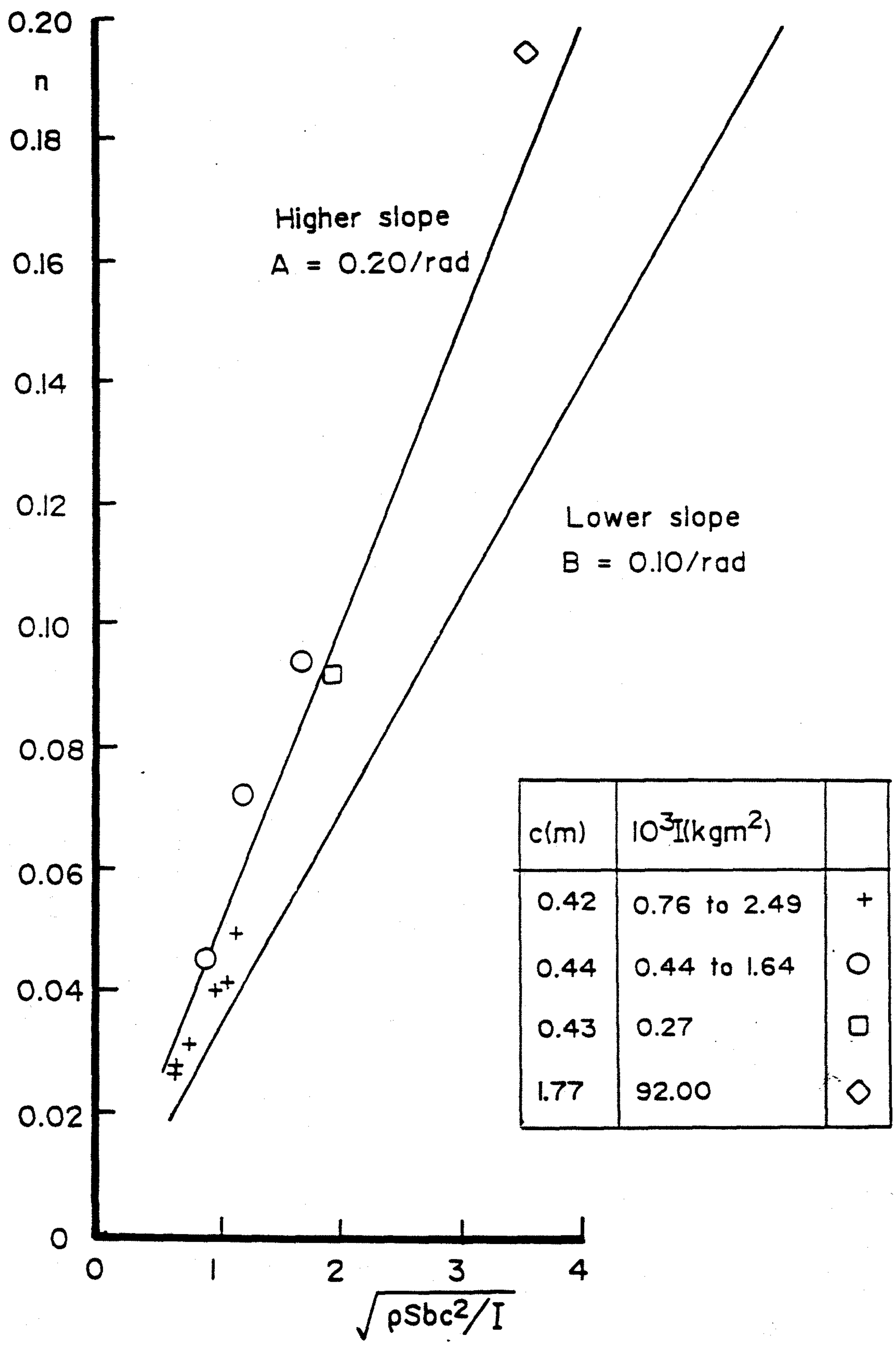

Fig 9 Variations of frequency parameter with interia parameter for 80° delta at $\alpha = 30°$