Geometry Nonlinear Optimal Feedback Tracing In Helicopter Positioning Control

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ABSTRACT

A new geometry nonlinear optimal feedback trace method, which based upon the optimal nonlinear feedback regulation theory, is presented in the paper. The method utilizes the concept of infinite hierarchy optimal control necessary condition (also the sufficient condition to some extent has been proved in this paper) and vector fields on manifold to simplifiedthe solution of the control law. It is shown that the optimal feedback law satisfies a system of first order, quasi–linear, partial defferential equations. Higher precision is obtained in helicopter positioning control. The method can eliminate the adjoint vector from a set of conditions. So it does not need to solve the Riccati equation, which complicated the solution processing. It will save a lot of computer time and obtain higher precision at the same time. It also will be able to deal with the control varible in restrains. Generally speaking, the local solution of the nonlinear functional analysis can be extended to the globle solution when applying to convex function. The significent of the paper's method is to looking for a globle solution in convex function. The detail and the results are presented in the paper.

The optimal tracing, under aerodynamics reaction alone and the steady power engine of a coaxil helicopter, is considered. Higher precision positioning control is obtained by the feedback law control.

1.Introduction

Many progress have been made by geometry nonlinear control in some fields. There have been more and more the examples in engineering. The reason is both the fundation theory forword and many nonlinear control problem need to solve. Reference [1–3] attempted the vehical control by geometry method. Satisfied results have been reported. Reference [4] presented a nonlinear optimal regulator. It does not need to solve the Riccati equation, which complex and lower the solution precision. The concept of infinit hierachy converses the problem into first order, quasi–linear partial differential equation. The optimal control will be obtained by simplicity analytical general algorithm of partial differential equation.

Since the tracing singnals are of time varying that the solution of the optimal tracing will be complicated. In fact, the regulator is the traced signal in a fixed point. The paper's method remains the advantages of optimal regulator without solving the Riccati equation, at the same time considering the time varing characteristic of the problem and the restrains of the control varibles.

For discribing the applied procession, numerical example of a helicopter positioning control appended to the paper. Since the aerodynamic fields of the coaxil helicopter is very complex, so it is a typical multi–axis control problem. The requirement for robust and tracing sentivity is higher than others. As a test for tracing controller, satisfying results has been obtained by the paper's method.

2. General Model

Definiting following Bolza system optimal model:

$$b(u) = \dot{x} - F(x,u) = 0.$$

$$c(u) < 0.$$

$$J = \Phi(x) + \int_0^T F^0(x,u)dt \qquad (1)$$

where $x \in R^n$, $u \in R^m$, $F = \{F^1, F^2, ..., F^n\}$. Expending 1 dimension x^0 to the state vector, then the Bolza form converse to a Mayer one. Definites:

$$\tilde{x} = \{x^0, ..., x^n\},\ x^0(0) = 0,$$

$$\tilde{F} = \{F^0, ..., F^n\}$$

The Hamiltonian as following:

$$H = \sum_{k=0}^{n} p_k F^k \qquad (2)$$

where p is the covector, $p^0 \equiv 1$. Then it is:

$$H = < p\ ,\ \tilde{F} > \qquad (3)$$

3. Infinit hierachy optimal principle

3.1 Necessary condition of the optimal control

Considering optimal control problem, the Hamiltonian as (2), According to the maximum principle, the necessary condition is:

$$\frac{\partial H}{\partial u} = 0. \qquad 0 \leqslant t \leqslant T$$

or it written as

$$H_{u_i} = < p\ ,\ \tilde{F}_{u_i} > = 0.$$

3.2 Necessary condition of Infinit hierachy

Appling the necessary condition of the optimal control in last section to the time varying Hamiltonian. Notes that the optimal orbits is the curve along the manifold, then its necessary condition is:

$$\frac{dH_{u_i}}{dt} = 0.$$

That is to say, the Hamiltonian along the optimal control orbits is constant at any time. Notes that the Hamilton condition is:

$$\dot{x} = \frac{\partial H}{\partial p}$$

$$\dot{p} = -\frac{\partial H}{\partial x}$$

And defining a new vector field operator:

$$A = \frac{\partial}{\partial t} + \tilde{F}$$

it can be induced:

$$\frac{dH_{u_i}}{dt} = < p ,[A,\tilde{F}_{u_i}]> = 0.$$

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$$i = 1,...,m$$

$$\frac{d^k H_{u_i}}{dt^k} = < p ,[A,...[A,\tilde{F}_{u_i}]...]> = 0.$$

Where [.,.] is the general comutator, [...[.,.]..] indicates the multi operating of the comutator.

If the restrain condition satified that Kuhn – Tucker sufficient condition :

$$\nabla J + \sum \lambda_i \nabla c_i + \sum \mu_i \nabla b_i = 0.$$

$$\lambda_i c_i = 0.$$

$$\lambda_i \geqslant 0. \qquad i = 1,...s$$

$$\nabla c_i^T Y < 0. \qquad (\lambda_i = 0.)$$

$$\nabla c_i^T Y = 0. \qquad (\lambda_i > 0.)$$

$$\Delta b_{i\sim}^T Y = 0.$$

If any nonzero vector Y satisfies:

$$Y^T \Delta L Y > 0.$$

Then the above condition is also sufficient.

4.Infinite hierachy optimal tracing control law:

For the limitation of the paper lenth, here we simplity presents an optimal tracing control law, only square case has been considered, (that is the number of state equal to the number of control). See [5] in detail.

A guidance of the frontal discussion is, with the expression of the manifold, vector field and the comutator, a simple expression of optimal hierachy control can be induced. To the 0 and first order vector fields,

$$H_u = < p , \tilde{F}_u > = 0.$$

$$\frac{dH_{u_i}}{dt} = < p ,[A,\tilde{F}_{u_i}]> = 0.$$

we have

$$\begin{bmatrix} \tilde{F}^1_{u_1} & \dots & \tilde{F}^1_{u_n} & [A,\tilde{F}_{u_i}]^1 \\ \dots & \dots & \dots & \dots \\ F^N_{u_1} & \dots & F^N_{u_n} & [A,\tilde{F}_{u_i}]^N \end{bmatrix} = 0.$$

$$i = 1,\dots,m$$

The model as (1), and the corresponding Hamiltonian as (3), the time varying traced signal is $z \in R^n$, then the geometry infinit optimal tracing control law is the solution of following quasi-linear partial differential equation with the given initial condition

$$\{ F^0_{u^2} - F_{u^2} (I \odot F_u^{-T} F^0_u) \}\{ \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} F + \frac{\partial u}{\partial z} G \}$$
$$= F_{ut} F_u^{-T} F^0_u + (F^T F_{ux} - G^T F_{uz} - F_u^T F_x)$$
$$F_u^{-T} F^0_u - (F^T F^0_{ux} - G^T F^0_{uz} - F_u^T F^0_x)$$

where, $x \in R^n$,

$$\dot{z} = G(z,t)$$

$$\dot{x} = F(x,u)$$

Object function is the error of design tracing signal and the real signal, real one is the negetive feedback. $F = \{F^1,\dots\dots, F^n\}$, F^0 defined as in (1). $\odot$ is the Keronrcker product.

5.Example

The model of the coaxil helicopter as following

5.1 Dynamical equation

$$\frac{dU}{dt} = \frac{F_x}{m} + VR - WQ - g sin\theta$$

$$\frac{dV}{dt} = \frac{F_y}{m} + WP - UR - g sin\varphi cos\theta$$

$$\frac{dW}{dt} = \frac{F_z}{m} + UQ - VP - g cos\varphi cos\theta$$

$$I_x \frac{dP}{dt} = L - (I_z - I_y)RQ$$

$$I_y \frac{dQ}{dt} = M - (I_x - I_z)PR$$

$$I_z \frac{dR}{dt} = N - (I_y - I_x)PQ$$

5.2 Motion equation

$$\frac{d\varphi}{dt} = P + Q\sin\varphi\tan\theta + R\cos\varphi\tan\theta$$

$$\frac{d\theta}{dt} = Q\cos\varphi - R\sin\varphi$$

$$\frac{d\psi}{dt} = (Q\sin\varphi + R\cos\varphi)\sec\theta$$

The above equations can be represented by:

$$\dot{x} = f(x) + g(x)u$$

$$y = h(x)$$

5.3 Hovering position optimal tracing control law

Since it is the first order vector field on the manifold, so the only first hierachy is considered. For the difficultes of determining the Cauchy condition, the designing utilize the hierachy optimal results as the traced signal, so

$$\frac{dv}{dt} = -2R^{-1}g(x)^T Q(z(t) - h(x)) \qquad (4)$$

Where R is the control varibles weighting matrix, Q is the state varible's.

It is obviously that there is a similar relation between linear and nonlinear optimal control. That is the correlationship between the two control systems.

According to the decoupling with stabilization algorithm [5], the system control is following the hierachy optimal results, the expected state can be obtained. The stabilization term as following:

$$u = -\sum K y^{(i)} - L\{v - \nabla Wg/2\}$$

where, v is the solution of (4), W is the Lyapunov function of the system, ∇W is the gradient of the system Lyapunov function.

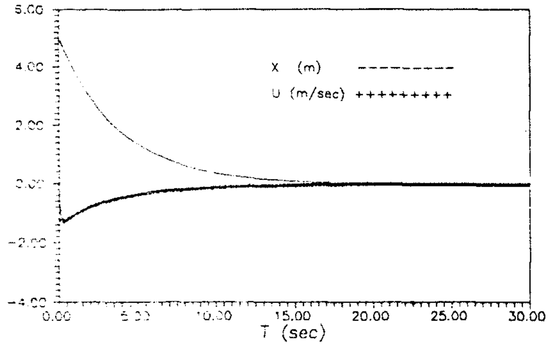
The results of simulation in the appendix.

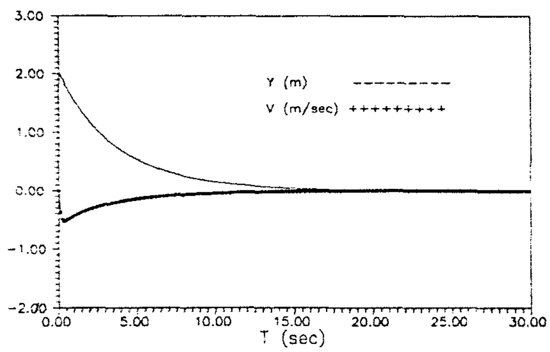
6. Conclusion

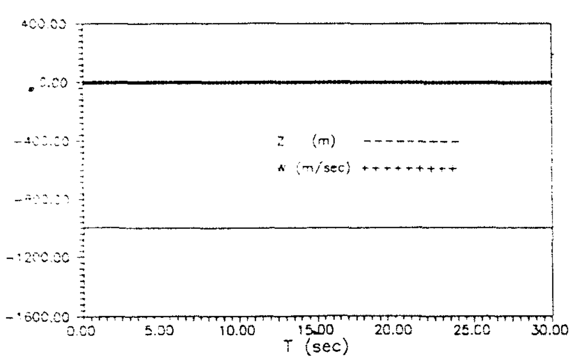
1). General engineering algorithm requires simplity, clear and accuracy. The optimal hierachy method satisfied the above condition. It is a proposal practical method to the complex synthesis control.
2). For the nonsquare case, the necessary condition leads to higher order partial differential equation.
3). The interaction aerodynamic force on coaxil helicopter is very complex, the hierachy control can overcome the difficulty effectively. Higher control precision is obtained.
4). Since the method just begining, a lot of discussion and analysis are required before practice.

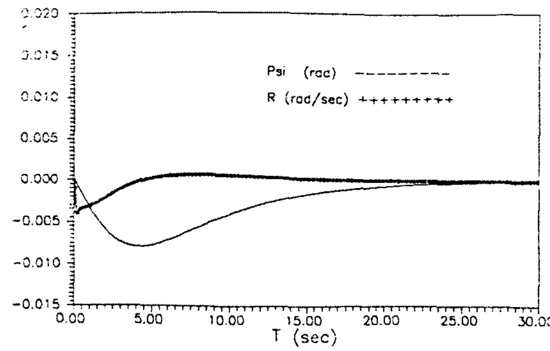
Appendix

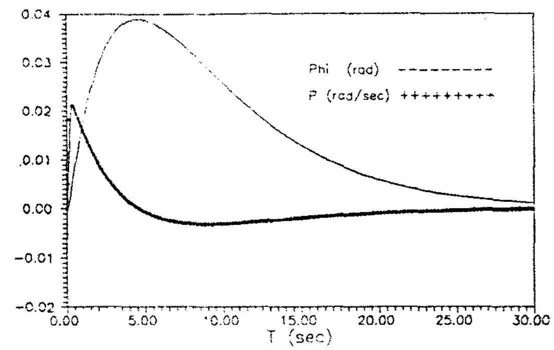
The begining value in positioning flight are : (the earth coordinate) x = 5m, y = 2m, z = −995m, velocities and angular velocities are 0. Expected coordinate are x = y = z = 0m, velocities and angular velocities are 0.

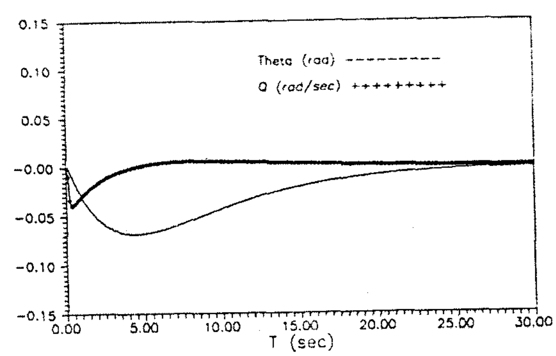


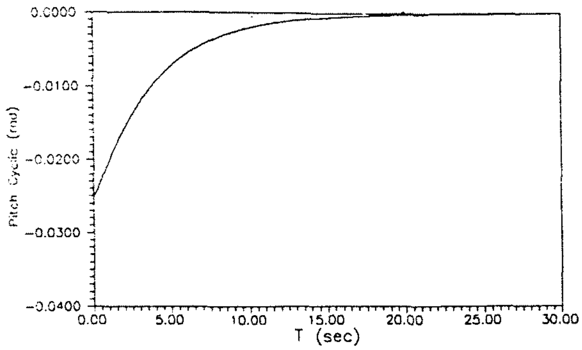


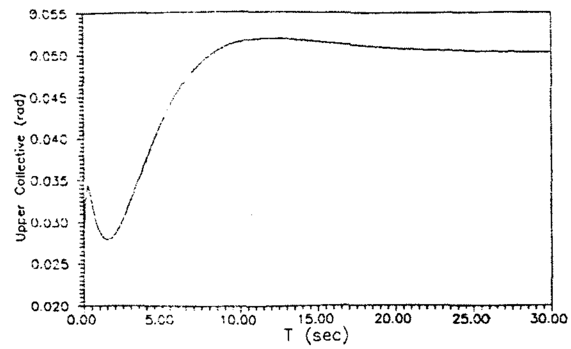


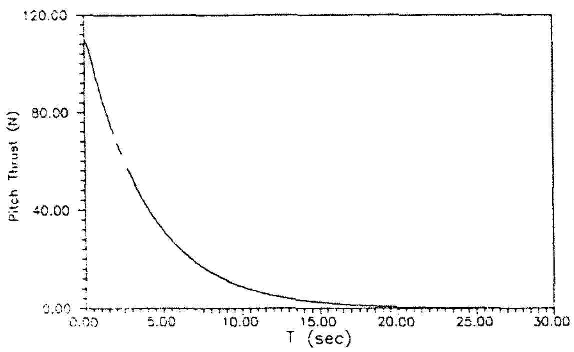


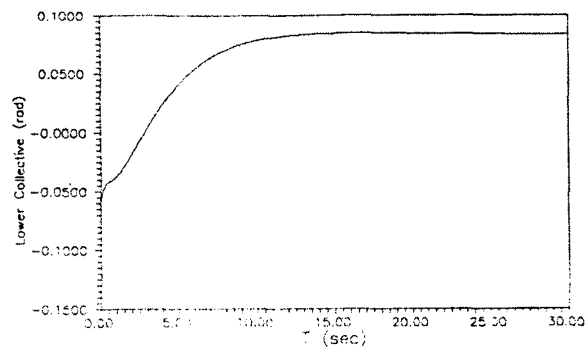





Pitch Cyclic (rad)
0.0000
-0.0100
-0.0200
-0.0300
-0.0400
0.00 5.00 10.00 15.00 20.00 25.00 30.00
T (sec)


Upper Collective (rad)
0.055
0.050
0.045
0.040
0.035
0.030
0.025
0.020
0.00 5.00 10.00 15.00 20.00 25.00 30.00
T (sec)


Pitch Thrust (N)
120.00
80.00
40.00
0.00
0.00 5.00 10.00 15.00 20.00 25.00 30.00
T (sec)


Lower Collective (rad)
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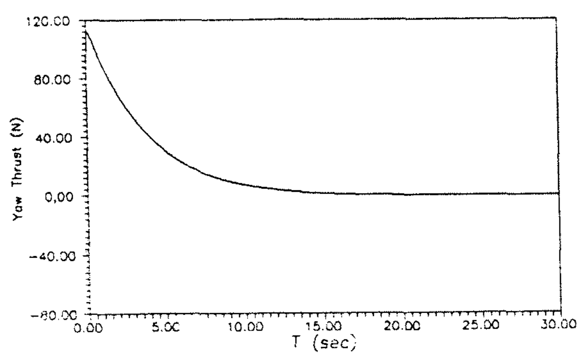
0.0500
-0.0000
-0.0500
-0.1000
-0.1500
15.00 20.00 30.00
(sec)


Yaw Thrust (N)
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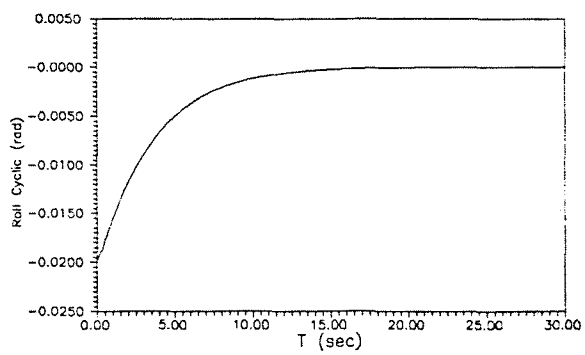
80.00
40.00
0,00
-40.00
-80.00
0.00 5.00 10.00 15.00 20.00 25.00 30.00
T (sec)


Roll Cyclic (rad)
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-0.0000
-0.0050
-0.0100
-0.0150
-0.0200
-0.0250
0.00 5.00 10.00 15.00 20.00 25.00 30.00
T (sec)

Reference

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