PRACTICAL CONTROL LAWS FOR ONBOARD REAL-TIME GUIDANCE OF OPTIMAL AIRCRAFT TRAJECTORIES

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Abstract

The objective of this paper is to develop an admissible real-time guidance program (based on a precomputed optimal trajectory) with elevator deflection as control, to transfer the aircraft from an initial state to a final state in a given time, while maximizing a performance index and respecting practical physical constraints.

The approach presented here assumes the existence of a library of optimal maneuvers generated by optimal control theory calculations performed on the ground and stored onboard. At the starting point of the maneuver the recorded values are updated according to maneuver to be performed. In order to construct the control history that brings the aircraft to the vicinity of optimal trajectory a feedback guidance law is proposed. The results are compared with optimal solutions obtained for the reduce dynamic model. It is shown that a better than 98% accuracy is achieved for the performance index.

Numerical examples are given to show the effectiveness of method for various maneuvers (zoom, dive and half-loop-ascent phase), with different time domain and performance index, to prove the effectiveness of the proposed control laws.

Nomenclature

C_D = drag coefficient
$C_{D_{\dot{\alpha},q,\delta_e}}$ = drag coefficient derivatives
C_{D_0} = zero-lift drag coefficient
C_L = lift coefficient
$C_{L_{\dot{\alpha},q,\delta_e}}$ = lift coefficient derivatives
C_m = pitching moment coefficient
$C_{m_{\dot{\alpha},q,\delta_e}}$ = pitching moment derivatives
$\bar{c}$ = mean aerodynamic chord
D = drag force
g = acceleration of gravity
h = altitude
I_y = Y body axis moment of inertia
K = induced drag coefficient
L = lift force
M = Mach number
M_y = aerodynamic pitching moment
M_T = pitching moment due to thrust
m = mass
n = normal load factor $(=L/W)$
q = pitch rate
S = aerodynamic reference area
T = thrust
t = time
v = velocity (true airspeed)
W = weight
x = downrange position
α = angle of attack
γ = flight-path angle
δ_e = elevator deflection
δ_T = throttle setting
ρ = density of atmosphere
$\dot{a}$ = da/dt (a arbitrary)

Subscripts

$(\ )_0$ = values at initial time
$(\ )_f$ = values at final time
$(\ )_{max}$ = maximum value
$(\ )_{min}$ = minimum value

Superscripts

$(\ )^r$ = reference values
$(\ )^0$ = prescribed values at initial time

Introduction

Guidance of aircraft involves the use of a variety of control laws that, in one way or another, uses measured state information to produce commands such that the motion of the guided vehicle to be changed.

In this paper an easily implementable feedback law for aerodynamic control of symmetric flight is sought. A practical approach has been used to obtain simplified, approximated closed-loop, nonlinear near-optimal guidance law applicable to a large class of optimal symmetric maneuvers performed in the vertical plane. Solutions are derived to satisfy the need for a real-time guidance law that can be easily implemented online within the computational capabilities of current microprocessor-based onboard systems.

The problem under consideration may be enclosed in the class of inverse simulation problems. Essentially, an inverse simulation problem seeks for the determination of the control inputs to a dynamic system that produces desired system outputs[1-3]. In Ref.2 a criterion proposed to classify the problems belonging to this field is the number of outputs (N_o) compared with the number of inputs (N_i). For nominal problems $N_o=N_i$. When $N_o<N_i$ the problem is called redundant. Regarding the problems in which $N_o>N_i$ Ref.2 outlines that "there is a little promise of developing a general solution technique". The problem addressed here belongs to this last class. The control laws advanced in this paper have not as main goal to restrict the vehicle to a multivariable reference trajectory defined by some of the state variables, but to achieve a global performance during the maneuver to be performed. To be more specific, we are looking for practical control laws capable to guide the vehicle to reach an extreme value of a certain performance index. Loosely speaking, the achievement of reference path becomes not a purpose but a means to realize the invoked goal.

The objective of this paper is to develop an admissible real-time guidance program (based on a precomputed optimal trajectory) with elevator deflection as control, to transfer the aircraft from an initial state to a final state in a given time, while maximizing a performance index and respecting practical physical constraints.

The approach presented here assumes the existence of a library of optimal maneuvers generated by optimal control theory calculations performed on the ground and stored onboard. At the starting point of the maneuver the recorded values are updated according to maneuver to be performed. The number of optimal trajectories required to form a typical optimal control library depends on the range of values of interest and the accuracy required. The solution for each optimal maneuver is an element of the library, which should be supplied by various applied computational methods. One of these is described in Ref.4. The reduced order optimal solutions supplied by the invoked work become nominal (reference) trajectories in the present one where the lift coefficient control must be removed by elevator deflection control. The implementation of the latter one leads to the arising of overshoot phenomena.

In this paper, the trajectories to be tracked, belong to an optimal class of symmetric maneuvers in vertical plane (defined above). In this case many practical situations require not a strict following but to reach a global performance. Traditionally, a reference trajectory in vertical plane is defined by altitude, velocity, flight-path angle and horizontal position. The effort for tracking all this variable is unjustified for many practical situations.

The inverse dynamic problems and techniques address problems in which the desired outputs are in fact assigned laws for the state variables. In many practical situations the imposed output are other flight related quantities, such as the load factor, or more general, a global performance expressed as an integral performance index. For example a zooming maneuver is performed mainly to achieve a maximum change of altitude over a fixed-time domain and not to track a reference flight profile (altitude vs. down-range position). A way to achieve real-time control laws is to identify common features of sets of maneuvers and to put them in wide classes. Once a class is specified, the particular form of the guidance law is in a closed connection with it. The main features of trajectory belonging to the class can be exploited to design an appropriate analytical form for the guidance law. One section of this study is dedicated to exemplify the ideas above. An analytical law is presented to generate near-optimal trajectories over fixed -time domains. The performance of the proposed control law is shown via simulation of some typical symmetric maneuvers.

In order to investigate the control law performance, the same controller gain is successfully applied to different reference trajectories. How the controller is capable of tolerating changing vehicle configuration parameters or start conditions dispersions, as well as how the correction term of feedback control function performs on reference trajectories that are different from the design trajectory are investigated by numerical simulations.

The computations were carried out on the IBM PC-AT using Fortran and IMSLLIB software. Numerical examples are given to show the effectiveness of method for various maneuvers (zoom, dive and half-loop- ascent phase), with different time domain and performance index to prove the effectiveness of the proposed control laws.

Design Goals

The purpose of this study is to devise approximate, analytical, nonlinear, near-optimal control laws (in terms of elevator deflection with respect to state) that can be implemented onboard to provide real-time guidance for an aircraft pursuing fixed-time optimal symmetric trajectories. In particular, the aim is to reach (with zero path inclination and admissible velocity) a minimum final altitude value in descent symmetric maneuvers or maximum final altitude in ascent symmetric maneuvers, at a prescribed final time.

For all the trajectories belonging to the invoked class the initial state is completely specified. From the final state only the flight-path angle is specified to define the maneuver type (according to Table 1). The solution is quite difficult to obtain if a complete dynamic system is considered. The equations can be simplified by using singular perturbations as in Ref.5 or they can be simplified by using classical approach of neglecting higher frequency dynamics.

Considering reduced dynamic model (RDM) optimal solution obtained and recorded for a given initial state, time domain and performance index, the goal is to design a near-optimal feedback guidance law with δ_e as control and full state equations to describe the aircraft symmetric motion. Therefore, the inputs are: **1)** the reference values for state and control variables recorded as function of time (or altitude) along the optimal trajectory on the RDM, **2)** a full state equations for symmetric flight and **3)** a practical physical constraint system.

On these input conditions we have to construct the δ_e history to achieve the same value of performance index as for RDM, respecting the constraints for state and control variables. This must be done such that aircraft follows optimal trajectory in a reasonable manner (that means: reasonable overshoots for load factor and angle of attack, limited dumping behavior at the ends of the maneuver).

It must be noted that in this particular case of class of maneuvers addressed in this paper thrust history is permanently at its upper bound (δ_T=$(\delta_T)_{max}$ along the whole maneuver for any initial state, time domain or performance index). This is one conclusion of Ref.4, and allows us to avoid the problem of considering it here.

A block diagram schematic of the simulated control system is shown in Fig.1. The abbreviation OCP is used for Optimal Control Problem. The block that builds the control law receives feedback of the state errors and state reference values. Using them the controller computes the necessary changes in δ_e^r. A saturation block limits the correction of δ_e ($\Delta\ \delta_e$-in Fig.1), to an admissible interval. The response of the system is integrated numerically in the time domain using a standard fourth-order Runge-Kutta algorithm. Neither dynamic sensor models nor actuator dynamics were used in this study.

Concluding, we are looking for a correction term ($\Delta\delta_e$) expressed in terms of actual and reference values of some of the state variables. The idea is to pick out (for tracking) from the available reference state variables those which are responsible for the achievement of integral performance index, while respecting the imposed final conditions. For the problem addressed in this study we show that normal acceleration is the only one state variable employed to construct the correction term.

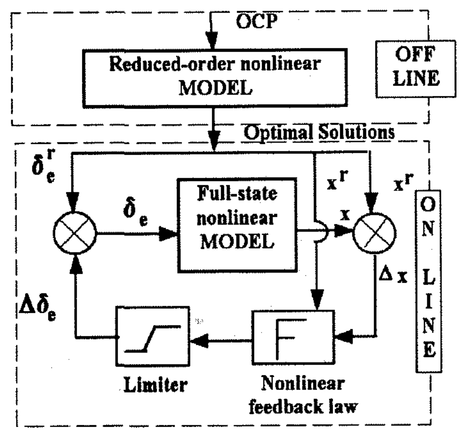

Fig.1.Block diagram schematic of the simulated feedback system

Maneuvers

The maneuvers are defined by specifying initial and final conditions, duration and performance index. All the maneuvers described in this paper are initiated with the aircraft in straight and level flight with normal attitude (zero flight-path-angle) or inverted attitude (flight-path angle = 180 deg.). These values are also valid for final conditions (see Table 1). The choice of this particular values is made for practical reasons and does not reduce the generality of the problem.

Dynamic Models

General modelling assumptions adopted for both of the dynamic models are presented in Appendix A. Here are pointed out the particularities of each.

Reduced-Order Model

The RDM for aircraft symmetric flight incorporates the assumption of thrust-along-the-path (angle between velocity and thrust is zero). It is also assumed that the angle-of-attack is sufficiently small so that the component of thrust normal to the flight-path angle is much smaller than the lift. The controls (C_L and δ_T) are allowed to vary instantaneously in an imposed admissible range.

Table 1. Particular values to define the type of maneuver

$(\gamma)_0$ [°]	$(\gamma)_f$ [°]	$(h)_f$	Maneuver
0.	0.	max.	zoom
0.	0.	min.	dive
0.	180.	max./min.	half-loop (climbing phase)
180.	0.	max./min.	half-loop (descent phase)
0.	-180.	max./min.	inverted half-loop (descent phase)
-180.	0.	max./min.	inverted half-loop (climbing phase)

The RDM for aircraft symmetric flight incorporating the assumptions pointed out before is given by:

$$\dot{v}=g\left(\frac{T-D}{W}-\sin\gamma\right)$$
$$\dot{\gamma}=\frac{g}{v}\left(\frac{L}{W}-\cos\gamma\right) \quad (1)$$
$$\dot{h}=v\ \sin\gamma$$
$$\dot{x}=v\ \cos\gamma$$

where:

$L=\frac{\rho}{2}Sv^2C_L,\ D=\frac{\rho}{2}Sv^2C_D,\ T=\tau(M,h)\delta_T$

$\rho=\rho(h)$ (according to standard atmosphere).

Control Variables Constraints

The two control variables are constrained by physical considerations. The lift coefficient is bounded above and below by both the stall and structural limits

$$C_L^{(-)}\leq C_L\leq C_L^{(+)} \quad (2)$$

where:

$$C_L^{(-)}=\max\left(C_{L_{\min}}(M),\frac{n_{\min}(h)W}{(\rho/2)Sv^2}\right)$$
$$C_L^{(+)}=\min\left(C_{L_{\max}}(M),\frac{n_{\max}(h)W}{(\rho/2)Sv^2}\right) \quad (3)$$

The Mach dependent extreme values of C_L refer the warning limits for stall in terms of Operational Flight Envelope (OFE), according to MIL-F-8785B (ASG) (that means, lift coefficient values for 90% of difference between α_{stall} and α for zero-lift). The altitude dependent extreme values of load factor refer, also, the extreme values reached within the OFE. These constraints are imposed to provide more feasibility to the solution of OCP of RDM because along optimal trajectories the control history has usually segments on which it achieves its bounds (and permissible bounds values to be achieved for an unlimited time interval are those of OFE).

For throttle setting, also, an inequality constraint is considered:

$$\delta_{T_{\min}}\leq\delta_T\leq\delta_{T_{\max}} \quad (4)$$

Full-State Nonlinear Model

With no time delays for control system the aircraft equations of motion in vertical plane can be written:

$$\dot{v}=g\left(\frac{T\cos\alpha-D}{W}-\sin\gamma\right)$$
$$\dot{\gamma}=\frac{g}{v}\left(\frac{L+T\sin\alpha}{W}-\cos\gamma\right)$$
$$\dot{q}=\frac{1}{I_y}(M_y+M_T) \quad (14)$$
$$\dot{\alpha}=q-\dot{\gamma}$$
$$\dot{h}=v\ \sin\gamma$$
$$\dot{x}=v\ \cos\gamma$$

where L, D, T, have the same meaning as for RDM, and:

$$M_y=\frac{\rho}{2}S\bar{c}v^2C_m$$
$$C_m=C_{m_0}(\alpha,M)+C_{m_q}q+C_{m_{\dot{\alpha}}}\dot{\alpha}+C_{m_{\delta_e}}\delta_e$$

C_L that is control variable for the RDM here becomes function of state and control variables. Its expression is:

$$C_L=C_{L_0}(\alpha,M)+C_{L_q}q+C_{L_{\dot{\alpha}}}\dot{\alpha}+C_{L_{\delta_e}}\delta_e$$

Control Variable Constraints

The inequality constraints on control variables are taken into account to assuring the feasibility of their action.

$$\delta_{e_{\min}} \le \delta_e \le \delta_{e_{\max}} \tag{6}$$

$$\delta_{T_{\min}} \le \delta_T \le \delta_{T_{\max}} \tag{7}$$

State Variable Constraints

To restrict the flight within the limits of Permissible Flight Envelope (PFE) the following state variable constraints are considered:

$$\alpha_{stall}^{(-)} \le \alpha \le \alpha_{stall}^{(+)} \tag{8}$$

$$n_{\min} \le n \le n_{\max} \tag{9}$$

The stall limits are Mach dependent functions. The load factor limits are altitude dependent functions, too. The values accepted for practical situations must be within the PFE.

Near-Optimal Laws

General Requirements

A reliable control system must be able to restrict the airplane to a flight path inside the structural and stall limits subjected to unexpected parameter variations. That means, the control system must be able to respond reasonably to off-nominal starting conditions and must be tolerant to aerodynamic uncertainties and atmospheric disturbances. Moreover, the control system must have modest computational requirements that are within the capabilities of an onboard computer (regarding amount of data to be stored and calculation time).

The main goal of this section is to propose a near-optimal control law in terms of elevator deflection with respect to state.

Near-optimal guidance law is obtained by tracking an optimal trajectory, that is an optimal solution of reduced model of the vehicle dynamics.

In order to construct the δ_e -history that brings the aircraft to the vicinity of optimal trajectory a feedback guidance law is proposed:

$$\delta_e = \delta_e^r + \Delta\delta_e \tag{10}$$

where δ_e^r is obtained as a solution of the algebraic system (11):

$$\begin{cases} \dot{q} = \dfrac{1}{I_y}(M_y + M_T) = 0 \\ \dot{\alpha} = q - \dot{\gamma} = 0 \qquad \rightarrow \alpha^r, q^r, \delta_e^r \\ C_L^r = C_{L_0}(\alpha, M) + C_{L_q} q + C_{L_{\delta_e}} \delta_e \end{cases} \tag{11}$$

The nonlinear algebraic system (11) is solved by Newton scheme to obtain the δ_e^r vs. time (or altitude) schedule. This operation can be done off-line, as is suggested in Fig.1. In (11) C_L^r is the optimal control recorded as time (or altitude) function from the reference (optimal) trajectory.It is obviously that the RDM (1) was obtained from the full-state system (5) by neglecting the α and q dynamics, that means, setting to zero their derivatives. The solution of the algebraic system (11) may be interpreted as quasi-steady-state solution. A guidance law based only on this values will generate a non acceptable behavior at the trajectory ends. This remains true even for the case when no aerodynamic or atmospheric disturbances are considered.

The main cause responsible for this behavior is the large discrepancy existing between the reference (optimal) initial value of normal acceleration (its maximum/ minimum value) and the prescribed initial conditions ($n^0 = \pm 1g$). That because, out of the performed maneuver the aircraft must have a straight-line course. Inspecting the optimal solutions obtained for the class of symmetric maneuvers addressed here, common behavior at the ends of them appears.

Different nature of the certain control program tasks near the ends of the interval and over the rest of the interval is obviously. The requirements to satisfy the initial and final conditions dominate at the ends of the interval, while the maximization of absolute value of altitude change is dominant over the rest of the interval.

By contrast with the original variable n, starting at initial time from a prescribed value ($n^0 = \pm 1g$), the reference value n^r is not free to start from n^0. In this particular class of optimal maneuvers the discrepancy between n^r and n^0 is large. The best one can expect is that during an initial time interval n approaches n^r and then, during the rest of the interval remains close to n^r. If that happens, the value achieve for performance index will be near the optimal one. The question is if it is possible to construct an analytical and nonlinear form for $\Delta\delta_e$ in order to achieve the goal expressed above. Let us consider the following form:

$$\Delta\delta_e = \mathrm{arctg}[-K_n n^r (n^r - n)] \tag{12}$$

Measuring the normal acceleration errors, the controller continuously adjusts the δ_e to correct the actual trajectory and to match the reference. The analytical form chosen for the correction term produces a qualitative desired behavior. To achieve the admissible range of control input a constant factor K_n is employed. In a separate section it is discussed the way of choosing the value for K_n. The guidance law tracks a stored reference of the load factor. In flight the error of normal acceleration and its reference value are used to construct the guidance law in order to eliminate tracking errors. From the following section one can see how efficient is the proposed law.

The reference trajectory is selected accordingly to started conditions, imposed final time, and type of maneuver to be performed. In fact, selecting a reference trajectory it means selecting a load factor vs. time (or altitude) schedule and the appropriate value for K_n, stored onboard. Thus, measuring only the load factor becomes sufficient to guide the airplane to the desired altitude and flight-path angle during an imposed time interval.

The discretisation interval (sampling interval) is assumed to be dictated by the desired accuracy (accepted error) in the phase of linear interpolating done onboard. The simulation provides its own time base, using a fourth-order Runge-Kutta integrator on Eqs(5). The force coefficients and their derivatives may be updated every moment or every Δt (0.1s).

An Optimal Value for K_n

To select an optimal value for K_n, an optimization problem can be stated as follows: for given initial state, final time and maneuver to be performed, find a constant value for K_n in (12) which maximizes the absolute value of altitude change, while simultaneously respecting the state and control variable constraints.

The performance index value achieved by simulation on full-state nonlinear system is compared with value on the reference trajectory. K_n is then modified to correct the actual value. This task is done on the ground (off-line). Feedback gains are scheduled with initial dynamic pressure, mission type and time domain to establish a full envelope control law. Starting points are selected at various altitudes and Mach numbers to capture a board range of dynamic pressures.

Strictly speaking theoretically, the value find out by solving the optimization problem described above is valid only for the particular reference trajectory addressed. However, for particular applications, an average value for K_n, that satisfy a large range of missions is of certain interest. The following section shows the existence of such values.

Owing to the analytical form of the resulting feedback control law, the guidance program is suitable for implementation in an airborne digital computer.

Simulation Results

Simulation results presented in this section are chosen in order to affirm the idea of efficiency and generality of the proposed law. The control law was implemented in Fortran. The input data describing the aircraft for nonlinear simulation is given in Appendix B of Ref.4. The starting points are chosen over a large range of dynamic pressure and final time values. Also, the type of maneuver performed are very different.

Figure 2 contains the simulation results for a zooming maneuver. The maneuver is defined by following initial conditions: straight-level symmetric flight (n^0=1g, γ^0=0) at h^0=2,000m. and M^0=0.6. For the final conditions the only one imposed is γ=0. The imposed final time is 10 sec. The performance index is maximum value of final altitude. In all cases the solid line corresponds to the nonfeedback law, the dashed line to the feedback law. The markers are for values of reference (optimal) trajectory for the sample time of 1 sec.

Fig. 3 contains the simulation results for a zooming maneuver starting at h^0=7,000m. The other input data are the same as in Fig. 2. This value for initial altitude is chosen such that the dynamic pressure become sufficiently low. As it has been pointed out in Ref.4, the dangerous overshoots will be produced for normal acceleration along maneuvers performed at high dynamic pressure and for angle-of-attack along maneuvers performed at low dynamic pressure. This is the reason that suggests the idea to employ an angle-of-attack command system at low dynamic pressure and a normal acceleration at high dynamic pressure. Simulation results show that if the value of load factor is sufficiently far from 1g the proposed law shows good behavior concerning the nonviolation of stall constraints.

Fig. 4 shows the simulation results obtained for a diving maneuver with the same starting conditions, and final time and final flight-path angle as zooming. The performance index in this case is the minimization of the final altitude. The same feedback law with the same value for K was used. The results for feedback control law show a good behavior.

Fig. 5 shows the simulation results obtained for ascent half-loop maneuver with the same starting conditions and final time as zooming and diving maneuvers presented above. The final flight-path angle is 180 deg. (accordingly with Table 1) and the performance index is to maximize the final altitude. The employed feedback law is the same. The imposed final time is 15 sec.

The integration step of the equations of motion was equal to 0.05 sec. The CPU time required was about 50% of maneuver duration. This value assures the requirement of real time computation. To address the problem of control law sensitivity each parameter was perturbed and appropriate performance index was computed. The parameters are the force (moment) coefficients and their derivatives and the initial conditions.

The results of the simulation study can be

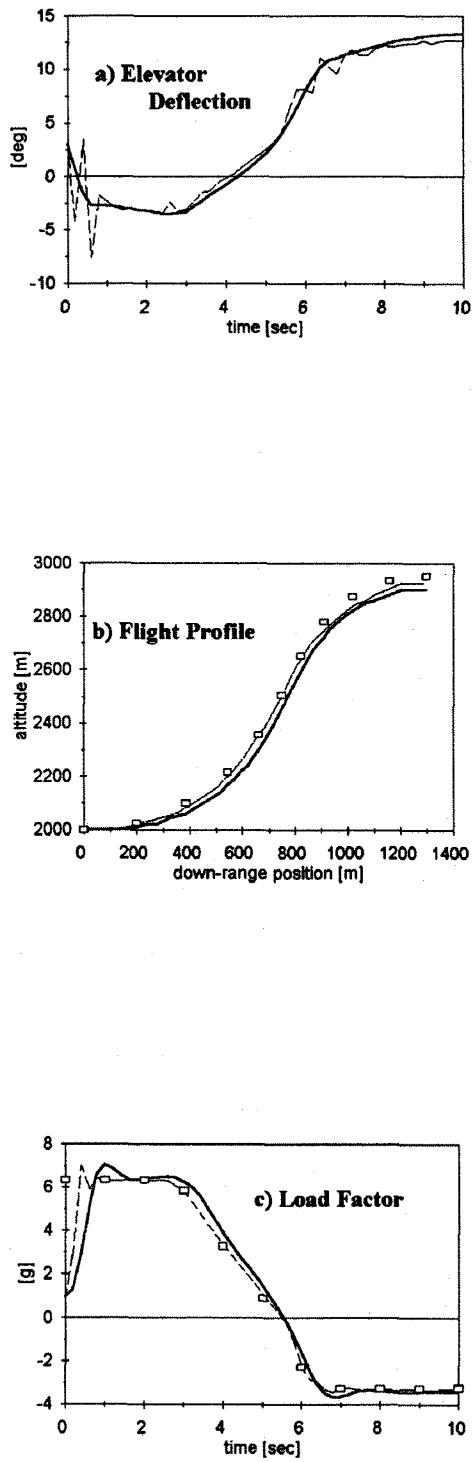

Fig.2 Simulation results for optimal zooming maneuver performed at high dynamic pressure

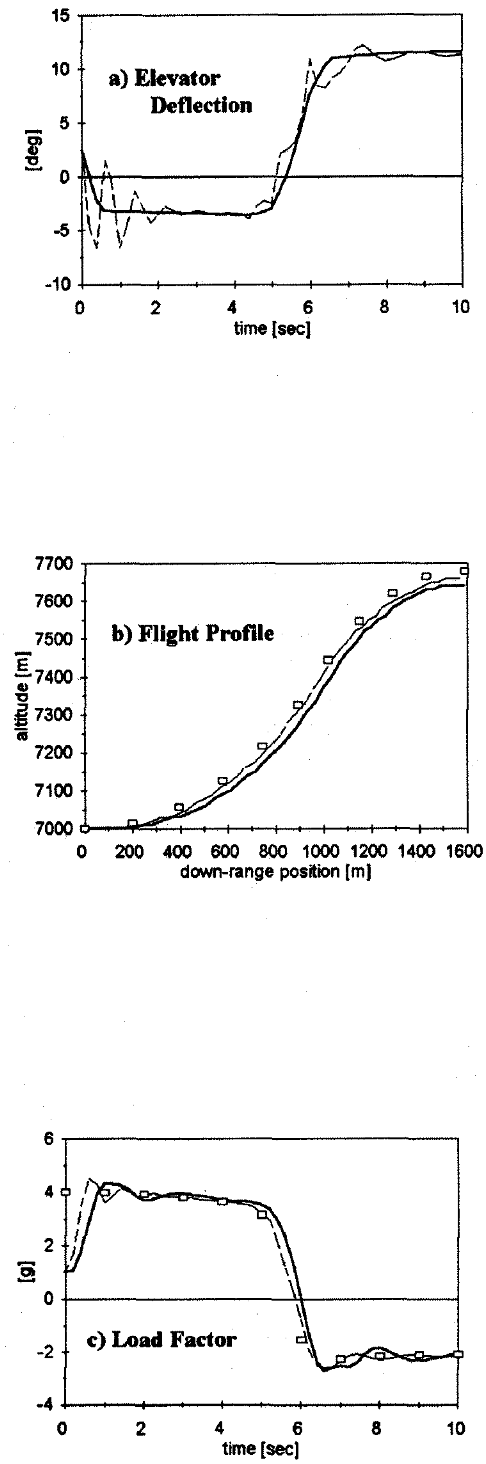

Fig.3 Simulation results for optimal zooming maneuver performed at low dynamic pressure

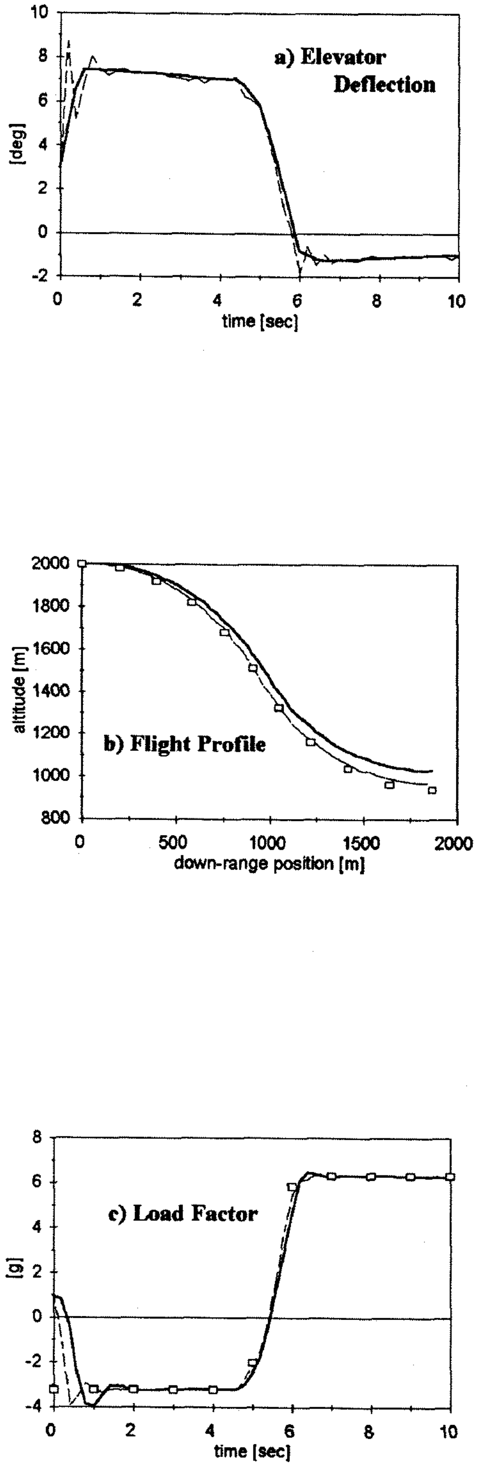

Fig.4 Simulation results for optimal diving maneuver

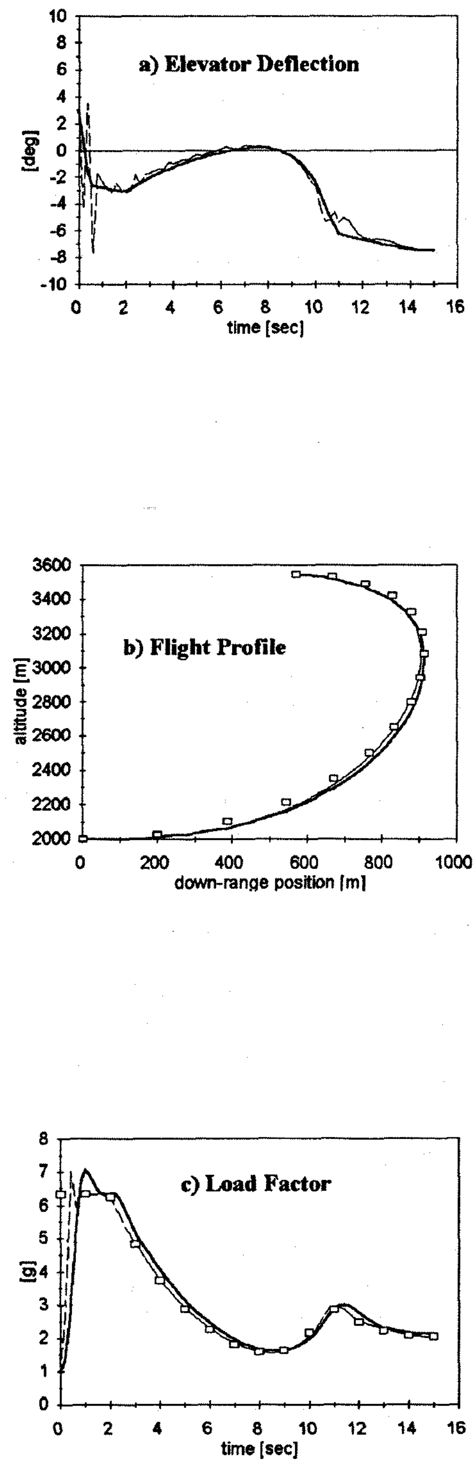

Fig.5 Simulation results for optimal half-loop-ascent phase maneuver

summarized as follows:

1) The control law was not sensitive to variations in aircraft parameters

2) Initial condition variations did not appreciablely degrade the control law performance

3) K_n remains constant along an optimal maneuver

4) Can be found values for K_n parameter to satisfy a broad class of optimal maneuver (all simulation results show in Figs. 2-5 are performed with the same value for K_n)

5) The imposed final time did not influence the control law (Fig.5 in comparison with Figs 2-4)

6) Good accuracy is achieved for performance index

Conclusions

In this study, we have attempted to offer a practical control law that will force the full-state nonlinear dynamic system for symmetric flight, to follow a desired trajectory with the fidelity suitable for engineering analysis. The proposed law allow us to meet the objective stated in the introductory section. The results obtained here are easy to follow and readily applicable to practical control problems concerning with optimal aircraft maneuvers in vertical plane.

The proposed approach has the following advantages: the onboard numerical manipulations and storage requirements are modest; real-time computational can be achieved; the results obtained are nearly the same as optimum solutions; extension to other performance indices and other forms of terminal constraints should be possible; the control laws have a general character in the sense that their form remains invariant for any conventional aircraft.

The results in this paper show that the proposed control laws respond efficiently to several parameter changes applied simultaneously. The simulation results show that the performance of controller is also satisfactory on reference trajectories that are different from the design trajectory.

Very little can be said about the stability of the overall nonlinear systems, therefore it appears necessary to conduct a stability analysis. Further work is needed to ascertain the usefulness of the propose control laws.

Other interesting way to expand the range of validity is to include delays in the control system.

Appendix A

Aerodynamics

The aerodynamic forces and moments were obtained through interpolation of a complete set of aerodynamic coefficient and derivative tables for a subsonic training and ground attack aircraft. The data base provided for α ranging between -14 and +14 deg., Mach number ranging between 0.2 and 0.85 and altitude to 14, 000 m.

The upper and lower bounds on lift coefficient are Mach-altitude dependent. The drag coefficient is modelled as a parabolic function of the control C_L:

$$C_D = C_{D_0}(M) + K(M) C_L^2$$

where C_{D_0} and K are supplied in tabular form and are interpolated as cubic-spline functions of Mach number.

Propulsion

The aircraft thrust is assumed to be given as a tabular function of Mach number and altitude for every value of throttle setting. A linear variation in throttle is assumed between $\delta_T=0$ (zero-thrust setting) and $\delta_T=1$ (maximum-thrust setting).

Atmosphere

Atmospheric properties are taken according to standard atmosphere and are supplied in tabular form as functions of altitude. The sonic velocity and the natural logarithm of the air density are interpolated as cubic-spline functions of altitude. The gravitational acceleration is assumed to be constant over the altitude range covered during maneuver to be performed.

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