

LATERAL MOTION OF AIRCRAFT ON THE RUNWAY

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Abstract

Soon, the Hungarian Army should change its park of military aircraft used till today. The infrastructure conditions do not allow the operation of all types of fighters. Lack of financial funds does not give opportunity for the quick development of airfields in future. This is the reason, why motion of the aircraft on the runway should be investigated.

The author of the lecture has developed a method for analyzing the characteristics of lateral motion and stability of aircraft on the runway during takeoff and landing for evaluation purposes. With the application of this method the investigated aircraft can be graded on the basis of the effects of bad runway surface conditions, rolling friction of wheels, cross winds, etc.

Forces and moments during takeoff and landing

Straight movement of aircraft on three wheels can be described with the system of equations well-known from flight mechanics.

Let us suppose: during takeoff and landing, as a result of lateral disturbance the aircraft turns to the left which causes a slip angle of  $(-\beta)$ . For this reason, forces and moments appear on the aircraft in the horizontal plane, (Fig.1.) Forces can be divided into two groups:

-sideways, frictional  $F_z$  forces.

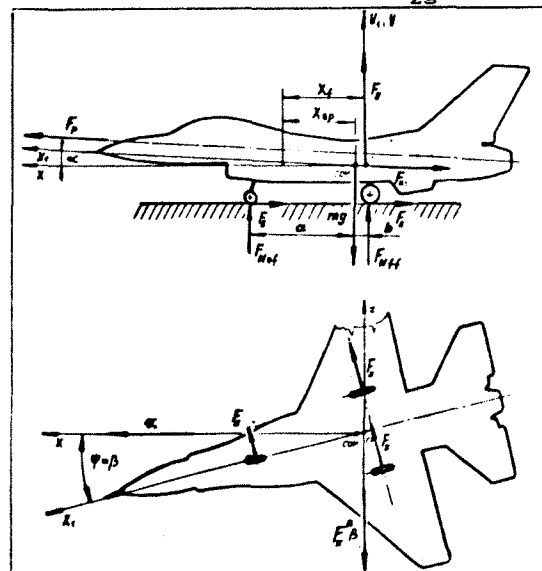


Figure 1.

Until the sideways aerodynamics  $F$  force is smaller, than the sideways frictional  $F_z$  force, the aircraft does not deflect its path, the forces neutralize each other.

Appearance of the angle of yaw  $(\Psi)$ , leads to sideways forces on the wheels, which affect perpendicular to the plane of rotation of the wheels,  $(\psi)$  (Fig.2.).

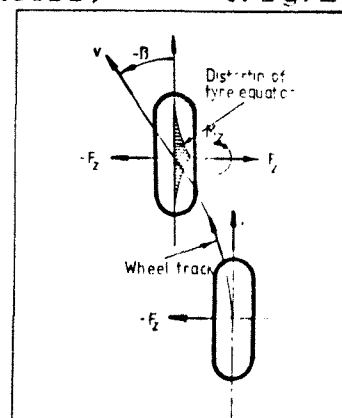


Figure 2.

$C_{zk}$  the coefficient of side forces for rolling wheel which is sliding, originates from the relation of the side forces  $F_y$  and perpendicular forces  $F_z$ . There is a determinate connection between  $C_{zk}$  and  $\beta$ ,<sup>(1)(8)</sup> (Fig. 3.).

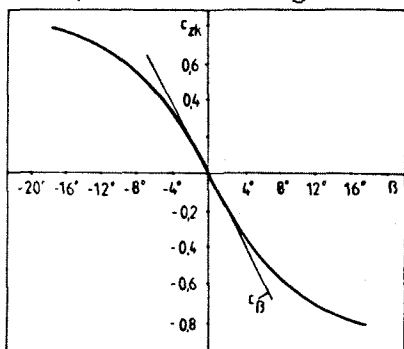


Figure 3.

One of the important characteristics in the control of aircraft in its path is side slip stiffness ( $C_{\beta}$ ), which can be determined from the gradient of the curve.

The motion of the aircraft on the runway is the result of the effect of friction force between the wheel tyre and surface. Fig. 2 shows such a wheel, which initially has its track parallel with the centerline of the runway. When a supposed  $-F_z$  force acts on the rolling wheel then in the plane of connection appears an  $F_z$  force - which in equal is size and opposite in direction - which prevents the airplane rolling off the runway.

If the wheel rolls in the  $v_x$  direction with an angle of  $(-\beta)^x$  compared to the tyre equator, then the points of the tyre - crossing the connection zone - became compressed transversally. This deformation results in the appearance of the readjusting forces which means the resultant of the  $F_z$  forces.

But the readjusting  $F_z$  force has

a limit value which it can take. This value depends on the local friction and distribution of perpendicular load,<sup>(5)(9)</sup>:

$$F_{z, s. max.} = \mu_z (mg - F_y - F_p \alpha_p)$$

where:  $\mu_z$  - coefficient of friction direction z.

The side force of the wheel is proportional to the load of the wheel,<sup>(6)</sup> The sum of the side forces is,<sup>(6)</sup>:

$$F_{zk} = (F_y^{Nff} + F_p^{Nff}) C_{zk}^{\beta} \beta = C_{zk}^{\beta} (mg - F_y - F_p \alpha_p) \beta$$

Use of the empirical formula by BLANDOV gives,<sup>(4)</sup>:

$$F_{zk} = 1,6 \mu_z F_{No} \sin\left(\frac{\pi \beta}{2\beta_{kr}}\right) \sin\left(\frac{\pi F_{No}}{5F_{No}}\right)$$

in a simpler formula to determine  $F_z$ :

$$F_{zk} = \mu_z \sin \frac{\pi \beta}{2\beta_{kr}}$$

where:  $\mu_z = \mu_{z0} (1 + a_1 v + a_2 v^2)$

In the case of big slip angles, friction seems not to be enough to compensate for the readjusting force and this causes considerable slippage - first at the rearward section of the contact surface, where the transversal displacement is the largest - then, as the slip angle moves to the frontal section of the contact surface as well,<sup>(1)</sup>, (see Fig. 4.).

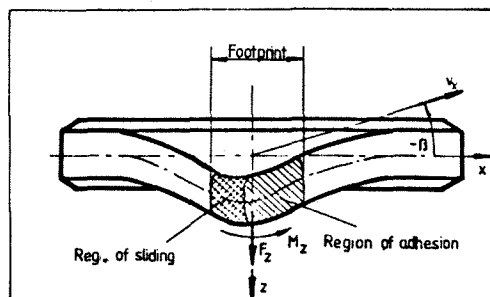


Figure 4.

The most important operational features, that affect on the side force are,<sup>(9)</sup>:

- perpendicular load  $F_N$
- slip angle.

Fig. 5. shows the relation between the side force and slip angle, in three case of perpendicular loads.

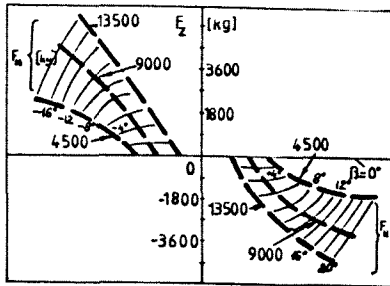


Figure 5.

The situation means that side force  $F$  is not in linear relation with values  $F_N$  and  $\beta$ , hence by the reduction of radial load the side slip stiffness of wheel reduces as well thus the linear relation between  $F/F_N$  and  $\beta$  is no longer valid. For this reason the airplane is more difficult to steer in its path with reduced radial load than with normal load, especially on a snowy and icy runway.

The pressure of balloon also has influence on the side force  $F_z$ , since the larger pressure increases its stiffness both sideways and vertically, thus acts opposite to the growth of the sideway force, (6). The increment of the balloon pressure, the growth in the sideway force is less than expected. (For example on Fig. 6. it is seen, that increasing the tyre pressure by 25% the sideway force increases only by 10%, (1)(6).

Efficiency of the tyre's influence in directional stability is reduced on snowy and icy runways, (2). Fig. 7. shows the changes in the coefficients of the sideway forces as a function of the slip angle: 1- dry, 2- wet supporting surface.

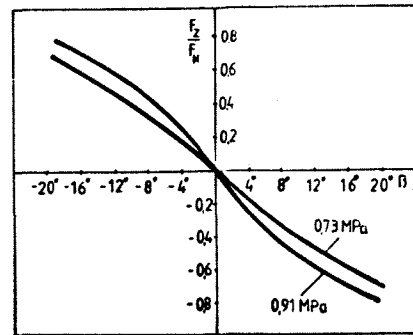


Figure 6.

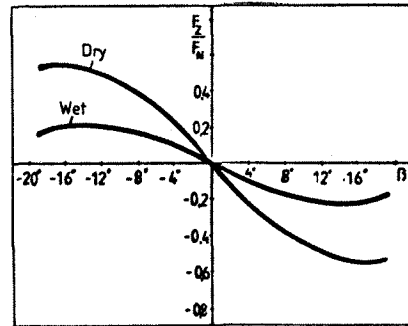


Figure 7.

The materials and the shape of the tyre has a major influence on the side forces as well, (3), (5). Fig. 8. shows how the side forces differ in case of the tyre shape A and B, (10).

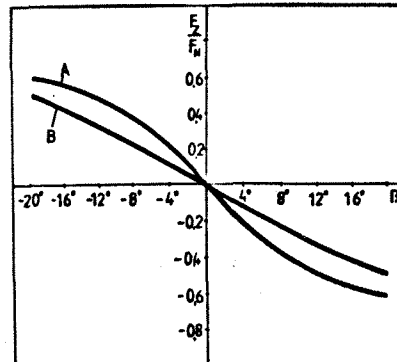


Figure 8.

In the case of a sliding rolling wheel, besides the side force  $F_z$ , a so called  $M_z$  corner moment appears as well which is also called readjusting moment, because it acts against slippage, (see Fig. 4.)

The attachment points of the steerable wheels must take this mo-

ment up. The cornering moment plays a part in the origin of the "simmi". The  $M_z^k$  moment is in a complicated relationship with the slip angle  $\beta$  and with the vertical load  $F_N$ . Their relationship is shown in Fig. 9. (1)

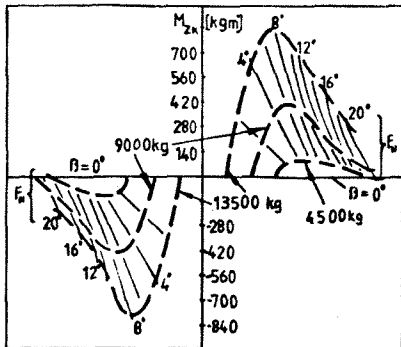


Figure 9.

With a given  $F_N$  vertical load the cornering moment increases as a function slip angle till it takes up its maximum value which is 6-8° in the case of normal operation.

By further increment of the slip angle the cornering moment intensively decreases then by larger  $\beta$  angles it changes its sign.

#### Forces resulting from braking of the wheel

The wheel's behavior during braking is similar in several aspects to the behavior during sideway slipping during rolling. In both cases, the nature of the origin of the force is determined by the stiffness of the tyre and friction.

The angular velocity  $\omega$  of the wheel rolling not under braking reduces to  $\omega^0$  under braking. As the braking moment increases the  $\omega^1 - \omega^0$  difference also increases. For their comparison, the formula has been introduced (1), (11):

$$S = \frac{\omega^0}{\omega^1} - 1, \text{ which is called relative roll-over.}$$

As a consequence of braking, because of the action-reaction bet-

ween the tyre and the runway surface an  $F_x$  longitudinal force appears, which slows the plane down.

The  $F_x/F_N$  relationship is well known as the coefficient of friction between the tyre and runway surface. A typical relationship between  $F_x/F_N$  and  $S$  is shown in Fig. 10., where  $S=0$  represents free rolling of the wheel and  $S=1$  corresponds to the blocking of the wheel.

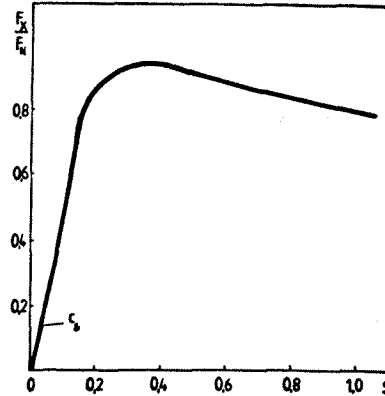


Figure 10.

The sleepers of the curve at  $S=0$  is determined by the longitudinal stiffness of the tyre which is represented by  $C_s$ . This stiffness determines the growth of the force at the beginning of the curve.

Experimental results have shown that tyre with large longitudinal stiffness the longitudinal  $F_x$  force increases. (1), (5).

The above mentioned braking procedure is the result of the connection between tyre stiffness and friction of which is the function of several variables such as the characteristics of the geometry of the runway or the cord - material used in the tyre. (6).

During the braking the velocity of the movement and the amount of dirt on the runway has the most influence on friction. (3).

In the works of Brewers we find a simple linear relationship for

the coefficient of friction for the wheel under braking as a function of the sleeding speed (when the whole surface is sliding).

$$\mu^x = \mu (1 + a_s v_s)$$

where  $v_s = S v$ ;

$a_s$  is a coefficient determined by experimental methods.

The effect of the amount of dirt on the runway on the wheel under braking is shown in Fig.11. where  $F^x/F^N$  is given as a function of  $S$ , 1-dry, 2-wet, 3-icy runway, (1)(5).

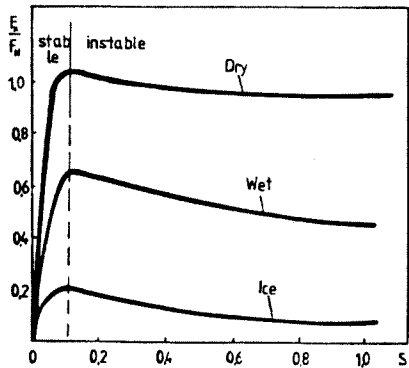


Figure 11.

The 2. curve Fig.11. is for wet run ways with specific amount of water on it. The effect of the thickness of water (on the runway) on the  $F^x/F^N$  relation as a function of  $S$  is shown in Fig.12. for 0.38, 1.00, 1.90 mm water layer thickness.

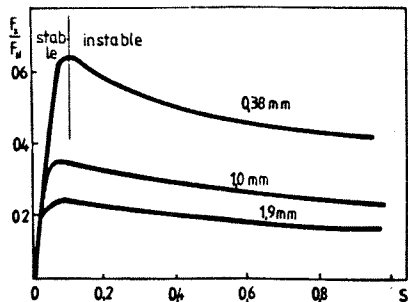


Figure 12.

The structure of tyre affects on the longitudinal force trough its stiffness, this is why Fig.13. is interesting, where the  $F^x/F^N$  is shown as a function of  $S$ . Cord plies are placed: 1-radially; 2-belted biased; 3-biased, (1)(10).

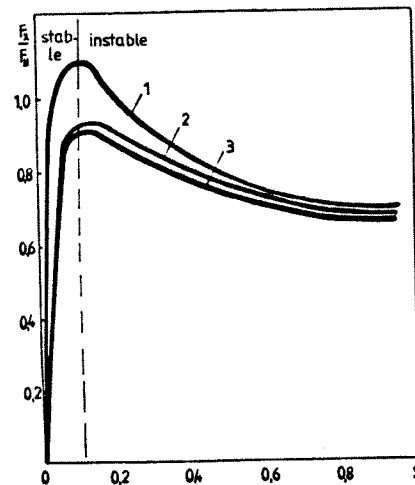


Figure 13.

Rolling of the wheel under braking and slipping

Not long ago has a new model appeared for the description of the effect of simultaneous braking and slipping on the wheel. Fig.14. shows a footprint which originates as a result of the  $F^x$  and  $F^N$  forces. When the external part of which is "free of tension" gets into the contact zone, it gets under the effect of the frictional force and stays at rest in relation to the ground.

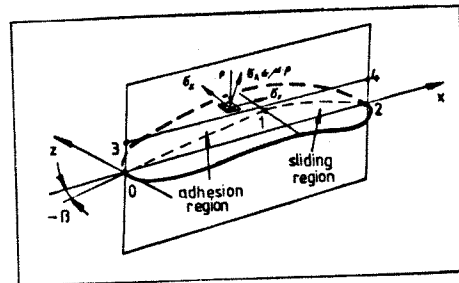


Figure 14.

Depending on the movement on the contact zone the given section starts sliding both longitudinal-ly and laterally because of the tensions  $\alpha^x$  and  $\alpha^y$  and their sum  $\alpha^R$ . Their value depends on  $S, \beta$  and the stiffness of the tyre. Until  $\alpha^R$  does not exceed the available " $\mu^R p$ " friction considerable "sli-

ding" does not occur, but above this limit it does.

The values of the lateral  $F_z$  and longitudinal  $F_x$  forces can be calculated by the integration of the respective values of  $\alpha$  tension on the contact surface. The sign used in Fig.14. 1-region of adhesion; 2-region of sliding.

The values of  $F_x$  and  $F_z$  as a function of the relative sliding and slipping angle can be found in Brewer's work. The final results are:

$$\mu_x = \frac{F_x}{F_N} = \frac{\bar{S}}{S_R} \frac{F_R}{F_N}$$

$$\mu_z = \frac{F_z}{F_N} = \frac{\bar{\beta}}{S_R} \frac{F_R}{F_N}$$

where:  $\bar{S} = \frac{C_s S}{\mu_o F_N (1-S)}$

$$\bar{\beta} = \frac{C_\beta \text{tg}\beta}{\mu_o F_N (1-S)}$$

$$\bar{S}_R = (\bar{S}^2 + \bar{\beta}^2)^{0.5}$$

$$F_R = \begin{cases} \mu_o F_N \bar{S}_R & \bar{S}_R < 0.5 \\ \mu_o F_N \left(1 - \frac{1}{4\bar{S}_R}\right) & \bar{S}_R > 0.5 \end{cases}$$

Fig.15. is based on experimental work and shows the mutual relationship of the lateral and longitudinal forces. In the case of "o" relative longitudinal overturn there is a sideways force and the tyre which is proportional to the angle  $\beta$ .

Adding the braking moment, longitudinal sliding takes place as well, because there is an overturn to the gives direction. The summing of the longitudinal and la-

teral tensions gives  $\alpha$  resultant tension, which exceeds<sup>R</sup> the limiting value of the available friction.

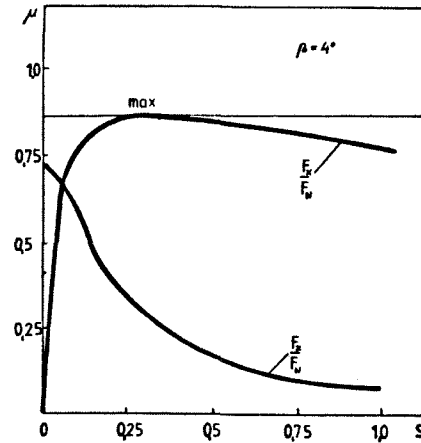


Figure 15.

For this reason, at the rearward section of the contact zone, the contact weakens. Because of the resulting sliding, the deformation of the lateral displacement lessens, which is revealed in the reduction of the lateral force. The further increase of the longitudinal sliding under breaking causes a larger zone to take place in the sliding, which results in the reduction of the lateral force.

On Fig. 16. we can see one of a series of graphs, with  $\beta=4^\circ$  slipping angle. If we draw further graphs with different values of  $\beta$ , we get Fig.17. which gives a considerably good relationship between  $F_x$ ,  $F_z$ ,  $\beta$  and  $S$ .

The traction envelope of the graphs in Fig.17. is shown in Fig. 18., which is sometimes called limiting relationship, ( curve 1).

This comprises all the theoretically possible relationships between lateral and longitudinal forces for the steering of the aircraft in its path.

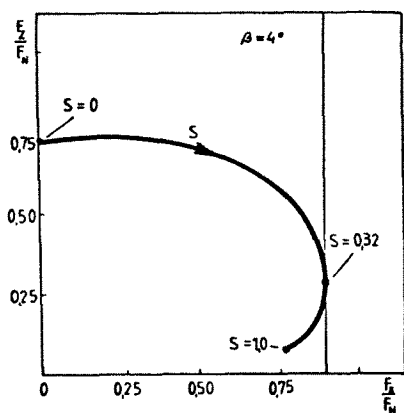


Figure 16.

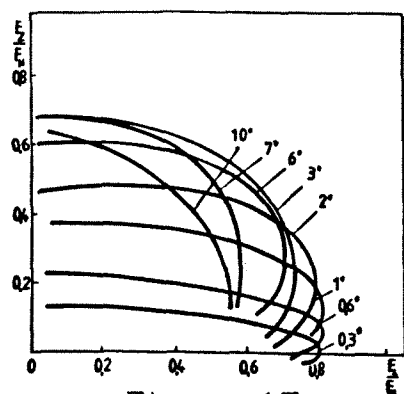


Figure 17.

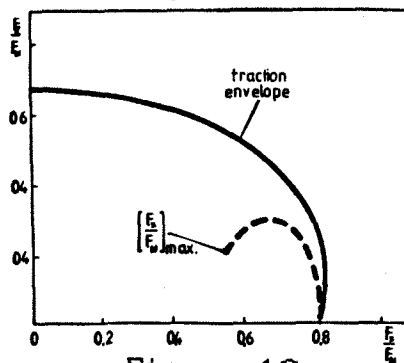


Figure 18.

In practice this limiting connection reduces considerably due to the fact that modern ABS systems attain the maximum coefficient of friction between the wheel and the runway.

Although this value is more than enough for the steering of the aircraft under normal conditions.

In Fig.19. it is seen that the limiting contact of the wheel to the runway suddenly decreases on wet runway, which result in the loss of control in the steering of the aircraft in its path.

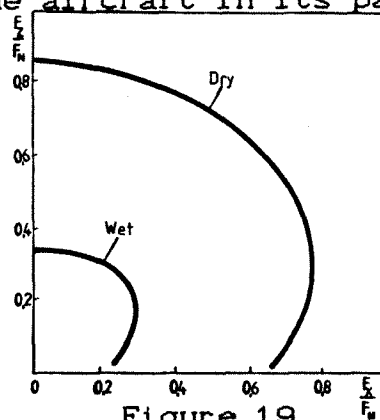


Figure 19.

### Conclusion

The lateral motion of aircraft is influenced by the following parameters:

- coefficient of friction of the surface of the runway;
- rolling resistance from deformation of the ground in case of the use of grass airfields;
- longitudinal and lateral gradient of iced, snowed, short runways, and highway;
- the strength and direction of crosswinds;
- material of the tyre, form ply pressure of balloon, distribution of load, braking system of the wheel;
- directional stability of the aircraft;
- geometrical construction of aircraft;
- proportion of longitudinal stability and wing span;
- wing dihedral angle;
- angle of sweep back;
- interference of the wing, tail and fuselage;
- area of tail;
- statically moment of the tail.

In literature <sup>(9)</sup> a relationship is given by which the velocity of

lateral sliding can be calculated.

The possibility of the airplane turning over can be investigated during take-off and landing in the case of the sudden braking of one of the mainwheels, or during the case of a serious damage to a wheel or the case of one of the wheels running onto softer ground.

#### Applied signs

$m$	-mass of aircraft;
$v$	-velocity of aircraft;
$F$	-thrust;
$\alpha^p$	-building angle of engine;
$F^p$	-drag;
$F^{xc}$	-force of friction;
$F^a$	single nose wheel load;
$F^{Nof}$	single main wheel load;
$F_y^{Nff}$	-lift;
$\Psi$	-angle of yaw;
$F$	-aerodynamics side force;
$a^z$	-distance between center of mass and nosewheel;
$b$	-distance between center of mass and mainwheel;
$x_f$	-aerodynamical focus;
$x_f$	-center of mass;
$C^{sp}$	-traction stiffnes;
$C^s$	-lateral stiffnes of tyre;
$F_x^\beta$	-longitudinal force of balloon;
$F$	-side force of balloon;
$F^z$	-radial load of wheel;
$F^N$	-resulting force;
$M^R$	-braking moment;
$M^z$	-corner moment;
$v^{zk}$	-longitudinal slip velocity;
$v^x$	-side slip velocity;
$p^s$	-specific pressure in contact zone;
$S$	-relative overturn;
$\bar{S}$	-conversion factor of overturn;
$S_R$	-resulting vector of overturn;
$\beta$	-slip angle;
$\bar{\beta}$	-conversion factor of slip angle;
$\omega_o$	-angular velocity of wheel without braking;
$\omega$	-angular velocity of wheel

	under braking;
$\mu$	-coefficient of friction;
$\sigma_x$	-longitudinal tension of connection;
$\sigma_y$	-normal tension of connection;
$\sigma_R$	-resulting tension of connection.

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