SOLUTIONS OF GENERALIZED PROPORTIONAL NAVIGATION WITH MANEUVERING AND NONMANEUVERING TARGETS

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Abstract

In this generalized proportional navigation, the commanded acceleration is applied in a direction with a bias angle to the normal direction of line-of-sight and its magnitude is proportional to the product of closing speed and line-of-sight rate between interceptor and its target. Some solutions of generalized proportional navigation were obtained before under the assumption that the magnitude of commanded acceleration is proportional to the line-of-sight rate only. Now in this article, the exact and complete closed-form solutions are derived under this modified guidance scheme for both maneuvering and nonmaneuvering targets. Some related important characteristics, such as capture capability and energy cost, are investigated and discussed. The variation of bias angle will induce the change of capture area and energy cost required. Also, a typical example of target maneuver is introduced to describe the effect of target maneuver easily. It shows that the target maneuver will decrease the capture area and increase the energy cost for effective intercept of target.

I. Introduction

The proportional navigation has been widely used as the guidance scheme in homing phase of flight for most missile systems. In pure proportional navigation (PPN), the commanded acceleration is applied in the direction normal to pursuer's velocity, and its magnitude is proportional to the angular rate of line-of-sight (LOS) between pursuer and its target [1,2,3,4]. In traditional true proportional navigation (TPN), the commanded acceleration is applied in a direction normal to the LOS, and its magnitude is proportional to the LOS rate [5,6]. Then a modified TPN was submitted, in which the commanded acceleration is applied in the normal direction of LOS and its magnitude is proportional to the product of LOS rate and closing speed between pursuer and target [7,8,9]. Furthermore, generalized proportional navigation (GPN) and ideal proportional navigation (IPN) were presented recently, in which the commanded acceleration is applied in a direction with a fixed bias angle to the normal direction of LOS and normal to the relative velocity between pursuer and target respectively [10,11,12,13].

In conventional GPN, some solutions were obtained before under the assumption that the magnitude of the commanded acceleration is proportional to LOS rate only [10,11,12]. Now in this article, we try to derive an exact and complete closed-form solution of a modified GPN for both maneuvering and nonmaneuvering targets, in which the magnitude of commanded acceleration is regulated proportional to the product of LOS rate and closing speed. It can be solved as a function of deflection angle of LOS, in general. Some important and significant characteristics related to the system performance, such as capture capability and energy cost, are investigated and discussed in detail.

II. Solution for a nonmaneuvering target

Consider a pursuer of speed V_M is pursuing a non-maneuvering target with speed V_T in exoatmospheric flight under the guidance law of GPN. The commanded acceleration is given in a direction with a bias angle β to the normal direction of LOS and its magnitude is proportional to the product of closing speed and LOS rate, i.e.,

$$\begin{aligned}\vec{a}_c &= \lambda\dot{r}\dot{\theta}(\cos\beta\,\vec{e}_\theta + \sin\beta\,\vec{e}_r) \\ &= \dot{r}\dot{\theta}(\lambda_\theta\vec{e}_\theta + \lambda_r\vec{e}_r) \end{aligned} \tag{1}$$

Here λ is the effective proportional navigation constant, $\dot{r}$ is the range rate and $\dot{\theta}$ is the LOS rate between pursuer and target. And the relative motion can be described in a polar coordinate system as (see Fig. 1)

$$\begin{aligned}\vec{v} = \vec{V}_M - \vec{V}_T &= v_r\vec{e}_r + v_\theta\vec{e}_\theta \\ &= \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta\end{aligned}$$

Then the equations of relative motion are

$$v_r' = v_\theta + \lambda_r v_r \tag{2a}$$

$$v_\theta' = -v_r + \lambda_\theta v_r \tag{2b}$$

Where primes denote differantiation with respect to θ. Thus the solutions can be obtained as

$$v_r = v_{r_0}\{[\frac{B_1}{B_1-B_2} + \frac{1}{(B_1-B_2)A}]e^{B_1\theta} - [\frac{B_2}{B_1-B_2} + \frac{1}{(B_1-B_2)A}]e^{B_2\theta}\} \tag{3a}$$

$$v_\theta = v_{\theta_0}\{[-\frac{B_2}{B_1-B_2} + \frac{(\lambda_\theta-1)A}{B_1-B_2}]e^{B_1\theta} + [\frac{B_1}{B_1-B_2} - \frac{(\lambda_\theta-1)A}{B_1-B_2}]e^{B_2\theta}\} \tag{3b}$$

Here v_{r_0} and v_{θ_0} are the initial conditions of v_r and v_θ respectively. A is a constant $(= \frac{v_{r_0}}{v_{\theta_0}})$, and B_1, B_2 can be derived as a function of λ_r and λ_θ

$$B_1, B_2 = \frac{\lambda_r \pm \sqrt{\lambda_r^2 + 4(\lambda_\theta - 1)}}{2} = \frac{\lambda \sin\beta \pm \sqrt{\lambda^2 \sin^2\beta + 4(\lambda\cos\beta - 1)}}{2}$$

Then the final deflection angle of θ till intercept can be derived from the final condition $v_\theta(\theta_f) = 0$ as

$$\theta_f = \frac{1}{B_1 - B_2} \ln \frac{B_1 - (\lambda_\theta - 1)A}{B_2 - (\lambda_\theta - 1)A} \tag{4}$$

Here a positive θ_f is considered with $A < 0$. We find that the denominator and the numerator in Eq. (4) must be positive for effective intercept of target, thus the capture criterion can be obtained as

$$\lambda_\theta > 1 \tag{5a}$$

and

$$\lambda_r > \frac{(\lambda_\theta - 1)A^2 - 1}{A} \tag{5b}$$

where both constraints must be satisfied simultaneously for effective intercept of target. It can be simplified to

$$\lambda > \frac{A^2 + 1}{A(A\cos\beta - \sin\beta)} \tag{5c}$$

When β is equal to zero, it has the same results as that of TPN [8]. And the cumulative velocity increment required can be obtained as

$$\Delta V = \int_0^T |a_c| dt = |\int_0^{\theta_f} \lambda v_r d\theta| = \frac{\lambda}{\lambda_\theta - 1} |v_{\theta_0}| \tag{6}$$

which is related to the corresponding propellant mass required for effective intercept in exoatmospheric flight. It shows that the energy cost is proportional to the component of relative velocity along the normal direction of LOS and independent to the component of relative velocity along LOS.

The relation between range r and LOS deflection angle θ can be derived as

$$\begin{aligned} r &= r_0 e^{\int_0^\theta \frac{v_r}{v_\theta} d\alpha} \\ &= r_0 \{[\frac{-B_2}{B_1 - B_2} + \frac{(\lambda_\theta - 1)A}{B_1 - B_2}] e^{B_1\theta} + [\frac{B_1}{B_1 - B_2} - \frac{(\lambda_\theta - 1)A}{B_1 - B_2}] e^{B_2\theta}\}^{\frac{1}{\lambda_\theta - 1}} \\ &= r_0 (\frac{v_\theta}{v_{\theta_0}})^{\frac{1}{\lambda_\theta - 1}} \end{aligned} \tag{7}$$

and the LOS rate $\dot{\theta}$ can be obtained as

$$\begin{aligned} \dot{\theta} &= \frac{v_\theta}{r} = \dot{\theta}_0 (\frac{v_\theta}{v_{\theta_0}})^{\frac{\lambda_\theta - 2}{\lambda_\theta - 1}} = \dot{\theta}_0 (\frac{r}{r_0})^{\lambda_\theta - 2} \\ &= \dot{\theta}_0 \{[\frac{-B_2}{B_1 - B_2} + \frac{(\lambda_\theta - 1)A}{B_1 - B_2}] e^{B_1\theta} + [\frac{B_1}{B_1 - B_2} - \frac{(\lambda_\theta - 1)A}{B_1 - B_2}] e^{B_2\theta}\}^{\frac{\lambda_\theta - 2}{\lambda_\theta - 1}} \end{aligned} \tag{8}$$

Thus $\dot{\theta}$ approaches zero when $\lambda_\theta > 2$ and approaches infinity when $\lambda_\theta < 2$ during intercept period.

III. Solution for a maneuvering target

For the case of maneuvering target, the equations of relative motion can be written as

$$v_r' = v_\theta + \lambda_r v_r - f_r(\theta) \tag{9a}$$

$$v_\theta' = -v_r + \lambda_\theta v_r - f_\theta(\theta) \tag{9b}$$

where $f_r(\theta)$ and $f_\theta(\theta)$ are corresponding to target maneuver in two components of polar coordinate respectively, and are given as continuous functions of θ. Then the general solutions of Eqs. (9) are

$$v_r = m\frac{e^{B_1\theta} - e^{B_2\theta}}{B_1 - B_2} + n\frac{B_1 e^{B_1\theta} - B_2 e^{B_2\theta}}{B_1 - B_2} + F_r(\theta_f - \theta) \tag{10a}$$

$$v_\theta = m\frac{-B_2 e^{B_1\theta} + B_1 e^{B_2\theta}}{B_1 - B_2} + n(\lambda_\theta - 1)\frac{e^{B_1\theta} - e^{B_2\theta}}{B_1 - B_2} + F_\theta(\theta_f - \theta) \tag{10b}$$

with

$$\begin{aligned} F_r(\theta_f - \theta) = \int_0^{\theta_f - \theta} \{ &\frac{1}{B_1 - B_2}[B_1 e^{B_1(\theta_f - \theta - \alpha)} - B_2 e^{B_2(\theta_f - \theta - \alpha)}] f_r(\theta_f - \alpha) \\ &- \frac{1}{B_1 - B_2}[e^{B_1(\theta_f - \theta - \alpha)} - e^{B_2(\theta_f - \theta - \alpha)}] f_\theta(\theta_f - \alpha)\} d\alpha \end{aligned}$$

$$\begin{aligned} F_\theta(\theta_f - \theta) = \int_0^{\theta_f - \theta} \{ &-\frac{\lambda_\theta - 1}{B_1 - B_2}[e^{B_1(\theta_f - \theta - \alpha)} - e^{B_2(\theta_f - \theta - \alpha)}] f_r(\theta_f - \alpha) \\ &+ \frac{1}{B_1 - B_2}[-B_2 e^{B_1(\theta_f - \theta - \alpha)} + B_1 e^{B_2(\theta_f - \theta - \alpha)}] f_\theta(\theta_f - \alpha)\} d\alpha \end{aligned}$$

Here θ_f is the final deflection angle of LOS till intercept. Thus the integration constant m and n can be computed from initial condition, i.e.,

$$m = v_{\theta_0} - F_\theta(\theta_f) \tag{11a}$$

$$n = v_{r_0} - F_r(\theta_f) \tag{11b}$$

Also the terminal constraint $v_\theta(\theta_f) = 0$ must be satisfied for effective intercept, i.e.,

$$m(-B_2 e^{B_1\theta_f} + B_1 e^{B_2\theta_f}) + n(\lambda_\theta - 1)(e^{B_1\theta_f} - e^{B_2\theta_f}) = 0 \tag{12}$$

Then m, n and θ_f can be solved from Eqs. (11) and (12). And the capture criterion can be obtained from the constraint $v_r(\theta_f) < 0$ for effective intercept of target, i.e.,

$$m(e^{B_1\theta_f} - e^{B_2\theta_f}) + n(B_1 e^{B_1\theta_f} - B_2 e^{B_2\theta_f}) < 0 \tag{13}$$

in which the solution of m, n and θ_f in Eqs. (11) and (12) exist. It is found that the induced total deflection angle θ_f for a maneuvering target will be normally larger than that induced for a nonmaneuvering target and the effective proportional navigation constant with a maneuvering target must be selected larger than that required with a nonmaneuvering target for effective intercept. From Eqs. (12) and (13), it can be simplified to the results in Eqs. (5) for a nonmaneuvering target with $f_r = f_\theta = 0$. And the total cumulative velocity increment required till intercept can be expressed as

$$\begin{aligned} \Delta V = \int_0^T |a_c| dt = |\int_0^{\theta_f} \lambda v_r d\theta| &= \frac{\lambda}{\lambda_\theta - 1} |v_{\theta_0} - \int_0^{\theta_f} f_\theta(\theta) d\theta| \\ &= \frac{\lambda}{\lambda_\theta - 1} |v_{\theta_0} - \int_0^T a_{T_\theta} dt| \end{aligned} \tag{14}$$

in which a_{T_θ} is the component of target acceleration along the direction normal to LOS. From Eq. (14), we find that the energy expenditure is related to v_{θ_0} and a_{T_θ}. Thus effective target maneuver will usually increase the energy cost required. The relation between range-to-go r and LOS deflection angle θ can be expressed as

$$
\begin{aligned}
r &= r_0 e^{\int_0^\theta \frac{v_r}{v_\theta} d\alpha} \\
&= r_0 e^{\int_0^\theta \frac{m(e^{B_1\alpha} - e^{B_2\alpha}) + n(B_1 e^{B_1\alpha} - B_2 e^{B_2\alpha}) + (B_1 - B_2) F_r(\theta_f - \alpha)}{m(-B_2 e^{B_1\alpha} + B_1 e^{B_2\alpha}) + n(\lambda_\theta - 1)(e^{B_1\alpha} - e^{B_2\alpha}) + (B_1 - B_2) F_\theta(\theta_f - \alpha)} d\alpha}
\end{aligned}
\tag{15}
$$

and the LOS rate can be also obtained as

$$
\begin{aligned}
\dot{\theta} &= \frac{v_\theta}{r} \\
&= \dot{\theta}_0 \frac{\frac{m}{B_1 - B_2}(-B_2 e^{B_1\theta} + B_1 e^{B_2\theta}) + \frac{n(\lambda_\theta - 1)}{B_1 - B_2}(e^{B_1\theta} - e^{B_2\theta}) + F_\theta(\theta_f - \theta)}{v_{\theta_0} e^{\int_0^\theta \frac{m(e^{B_1\alpha} - e^{B_2\alpha}) + n(B_1 e^{B_1\alpha} - B_2 e^{B_2\alpha}) + (B_1 - B_2) F_r(\theta_f - \alpha)}{m(-B_2 e^{B_1\alpha} + B_1 e^{B_2\alpha}) + n(\lambda_\theta - 1)(e^{B_1\alpha} - e^{B_2\alpha}) + (B_1 - B_2) F_\theta(\theta_f - \alpha)} d\alpha}}
\end{aligned}
\tag{16}
$$

Now a special case of target maneuver is considered to illustrate the system performance in detail. Let $f_r = c_r v_r$ and $f_\theta = c_\theta v_r$, i.e., target maneuvers in a fixed direction relative to LOS and its magnitude is also proportional to the product of LOS rate and closing speed. Here $c = \sqrt{c_r^2 + c_\theta^2}$ is the factor of target maneuver. Thus the solution in this case can be simply derived as

$$
v_r = v_{r_0}\{[\frac{B_3}{B_3 - B_4} + \frac{1}{(B_3 - B_4)A}]e^{B_3\theta} - [\frac{B_4}{B_3 - B_4} + \frac{1}{(B_3 - B_4)A}]e^{B_4\theta}\} \tag{17a}
$$

$$
v_\theta = v_{\theta_0}\{[-\frac{B_4}{B_3 - B_4} + \frac{(\lambda'_\theta - 1)A}{B_3 - B_4}]e^{B_3\theta} + [\frac{B_3}{B_3 - B_4} - \frac{(\lambda'_\theta - 1)A}{B_3 - B_4}]e^{B_4\theta}\} \tag{17b}
$$

with

$$
B_3, B_4 = \frac{\lambda'_r \pm \sqrt{\lambda'^2_r + 4(\lambda'_\theta - 1)}}{2}
$$

Here $\lambda'_r = \lambda_r - c_r$, $\lambda'_\theta = \lambda_\theta - c_\theta$ and the capture criterion can be obtained as

$$
\lambda'_\theta > 1 \tag{18a}
$$

and

$$
\lambda'_r > \frac{(\lambda'_\theta - 1)A^2 - 1}{A} \tag{18b}
$$

or

$$
\lambda > \frac{A^2 + 1}{A(A\cos\beta - \sin\beta)} + c \tag{18c}
$$

when target maneuvers in the same direction as that of the commanded acceleration. Therefore, a larger λ is required with a larger target maneuver for effective intercept. And the cumulative velocity increment required is

$$
\Delta V = \int_0^T |a_c| dt = |\int_0^{\theta_f} \lambda v_r d\theta| = \frac{\lambda}{\lambda'_\theta - 1}|v_{\theta_0}| \tag{19}
$$

Then a larger energy cost is induced with a larger target maneuver. The response of range-to-go and LOS rate can be written as

$$
\begin{aligned}
r &= r_0 e^{\int_0^\theta \frac{v_r}{v_\theta} d\alpha} \\
&= r_0\{[\frac{-B_4}{B_3 - B_4} + \frac{(\lambda'_\theta - 1)A}{B_3 - B_4}]e^{B_3\theta} + [\frac{B_3}{B_3 - B_4} - \frac{(\lambda'_\theta - 1)A}{B_3 - B_4}]e^{B_4\theta}\}^{\frac{1}{\lambda'_\theta - 1}} \\
&= r_0(\frac{v_\theta}{v_{\theta_0}})^{\frac{1}{\lambda'_\theta - 1}}
\end{aligned}
\tag{20}
$$

and

$$
\begin{aligned}
\dot{\theta} &= \frac{v_\theta}{r} = \dot{\theta}_0(\frac{v_\theta}{v_{\theta_0}})^{\frac{\lambda'_\theta - 2}{\lambda'_\theta - 1}} = \dot{\theta}_0(\frac{r}{r_0})^{\lambda'_\theta - 2} \\
&= \dot{\theta}_0\{[\frac{-B_4}{B_3 - B_4} + \frac{(\lambda'_\theta - 1)A}{B_3 - B_4}]e^{B_3\theta} + [\frac{B_3}{B_3 - B_4} - \frac{(\lambda'_\theta - 1)A}{B_3 - B_4}]e^{B_4\theta}\}^{\frac{\lambda'_\theta - 2}{\lambda'_\theta - 1}}
\end{aligned}
\tag{21}
$$

respectively. In this typical example, it shows that the LOS rate approaches zero when $\lambda'_\theta > 2$ and approaches infinity when $\lambda'_\theta < 2$ during intercept period.

IV. Discussion

In this modified GPN the commanded acceleration is regulated proportional to the product of closing speed and LOS rate, therefore a comprehensible solution is obtained, which is more significant than those obtained before. For the case of a nonmaneuvering target, λ_r must be greater than $\frac{(\lambda_\theta - 1)A^2 - 1}{A}$ with $\lambda_\theta > 1$ for effective intercept of target, which can be transformed to $\lambda > \frac{A^2 + 1}{A(A\cos\beta - \sin\beta)}$ with $A = \cot\phi_0$, as shown in Fig. 2. Successful intercept of the target is restricted to the upper side of each curve representing the capture boundary of different bias angles, in which $|\phi_0 + \beta|$ and $|\phi_0|$ must be greater than $\frac{\pi}{2}$ no matter what the proportional navigation constant is. We find that from Eq. (8) the value of λ_θ, which must be greater than one for effective intercept of target, determines whether the LOS rate approaches zero or infinity during intercept period. The final LOS rate approaches infinity with $\lambda_\theta < 2$ and approaches zero with $\lambda_\theta > 2$. Thus the λ_θ must be chosen to be greater than 2 due to the limitation of the commanded acceleration in actual application. The solutions of v_θ and $\dot{\theta}$ can be derived as a exponential function of range r, as shown in Eqs. (7) and (8). Thus a similarity solution can be realized to describe the system performance easily. The same solutions of TPN as derived before can be obtained here from GPN with $\beta = 0$ [8]. The bias angle affects not only the capture capability but also the energy cost required for effective intercept of target, as depicted in Fig. 3. It shows that a higher energy cost is required with a larger bias angle and TPN has the minimum energy expenditure absolutely in this modified GPN. If the critical condition of capture criterion is considered (i.e., $\lambda_r = \frac{(\lambda_\theta - 1)A^2 - 1}{A}$), then the solution for rendezvous guidance with proportional navigation can be derived [14].

For the case of a maneuvering target, in general, a larger proportional navigation constant is required for effective intercept of target and a higher energy cost is induced during intercept course. It is simply illustrated with a typical example as described in previous section. Thus target maneuver decreases the capture area and increases the energy cost required for effective intercept of target, as depicted in Fig. 4 and Fig. 5 respectively. The energy cost is affected only by v_{θ_0} and f_θ and is not affected by f_r. Thus it is more effective to maneuver in the direction normal to LOS for both interceptor and target during intercept course.

V. Conclusion

In this article, the exact and complete closed-form solutions of the modified generalized proportional navigation with maneuvering and nonmaneuvering targets are derived. Under this guidance scheme, the magnitude of commanded acceleration is proportional to the product of LOS rate and closing speed, which is different from those obtained before. Also, some significant characteristics, such as capture capability and energy cost, are investigated and discussed in detail under the effect of bias of the commanded acceleration. Then in advance, a special case of target maneuver is introduced to describe the effect of target maneuver easily. The target maneuver will usually decrease the capture area and increase the energy cost for effective intercept of target. Finally, we find that true proportional navigation is not only a special case but also the optimal solution of generalized proportional navigation, and that the rendezvous guidance with proportional navigation is the critical solution of generalized proportional navigation on the capture boundary.

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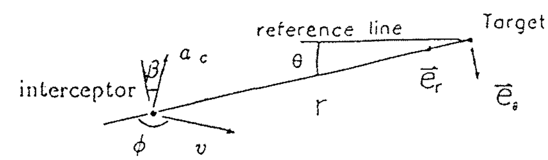

Fig. 1 Planar pursuit geometry

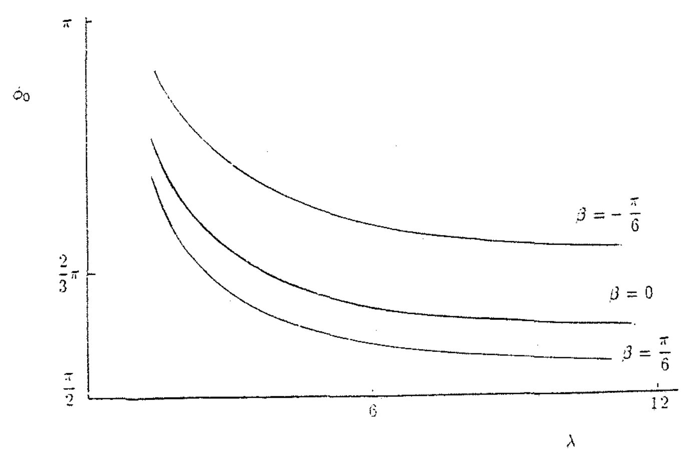

Fig. 2 Capture boundary of GPN

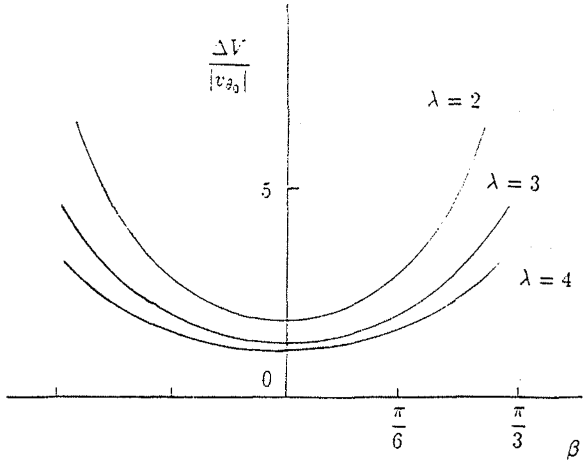

Fig. 3 ΔV vs. β for different values of λ

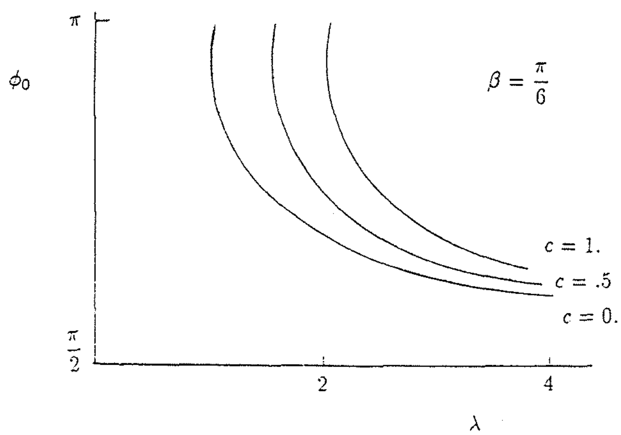

Fig. 4 Capture boundary with a maneuvering target

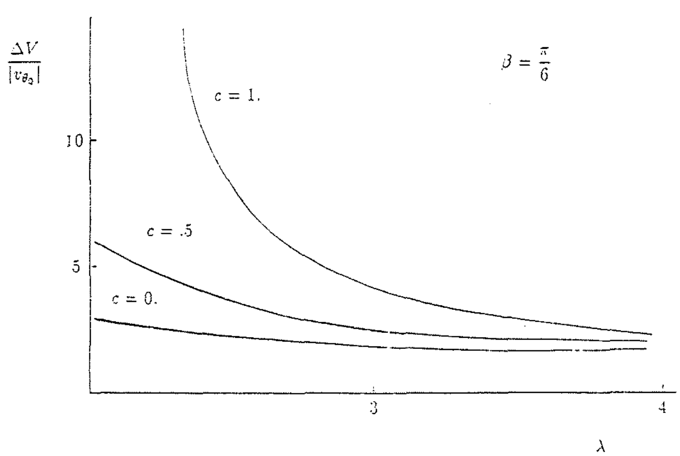

Fig. 5 ΔV vs. λ with a maneuvering target