ICAS-94-7.1.4

NONLINEAR DECOUPLING CONTROL AND AIRCRAFT AGILITY MANEUVERS

Zhou Zhiqiang* and Gao Hao**
Northwestern Polytechnical University, Xi'an, 710072, Shaanxi, P.R.China

Abstract

In this paper the nonlinear decoupling control of aircraft motion is deeply studied with differential geometric control theory. The study will provide a new theoretical approach to investigate aircraft agility、 the direct force control and post–stall maneuver of modern combat aircraft.

At the begining of the paper, the decoupling theory for nonlinear multivariable systems based on differential geometry control theory is introduced. The influence of the position exchange in components of input and in those of decoupling variable on decoupling law of the nonlinear system is discussed. The calculation of equilibrium points for nonlinear decoupling systems is discussed.

With the decoupling theory mentioned above, three modes of decoupling controls for aircraft nonlinear motion are studied. Three basic modes of nonlinear direct force control and two types of extraordinary manoeuvres are realized through the nonlinear decoupling controls. For the engineering application linear approximation of the aircraft nonlinear decoupling law is studied. The error made by the approximation is acceptable.

The calculation results show that all three agility maneuvers can be realized by the aircraft decoupling control thoroughly.

Introduction

With the research on the fourth–generation combat aircraft and the appearence of the all–aspect missile, the fighter with the ability of the decoupling between heading angle and flight path will enhance the nose pointing capability and take the superiority in the air combat. The decoupling control of the aircraft motion is one of the means to increase the aircraft agility.

Nine fundamental agility maneuvers for flight test to quantify aircraft agility have been developed by AFFTC (Air Force Flight Test Center). In this paper the aircraft decoupling control modes are used to realize three maneuvers, which, to a certain degree, are equal to the fundamental agility maneuvers proposed by AFFTC.

Synthesis of Nonlinear Decoupling Control Law

The nonlinear state equation of the aircraft motion can be described by

$$\begin{cases} \dot{x} = A(x) + B(x)u \\ y = C(x) \end{cases} \qquad (1)$$

where $A(x)$、 $B(x)$ and $C(x)$ in (1) are a $(n \times 1)$ vector、 $(n \times m)$ matrix and $(m \times 1)$ vector as functions of state variable x respectively.

Consider a nonlinear control law, which is composed of a nonlinear feedback of state variable and a nonlinear input transformation as follows

$$u = F(x) + G(x)v \qquad (2)$$

where $F(x)$ and $G(x)$ are $(m \times 1)$ and $(m \times m)$ matrices respectively, and where $G(x)$ is assumed to be nonsingular.

Applying the control law (2) to the system (1) gives the closed –loop system

$$\begin{cases} \dot{x} = \hat{A}(x) + \hat{B}(x)v \\ y = C(x) \end{cases} \qquad (3)$$

where

$$\begin{cases} \hat{A}(x) = A(x) + B(x)F(x) \\ \hat{B}(x) = B(x)G(x) \end{cases}$$

Dr. Zhou Zhiqiang, Lecturer, Aircraft Engineering Department

Professor Gao Hao, Aircraft Engineering Department

Copyright © 1994 by ICAS and AIAA. All rights reserved.

Based on the differential geometric control theory [1], the nonlinear decoupling control problem of nonlinear system (1) by means of a static feedback (2) is solvable if and only if the following $m\times m$ matrix

$$B^*(x) = \begin{bmatrix} L_{b_1}L_A^{r_1-1}c_1(x), & \cdots, & L_{b_m}L_A^{r_1-1}c_1(x) \\ \vdots & & \vdots \\ L_{b_1}L_A^{r_m-1}c_m(x), & \cdots, & L_{b_m}L_A^{r_m-1}c_m(x) \end{bmatrix} \tag{4}$$

which is defined for all x in a neighborhood of an initial point x_o, is nonsingular for all x in the neighborhood of the point x_o.

A multivariable nonlinear system of the form (1) has a vector *relative degree* $\{r_1, r_2, \cdots, r_m\}$ at a point x_o if

i) $L_{b_j}L_A^k c_i(x)=0$

for all $1<j<m$, for all $1<i<m$, for all $k<r_i-1$, and for all x in a neighborhood of x_o,

ii) The $m\times m$matrix $B^*(x)$is nonsingular at $x=x_o$.

If $B^*(x)$ is nonsingular at x_o, $F(x)$ and $G(x)$in the nonlinear decoupling law (2) are given by

$$\begin{cases} F(x) = -B^{*^{-1}}(x)[A^*(x)+D^*(x)] \\ G(x) = -B^{*^{-1}}(x)\wedge \end{cases} \tag{5}$$

where $\wedge = diag\{\lambda_1, \lambda_2, \cdots, \lambda_m\}$, $(\lambda_i\neq 0,\ i=1, 2, \cdots, m)$ is a constant matrix, and

$$A^*(x) = (L_A^{r_1}c_1(x)\cdots L_A^{r_i}c_i(x)\cdots L_A^{r_m}c_m(x))^T$$

$$D^*(x) = \begin{bmatrix} \alpha_{1r_1-1}L_A^{r_1-1}c_1(x)+\cdots+\alpha_{11}L_Ac_1(x)+\alpha_{10}c_1(x) \\ \vdots \\ \alpha_{ir_i-1}L_A^{r_i-1}c_i(x)+\cdots+\alpha_{i1}L_Ac_i(x)+\alpha_{i0}c_i(x) \\ \vdots \\ \alpha_{mr_m-1}L_A^{r_m-1}c_m(x)+\cdots+\alpha_{m1}L_Ac_m(x)+\alpha_{m0}c_m(x) \end{bmatrix}$$

where $\alpha_{ir_i-1}, \alpha_{ir_i-2}, \cdots, \alpha_{i1}, \alpha_{i0}$ $(i=1, 2, \cdots, m)$ are constants, which can be chosen to specify the dynamic behavior of the decoupled closed–loop system. The output vector y satisfies the following differential equation

$$y_i^{(r_i)}+\alpha_{ir_i-1}y_i^{(r_i-1)}+\alpha_{ir_i-2}y_i^{(r_i-2)}+\cdots+\alpha_{i1}y_i^{(1)}+\alpha_{io}y_i=\lambda_i v_i \quad (i=1, 2, \cdots, m) \tag{6}$$

The following two theorems obtained by the author give some features of the decoupled system.

Theorem 1 when the component y_i $(i=1, 2, \cdots, m)$ of $y(x)$ exchanges its position, the feedback matrix $F(x)$ in (5) remains unchanged, the column of matrix $G(x)$ in (5) exchanges its position in the same way as y_i; $\hat{A}(x)$ in the closed–loop system (3) remains also unchanged and the column of matrix $\hat{B}(x)$ exchanges its position similar to $G(x)$.

Theorem 2. When the component u_i $(i=1, 2, \cdots, m)$ of u exchanges its position, the row of the matrices $F(x)$ and $G(x)$ in (5) exchanges its position, respectively, in the same way as u_i; matrices $\hat{A}(x)$ and $\hat{B}(x)$ remain unchanged.

Equilibrium Points of the Nonlinear Decoupling System

Let x_e is one of the equilibrium points of the nonlinear system (1). u_e is the input of the system (1) corresponding to x_e. For the nonlinear decoupled closed–loop system (3), v_e of the new input v corresponding to the equilibrium x_e has been obtained by author and given by

$$v_e = \wedge^{-1}[B^*(x_e)u_e + A^*(x_e) + D^*(x_e)] \tag{7}$$

or

$$v_e = \wedge^{-1}B^*(x_e)[u_e - F(x_e)] \tag{8}$$

The equilibrium value y_e of the output y is given by

$$y_e = C(x_e).$$

Then v_e can be also obtained by

$$v_{ie} = \alpha_{io} y_{ie} / \lambda_i \quad (i = 1, 2, \cdots, m) \tag{9}$$

The equilibrium points of the nonlinear decoupled system were studied by author for first time. The study has not only theoretical significance, but also meaningful for engineering problems. Because the engineering problems are normally complicated, the nonlinear decoupling control law (5) can be obtained mostly by numerical calculation with computers. Equation (7) can be used to verify correctness of the numerical results.

Nonlinear Decoupling Control Modes

According to the above mentioned theory, three modes of aircraft decoupling control are investigated. The nonlinear decoupling laws of an aircraft for these modes described as follows are derived.

$$\text{mode A:} \quad y = \begin{bmatrix} r \\ \theta \end{bmatrix}, \quad u = \begin{bmatrix} \delta_f \\ \delta_e \end{bmatrix} \tag{10}$$

$$\text{mode B:} \quad y = \begin{bmatrix} \alpha \\ \theta \end{bmatrix}, \quad u = \begin{bmatrix} \delta_f \\ \delta_e \end{bmatrix} \tag{11}$$

$$\text{mode C:} \quad y = \begin{bmatrix} \alpha \\ \beta \\ \varphi \end{bmatrix}, \quad u = \begin{bmatrix} \delta_e \\ \delta_r \\ \delta_a \end{bmatrix} \tag{12}$$

Six degrees of freedom equations for a rigid body aircraft are employed to obtain the decoupling control laws for the modes.

The decoupling control law (5) for the aircraft at large maneuvers is nonlinear and can be generally obtained only through numerical calculations. If the on–line decoupling control law is required, large amount of numerical calculation will be the main obstacle for the realization. One way to reduce the amount of the calculation is the search of an approximate solution of the nonlinear decoupling control law.

Let $\tilde{F}$ and $\tilde{G}$ are the approximate solutions of $F(x)$ and $G(x)$ respectively, then $\tilde{F}$ and $\tilde{G}$ are given by

$$\begin{cases} \tilde{F} = \left. \dfrac{\partial F(x)}{\partial x} \right|_{x = x_e} \\ \tilde{G} = G(x)\big|_{x = x_e} \end{cases} \tag{13}$$

where x_e is one of the equilibrium points of the decoupled closed–loop system (3).

Agility Maneuvers of Aircraft with Decoupling Control

Advances in weapons and avionics systems are dramatically affecting fighter maneuvering tactics and are decreasing the validity of currently accepted measures of merit for evaluating fighter aircraft performance. The all–aspect, short–range missile, in particular, has changed the tactics of maneuvering during close–in, visual combat. The concept of *agility* has evolved as a result of deficiencies in traditional measures of merit for adequately predicting fighter aircraft effectiveness in the modern air combat arena. The AFFTC (Air Force Flight Test Center) defination of aircraft agility [4] is the ability of change aircraft attitude and flight path with quickness and precision. Therefore, aircraft agility implies not only aircraft maneuvering capability, but aircraft control in concert with this capability. The aircraft agility metrics and the aircraft agility design parameters are proposed by AFFTC. Nine fundamental agility maneuvers for flight test to quantify aircraft agility have been also devloped by AFFTC [4] .

The modes *A* and *B* can be used to realize the three basic modes A_n、 α_1 and α_2 of the nonlinear direct lift control thoroughly. The aircraft decoupling control modes *A*、 *B* and *C* are used to realize three maneuvers, which , to a certain degree, are equal to the fundamental agility maneuvevs proposed by AFFTC.

The first maneuver is the longitudinal path angle and pitch angle decoupled flight, which can be realized by mode *A*. The second maneuver is called level yaw [4], in which the maximum angle of sideslip will be achieved and the initial airspeed and *1–g*, wingslevel flight are maintained. The mode *C* can be used to realize the maneuver. The third manenver is roll angle capture, which can also be realized by mode *C*.

As an example, the combat aircraft *F–16* with a direct lift control surface is employed. The required data of *F–16* are taken from [3]. The time histories of the second maneuves for example aircraft *F–16* are given by the curves" 1" in Fig.1. The constants in the decoupling mode *C* which is used to realized the third maneuver are given as follows.

$\alpha_{10}=12.5$, $\alpha_{11}=5.0$, $\alpha_{20}=8.0$, $\alpha_{21}=4.0$,
$\alpha_{30}=13.0$, $\alpha_{31}=6.0$, $\lambda_1=12.5$, $\lambda_2=8.0$,
$\lambda_3=13.0$

The equilibrium point for the decoupling mode *C* is

$V=253$ m/s, $\alpha=2.2°$, $q=0$ deg/s
$\theta=2.2°$

$\beta=0.0°$, $\varphi=0.0°$, $p=0$ deg/s, $r=0$ deg/s

$\delta_e=1.21°$, $\delta_r=0°$, $\delta_a=0°$

$v_1=2.2°$, $v_2=0°$, $v_3=0°$

The step input for v in Fig.1. is given by $\Delta v=(0°, 5°, 0°)^T$. All curves in Fig.1 are increments of the state variables and deflection angles relative to the equilibrium point. The figure shows that the decoupling mode *C* can realize the maneuver thoroughly and the deflections of δ_e, δ_a and δ_r must move cooperatively, so the decoupling control system is needed to reduce the burden of the pilot during the maneuver.

The time histories of level yaw created by the approximate decoupling control law $\tilde{F}$ and $\tilde{G}$ for the example aircraft *F–16* is presented by curves " 2" in Fig.1. It can be seen from the figure that the error made by the approximation is acceptable. $\tilde{F}$ and $\tilde{G}$ in the approximate control law are constant matrices. Therefore, the amount of numerical calculations can be dramatically reduced.

Reference

1. Alberto Isidori:"Nonlinear Control Systems–An Introduction" 2nd Edition Springer–Verlag, 1989
2. S.H.Lane & R.F.Stengel:"Flight Control Design Using Non–linear Inverse Dynamics" Automatica, 1988, Vol.24, No.4
3. R.H.Hoh et al."Development of Handling Quality Criteria for Aircraft with Independent Control of Six Degrees of Freedom" AD / A100045, 1981
4. Stuart L.Butts et al.:"Flight Testing for aircraft Agility" AIAA–90–1308–CP

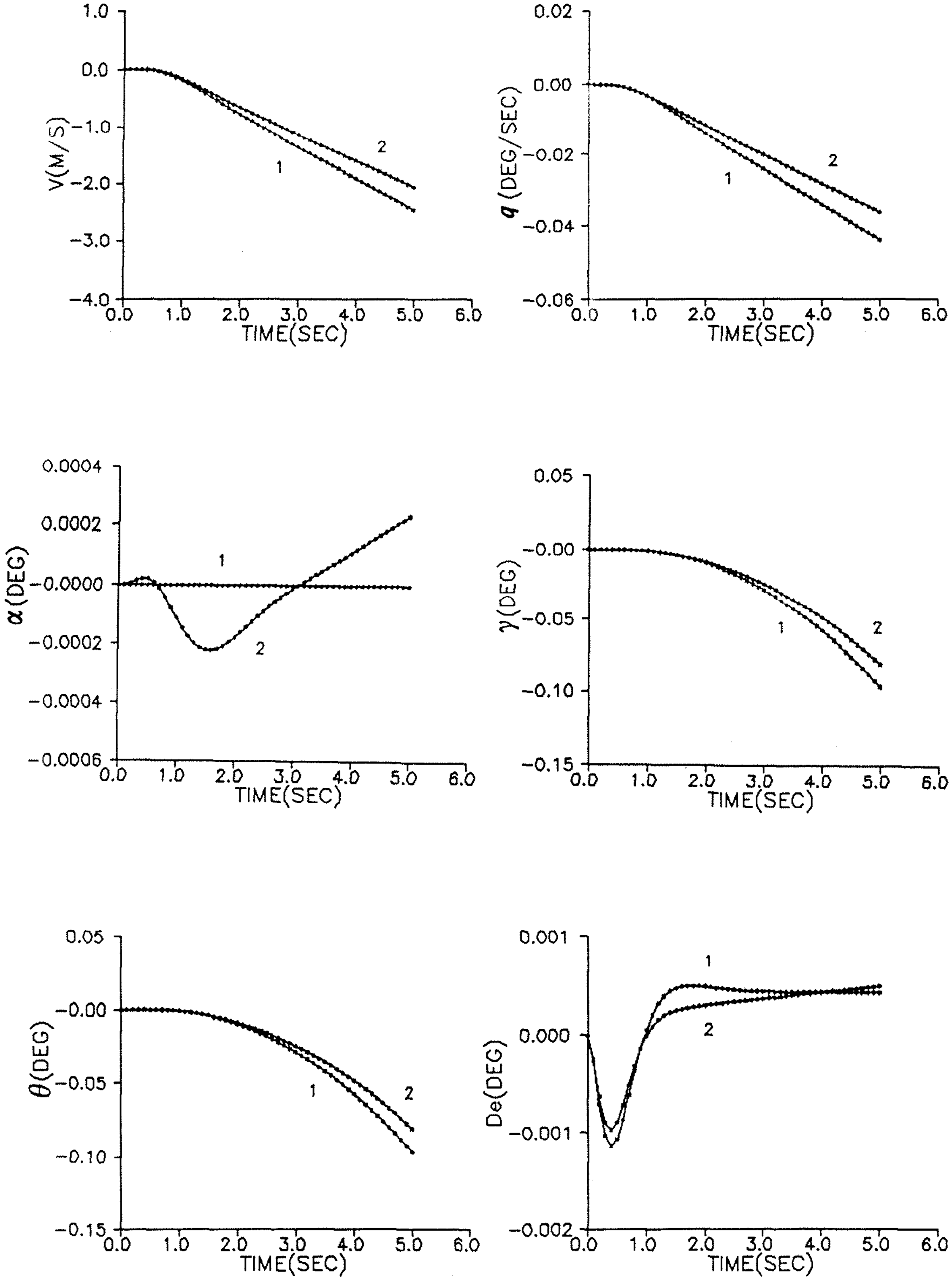

Fig.1. Time histories for level yaw maneuver by nonlinear decoupling control.
"1"··· exact solution "2"··· approximate solution

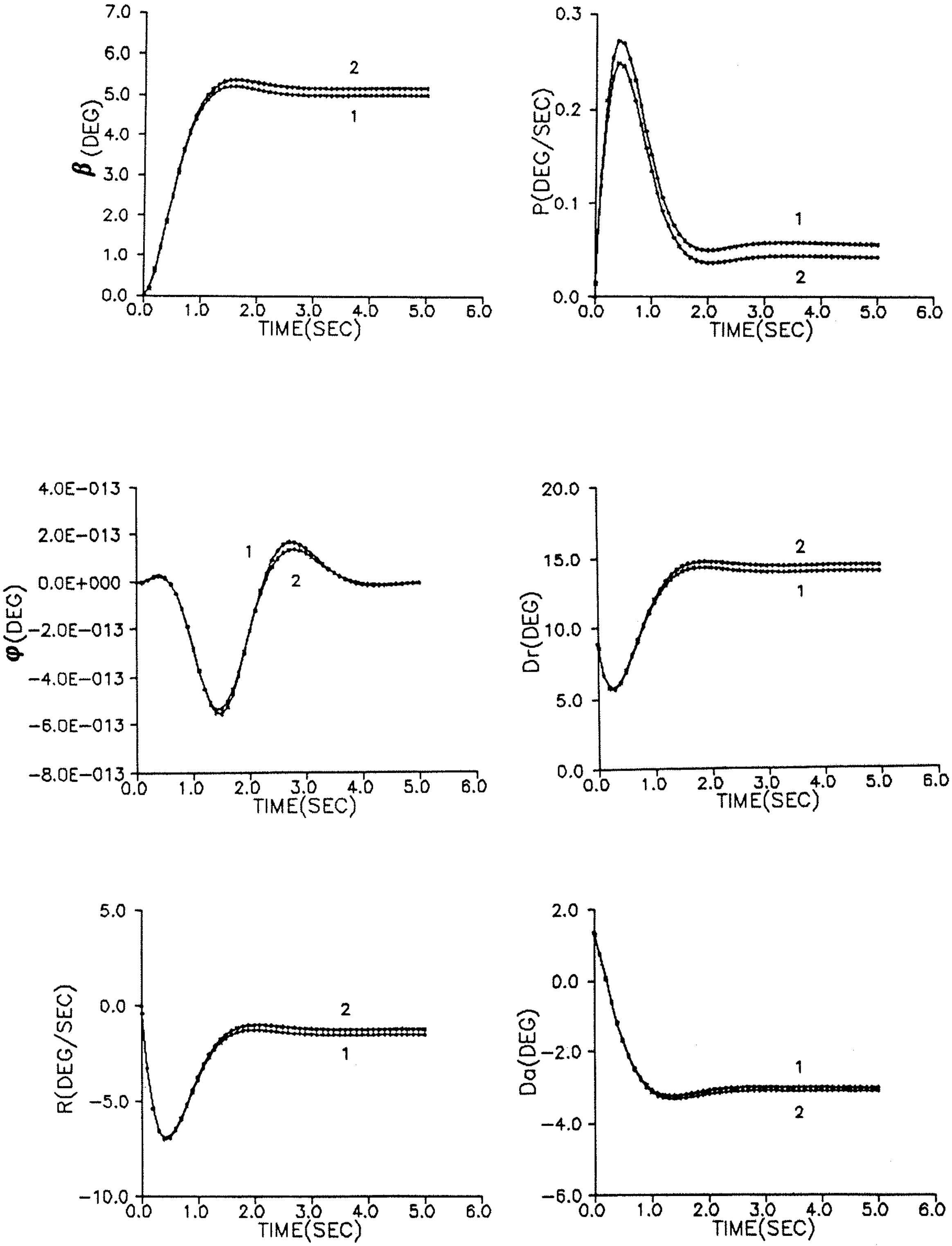

Fig.1. Concluded