AEROELASTIC ANALYSIS OF AN AIRCRAFT WITH STAND-BY ACTUATOR USING STATE-SPACE APPROACH

Yan Mursal [1], Harijono Djojodihardjo [2] and Risdaya Fadil [3]
PT. Industri Pesawat Terbang Nusantara
Jalan Pajajaran 154, Bandung, Indonesia

Abstract

The aircraft analyzed in this paper is driven by a pair of hydraulically powered servo actuators. One actuator is normally in an active mode and the other is normally in a passive stand-by mode. In a failure case, in which the active mode actuator failed, the stand-by actuator must provide sufficient damping in order that the airplane can still maintain flutter free condition. For this purpose, a method for aero-servo-elastic analysis of an airplane with a stand-by actuator using state space approach has been developed. The actuator used in this analysis was assumed to exhibit linear transfer function and viscous damping behaviour. The eigen-mode, eigen-frequencies and generalized mass were calculated with MSC/NASTRAN software package. The generalized unsteady aerodynamic forces were calculated using Doublet Lattice Method. All of these data then will be used to set-up the equations of motion of the airplane with stand-by actuator. This set of equations will be formulated as an eigenvalue problem in a state space form using MATLAB package software program. The generalized aerodynamic forces were expressed in Laplace domain using commonly used Pade approximation technique. Flutter analysis was carried out for several combination of altitudes and mass configurations. The results of this analysis is the impedance curve boundary that will be used as a design criteria of the actuator in the stand-by mode. Some of the results were also presented as examples.

Notation

$[\hat{a}_1], [\hat{a}_2], [\hat{a}_3], [\hat{a}_4]$	Intermediate step in equations
$[A_0], [A_1], [A_2]$	Pade fit coefficient
$[D], [N_0], [N_1]$	Pade fit coefficient
$[\hat{C}]$	Generalized viscous damping
$[I]$	Identity matrix
$[\hat{K}]$	Generalized stiffness
$L(\ )$	Laplace operator
$[\hat{M}]$	Generalized mass
$\bar{q}$	Dynamic pressure
$\{q\}$	Generalized coordinate in the s domain
$\{q_1\}$	Generalized coordinate in the time domain
$\{q_2\}$	Generalized velocity in the time domain
$\{q_3\}, \{q_4\}$	State due to Pade approximation in the time domain
$[Q]$	Generalized aerodynamic force coefficient matrix
s	Laplace operator
a_0, a_1, a_2, a_3, a_4	Approximation coefficient
$A(k)$	Real part of generalized aerodynamic force for the k^{th} reduced frequency
A_0, A_1, A_2	Approximation coefficient
$[A_0], [A_1], [A_2]$	Approximation coefficient in matrix form
$\{A\}$	Vector of unknown coefficient a_1, a_2, a_3 and a_4
b	Reference semi chord
b_3, b_4	Approximation coefficient
$B(k)$	Imaginary part of term of the generalized aerodynamic force matrix for the reduced frequency k.
N_0, N_1	Numerator coefficient
$[N_0], [N_1]$	Matrix of numerator coefficients
$Q_{ij}(k)$	Generalized aerodynamic force for the i^{th} integration mode and j^{th} pressure mode at the specific reduced frequency k.
R_0, R_1	Denominator coefficient
$[R_0], [R_1]$	Matrix of denominator coefficient
v	Free stream velocity
ω	Circular frequency
A	Plant matrix of combined system
A_m	Plant matrix of final moment equation
B_m, C_m, D_m	Coefficient matrix of final moment equation
f	Time dependent actuator force on control surface arm.
$\hat{f}$	Time dependent actuator generalized force
h	Control surface arm distance
K	Numerator coefficient (stiffness)
m	Time dependent actuator moment on control surface arm.
$\{W\}$	State vector of equations of motion with airplane, control surface forces, and aerodynamic forces.

1 Head of Structural Dynamics Dept., Development of Methods, Technology and Production, PT. IPTN.
2 Professor of Aeroelastomechanics, Institute of Technology Bandung, Indonesia.
3 Head of Aeroelastic Dept., Aircraft Design Division, PT. IPTN.

x_1, x_2, x_3	Displacement of airplane at actuator attachment in the direction of actuator displacement.
$\Delta\theta$	Control surface angle with respect to main aerodynamic surface.
ϕ_a	Airplane modal angle (about hinge line) at actuator attachment.
ϕ_c	Control surface modal angle (about hinge line) at actuator attachment.
$\Delta\phi$	Control surface modal angle relative to airplane modal angle (ϕ_c-ϕ_a)
$\vert\ \vert$	Absolute value of the argument.
C	Viscous damping constant
F	Frequency domain force in the idealized system
x	Time domain distance spring has compresses
Δx	Time domain distance idealized system has compressed
X	Frequency domain . distance spring has compressed
ΔX	Frequency domain distance idealized system has compressed

I. Introduction

PT. IPTN (Indonesian Aircraft Industry) is developing a new aircraft designated the N-250. It has a high wing and T-tail configuration, will accomodate 64 passengers at 32 inches seat pitch or 68 passenger at 30 inches seat pitch, and powered by two six bladed Dowty Rotol Propellers where each propeller is driven by an Allison GMA 2100C Engine. This airplane will have an Electrically Controlled Hydraulically Powered flight control system where each control surface (Rudder, Elevators and Ailerons) is equipped with two actuators. The use of electrically controlled and hydraulically powered flight control systems in the aircraft requires particular attention in its aeroelastic behaviour.

An important aspect in the aeroelastic analysis of N-250 Airplane, in particular to meet certification requirements in aeroelasticity, addresses failure cases. There are several failure cases in the flight control system that must be taken into account in order to meet the regulation stipulated in FAR 25.629 (Amendment 25-77 dated 29 July 1992) ; these are:

a. Any single failure, or malfunction, or combinations thereof, in the flight control system, and any single failure in any flutter damper system.

b. Any single failure of the stability augmentation system, or any other automatic or power operated system.

c. Control surfaces, including tabs, should be investigated for nominal conditions and for failure modes that include single structural failures (such as actuator disconnects, hinge failures, or in the case of aerodynamic balance panels, failed seals), single and dual hydraulic system failures and any other combination of failure not shown to be extremely improbable. Where other structural components contribute to the flutter stability of the system, failures of those components should be considered for possible adverse effects.

The flight control system is considered to be operating in one of two modes, normal flight or failure condition.

1) In normal flight, one actuator has Power On and the other one is in standby. In standby a bypass valve is actuated allowing fluid to flow to each side of the actuator piston through an orifice sized to give damping required to prevent flutter, in case the Power On actuator fails. The schematic of the system is given in Figure 1. Therefore in the Finite Element Model, under normal flight conditions each control surface has one actuator which is represented as stiffness only and another one which is represented as a stiffness and damper in series. The stiffness is represented by the stiffness of the back up structure and the actuator in series. The actuator sustem can be modeled as depicted in Figure 2.

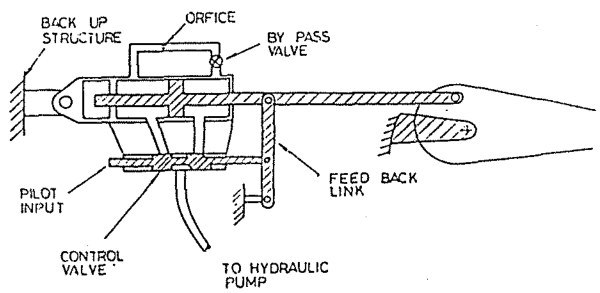

Figure 1. Schematic of the actuator system

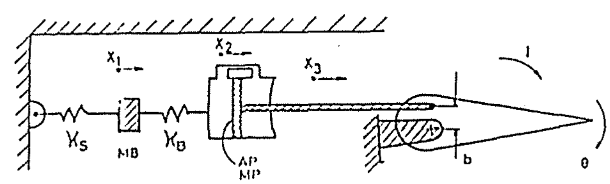

Figure 2.

$$F\ damp = Ap \cdot P = Cd\,(X3 - X2)^n$$

n = 1.0 linear damper
n = 2.0 velocity square damper

Ks	=	Back up (Reaction Link) Structure Stiffness
Kb	=	Actuator Stiffness including hydraulic Bulk Modulus
Ap	=	Piston Area
Mb	=	Mass of Actuator Body
Mp	=	Mass of Piston
Cd	=	Damper Coefficient

2) In failure condition, The Federal Aviation Regulation (FAR) of USA requires the flight control surfaces to be flutter free with adequate damping up to the Design Dive Speed (Vd) for any single failure, two hydraulic failures or a combined hydraulic and structural failure.

This paper will describe the modeling idealization of an airplane in case the active actuator fails, it means one hydraulic and one structural failure. In this condition the stand-by actuator will take part and is represented in the FE Model by stiffness and damping in series. The natural frequencies, mode shapes as well as the generalized unsteady aerodynamics were calculated by MSC/NASTRAN package, the results then transfered to the MATLAB software. The equation of motion of the airplane without stand-by actuator is setup in S-domain. In order to do that, the generalized unsteady aerodynamics must be transformed in to Laplace domain using commonly used Pade approximation technique. These were done in the next first two chapters.

The actuator transfer function is derived as presented in Chapter IV. The assumption made is that the actuator can be represented as spring and damper in series. The same procedure is followed in Chapter V to setup the equation of motion of the aircraft with stand-by actuator.

The flutter analysis is performed for several "a" values, see equation (30) in Chapter IV. With those analysis, the minimum damping coefficient required, C, can be obtained from the most critical V-g diagram.

II. A State Space Form For An Aeroelastic Airplane

In the following chapter will derive the set of equations that represent the airplane with unsteady aerodynamics written in a state space or first order form.

Modern control theory and state space methods have been convenient and hence popular inobtaining the solution of dynamic problems. This seems to be the case because of increased ability of quality sofwares (such as MATLAB, MATRIXx, MSC/NASTRAN, STICM, EASY5, MATOP, DIGICON, and a lot more) that will solve these first order matrix forms. These softwares were mainly developed to solve control problems but the numerical approach to the solution of the mathematics is very robust and will work on a wide range of problems. MATLAB is such a matrix algebra solver computer program and has been considered to be very convenient.

Unsteady aerodynamic computer codes such as the doublet lattice codes in MSC/NASTRAN computes the aerodynamics in the reduced frequency domain. This is ideally suited for a k type flutter solution but does not lend itself to a time domain or s domain system of equations. Recently, an approximation using a form of Pade approximation has become popular in the aeroelastic and aeroservoelastic analyses. With the unsteady aerodynamics written in this form a state space formulation is possible. In this state space form, s domain, time domain, and z domain transformations are not only possible but simple. At this point the airplane can be treated as any other control block and combined with other control blocks using common block diagram algebra.

The equations of motion of an airplane with unsteady aerodynamics can be expressed as a set of generalized coordinates in the s domain. This expression is

$$s^2[\hat{M}]q + s[\hat{C}]q + [\hat{K}]q = \bar{q}[\hat{Q}]q + \{\hat{F}\} \tag{1}$$

where the {F} are the other forces (internal or external) acting on the airplane and may or may not be needed in more extensive equation development. These forces could be gust forces for a gust loads analysis, control surface forces caused by inputs of the flight control system, landing gear forces for taxi analysis, discrete velocity square damping terms, or many others.

The generalized aerodynamic forces of equation (1) can be approximated from equation (22). This is an approximation and some caution is needed for its use. With this approximation the aircraft equations of motion become:

$$\begin{aligned} s^2[\hat{M}]\{q\} + s[\hat{C}]\{q\} + [\hat{K}]\{q\} &= \\ &\bar{q}[A_0]\{q\} + s\bar{q}[A_1]\{q\} + s^2\bar{q}[A_2]\{q\} \\ &+ \bar{q}[N_0][D]^{-1}\{q\} + s\bar{q}[N_1][D]^{-1} + \{\hat{F}\} \end{aligned} \tag{2}$$

The next task is to put equation (1) into a time domain. This can be accomplished by a series of inverse Laplace transforms. Let

$$\{q_1\} = L^{-1}(\{q\}) \tag{3a}$$

$$\{q_2\} = L^{-1}(s\{q\}) \tag{3b}$$

$$\{\dot{q}_2\} = L^{-1}(s^2\{q\}) \tag{3c}$$

$$\{q_3\} = L^{-1}([D]^{-1}\{q\}) \tag{3d}$$

$$\{q_4\} = L^{-1}([sD]^{-1}\{q\}) \tag{3e}$$

$$\{\dot{q}_4\} = L^{-1}([s^2D]^{-1}\{q\}) \tag{3f}$$

$$\{f\} = L^{-1}(\{\hat{F}\}) \tag{3g}$$

when these Laplace transformations are substituted into the equations

$$[\hat{M}]\{\dot{q}_2\} + [\hat{C}]\{q_2\} + [\hat{K}]\{q_1\} = \bar{q}[A_0]\{q_1\} + \bar{q}[A_1]\{q_2\} + \hat{q}[A_2]\{\dot{q}_2\} + \bar{q}[N_0]\{q_3\} + \bar{q}[N_1]\{q_4\} + \{\hat{f}\} \quad (4)$$

and if similar terms are collected and the highest order derivative is put on the left hand side, the equations of motion become

$$\{\dot{q}_2\} = [\hat{a}_1]\{q_1\} + [\hat{a}_2]\{q_2\} + [\hat{a}_3]\{q_3\} + [\hat{a}_4]\{q_4\} + \left([\hat{M}] - [A_2]\right)^{-1}\{\hat{f}\} \quad (5)$$

$$[\hat{a}_1] = -\left([\hat{M}] - \bar{q}[A_2]\right)^{-1}\left([\hat{K}] - \bar{q}[A_0]\right).$$

$$[\hat{a}_2] = -\left([\hat{M}] - \bar{q}[A_2]\right)^{-1}\left([\hat{C}] - \bar{q}[A_1]\right)$$

$$[\hat{a}_3] = -\,\bar{q}\left([\hat{M}] - \bar{q}[A_2]\right)^{-1}[N_0]$$

$$[\hat{a}_4] = -\,\bar{q}\left([\hat{M}] - \bar{q}[A_2]\right)^{-1}[N_1]$$

Equation (5) is in the desired first order form but there are four unknowns. Three other differential equations must be obtained. These are arrived at by identity relationships. The first two of these identity relationships are easily obtained from equations (3) which define the Laplace transformations and are given by

$$\{\dot{q}_1\} = [I]\,\{q_2\} \quad (6a)$$

$$\{\dot{q}_3\} = [I]\,\{q_4\} \quad (6b)$$

The third identity relationship is less straight forward and involves several steps. These steps are :

1) A simple identity relationships is established in the s domain

$$\{q\} = [I]\,\{q\}$$

2) The right side is expanded by $[I] = [D]\,[D]^{-1}$

$$\{q\} = [D]\,[D]^{-1}\,\{q\}$$

3) The [D] (not the one that is inverted) is replaced by the matrix expression $s^2\,[I] + s\,[R_1] + [R_0]$

$$\{q\} = \left(\,s^2[I] + s[R_1] + [R_0]\,\right)[D]^{-1}\,\{q\} \quad (7)$$

4) The Laplace relationships of equations (3) are used to transform equation (7) into the time domain.

$$\{q_1\} = [I]\{\dot{q}_4\} + [R_0]\{q_3\} + [R_4]\{q_4\}$$

5) Finally the equation is solved for (q) for the desired form

$$\{\dot{q}_4\} = [I]\{q_1\} - [R_0]\{q_3\} - [R_4]\{q_4\} \quad (8)$$

Equations (5), (6a), (6b), and (8) are collected into a single first order simultaneous differential form

$$\begin{Bmatrix} \{\dot{q}_1\} \\ \{\dot{q}_2\} \\ \{\dot{q}_3\} \\ \{\dot{q}_4\} \end{Bmatrix} = \begin{bmatrix} 0 & [I] & 0 & 0 \\ [\hat{a}_1] & [\hat{a}_2] & [\hat{a}_3] & [\hat{a}_4] \\ 0 & 0 & 0 & [I] \\ I & 0 & -[R_0] & -[R_1] \end{bmatrix} + \begin{bmatrix} 0 \\ \left([\hat{M}] - [A_2]\right)^{-1} \\ 0 \\ 0 \end{bmatrix} \{\hat{f}\} \quad (9)$$

In a state space form this can be expressed as

$$\{\dot{X}\} = [A]\{X\} + [B]\{U\} \quad (10)$$

where [A] is called the plant matrix and has all the information that is needed to compute the stability of the linear system.

It should be noted that the approximation of the unsteady aerodynamics is nothing more than an approximation. The engineers needs to understand this problem, asumptions and limitation of the tools being used in the analysis.

III. A Simple Approximation For The Unsteady Aerodynamics In The S Domain

The generalized aerodynamic force coefficients are computed in the k (reduced frequency) domain and cannot be directly transformed in to the s domain. This approach is a variation of a technique that is commonly known as Pade approximation. This technique takes advantage of

$$ik = \frac{b}{v}\,s \quad (11)$$

Many variations of this fit is possible. The unsteady aerodynamic force matrices as a function of k can be examined one term at a time and the fit accomplished in that manner. The coefficients of the fit can then be assembled in a matrix form. Let

$$A(k) = \Re(\hat{Q}_{ij}(k)) \quad (12a)$$

$$B(k) = \Im(\hat{Q}_{ij}(k)) \quad (12b)$$

where i is the integration mode and j is the pressure mode.

Construction of the matrices for different numbers or terms is straight forward once the basic approach is understood. This is in the form of the approximation for the fit is made

$$A(k) + iB(k) = a_0 + (ik)a_1 + (ik)^2 a_2 + \frac{(ik)a_3}{(ik) + b_3} + \frac{(ik)a_4}{(ik) + b_4} \quad (13)$$

where the a's and b's of equation (13) are not known.

If the b's are assigned a value (say $b_3 = 0.5$ and $b_4 = 0.5$),then a solution of the a's is possible (and described below). After the solution is obtained, graphical techniques can be employed to check if the fit was good. If the fit is not good,the b's can be adjusted until a good fit is obtained.

Equation (13) is separated into real and imaginary parts as

$$A(k) = a_0 - k^2 a_2 + \frac{k^2 a_3}{b_3^2 + k^2} + \frac{k^2 a_4}{b_4^2 + k^2} \tag{14a}$$

$$B(k) = k a_1 + \frac{a_3 b_3 k}{b_3^2 + k^2} + \frac{a_4 b_4 k}{b_4^2 + k^2} \tag{14b}$$

Note that as k approaches zero the value of the function becomes

$$A(0) = a_0 \quad ; \quad B(0) = 0$$

The low reduced frequency response is often very important for most aeroservoelastic analyses. Thus, the a term is often assigned to the real part of generalized aerodynamic force evaluated at the lowest (very nearly zero) reduced frequency.

$$a_0 = A(k_1) \tag{15}$$

The imaginary part of the generalized aerodynamic force is zero at k=0 and very small for small values of k. Variations of this assigment of the a_1 term are possible and well within the range of acceptable results.

With the a_0 term assigned, equation (14a) is put into a slightly different form with the unknown a's on the right hand side of the equation.

$$A(k) - a_0 = k^2 a_2 + \frac{k^2 a_3}{b_3^2 + k^2} + \frac{k^2 a_4}{b_4^2 + k^2} \tag{16}$$

The generalized aerodynamic force can be found for a range of reduced frequencies that bound the problem. As with any fit method it is important that the range of reduced frequencies be chosen such that the analysis does not have to extrapolate the data. This curve fitting procedures does not extrapolate very well. On the same note it is also not reasonable to include too large a range of reduced frequencies because 1) the unsteady aerodynamic theory does not hold well for high reduced frequencies and 2) the fit approach puts emphasis on the larger values of the generalized aerodynamic force. Thus it is important that only the needed range of reduced frequencies be chosen for the problem.

The reduced frequency distribution should also be selected with some care. One rule of thumb that the author likes to use is that the delta k should double with each increase in k. Which ever method that is chosen the distribution of reduced frequencies MUST be such that the character of the curve is captured and there is no doubt of the shape of this smooth well behaved curve. Graphic methods can be employed to assure this requirement in the easily stages of the unsteady aerodynamic development.

Equations (14b) and (16) are evaluated at several reduced frequencies (the same that were used to build the generalized aerodynamic forces) and can be put into the following form:

$$\begin{Bmatrix} A(k_1)-a_0 \\ A(k_2)-a_0 \\ A(k_3)-a_0 \\ \vdots \\ B(k_1) \\ B(k_2) \\ B(k_3) \\ \vdots \end{Bmatrix} = \begin{bmatrix} 0 & -k_1^2 & \frac{k_1^2}{b_3^2+k_1^2} & \frac{k_1^2}{b_3^2+k_1^2} \\ 0 & -k_2^2 & \frac{k_2^2}{b_3^2+k_2^2} & \frac{k_2^2}{b_4^2+k_2^2} \\ 0 & -k_3^2 & \frac{k_3^2}{b_3^2+k_3^2} & \frac{k_3^2}{b_4^2+k_3^2} \\ \vdots & \vdots & \vdots & \vdots \\ k_1 & 0 & \frac{b_3 k_1}{b_3^2+k_1^2} & \frac{b_4 k_1}{b_4^2+k_1^2} \\ k_2 & 0 & \frac{b_3 k_2}{b_3^2+k_2^2} & \frac{b_4 k_2}{b_4^2+k_2^2} \\ k_3 & 0 & \frac{b_3 k_3}{b_3^2+k_3^2} & \frac{b_4 k_3}{b_4^2+k_3^2} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{Bmatrix} \tag{17}$$

where the only unknowns are the coefficients a_1, a_2, a_3 and a_4. This equation is in the form of an over determined system of equations. In a short notation this is often written as

$$\{Y\} = [H]\{A\} \tag{18}$$

and solved in a least squares sense (also call a pseudo-inverse method) for the unknown

$$\{A\} = \left([H]^T[H]\right)^{-1}[H]^T\{Y\} \tag{19}$$

The approximation in equation (13) is in a form that is easily used to solve the coefficients but needs some changes to arrive at the final form that is desired. The relationship of ik and s from equation (11) in equation (13) to obtain

$$A(s) + iB(s) = a_0 + \left(\frac{b}{v}\right) s a_1 + \left(\frac{b}{v}\right)^2 s^2 a_2 + \frac{\left(\frac{b}{v}\right) s a_3}{\left(\frac{b}{v}\right) s + b_3} + \frac{\left(\frac{b}{v}\right) s a_4}{\left(\frac{b}{v}\right) s + b_4} \tag{20}$$

The common denominator is obtained for the lag terms making a second order numerator and a second order denominator. This second order form is reduced (by polynomial division) to the normalized (coefficient of the highest denominator term is one) form

$$A(s) + iB(s) = a_0 + a_3 + a_4 + \left(\frac{b}{v}\right)sa_1 + \left(\frac{b}{v}\right)^2 s^2 a_2$$

$$+ \frac{-s\left(\frac{v}{b}\right)(b_4a_3+b_3a_4)-\left(\frac{v}{b}\right)^2 b_3b_4(a_3+a_4)}{s^2+s\left(\frac{v}{b}\right)(b_3+b_4)+\left(\frac{v}{b}\right)^2 b_3b_4} \qquad (20a)$$

which can be written in a simplified notation

$$A(s) + iB(s) = A_0 + sA_1 + s^2A_2 + \frac{sN_1+N_0}{s^2+sR_1+R_0} \qquad (21)$$

where

$$A_0 = a_0 + a_3 + a_4$$

$$A_1 = \left(\frac{b}{v}\right)a_2$$

$$A_2 = \left(\frac{b}{v}\right)^2 a_3$$

$$N_2 = -\left(\frac{v}{b}\right)(b_4a_3+b_3a_4)$$

$$N_0 = -\left(\frac{v}{b}\right)^2 b_3b_4(a_3+a_4)$$

$$R\text{-}1 = \left(\frac{v}{b}\right)(b_3+b_4)$$

$$R_0 = \left(\frac{v}{b}\right)^2 b_3b_4$$

Equation (21) is in scalar form, however, the generalized aerodynamic forces are normally used in a matrix form. The elements of the A_0, A_1, A_2, N_0 and N_1 matrices (i integration mode and j pressure mode) are made up of the corresponding term of the A_0, A_1, A_2, N_0, and N_1 scalars which were obtained in the corresponding unsteady aerodynamic fit of the (i) integration mode and (j) pressure mode. The R_0 and R_1 matrices are diagonal with the scalars values R_0 and R_1 respectively. The matrix equivalent form of equation (21) is

$$[\hat{Q}] = [A_0] + s[A_1] + s^2[A_2] + [N_0][D]^{-1} + s[N_1][D]^{-1} \qquad (22)$$

where

$$[D] = s^2[I] + s[R_1] + [R_0] \qquad (23)$$

Equation (22) is in the final desired form.

IV. Formulation Of A Viscous Damper In Series With A Spring

The purpose of this chapter is to derive the equations of a viscous damper in series with a spring. The force in a spring is related to the spring constant, and the displacement across the spring. The spring force is

$$f = K(x_2-x_1) \qquad (24)$$

The force in a viscous damper is related to the damping coefficient, C, and the velocity across the damper. The damper force is

$$f = C(\dot{x}_3-\dot{x}_2) \qquad (25)$$

Since this is a series connection, the forces are the same for each of the components. The forces in equations (24) and (25) are the same and can be set equal

$$K(x_2-x_1) = C(\dot{x}_3-\dot{x}_2) \qquad (26)$$

To arrive at the desired form several steps are needed. Equation (26) can be simplified by substituting

$$x = x_2-x_1 \quad ; \quad \Delta x = x_3-x_2$$

This simplified equation becomes

$$Kx = C(\Delta\dot{x}-\dot{x}) \qquad (27)$$

A Laplace transformation of the variables in equation (27) can put this equation into a frequency domain. The pertinent transformations are :

$$X = L(x)$$
$$sX = L(\dot{x})$$
$$\Delta X = L(\Delta x)$$
$$s\Delta X = L(\Delta\dot{x})$$
$$F = L(f)$$

With the Laplace Transformations, equ.(27) becomes

$$KX = sC(\Delta X-X) \qquad (28)$$

Equations (28) can be solved for X

$$X = \frac{sC}{sC+K}\Delta X \qquad (29)$$

or

$$X = \frac{s}{s+a}\Delta X \qquad (30)$$

where

$$a = \frac{K}{C}$$

Finally, the force in the system is the stiffness times the displacement across the spring

$$F = \frac{sK}{s+a}\Delta X \qquad (31)$$

Equation (31) is in the desired form.

V. Linear State Space Approach To Airplane With A Stand By Actuator (Viscous Damper and Spring in Series)

Next we will derive the equations of motion of the aircraft equipped with a standby linearized actuator. These equations are put in a suitable form for the determination of stability. This set of equations will be formulated as an eigenvalue problem using state space methods so that the existing software, MATLAB, can be used for the solution and stability determination.

The need for a model of an actuator in the standby mode comes from the way the flight control are integrated into the airplane. The actuator can be "on", "standby", or (failed) "disconnected". In the "on" condition the actuator is modeled with stiffness and damping in series. In the "disconnect" condition the actuator is in a "free" condition with no stiffness or damping (both structural and hydraulic failures).

Previous section formulates the airplane in a state space form with a generalized force as the input. In this section the equations of the actuator force are written and combined with the airplane equations. The actuator moment (output) will be written in terms of the aileron rotation (input). The airplane aileron response (output) will be written in terms of actuator moments (input). These two blocks will be combined into a single system of equations from which the stability will be seen from the eigensolution.

The actuator in the standby mode looks like a velocity squared damper in series with a linear spring. The model for this formulation of the actuator was selected as simply a mathematical curve, because it was desired to have an impedance curve boundary to be used as design criteria of the actuator in the standby mode. The curve selected as the mathematical model of the actuator is

$$f = \frac{Ks}{s+a}\,\Delta x \qquad (32)$$

Though less important, this curve does have some basis in a physical system. This physical system is a viscous damper in series with a spring which is not like the actual actuator. In the next section we will derive the impedance of a viscous damper in series with a spring in the same form as equation (32).

The actuator force acts over an arm (which is a distance to the hinge line) to create a moment on the control surface. This actuator moment is

$$m = hf \qquad (33)$$

A similar relationship exists that relates the actuator displacement and the control surface relative angle (see equation (39) below)

$$\Delta x = h\Delta\theta \qquad (34)$$

A substitution of equations (33) and (34) into (32) gives a relationship of the moment and the control surface relative angle which is

$$m = \frac{h^2 Ks}{s+a}\,\Delta\theta \qquad (35)$$

Transforming to the time domain this can be expressed as a linear first order differential equation

$$\dot{m} + am = h^2 K \Delta\dot{\theta} \qquad (36)$$

Solving for the highest order differential

$$\dot{m} = -am + h^2 K \Delta\dot{\theta} \qquad (37)$$

In a state space form where the input is the relative angular velocity and the output is the generalized moment, equation (37) becomes

$$\dot{m} = A_m m + B_m \Delta\dot{\theta} \qquad (38a)$$

$$m = C_m m + D_m \Delta\dot{\theta} \qquad (38b)$$

where

$$A_m = -a$$
$$B_m = h^2 K$$
$$C_m = 1$$
$$D_m = 0$$

The relative angular velocity, $\Delta\theta$, can be made as the angle on the wing side of the actuator and angle at the control surface side.

$$\Delta\dot{\theta} = \dot{\theta}_c - \dot{\theta}_a \qquad (39)$$

Since the mode shapes and generalized velocity is available these angles can be computed and the expression becomes

$$\Delta\dot{\theta} = (\lfloor\phi_c\rfloor - \lfloor\phi_a\rfloor)\,\{q_2\} \qquad (40)$$

or for shorter notation

$$\Delta\dot{\theta} = \lfloor\Delta\phi\rfloor\,\{q_2\} \qquad (41)$$

The airplane was put into a first order differential form in equation (9). This form is

$$\begin{Bmatrix} \{\dot{q}_1\} \\ \{\dot{q}_2\} \\ \{\dot{q}_3\} \\ \{\dot{q}_4\} \end{Bmatrix} = \begin{bmatrix} 0 & [I] & 0 & 0 \\ [\hat{a}_1] & [\hat{a}_2] & [\hat{a}_3] & [\hat{a}_4] \\ 0 & 0 & 0 & [I] \\ I & 0 & -[R_0] & -[R_1] \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{Bmatrix} + \begin{bmatrix} 0 \\ ([\hat{M}]-[A_2])^{-1} \\ 0 \\ 0 \end{bmatrix} \{\hat{f}\} \qquad (42)$$

The generalized force can be expressed in terms of the actuator moment as

$$\{\hat{f}\} = \lfloor\phi_a\rfloor^T m \qquad (43)$$

Combining equations (38a),(38b), (39),(41), and (43),

$$\begin{Bmatrix} \dot{m} \\ \{\dot{q}_1\} \\ \{\dot{q}_2\} \\ \{\dot{q}_3\} \\ \{\dot{q}_4\} \end{Bmatrix} = \begin{bmatrix} -a & 0 & h^2K\lfloor\Delta\phi\rfloor & 0 & 0 \\ 0 & 0 & [I] & 0 & 0 \\ \{*\} & [\hat{a}_1] & [\hat{a}_2] & [\hat{a}_3] & [\hat{a}_4] \\ 0 & 0 & 0 & 0 & [I] \\ 0 & [I] & 0 & -[R_0] & -[R_1] \end{bmatrix} \begin{Bmatrix} m \\ \{q_1\} \\ \{q_2\} \\ \{q_3\} \\ \{q_4\} \end{Bmatrix} \quad (44)$$

where

$$\{*\} = \left([\hat{M}]-[A_2]\right)^{-1} \lfloor\phi_a\rfloor^t$$

or in a short notation

$$\{\dot{X}\} = [A]\{X\} \quad (45)$$

The eigenvalues can be obtained in the standard form

$$|[A]-\lambda[I]| = 0 \quad (46)$$

These eigenvalues of the A matrix look like the roots of the characteristic equation. For the complex conjugate (oscillatory) roots

$$\zeta = \Re(\lambda)/|\lambda| \quad (47a)$$

$$\omega = |\lambda| \quad (47b)$$

Plots of the velocity vs damping and velocity vs frequency and/or root locus give an understanding of the stability boundary and stability characteristics.

Results and Discussions

Using the equations developed in preceding sections, computations have been performed for all the failure cases. It should be noted, however, that in the present paper, the method has been developed for the most critical case 3.

MSC/NASTRAN software package has been utilized to calculate the generalized mass M, mode shapes, natural frequencies and the generalized unsteady aerodynamics (using doublet lattice scheme). For the calculation of flutter solution using p-method, MATLAB software package has been utilized. Solutions of flutter equation using both. MSC/NASTRAN PK and MATLAB p-methods, are satisfactorily close [1].
Some of the results, indicating the flutter solution in frequency velocity and damping velocity diagrams, are exhibited in Figures 3 to 8, for various values of stiffness to viscous damping ratio a. The results show that all cases being considered meet the flutter free envelope requirements, i.e. the flutter velocity is larger than V_D, for damping ratio of 0.03%.

Concluding Remarks

A method for aero-servo-elastic analysis of an airplane with a stand-by actuator using state space approach has been developed. For the particular aircraft considered, one actuator is normally in an active mode and the other is normally in a passive stand-by mode. In a failure case, in which the active mode actuator failed, the stand-by actuator must provide sufficient damping in order that the airplane can still maintain flutter free condition. The actuator used in this analysis was assumed to exhibit linear transfer fuction and viscous damping behaviour. The eigen-mode, eigen-frequencies and generalized mass were calculated with V-g method available in standard software packages. The calculation of flutter solution is performed using p-method. The procedure has been utilized in the design to meet certification requirements. The computational results show that flutter free requirements up to V_D can be met in all failure cases, for damping ratio of 0.03%, and for various values of stiffness to viscous damping ratio.

References

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2. Moore, John E. and Yan Mursal, *A Simple Approximation for the Unsteady Aerodynamics in the S-Domain*, Internal Report, Department of Aeroelasticity, PT IPTN, Bandung, 1992.

3. Dotson, Benny F. and Irianto, Bambang, *Notes on N-250 Flight Control System*, Technical Notes, Department of Aeroelasticity, PT IPTN, Bandung, 1992.

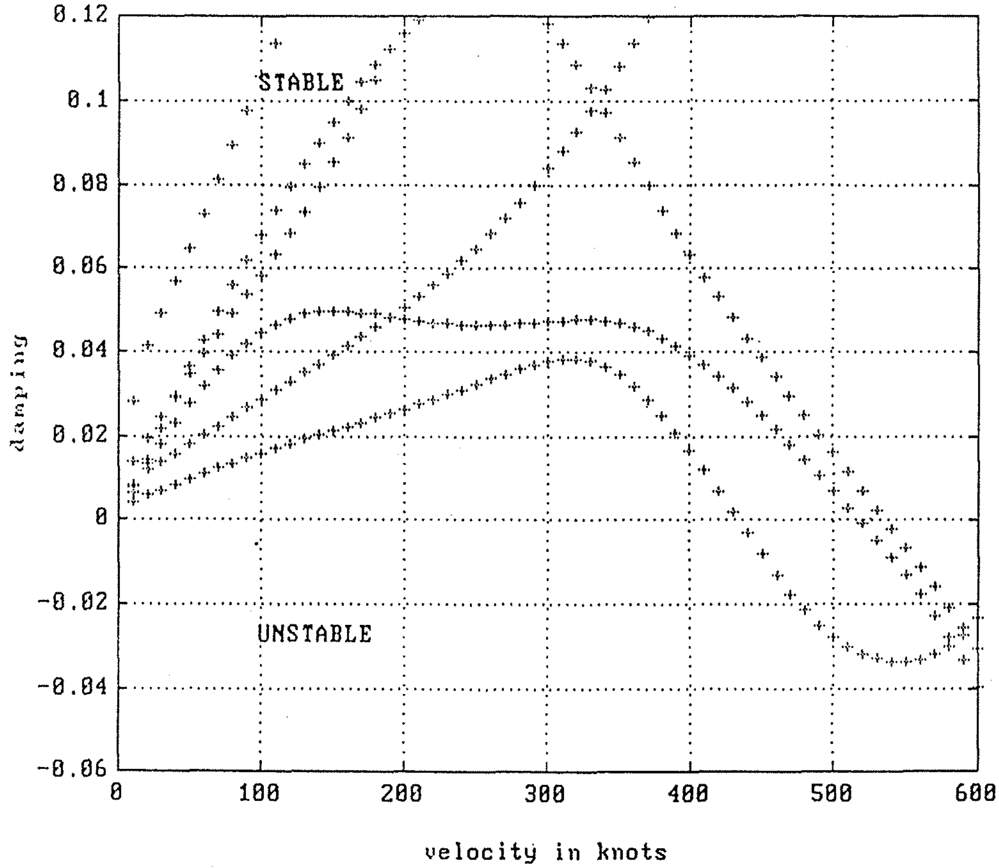

Figure 3. Velocity - Damping Diagram for a = 75 (MATLAB)

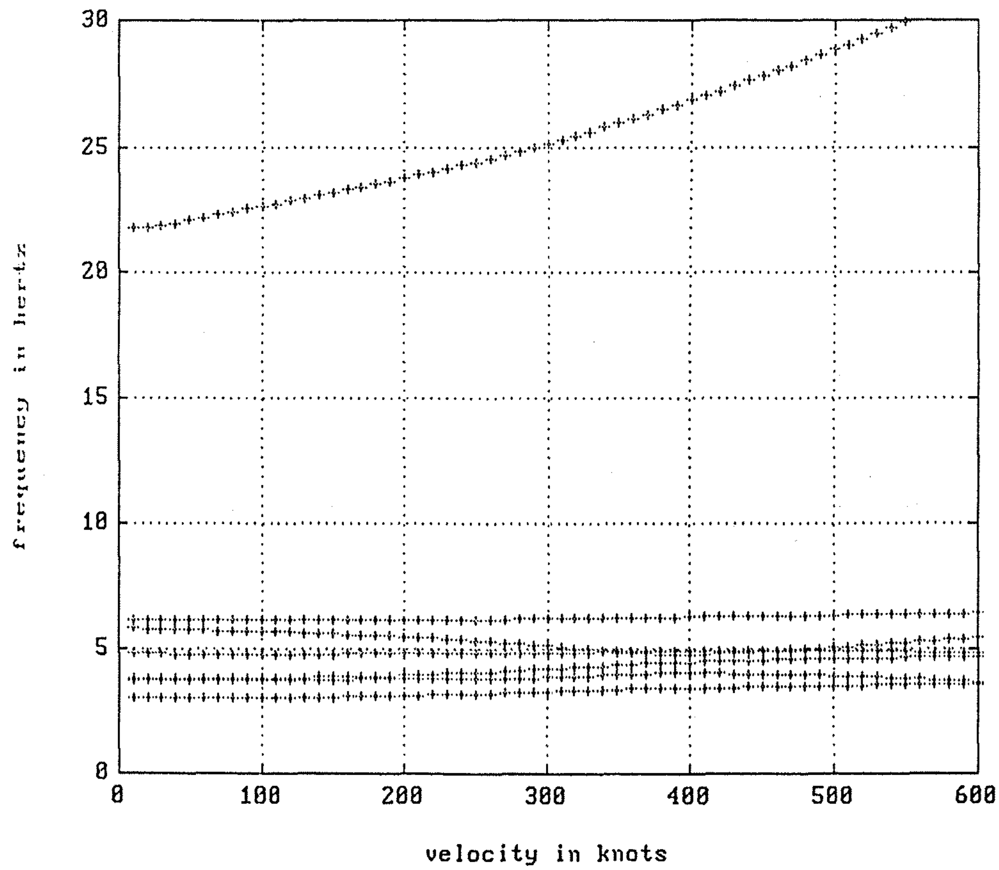

Figure 4. Velocity - Frequency Diagram for a = 75 (MATLAB)

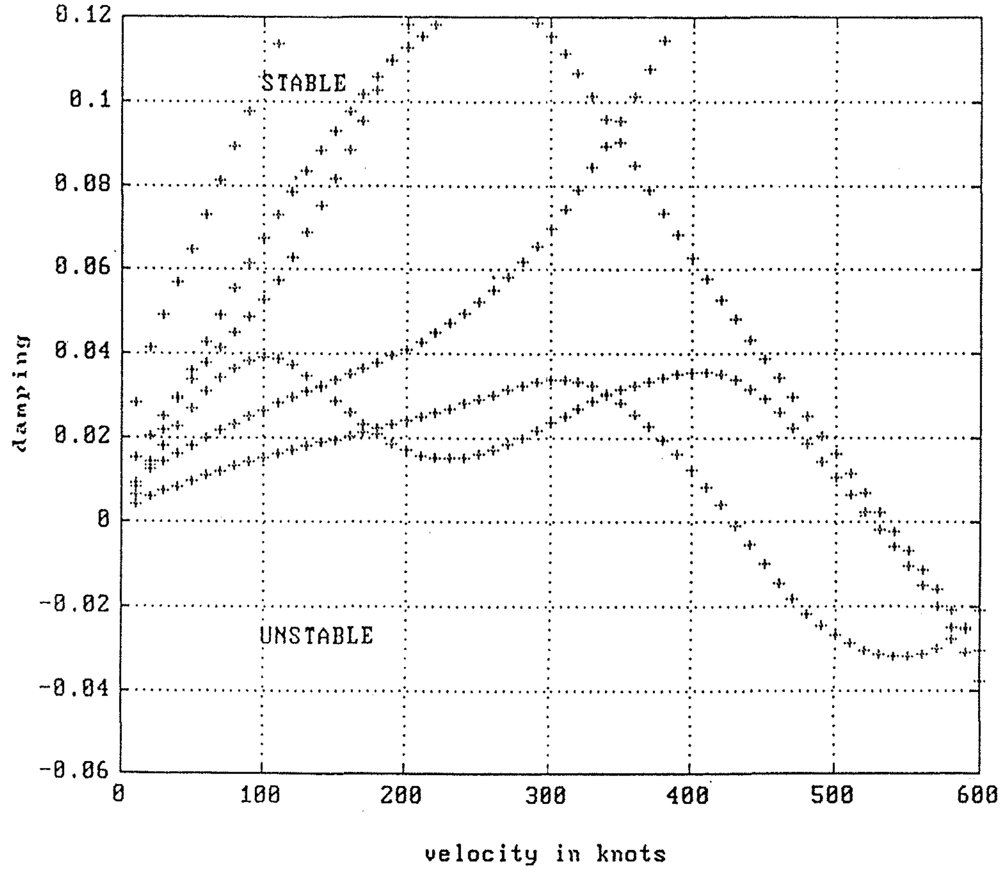

Figure 5. Velocity - Damping Diagram for a = 125 (MATLAB)

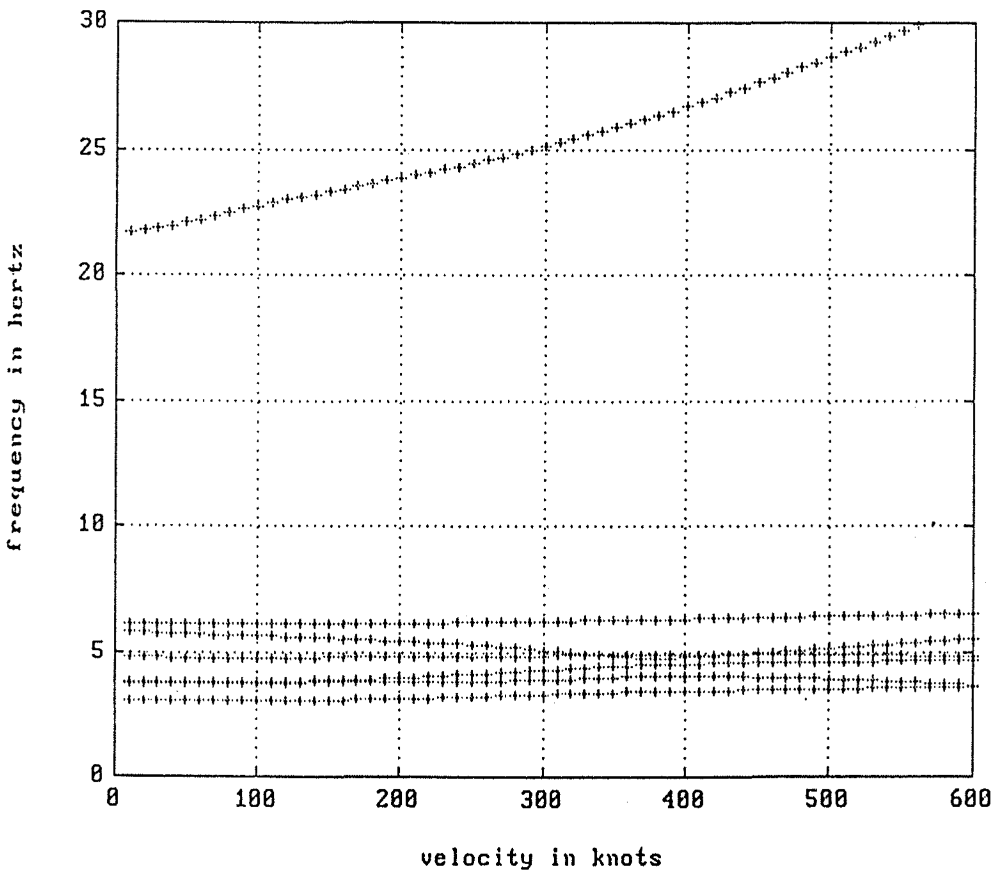

Figure 6. Velocity - Frequency Diagram for a = 125 (MATLAB)

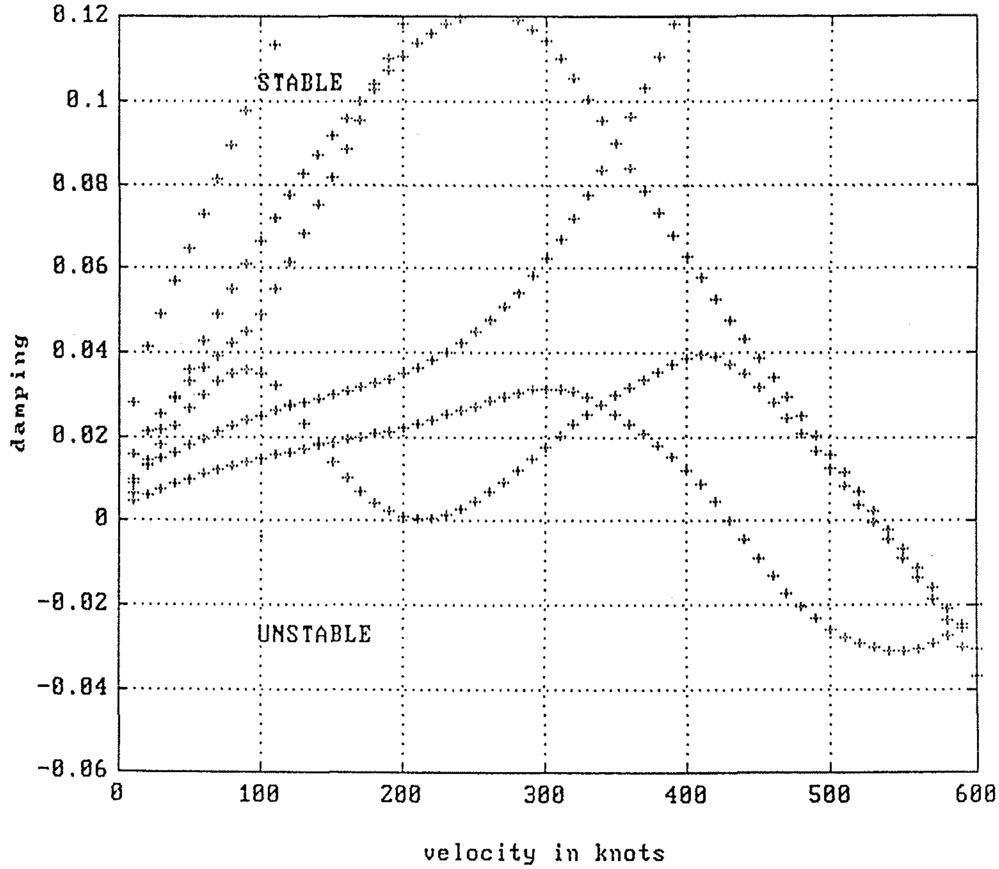

Figure 7. Velocity - Damping Diagram for a = 160 (MATLAB)

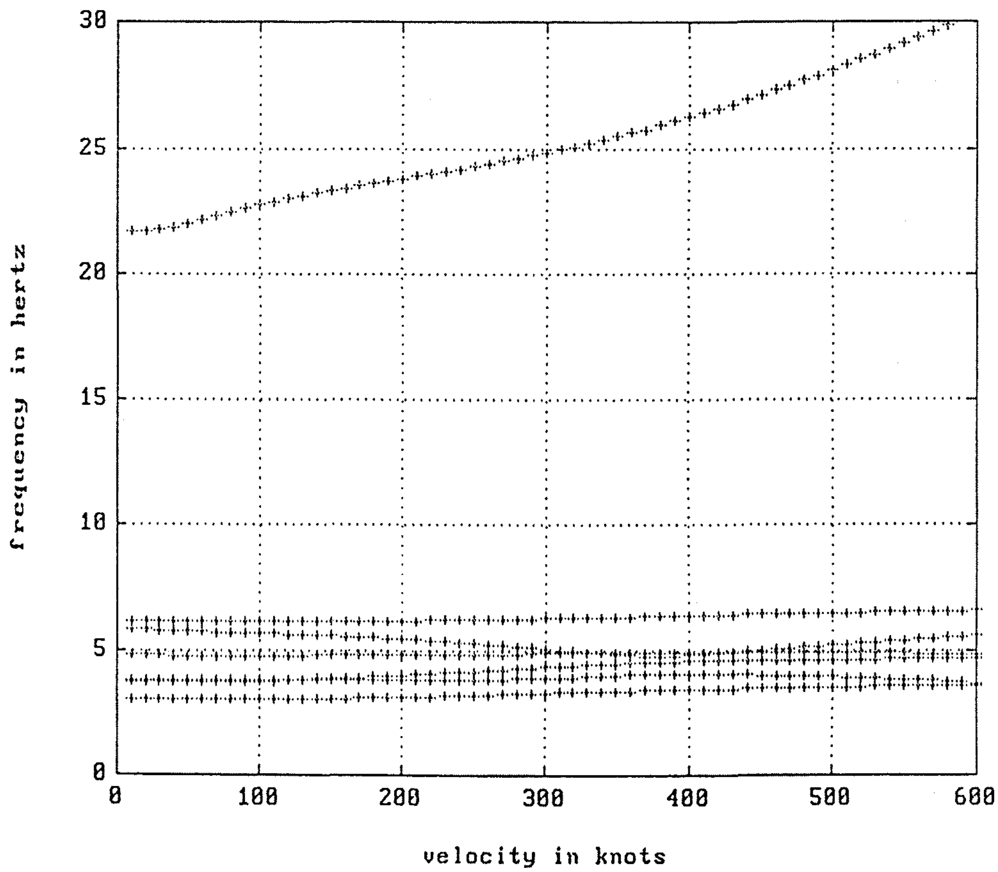

Figure 8. Velocity - Frequency Diagram for a = 160 (MATLAB)