REDUCTION IN THE VARIATION OF AIRCRAFT RESPONSE CHARACTERISTICS DURING OPTIMAL TRAJECTORY MANOEUVRES

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Abstract:-Aircraft trajectory optimisation frequently results in significant variation in the short period response of the aircraft throughout the optimal manoeuvre. The optimisation problem of acquiring maximum height in a fixed time while minimising a function of drag and satisfying a desired terminal constraint on velocity is considered . The resultant two-point boundary value problem is solved by a combination of the methods of steepest descent and quasilinearisation. The variation of the short-period dynamics on the optimal trajectory is investigated and in this example the steady-state gain, damping ratio, natural frequency and lead time constant vary by factors of up to ten to one. Scheduling of a command stability augmentation system with respect to auxiliary variables such as dynamic pressure, mach number and height, reduces this variation . It is shown that on the optimal trajectory the gain scheduling is not single- valued, resulting in a complex non-linear gain adjustment algorithm. A unique relationship between aircraft parameter variations and controller gains is determined and the combination of these provides a uniform pitch rate response characteristic throughout the optimal trajectory. The paper investigates the use of a quasilinearisation based algorithm for the on-line identification and tracking of the aircraft parameters. The subsequent adaptation and re-optimisation of the controller is performed to minimise the error between a desired optimal transient pitch rate response and the actual system response. This re-optimisation of system performance is achieved using an on-board digital model of the identified aircraft.

Introduction

This paper investigates the variation of aircraft response characteristics during an optimal trajectory manoeuvre. The optimal climb manoeuvre of maximising height acquired in a fixed time while satisfying a desired terminal constraint on the final velocity and minimising a function of drag has been chosen as the starting point for this investigation. This optimal manoeuvre has been specifically chosen as the aircraft encounters a significant portion of the flight envelope in performing the task. Also the solution of a similar optimisation problem is available[1,2] and it has therefore been possible to verify optimisation software developed for use in this investigation. In particular the variation of the aircraft parameters defining the small perturbations equations of motion representing the short period pitch response of the aircraft are investigated throughout the optimal trajectory manoeuvre. It is shown that the aircraft parameters are not in general single valued with respect to auxiliary variables such as dynamic pressure, mach number etc.; hence it is difficult to determine a satisfactory gain scheduling control law for a command stability augmentation system which will provide a uniform response characteristic throughout the manoeuvre. An on-line identification scheme is investigated to identify and track the parameters during the manoeuvre and it is these identified parameter values which are used to adapt the C.S.A.S. parameters in place of the normal auxiliary variables.

A relationship between aircraft and controller parameters is obtained which significantly reduces the variation in aircraft closed-loop pitch rate to pitch rate demand response throughout the trajectory. This closed loop adaptive system operates within the transient response time of the aircraft and maintains the transient response uniform for subsequent command inputs. It should be noted however that a finite identification period is required to establish the aircraft parameters and adapt the C.S.A.S. During this period the system transient response can deviate from the desired nominal transient response. At the end of the identification interval the adapted system dynamics are defined and this information is used to augment the control to correct these deviations in the response and return the transient response to the nominal desired transient response in an optimised manner. The augmented control which minimises an error function between desired nominal and actual transient response can also be operative during the identification and adaptation interval. This optimal adaptive controller reduces the deviations in the initial transient response from the nominal when there may be substantial mismatch between the dynamics of the closed loop adaptive system and those of the nominal transient response.

Climb Optimisation

The necessary conditions for optimal control to minimise a generalised cost functional of the form

$$J=\phi(\underline{x}(t_f),t_f)+\underline{\gamma}^T\psi(\underline{x}(t_f),t_f)+\int_{t_0}^{t_f}\{L(\underline{x},\underline{u},t)+\underline{\lambda}^T(f(\underline{x},\underline{u},t)-\dot{\underline{x}})\}dt$$

subject to dynamic constraints $\dot{\underline{x}}(t) = f(\underline{x},\underline{u},t)$ and specified terminal conditions are given below.

The Euler Lagrange equations:

$$\dot{\underline{\lambda}} = -\left(\frac{\partial f}{\partial x}\right)^T \underline{\lambda} - \left(\frac{\partial L}{\partial x}\right)^T$$

The optimality condition for unconstrained controls

$$\frac{\partial L}{\partial u} + \underline{\lambda}^T\left(\frac{\partial f}{\partial u}\right) = 0$$

The co-state terminal conditions

$$\underline{\lambda}^T(t_f) = \left(\frac{\partial \phi}{\partial x}\right)_{t_f} + \underline{\gamma}^T\left(\frac{\partial \psi}{\partial x}\right)_{t_f}$$

A fixed time problem has been considered in this instance for ease of computation. The actual cost functional chosen to be minimised for this maximum height in a fixed time problem was chosen as

$$J = -h(t_f) + \gamma_1(V(t_f) - 968.) + \frac{1}{2}\int_{t_0}^{t_f}\alpha^2 dt.$$

The initial conditions used for the state equations were as defined in the boundary conditions for the study. The optimisation period used was 332.0 sec. The system states were unconstrained to simplify the problem. The variation in engine thrust characteristics and aerodynamic data with Mach number are as shown in (Figs 16-17) and an interpolation procedure was used to generate the appropriate values and required partial derivatives at each time step in the integration process. From the aircraft forces diagram (Fig. 1) and applying the above necessary conditions the state and co-state equations are as follows.

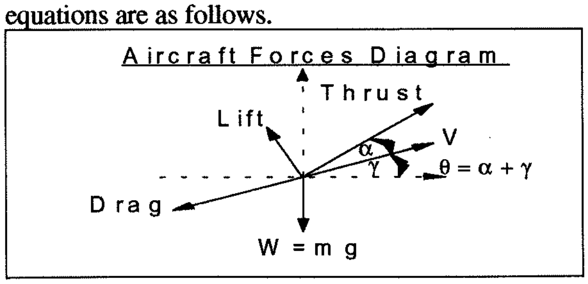

FIGURE 1

The State and Co-state Equations

$$\dot{V} = \frac{1}{m}\{T\cos\alpha - D - mg\sin\gamma\}$$

$$\dot{\gamma} = \frac{1}{mV}\{L + T\sin\alpha - mg\cos\gamma\}$$

$$\dot{h} = V\sin\gamma$$

$$\dot{x} = V\cos\gamma$$

$$\dot{m} = -\frac{T}{mg}$$

$$\dot{\lambda}_V = \frac{\lambda_V}{m}\left(\frac{\partial D}{\partial V} - \frac{\partial T}{\partial V}\cos\alpha\right) + \frac{\lambda_\gamma}{mV^2}\{(L + T\sin\alpha - mg\cos\gamma) - V\left(\frac{\partial T}{\partial V}\sin\alpha + \frac{\partial L}{\partial V}\right)\} - \lambda_h\sin\gamma - \lambda_x\cos\gamma + \frac{\lambda_m}{cg}\frac{\partial T}{\partial V}$$

$$\dot{\lambda}_\gamma = \lambda_V g\cos\gamma - \lambda_\gamma\frac{g}{V}\sin\gamma - \lambda_h V\cos\gamma + \lambda_x V\sin\gamma$$

$$\dot{\lambda}_h = \frac{\lambda_V}{m}\left(\frac{\partial D}{\partial h} - \frac{\partial T}{\partial h}\cos\alpha\right) - \frac{\lambda_\gamma}{mV}\left(\frac{\partial T}{\partial h}\sin\alpha + \frac{\partial L}{\partial h}\right) + \frac{\lambda_m}{cg}\frac{\partial T}{\partial h}$$

$$\dot{\lambda}_x = 0$$

$$\dot{\lambda}_m = \frac{\lambda_V}{m^2}(T\cos\alpha - D) + \frac{\lambda_\gamma}{m^2 V}(L + T\sin\alpha)$$

The Optimal Control

The optimal control obtained from the optimality condition is given by

$$\alpha - \frac{\lambda_V}{m}\{T\sin\alpha + \frac{\partial D}{\partial\alpha}\} + \frac{\lambda_\gamma}{mV}\{\frac{\partial L}{\partial\alpha} + T\cos\alpha\} = 0$$

The Boundary Conditions

$V(t_0) = 400.0\,ft.\sec^{-1}$	$\lambda_V(t_f) = \gamma_1$
$\gamma(t_0) = 0.0\,rad.$	$\lambda_\gamma(t_f) = 0.0$
$h(t_0) = 700.0\,ft.$	$\lambda_h(t_f) = -1.$
$x(t_0) = 0.0\,ft.$	$\lambda_x(t_f) = 0.0$
$m(t_0) = 1304.\,slugs$	$\lambda_m(t_f) = 0.0$

This complete set of equations constitute a non-linear two-point boundary value problem. The solution has been obtained by a combination of both steepest-descent and quasilinearisation iterative computational techniques. In the steepest-descent method a starting vector was chosen for the control and the state equations were integrated forward in time. At the end of the optimisation interval the terminal condition on $\lambda_V(t_f)$ was set to a weighted function of the error between the computed and desired terminal value of $V(t_f)$ and the co-state equations were integrated backwards in time using the solution of the state equations obtained in the forward integration. A new control vector was computed from

$$\alpha_{N+1} = \alpha_N - \tau\left[\alpha - \frac{\lambda_V}{m}\{T\sin\alpha + \frac{\partial D}{\partial\alpha}\} + \frac{\lambda_\gamma}{mV}\{\frac{\partial L}{\partial\alpha} + T\cos\alpha\}\right]_N$$

where τ controlled the step length along the gradient, and the process was repeated until the terminal error on $V(t_f)$ was within a small norm. To test the quasilinearisation programme which is required for the on-line identification process, the state and co-state equations were first linearised about the solution obtained from the steepest descent procedure which was then used as a starting vector for the quasilinearisation method of solution of the two-point boundary value problem. This process was iterated to convergence and a small improvement was obtained in the maximum acquired height with the terminal condition on $V(t_f)$ exactly satisfied. The results obtained by both methods of the optimum height versus Mach profiles are shown in (Fig 2.)

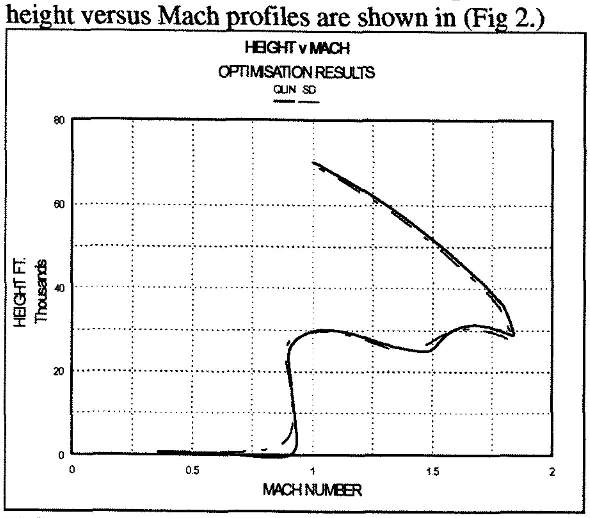

FIGURE 2.

From the results obtained in the optimisation process the nominal pitch attitude time history was computed from the optimum time profiles for flight path angle and angle of attack. Differentiation of this generated a desired pitch rate profile to be followed in order to fly the optimum manoeuvre. This signal was used as the excitation for the combined C.S.A.S. aircraft system to investigate the on-line identification of the aircraft parameters.

Parameter Variations on the Optimal Trajectory

Throughout the optimum trajectory at every time step in the integration procedure the parameters of the small perturbation pitch rate per elevator transfer function were computed.

$$\frac{q}{\eta} = \frac{K_0\omega_n^2(1 + sT)}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

The variation of the d.c. gain, natural frequency, damping ratio and lead time constant are shown with respect to dynamic pressure and Mach number in (Figs. 3-6)

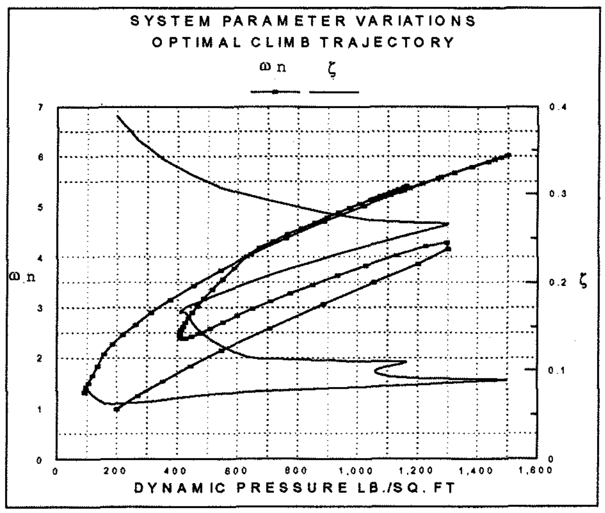

FIGURE 3

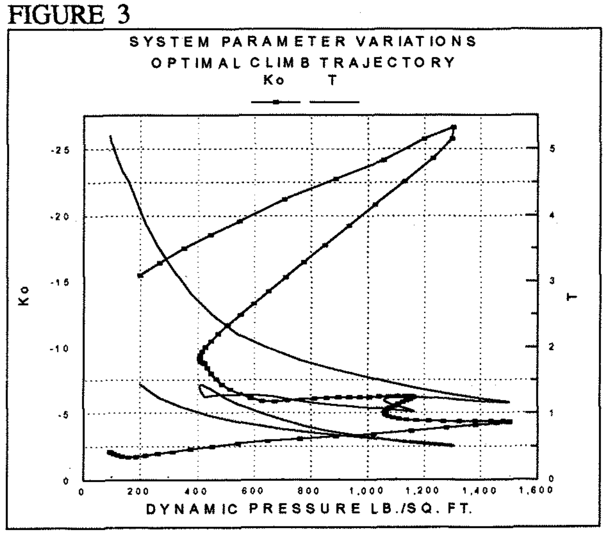

FIGURE 4

It is readily seen that during the optimal trajectory the d.c. gain K_0 varies by a ratio of approximately ten to one, the natural frequency by six to one and the lead time constant by ten to one, while the damping ratio varies from a value of 0.4 at the start of the trajectory down to about 0.06 towards the end. It is also seen that the parameter variations are very non linear and not single-valued . This increases the complexity of devising a simple gain scheduling algorithm for the C.S.A.S. .

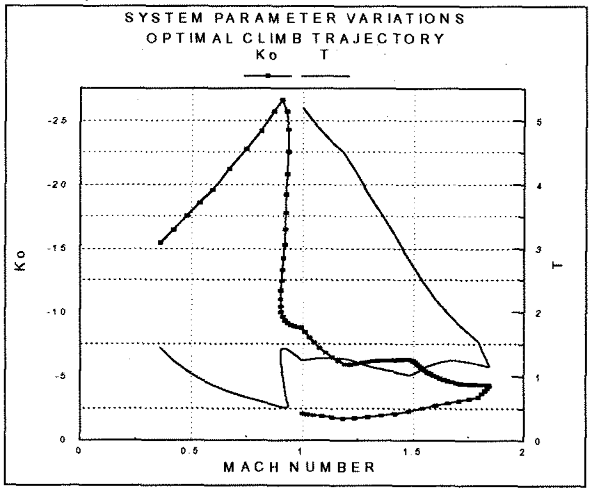

FIGURE 5.

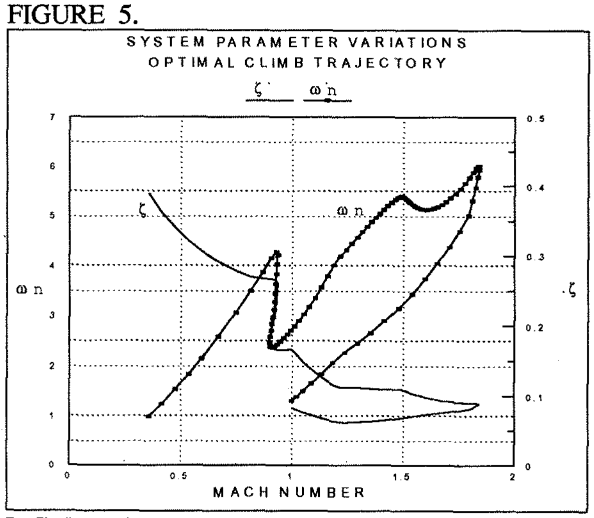

FIGURE 6.

Because of these difficulties an on-line adaptation of the C.S.A.S. with respect to continuously identified parameters was considered.

C.S.A.S. Structure.

A simple proportional plus lagged integral controller of the form of (1) was chosen as the C.S.A.S. for the purpose of the investigation.

$$\frac{\eta_D(s)}{q_{ERROR}(s)} = K_c\{1 + \frac{K_i}{s(1 + sT_c)}\} \qquad (1)$$

The relationship between controller and aircraft short period parameters used in this study is defined as

$$K_c \propto \frac{2\xi}{K_0\omega_n}, \quad T_c = \frac{1}{2\xi\omega_n}, \quad K_i = T_c\omega_n^2$$

where the constant of proportionality on K_c is selected to give the desired break frequency of the combined aircraft and C.S.A.S. closed loop response characteristic. For the purpose of the investigation this was set at 4 rad./sec. To on-line adapt the C.S.A.S. parameters, the aircraft parameters including the lead time constant were identified and tracked during the optimal trajectory. The envelope of transient closed loop pitch rate step responses of the adapted C.S.A.S. aircraft system is shown in (Fig 7) and the uniformity of response obtained is clearly demonstrated. The small variation in this envelope is caused by the ratio of the aircraft lead-time constant to the controller lag- time constant . For the purpose of comparison the envelope of transient responses obtained with a set of fixed parameter settings for the C.S.A.S. is shown in (Fig. 8). The controller parameters were set to the mid point of their adaptive range in this exercise. The spread of response is evident in both natural frequency and damping ratio and at some flight cases on the climb trajectory the system is unstable. Investigations have indicated that this response characteristic could be improved by scheduling the integrator gain as a function of Mach, however this is not exact and so the on-line identification and adaptation procedure has been implemented.

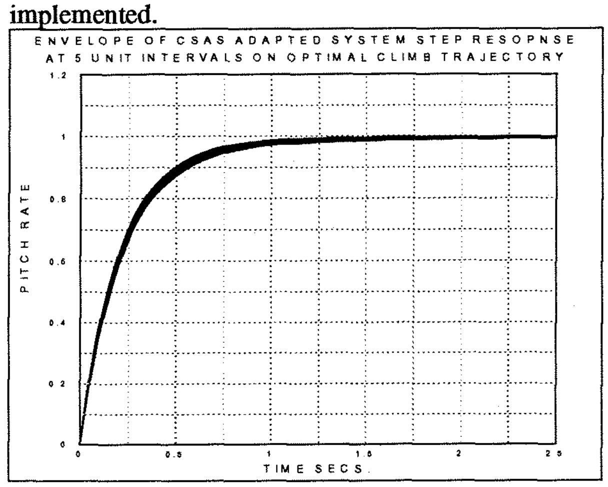

FIGURE 7

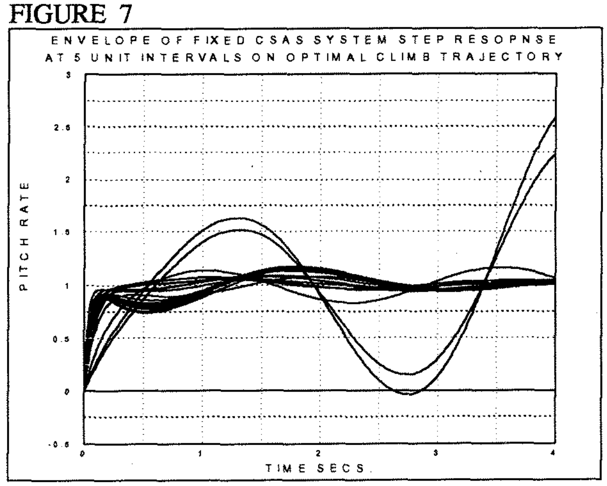

FIGURE 8

On-line Parameter Identification And Tracking.

The method of on-line identification selected is that of quasilinearisation[4, 5,6]. In general a non-linear system of the form $\dot{\underline{x}} = f(\underline{x}, \underline{u}, \underline{k}, t)$ where $\underline{k}$ represents the unknown time varying parameter set, is linearised using the Newton-Raphson algorithm

$$\begin{bmatrix} \dot{\underline{x}} \\ \dot{\underline{k}} \end{bmatrix}_{n+1} = [J(\underline{x}_n; \underline{k}_n)] \begin{bmatrix} \underline{x}_{n+1} - \underline{x}_n \\ \underline{k}_{n+1} - \underline{k}_n \end{bmatrix} + f(\underline{x}_n, \underline{u}_n, \underline{k}_n, t)$$

Although the actual system parameters are time varying, it is assumed for the purpose of identification that during the short time periods required for identification they are constant. At the end of each identification the best-fit constant values for the parameters is determined. The identification process is continuous and in this manner the unknown time varying parameters are tracked as piece-wise constant values. The results are similar to a discrete sampling of the time varying parameters. In the case under consideration of the identification of the short period dynamics of the aircraft four unknown parameters are required to be identified. Re-defining the aircraft system as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} k_3 \\ k_4 \end{bmatrix} u$$

with x_1 representing the pitch rate of the aircraft and u as the elevator input from the C.S.A.S. controller, the unknown parameters are now k_1 , k_2 , k_3 , k_4 where

$k_1 = 2\xi\omega_n$, $k_2 = \omega_n^2$, $k_3 = K_0\omega_n^2 T$, $k_4 = K_0\omega_n^2$

The linearised equations become

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{k}_1 \\ \dot{k}_2 \\ \dot{k}_3 \\ \dot{k}_4 \end{bmatrix}_{N+1} = \begin{bmatrix} -k_{1N} & 1 & -x_{1N} & 0 & u_N & 0 \\ -k_{2N} & 0 & 0 & -x_{1N} & 0 & u_N \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1N+1} - x_{1N} \\ x_{2N+1} - x_{2N} \\ k_{1N+1} - k_{1N} \\ k_{2N+1} - k_{2N} \\ k_{3N+1} - k_{3N} \\ k_{4N+1} - k_{4N} \end{bmatrix} + \begin{bmatrix} -k_{1N}x_{1N} + x_{2N} + k_{3N}u_N \\ -k_{2N}x_{1N} + k_{2N}u_N \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The identification task is now a boundary value problem where the initial conditions on the unknown parameters have to be selected such that the states of an identification model take on those of the aircraft states during the identification interval. As there are twice as many unknown parameters as there are states, two points in the identification interval are chosen , namely the mid point and end point of the interval from which the unknown initial conditions of the parameters are computed. The identification procedure commences with selecting a set of starting vectors for the coefficients of the Jacobian matrix. The obvious choice for initialising the iteration procedure is to use actual measurements of the aircraft system states and actual control input to the system. Starting vectors for the four unknown parameters are chosen as $k_i = g_i$ where the g_i are constants equivalent to the mid point of the range of the individual parameters. Expanding the linearised equations and making the above substitutions, the linearised equations become

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{k}_1 \\ \dot{k}_2 \\ \dot{k}_3 \\ \dot{k}_4 \end{bmatrix}_{N+1} = \begin{bmatrix} -g_1 & 1 & -x_{1s} & 0 & u_s & 0 \\ -g_2 & 0 & 0 & -x_{1s} & 0 & u_s \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1N+1} \\ x_{2N+1} \\ k_{1N+1} \\ k_{2N+1} \\ k_{3N+1} \\ k_{4N+1} \end{bmatrix} + \begin{bmatrix} g_1 x_{1s} \\ g_2 x_{1s} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The solution of these equations consists of a particular integration obtained with initial conditions on the unknown parameters of $k_i(0) = g_i$, together with a linear combination of ,in this instance four, sets of homogeneous solutions. The overall time solution during the identification interval is given by

$$\begin{bmatrix} x_{1m}(t) \\ x_{2m}(t) \\ k_1(t) \\ k_2(t) \\ k_3(t) \\ k_4(t) \end{bmatrix} = C_1 \begin{bmatrix} x_{1H1}(t) \\ x_{2H1}(t) \\ k_{1H1}(t) \\ k_{2H1}(t) \\ k_{3H1}(t) \\ k_{4H1}(t) \end{bmatrix} + C_2 \begin{bmatrix} x_{1H2}(t) \\ x_{2H2}(t) \\ k_{1H2}(t) \\ k_{2H2}(t) \\ k_{3H2}(t) \\ k_{4H2}(t) \end{bmatrix} + C_3 \begin{bmatrix} x_{1H3}(t) \\ x_{2H3}(t) \\ k_{1H3}(t) \\ k_{2H3}(t) \\ k_{3H3}(t) \\ k_{4H3}(t) \end{bmatrix} + C_4 \begin{bmatrix} x_{1H4}(t) \\ x_{2H4}(t) \\ k_{1H4}(t) \\ k_{2H4}(t) \\ k_{3H4}(t) \\ k_{4H4}(t) \end{bmatrix} + \begin{bmatrix} x_{1P.I.}(t) \\ x_{2P.I.}(t) \\ k_{1P.I.}(t) \\ k_{2P.I.}(t) \\ k_{3P.I.}(t) \\ k_{4P.I.}(t) \end{bmatrix}$$

EQUATION 2

It should be noted that the particular integration system as defined is a mathematical model of the aircraft system having an identical structure but with estimates for the unknown parameters. As the choice of the parameters are only estimates they will initially be incorrect and the model responses will not match the actual system state responses. These therefore have to be corrected by the linear combinations of homogeneous integrations. The initial conditions of the states of the model for the particular integration are set to the values of the actual system states pertaining at the start of the identification interval. The initial conditions for each set of homogeneous solution are defined as follows and are specifically chosen to simplify subsequent computation.

$$\begin{bmatrix} x_{1Hi} \\ x_{2Hi} \\ k_{1Hi} \\ k_{2Hi} \\ k_{3Hi} \\ k_{4Hi} \end{bmatrix}_{t=0} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{i=1} ; \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}_{i=2} ; \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}_{i=3} ; \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}_{i=4}$$

The complete sets of homogeneous integrations and the particular integration are computed simultaneously from the overall system of equations defined in (3).

The four weighting constants C_i of the sets of homogeneous solutions in (2) are computed from (4) where t_1 is the mid point of the identification interval and t_f is the end point.

$$\begin{bmatrix} \dot{x}_{1s} \\ \dot{x}_{2s} \\ \dot{x}_{1P.I.} \\ \dot{x}_{2P.I.} \\ \dot{x}_{1H1} \\ \dot{x}_{2H1} \\ \dot{x}_{1H2} \\ \dot{x}_{2H2} \\ \dot{x}_{1H3} \\ \dot{x}_{2H3} \\ \dot{x}_{1H4} \\ \dot{x}_{2H4} \end{bmatrix} = \begin{bmatrix} -k_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -g_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -g_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -g_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -g_2 & 0 & 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & -g_1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -g_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g_1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g_1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -g_2 & 0 \end{bmatrix} \begin{bmatrix} x_{1s} \\ x_{2s} \\ x_{1P.I.} \\ x_{2P.I.} \\ x_{1H1} \\ x_{2H1} \\ x_{1H2} \\ x_{2H2} \\ x_{1H3} \\ x_{2H3} \\ x_{1H4} \\ x_{2H4} \end{bmatrix} + \begin{bmatrix} k_3 u_s \\ k_4 u_s \\ g_3 u_s \\ g_4 u_s \\ -x_{1s} \\ 0 \\ 0 \\ -x_{1s} \\ u_s \\ 0 \\ 0 \\ u_s \end{bmatrix}$$

EQUATION 3.

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} x_{1H1}(t_1) & x_{1H2}(t_1) & x_{1H3}(t_1) & x_{1H4}(t_1) \\ x_{2H1}(t_1) & x_{2H2}(t_1) & x_{2H3}(t_1) & x_{2H4}(t_1) \\ x_{1H1}(t_f) & x_{1H2}(t_f) & x_{1H3}(t_f) & x_{1H4}(t_f) \\ x_{2H1}(t_f) & x_{2H2}(t_f) & x_{2H3}(t_f) & x_{2H4}(t_f) \end{bmatrix}^{-1} \begin{bmatrix} x_{1s}(t_1) - x_{1PI}(t_1) \\ x_{2s}(t_1) - x_{2PI}(t_1) \\ x_{1s}(t_f) - x_{1PI}(t_f) \\ x_{2s}(t_f) - x_{2PI}(t_f) \end{bmatrix}$$

EQUATION 4.

The corrected or identified values for the unknown parameters during the identification interval then become $k_i = C_i + g_i$.

As the small perturbation representation of the aircraft system dynamics is linear, convergence to the correct values of the identified parameters is single step and it is unnecessary to iterate the procedure to obtain convergence. If however a non-linear representation of aircraft dynamics had been chosen it would have been necessary to perform several iterations before a satisfactorily-converged identification is achieved. This would involve an extension of the lapse time to achieve identification; however this situation can be alleviated by time scaling equations (3) for subsequent iterations after the first, which of necessity is computed in real time in synchronism with the actual aircraft response. The identified values of the system parameters so determined by the above procedure become the starting estimates of the unknown parameters for the next identification interval. In this manner the identification process is continuous and the time varying parameters are tracked as constants during each identification interval. Any identification process requires persistency of excitation and this can be checked at each identification step by determining that the matrix of homogeneous solutions in (4) is non- singular. Should this prove not to be the case the values of the identified parameters are held at the last-identified values until persistency of excitation resumes. The results of the identification and tracking of the four aircraft parameters while being controlled by the adaptive system on the optimum climb trajectory are shown in (Figs 9-10). The actual aircraft parameters and the identified parameters are superimposed on each other and the accuracy of the identification and tracking of all four system parameters is evident.

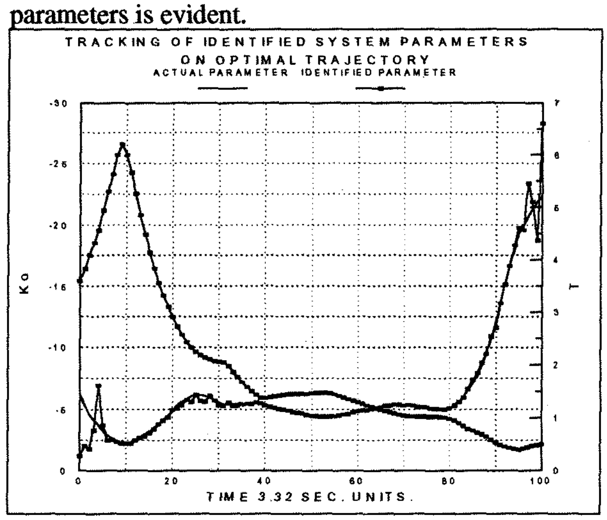

FIGURE 9

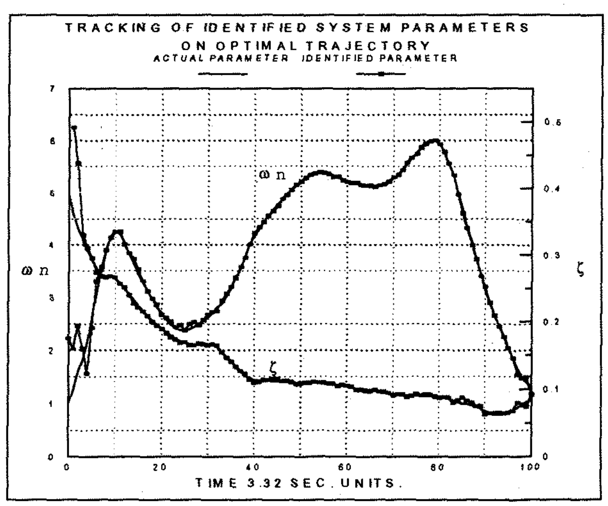

FIGURE 10

On-Line Adaptation Of C.S.A.S

At the end of each identification interval the controller parameters are updated as a function of the identified aircraft parameters. The controller parameter variations on the optimal climb trajectory are as shown in (Figs 11- 12). Also shown is the ratio of the aircraft lead time constant to the controller lag time constant throughout the manoeuvre. Although these are not matched exactly the deviation in this ratio from unity is small and the effect on the envelope of closed-loop aircraft /C.S.A.S. combination is negligible. By this technique the handling qualities of the aircraft in the pitch axis are maintained virtually uniform on the optimal climb trajectory. For the purpose of this investigation the identification interval was fixed at 200ms. which proved satisfactory. It would be perfectly feasible to select the identification interval as a function of the identified natural frequency of the system.

Augmented Control

As the identification and adaptation is occurring within the transient response time of the system it is interesting to investigate this further as a separate exercise. Starting with a nominal transient response characteristic for the closed-loop pitch rate response, then during the initial identification period and before identification and adaptation have occurred the actual response deviates from the nominal. This is due to the mismatch of the C.S.A.S. with the as yet unidentified aircraft. On adaptation the transient response continues from this point onwards with the now correct dynamics. This satisfies the handling criterion; however the actual transient response characteristic deviates from the nominal. (Fig 13). If the requirement is to minimise the error between the nominal and actual transient trajectory this may be done from the point of identification onward by minimising a quadratic function of this error subject to the now known dynamic constraints of the system. This optimisation would normally be performed off-line; however since an on-board tracking model of the aircraft exists this could be used to perform the optimisation and on-line generation of the additional augmented feed back control. The effect of applying this augmented control at the point of adaptation is shown in (Fig 14). Here two mismatched controllers are considered which represent a deviation by a factor of 100 from the nominal dynamics. The system response is forced back onto the nominal transient trajectory, by the augmented control, from the point of adaptation onwards. The resultant accelerations produced can be controlled by the introduction of state constraints. (Fig 15) shows that there is some benefit in using this optimal augmented control even during the initial identification period as the excursions away from the nominal transient response are significantly reduced.

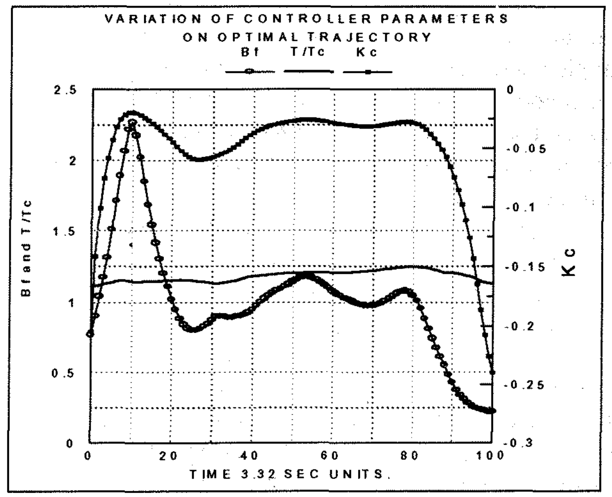

FIGURE 11

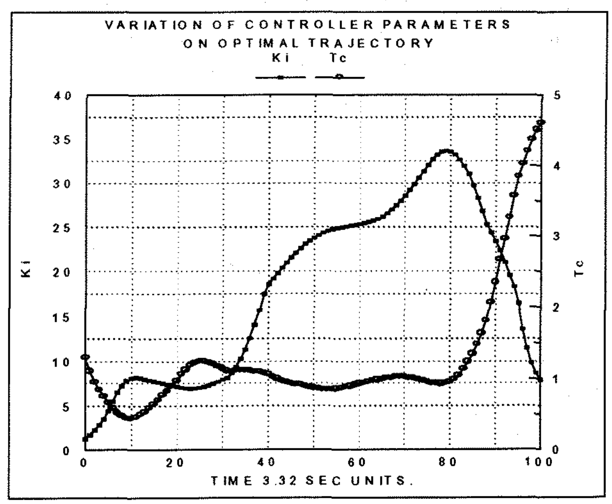

FIGURE 12

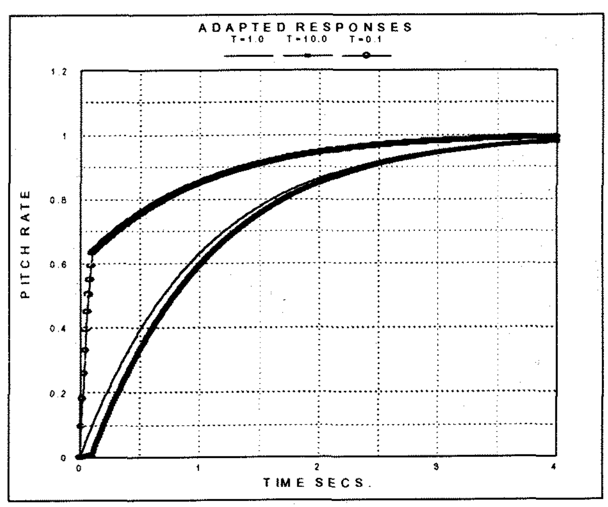

FIGURE 13

Conclusion

The variation of aircraft parameters on an optimal manoeuvre has been investigated and it has been shown that the design of a parameter scheduled controller using auxiliary variables is complex. An on- line identification and tracking model has been implemented using the normal commands to perform the manoeuvre, and without resorting to additional test signals for the purpose of parameter identification. Functions of the identified values of the aircraft parameters have been used to adapt the controller, in order to provide a virtually uniform response characteristic throughout the manoeuvre. The adaptation is continuous and operates within the transient response time of the system. An optimum controller augmentation has been studied which uses the identified parameter information to compensate for deviations from a nominal transient response arising during the identification interval. The augmented control returns the transient response trajectory to the nominal in an optimum manner. This augmented control also reduces deviations from the nominal transient response when there is mismatch between the controller and aircraft dynamics during the initial identification interval.

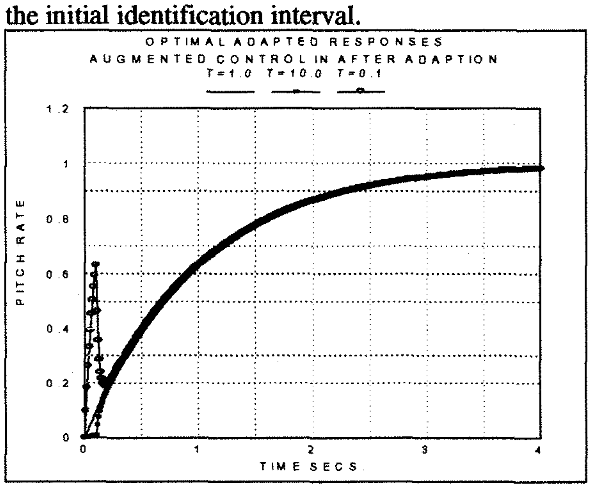

FIGURE 14

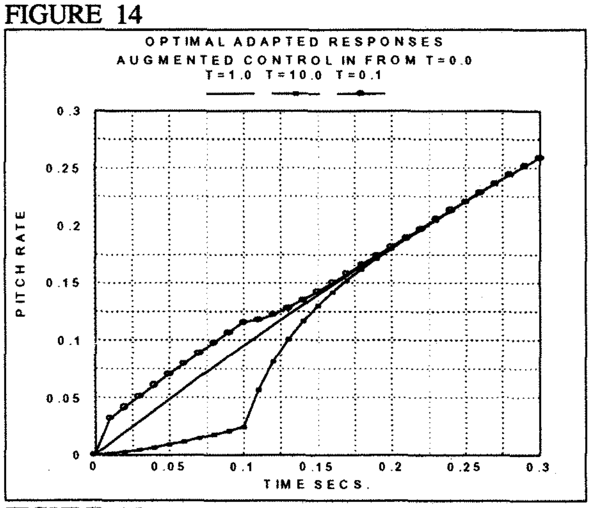

FIGURE 15

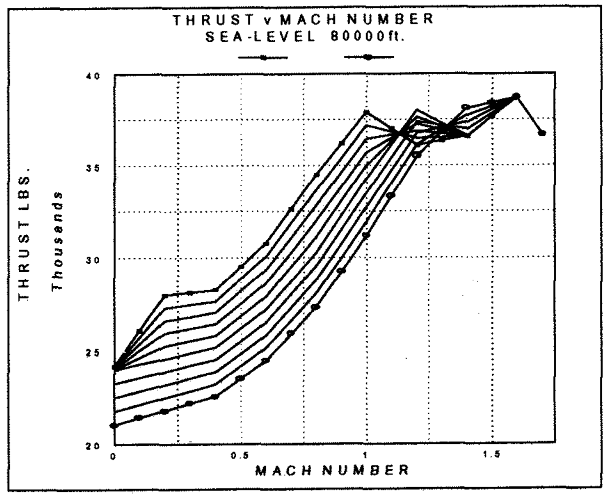

FIGURE 16

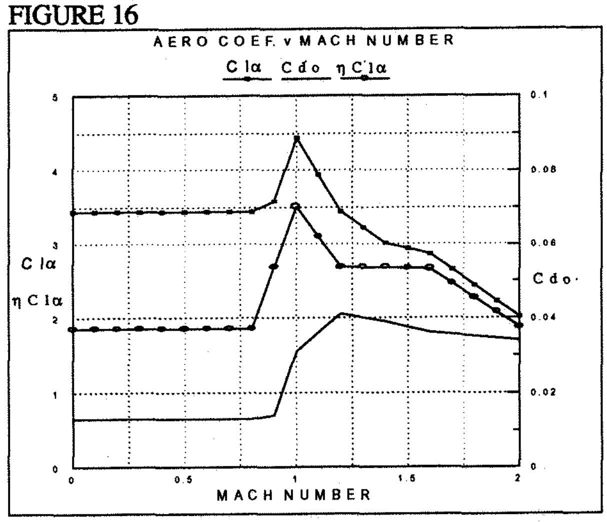

FIGURE 17

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