CALCULATION OF UNSTEADY AERODYNAMIC LOAD ON THREE DIMENSIONAL FINITE THICKNESS OSCILLATING WING

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Abstract

The problem of aerodynamic loads on finite thickness oscillating wing in three dimensional potential flow is revisited. A computer code for calculating the aerodynamic loads on finite thickness oscillating wing in three dimensional potential flow is developed using Boundary Element approach and velocity potential formulation. This method essentially utilizes distribution of yet unknown sources and doublet on the surface of the wing. In the numerical computation, quadrilateral surface panels on wing and wake surface have been incorporated. Wake geometry is assumed to be known and zeroeth order approximation has been carried out,i.e. every surface panels is assumed to have constant source and doublet strength. Essentially the method follows closely that of Geissler, but for the application of Kutta condition Kutta elements have been introduced and the evaluation of the wake integral have been carried out using predeterminded finite wake length. The source and doublet strengths are determined by solving the resulting system of linear equations. The method has been validated by comparison with standard work in the literature.

Nomenclature

x,y,z	global Cartesian coordinate
ξ,η,ς	local (panel) coordinate
s	countour coordinate
b	wing span
F	wing surface function
S	wing surface area
W	wake surface area
ℓ_0	root chord (reference) length
t	time
u	induced velocity vector
V	oscillatory movement velocity vector
U_∞	translatory movement velocity vector
Cp	pressure coefficient
α_s	static angle of attack
α'	oscillation amplitude
ω	oscillation frequency
Φ,φ	velocity potential
K	aerodynamic influence coefficient

Subscripts

s	steady
i	unsteady
u	wing upper
ℓ	wing lower
T	trailing edge
KE	Kutta element (panel)
q	source distribution
d	doublet distribution
a	pitching axis

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Introduction

Prediction of unsteady aerodynamic loads on an oscillating wing has been well treated in the literature by numerical solution of the integral equation of the potential flow representing the mathematical model of the physical state of affairs. Methods have been developed to solve the problem starting from simplified ones assuming linear aerodynamics and flat zero thickness geometries by utilizing the elegance of analytical approach to completely non-linear ones treating the problem exactly on the true boundaries of the wings and bodies taking advantage of the progress of computers and computational sciences by resort to numerical approaches. However, solution to separated flow problems appearing in practice does not seem to be well treated, particularly by using the convenience of potential flow model.

Several numerical methods for calculating unsteady loads on harmonically oscillating wings have been developed. Analytical two dimensional approach developed by Theodorsen[1] and three dimensional one developed by Kuessner[2] as solution of the Possio Integral Equation for the acceleration potential have been instrumental in understanding the basic physical

$$\Phi(x,y,z,t) = \varphi_\infty + \varphi_s + \bar{\varphi}_i \alpha' e^{i\omega t} + ... \quad (2)$$

where α' is the oscillating amplitude of first order. It is assumed that the oscillating amplitude is sufficiently small so that the second and higher terms in (2) can be neglected, at least in the first approximation.

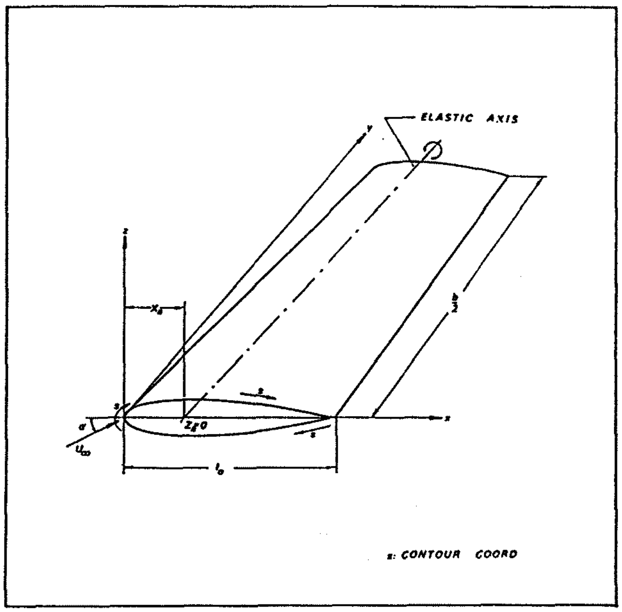

FIGURE 1. Discription of the problem

The kinematical boundary condition on the wing surface can be expressed by :

$$\left(\frac{D}{Dt}\right)_B F \equiv \frac{\partial F}{\partial t} + V \cdot \nabla F = 0 \quad (3)$$

where

V : free stream velocity

B : body

F : mathematical representation of the surface

Using equation (2), we can express the kinematical boundary condition as :

$$\left(\frac{D}{Dt}\right)_B F \equiv \frac{\partial F}{\partial t} + (u + U_\infty) \cdot \nabla F = 0 \quad (4)$$

where

$u = \nabla(\phi_s + \phi_i)$: induced velocity vector

$U = \nabla(\phi_\infty)$: translational ambient velocity vector

$\varphi_i = \bar{\varphi}_i \, \alpha' e^{i\omega t}$: first order unsteady part of the velocity potential

Dividing (4) by $|\nabla F|$ and using local coordinate system, one can obtain :

$$\frac{\partial \varphi_s}{\partial \zeta} + \frac{\partial \varphi_i}{\partial \zeta} = -\left(\frac{\partial \zeta}{\partial t} + \bar{\zeta} \cdot U_\infty\right) \quad (5)$$

where

$\zeta \equiv$ unit vector normal to the surface $= \frac{\nabla F}{|\nabla F|}$

$\frac{\partial \zeta}{\partial t} \equiv$ normal displacement of a surface point $= \frac{\partial F}{\partial t} \frac{1}{|\nabla F|}$

The left-hand side of (5) is the normal induced velocity due to the potential flow. The right-hand side is the prescribed normal velocity. The first term on the right-hand side represent free stream contribution, and can be expressed by:

$$\bar{\zeta} \cdot U_\infty = U_{\zeta s} + \bar{U}_{\zeta i} \alpha' e^{i\omega^* T} \quad (6)$$

and the second is contributed by the unsteady motion of the surface, and can be expressed by :

$$\frac{\partial \zeta}{\partial t} = \bar{V}_\zeta \alpha' e^{i\omega^* T} \quad (7)$$

Equation (6) represents the normal component of the translational velocity and equation (7) represents the normal component of the velocity due to oscillatory movement. ω^* is the reduced frequency and defined as $\omega \ell_0 / U_\infty$. T is the non dimensionalized time variable and defined as tU_∞ / ℓ_0.

Substitution of equations (2), (6) and (7) into equation (5) results in :

$$\frac{\partial \varphi_s}{\partial \zeta} + \alpha' e^{i\omega^* T} \frac{\partial \bar{\varphi}_i}{\partial \zeta} = -\left[U_{\zeta s} + (\bar{U}_{\zeta i} + \bar{V}_\zeta)\alpha' e^{i\omega^* T}\right] \quad (8)$$

This equation can be separated into steady and unsteady parts, i.e. :

Steady Part :

$$\frac{\partial \varphi_s}{\partial \zeta} = -U_{\zeta s} \quad (9)$$

Unsteady Part :

$$\frac{\partial \bar{\varphi}_i}{\partial \zeta} = -\left[\bar{U}_{\zeta i} + \bar{V}_\zeta\right] \quad (10)$$

Assuming small oscillation amplitude, the $\alpha' e^{i\omega^* T}$ term can be neglected.

The potential distribution on the surface of the wing and the wake can be established as the superposition of phenomena and can be used to verify other new and more general methods for the particular problems for which these analytical methods are valid. Further analytical development known as lifting surface methods, utilizing acceleration potential, have been contributed by Watkins, Runyan and Woolston[3] in the United States, Stark[4] in Sweden and Laschka[5] in Germany, and the latter has been regarded to give most accurate solution close to experimental ones for linear region. Further improvement has been contributed by Landahl[6]. These analytical approaches have been based on the series solution of the integral equation for the linearized problem with linearized boundary conditions. A new development that found wide application in the aircraft industry has later on been developed by Albano and Rodden[7], Giesing, Kalman and Rodden[8], and known as the doublet lattice method. The method is based on the solution of the linearized problem on the non-planar wing configuration, by treating the integral equation on surface panels on the wing, and in each panel analytical series representation of the pressure distribution has been assumed, utilizing development contributed by Laschka or Landahl. This method has been utilized in the NASTRAN package program which has found wide application. In 1969 the author has carried out another approach belonging to the time marching solution of the integral equation of unsteady flow, utilizing velocity potential formulation. Formulation and similar approach using velocity potential has been developed by Jones[9], Morino[10], Maskew[11] and Geissler[12][13]. This formulation has the advantage of treating the problem exactly on the boundary of the lifting configuration, and resort to numerical solution by utilising surface panels or boundary element approach. Since the technique has the promise of treating the problem exactly on the true boundary of the surface of the wing and body configuration, it is considered to be well appealing for further development in the treatment of more complicated separated flow case. It is with such motivation that the authors decided to review and further develop computational routine based on Geissler's approach[13] in this particular work. Furthermore, the method can be formulated to establish a boundary element computational routine; in what follows, only results for incompressible case is presented.

The present method has been developed using the boundary surface element approach. Boundary Element Approach was first introduced by Morino[10]. This approach has also been employed by Prananta & Djojodihardjo[14]. The method that will be elaborated here capitalizes on the use of velocity potential. Such approach has also been developed by W.P Jones[15], Hess[16], Djojodihardjo & Widnall[17], Djojodihardjo & Bunyamin[18], and Geissler[12], and reduces to both the use of singularities and the application of the kinematical boundary conditions on the true surface.

The present method is developed for incompressible potential flow which is governed by Laplace equation. It has been shown (ref.[12]) that it is possible to calculate steady and unsteady pressure distribution without linearization of the kinematical boundary condition or the Bernoulli equation. However, there exists connection between steady and unsteady parts that prompted us to solve both parts together. Essentially the method follows closely that of Geissler, but for the application of Kutta condition Kutta elements have been introduced and the evaluation of the wake integral have been carried out using predeterminded finite wake length.

Formulation of The Problem and The Integral Equation

Folowing the conventional approach, the flow around the wing can be assumed to be potential with uniform velocity U_∞. Figure 1 depicts the problem we are dealing with. The wake geometry is assumed to be known a priori. The inertial frame of reference is fixed on the wing, and coincides with the global coordinate system x,y,z. The wing is assumed to experience pitching oscillating movement about its mean angle of attack, α. In the numerical integration process, the wing and the wake surface are represented by a finite number of boundary elements, which for this work are further assumed to be flat panels. The related boundary conditions are specified at any points on these boundary elements.

Since the geometry is known, then the problem can be formulated to be the determination of the correct potential distribution such that the flow tangency boundary condition, or the kinematical boundary condition on the wing surface, is met, i.e. the induced velocity normal to a control point on the surface will cancel the normal velocity component of the free stream at that point. This condition will be elaborated futher.

For incompressible flow, the governing equation for the velocty potential reduces to Laplace equation, which is also valid for the disturbance velocity potential:

$$\nabla^2\Phi = 0 \qquad (1)$$

Following the approach of Geissler[8], the velocity potential can be described as a superposition of its steady and unsteady parts; the steady part consists of the undisturbed velocity potential $\varphi_\sim$ and the steady part of the disturbance potential, while the unsteady parts can further be represented by Fourier series.
Hence :

the potential of the singularities distribution on the surface. In general source and doublet singularities can be used. Sources distribution represents contribution due to thickness while doublet distribustion represents lift element of the wing. Thus :

$$\varphi = \varphi_q + \varphi_d \tag{11}$$

where:

φ_q : velocity potential due to source distribution
φ_d : velocity potential due to doublet distribution

The induced velocity potential at a point P(x,y,z) due to source distribution on the surface of the wing can be written as :

$$\varphi_q(x,y,z) = -\frac{1}{4\pi}\iint_s \frac{\sigma_q(x',y',z')}{r}\, dS \tag{12}$$

where:

σ_q : source strength at point P'(x',y',z')
S : wing surface area
r : distance from P(x,y,z) to P'(x',y',z')

The induced velocity potential at a point P(x,y,z) due to doublet distribution on the surface of the wing and the wake can be written as:

$$\varphi_q(x,y,z) = -\frac{1}{4\pi}\iint_s \sigma_d(x',y',z')\frac{\partial}{\partial\zeta'}\left(\frac{1}{r}\right) dS + -\frac{1}{4\pi}\iint_s \Delta\phi_{dw}(x',y',z')\frac{\partial}{\partial\zeta'}\left(\frac{1}{r}\right) dW \tag{13}$$

where

σ_d : doublet strength at point P(x,y,z)
$\Delta\phi_{dw}$: velocity potential jump across the wake
W : wake surface area

In equations (12) and (13), the distance between field (control) point P(x,y,z) and source point P'(x',y',z'), where the singularities are located, on the integration panel can be expressed by :

$$r = \left[(x-x')^2 + (y-y')^2 + (z-z')^2\right]^{\frac{1}{2}} \tag{14}$$

Analogous to equation (2), both the potential induced by source and doublet distributions can be separated into steady and unsteady parts, i.e. :

$$\varphi_q = \varphi_{qs} + \bar{\varphi}_{qi}\alpha' e^{i\omega' T} \tag{15}$$

and

$$\varphi_d = \varphi_{ds} + \bar{\varphi}_{di}\alpha' e^{i\omega' T} \tag{16}$$

Substitution of equations (15) and (16) into equation (11), (9) and (10) results in :

Steady Part :

$$\frac{\partial\varphi_{qs}}{\partial\zeta} + \frac{\partial\varphi_{ds}}{\partial\zeta} = -U_{\zeta s} \tag{17}$$

Unsteady Part :

$$\frac{\partial\bar{\varphi}_{qs}}{\partial\zeta} + \frac{\partial\bar{\varphi}_{ds}}{\partial\zeta} = -\left[\bar{U}_{\zeta i} + \bar{V}_\zeta\right] \tag{18}$$

By the use of Equations (12) and (13), and the application of the kinematical boundary condition, equations (17) and (18) can be written into a system of integral equations. This set of equations should be solved to obtain the distribution of the sources and doublets on the surface of the wing and the wake. By obtaining the solution, the aerodynamic characteristics of the oscillating wing can readily be obtained.

In order to obtain unique solutions of equations (17) and (18), another set of boundary conditions have to be applied. The corresponding boundary conditions are the Kutta-Joukowsky condition and the wake sheet dynamical boundary condition. This boundary condition will be discussed in the following paragraph.

Kutta-Joukowsky Condition and Wake Dynamical Boundary Condition

Kutta-Joukowsky Condition

The Kutta-Joukowsky condition states that the velocity vector of the fluid particles must be finite on the sharp trailing edge or cusped trailing edge.

Following the development in ref.[17][18], the Kutta-Joukowsky condition will be treated on added panels near the trailing edge (these panels will be called Kutta-Panels). These panels are assumed to be impermeable surfaces. The main problem is the determination of the direction of these panels. The direction of Kutta-Panels depends on wing planform and the distribution of circulation on the wing. It was shown in ref.[17][18] that the direction of the bisector of the trailing-edge is sufficiently accurate for the wings examined.

On the Kutta-Panels the kinematical boundary condition can be written to be:

Steady Part :

$$\left.\frac{\partial\varphi_{qs}}{\partial\zeta} + \frac{\partial\varphi_{ds}}{\partial\zeta}\right|_{KE} = -U_{\zeta s}\Big|_{KE} \tag{19}$$

Unsteady Part :

$$\left.\frac{\partial\bar{\varphi}_{qs}}{\partial\zeta} + \frac{\partial\varphi_{ds}}{\partial\zeta}\right|_{KE} = -\left[\bar{U}_{\zeta i} + \bar{V}_\zeta\right]\Big|_{KE} \tag{20}$$

where the subscript KE indicates that the kinematical boundary condition is applied on the Kutta-Panels.

Wake Dynamical Boundary Condition

Since the wake is assumed to be a stream surface, there is no pressure jump across the upper and lower surfasce of the wake. The mathematical formulation will be discussed in the following paragraph.

First, the Kelvin-Bernoulli equations for our problem can be written as:

$$\frac{\partial \Phi}{\partial t} + \frac{\nabla\Phi \cdot \nabla\Phi}{2} + \frac{p}{\rho} = \frac{(U_\infty + V)^2}{2} + \frac{p_\infty}{\rho} \qquad (21)$$

Using equation (21), the pressure coefficient can be expressed by:

$$C_p = \frac{(U_\infty + V)^2}{U_\infty^2} - \frac{2}{U_\infty^2}\frac{\partial \Phi}{\partial t} - \frac{\nabla\Phi \cdot \nabla\Phi}{U_\infty^2} \qquad (22)$$

The wake dynamical boundary condition is :

$$\Delta C_p = C_{p_u} - C_{p_l} = 0 \qquad (23)$$

Substituting equation (22) into equation (23), we obtain:

$$\frac{\partial(\Phi_u - \Phi_l)}{\partial t} - \frac{(\nabla\Phi\cdot\nabla\Phi)_u - (\nabla\Phi\cdot\nabla\Phi)_l}{2} = 0 \qquad (24)$$

We can use equation (2) to obtain :

$$\frac{\partial(\Phi_u - \Phi_l)}{\partial t} = i\omega^* \frac{U_\infty}{\ell_0} \Delta\bar{\varphi}_i \alpha' e^{i\omega^* T} \qquad (25)$$

where

$$\Delta\bar{\varphi}_i = \bar{\varphi}_{i_u} - \bar{\varphi}_{i_l} \qquad (26)$$

The second term of left-hand side of equation (24) can be written as :

$$(\nabla\Phi\cdot\nabla\Phi)_u - (\nabla\Phi\cdot\nabla\Phi)_l =$$

$$\left[(U_\infty + \nabla\phi_{s_u})^2 - (U_\infty + \nabla\phi_{s_l})^2\right] + 2\left[U_\infty \cdot \nabla(\Delta\bar{\varphi}_i) + \right.$$

$$\left. \bar{V}\cdot\nabla(\Delta\varphi_s) + \nabla\varphi_{s_u}\cdot\nabla\bar{\varphi}_{i_u} - \nabla\varphi_{s_l}\cdot\nabla\bar{\varphi}_{i_l}\right] \times \alpha' e^{i\omega * T} \qquad (27)$$

Using equations (25),(26) and (27) we can obtain, for the steady part :

$$(U_\infty + \nabla\varphi_{s_u})^2 = (U_\infty + \nabla\varphi_{s_l})^2 \qquad (28)$$

that is :

$$|\nabla\varphi_{s_u}| = |\nabla\varphi_{s_l}|$$

or

$$\nabla(\Delta\varphi_s) = 0 \qquad (29)$$

Therefore

$$\Delta\varphi_s = constant = (\Delta\varphi_s)_{TE} \qquad (30)$$

The last equation implies that the potential jump across the wake surface is constant and that its value is equal to the potential jump on related trailing edge.

Next we will derive the wake dynamical boundary condition for unsteady part. From equation (30) we can write :

$$\nabla\varphi_{s_u} = \nabla\varphi_{s_l} \qquad (31)$$

Using small perturbation condition, we can assume that:

$$(U_\infty + \nabla\varphi_s) \approx s U_\infty \qquad (32)$$

For the unsteady part, from equations (24) and (27) and the condition as implied by equation (32) :

$$s\cdot\nabla(\Delta\bar{\varphi}_i) = -\frac{i\omega^*}{\ell_0}\Delta\bar{\varphi}_i \;\; ; \;\; \frac{\partial\Delta\bar{\varphi}_i}{\partial s} = -\frac{i\omega^*}{\ell_0}\Delta\bar{\varphi}_i \qquad (33)$$

Integration of equation (33) results :

$$(\Delta\bar{\varphi}_i)_w = (\Delta\bar{\varphi}_i)_{TE} \exp - i\omega^* \left(\frac{s - s_{TE}}{\ell_0}\right) \qquad (34)$$

Numerical Solution

The solution of the integral equation for the unknown source and doublet strengths can be obtained by numerical approach. The surface of the wing and the wake are divided into boundary surface elements, and as a first approximation, is approximated by planar surface elements. More accurate representation can be made by using spherical surface elements. Alternatively, more accurate boundary surface elements using shape functions can also be employed. However, in this particular work, planar surface elements were employed. The numerical procedure is well known and essentially follows those given in refs.[17] and [18]. Following similar procedure, the problem of solving the integral equation can be reduced to problem of solving a set of linear system of equations.

To use the method, first the wing and wake surface are discretizised into large number of small quadrilateral panels. Fig.2 describes the discretization used in the calculation. Both the distribution of sources and doublet are assumed to be constant on each surface panel. Doublet strengths in the wake are related to their local trailing edge values as described by equations (30) and (34).

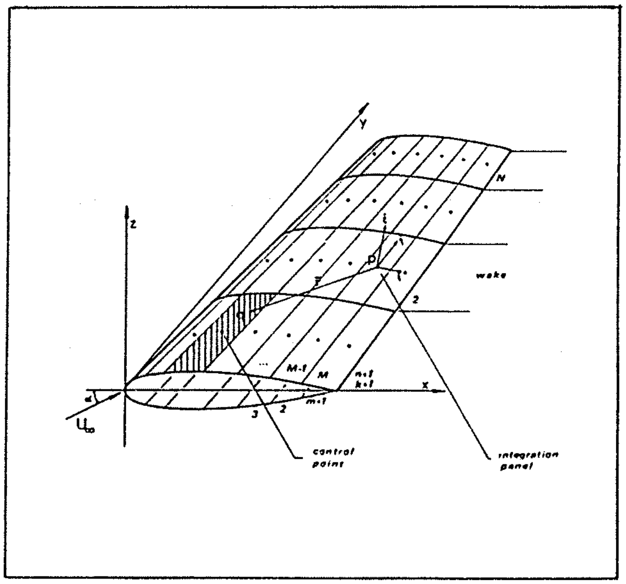

FIGURE 2. Discretization of surface of the wing and wake

Using this discretization, the surface integral for the velocity potential and the induced velocity of the sources and doublets are reduced to a sum of integrals over a single boundary element surface. Each integral is calculated by assuming sources or doublets of constant strength.

For control point i, the induced normal velocity due to a source distribution over the surface element is :

$$\left(\frac{\partial\varphi_q}{\partial\zeta}\right)_i = \frac{1}{4\pi}\sum_{j=1}^{M\cdot N}\sigma_{qj}\iint_S \frac{\partial}{\partial\zeta_i}\left(\frac{1}{r_{ij}}\right) dS \qquad (35)$$

where

- $\left(\frac{\partial\varphi_q}{\partial\zeta}\right)_i$: normal induced velocity on control point i due to distribution of sources
- r_{ij} : distance from control point i to integration panel j
- S : wing surface panel area j
- σ_{qj} : source strength on panel j
- M : chordwise panel number
- N : spanwise panel number

and the induced normal velocity due to a doublet distribution over the surface element is :

$$\left(\frac{\partial\varphi_d}{\partial\zeta}\right)_i = \frac{1}{4\pi}\sum_{j=1}^{M\cdot N}\sigma_{dj}\iint_{S_j}\frac{\partial^2}{\partial\zeta_i\,\partial\zeta_j}\left(\frac{1}{r_{ij}}\right) dS +$$

$$\frac{1}{4\pi}\sum_{j=1}^{M\cdot N}\iint_{W_k}\Delta\phi_{dW_k}\frac{\partial^2}{\partial\zeta_i\partial\zeta_j}\left(\frac{1}{r_{ij}}\right) dW_k \qquad (36)$$

where

- $\left(\frac{\partial\varphi_d}{\partial\zeta}\right)_i$: normal induced velocity on control point i due to distribution of doublets
- r_{ij} : distance from control point i and integration panel j
- S : wing surface panel area
- W : wake surface panel area
- σ_{dj} : doublet strength on panel j
- $\Delta\phi_{dW_k}$: potential jump on wake panel k

The surface integral terms on equations (35) and (36) are known as the aerodynamic influence coefficients. Refs.[12],[17],[18] show that these aerodynamic influence coefficients can be treated numerically.

Using equations (35) and (36) and also referring to ref.[12], the integral equation can be rewritten as :

Steady part :

$$\frac{1}{4\pi}\left[\sum_{j=1}^{M\cdot N}\sigma_{qj}K_{q\zeta_{ij}} + \sum_{j=1}^{M\cdot n}\sigma_{dj}K_{d\zeta_{ij}} + \sum_{k=1}^{N}\Delta\varphi_k W_{ik}\right] = U_{\zeta s} \qquad (37)$$

Unsteady part :

$$\frac{1}{4\pi}\left[\sum_{j=1}^{M\cdot N}\bar{\sigma}_{qj}K_{q\zeta_{ij}} + \sum_{j=1}^{M\cdot N}\bar{\sigma}_{dj}K_{d\zeta_{ij}} + \sum_{k=1}^{N}\Delta\varphi_k \bar{W}_{ik}\right] = -\left[\bar{U}_{\zeta_s} + \bar{U}_{\zeta}\right] \qquad (38)$$

The treatment of doublet distribution in the present method is similar to that of ref.[12]. Using similar approach, the equations (37) and (38) can further be written into :

Steady part :

$$\frac{1}{4\pi}\sum_{j=1}^{M\cdot N}\sigma_{qj}K_{q\zeta_{ij}} + \frac{1}{4\pi}\sum_{k=1}^{N}\sigma_k\left[\sum_{j=1}^{M}\left(s-\frac{s_M}{2}\right)_j K_{d\zeta_{ij}} + \frac{s_M}{2}W_{ik}\right] = -U_{\zeta_s} \qquad (39)$$

Unsteady part :

$$\frac{1}{4\pi}\sum_{j=1}^{M\cdot N}\bar{\sigma}_{qj}K_{q\zeta_{ij}} + \frac{1}{4\pi}\sum_{k=1}^{N}\bar{\sigma}_k\left[\sum_{j=1}^{N}\left(s-\frac{s_M}{2}\right)_j K_{d\zeta_{ij}} + \frac{s_M}{2}\bar{W}_{ik}\right] = -\left[\bar{U}_{\zeta_s} + \bar{U}_{\zeta}\right] \qquad (40)$$

where

- s : curvilinear coordinate of control point
- s_M : curvilinear coordinate of the trailling edge
- σ_k : doublet strength of section-k

$\bar{\sigma}_k$: doublet strength of section-k (complex)

W_{ik} : influence coefficient of wake strip-k as related to control point i

$\bar{W}_{ik}$: influence coefficient of wake strip-k as related to control point i (complex)

for i = 1 ... MxN.

The numerical approach of Kutta condition, equations (19) and (20) are carried out in the same manner as equations (39) and (40).
The treatment reduced to :

Steady part :

$$\frac{1}{4\pi}\left[\sum_{j=1}^{M\cdot N}\sigma_{qj}K_{q\zeta_{ij}}+\sum_{k=1}^{M\cdot N}\sigma_{dj}K_{d\zeta_{ij}}+\sum_{k=1}^{N}\Delta\varphi_k W_{ik}\right]\Bigg|_{KE} = -U_{\zeta_s}\Big|_{KE} \quad (41)$$

Unsteady part :

$$\frac{1}{4\pi}\left[\sum_{j=1}^{M\cdot N}\bar{\sigma}_{qj}K_{q\zeta_{ij}}+\sum_{j=1}^{M\cdot N}\bar{\sigma}_{dj}K_{d\zeta_{ij}}+\sum_{k=1}^{N}\Delta\varphi_k \bar{W}_{ik}\right]\Bigg|_{KE} = -\left[\bar{U}_{\zeta_s}+\bar{V}_{\zeta}\right]\Big|_{KE} \quad (42)$$

Equations (39), (40), (41) and (42) form the system of linear equations. The unknowns of this system of linear equations are the strengths of the sources and doublets at each boundary surface element.

The system of linear equations can be arranged further into the following symbolic matrix representations :

Steady part :

$$\begin{bmatrix} A'_{ij} & B'_{ik}+C'_{ik} \\ A'_{ij} & B'_{lk}+C_{lk} \end{bmatrix}\begin{Bmatrix}\sigma_{qj}\\ \sigma_{dk}\end{Bmatrix} = \begin{Bmatrix} D_i \\ D_l\end{Bmatrix} \quad (43)$$

Unsteady part :

$$\begin{bmatrix} A'_{ij} & B'_{ik}+\bar{C}'_{ik} \\ A'_{ij} & B'_{lk}+\bar{C}_{lk} \end{bmatrix}\begin{Bmatrix}\bar{\sigma}_{qj}\\ \bar{\sigma}_{dk}\end{Bmatrix} = \begin{Bmatrix} \bar{D}_i \\ \bar{D}_l\end{Bmatrix} \quad (44)$$

where

i,j = 1,2,3, ... , MxN

k,l = 1,2,3, ... , N

A'_{ij} : matrix of source influence coefficients of singularity distribution at panel j on the wing on control point i = $\frac{1}{4\pi}K_{q\zeta_{ij}}$

B'_{ik} : matrix of doublet influence coefficients of singularity distribution at panel k on the wing on control point i = $\frac{1}{4\pi}K_{d\zeta_{ik}}$

C'_{ik} : matrix of doublet influence coefficient of singularity distribution at wake strip k on control point i = $\frac{1}{4\pi}W_{ik}$

A'_{ij} : matrix of source influence coefficient of singularity distribution at panel j on Kutta control point ℓ = $\frac{1}{4\pi}K_{q\zeta_{ij}}$

B'_{lk} : matrix of doublet influence coefficient of singularity distribution at wing panel k on Kutta control point ℓ = $\frac{1}{4\pi}K_{d\zeta_{lk}}$

C'_{lk} : matrix of doublet influence coefficient of singularity distribution at wake strip k on Kutta control point ℓ = $\frac{1}{4\pi}W_{lk}$

D_i : matrix of prescribed normal velocity on control point i

D_l : matrix of prescribed normal velocity on Kutta control point ℓ

σ_{qj} : vector of source strength on panel j

σ_{dk} : vector of doublet strength on panel k

Discussion of Results

A numerical investigation has been made for rectangular wing with a NACA 0012 airfoil. The wing has an aspect ratio of four. Numerical results are evaluated for the case of pitching oscillation about the wing quarter chord-axis. The experimental data was taken from ref.[12]. Comparison of the results with experimental data and computational results of Geissler has been made.

Figure 3 shows the steady pressure distribution for static angle of attack of three degrees. Similar to Geissler result, the calculation is valid for up to 80% of wing half-span when compared with Geissler experiment and results. This behavior is similar for unsteady cases.

Figure 4 shows the unsteady pressure distribution for zero static angle of attack. The real part does not make significant differences the with experimental data and Geissler results. The imaginary part displays some discrepancies compared to Geissler results. But for 55% of the wing half-span, the imaginary part of our approach displays more accurate results compared to Geissler's experimental data. This behaviour are similar for other cases.

Figure 5 shows the unsteady pressure distribution for static angle of attack of three degrees.

Figure 6 shows the effect of trailing edge panel's chordwise length to Kutta control point coordinate. The coordinate is specific for every case. The equation for determining the coordinate of the Kutta control point has been obtained, for symetrical airfoil, by numerical investigation.

Figure 7 shows the error of calculated pressure distri-bution due to unsuitable coordinates of Kutta control points. We can see that for symetrical airfoil, there is a matching coordinate point for Kutta control point.

Figure 8 gives the mean error of pressure distribution due to the use of finite discretizised wake. The mean errors are compared with infinite discretizised wake, which is simulated by a wake of 25 chord-length.

Conclusions

A boundary element computational method based on panel method has been developed for the calculation of unsteady pressure distribution over three dimensional oscillating finite wing. The method presented here, although based on Geissler's approach, has utilized different approach in the treatment of the wake, in that it employs the use of finite discretizised wake and in the method of solving the integral equation. In solving the system of integral equation, use has been made of the method suggested by Sloof[19]. Using Sloof method, qualitatively, computer time can be shorter than Geissler method.

It apprears that this methods is very sensitive to the application of Kutta-Jowkowsky condition. This aspect will be the subject of further work. Work is also in progress in the application of this method to other wing planforms and airfoil sections.

References

[1] Theodorsen, T., *General Theory of Aerodynamic Instability and the Mechanicsm of Flutter*, NACA Report 496, 1935.

[2] Kuessner, H.G., *Allgemeine Tragflaechen-theorie*, Luftfahrtforschung, Vol.17, 1940, pp.370-378.

[3] Watkins, C.E., Runyan, H.L. and Woolston, D.S., *On the Kernel Function of the Integral Equation Relating the Lift and Downwash Distributions of Oscillating Finite Wing in Subsonic Flow*, NACA TN 3131, January 1954.

[4] Stark, V.J.E., Aerodynamic Forces on a Combination of a Wing and a Fin Oscillating in Subsonic Flow, SAAB Aircraft Co., Linkoping, TN 54, 1964.

[5] Laschka, B., *Zur Theorie der harmonisch schwingenden tragenden Flaeche bei Unterschall-stroemung*, Zeitschrift fuer Flugwissenschaften, Vol.11, No.7, 1963, pp.265-292.

[6] Landahl, M.T., *Kernel Function for Nonplanar Oscillating Surfaces in a Subsonic Flow*, AIAA Journal, Vol.5, No.5, 1967, pp. 1045-1046.

[7] Albano, E. and Rodden,W.P., *A Doublet Lattice Method for Calculating Lift Distributions on Oscillating Surfaces in Subsonic Flowa*, AIAA Journal, Vol.7, No.2, 1969, pp.279-285 and No.11, 1969, 9.2192.

[8] Giesing, J.P., Kalman, T.P., and Rodden, W.P., Subsonic Unsteady Aerodynamics for General Configurations, Part I, Direct Application of the Nonplanar Doublet-Lattice Method, US Air Force Flight Dynamics Laboratory, AFFDL-TR-71-5, 1971.

[9] Jones, W.P., and Moore, J.A., *Simplified Aerodynamic Theory of Oscillating Thin Surfaces in Subsonic Flow*, AIAA Journal, Vol.11, Sept.1973.

[10] Morino, L., *A General Theory of Unsteady Compressible Potential Aerodynamics*, NASA Contractor Report, CR-2464, 1974.

[11] Maskew. B., *Program VSAERO - A Computer Program for Calculating the Nonlinear Aerodynamic Characteristics of Arbitrary Configurations User's Manual*, Analitical Methods Inc., Washington 96052, July 1983.

[12] Geissler, W., *Berechnung der Druckverteilung an Oszillierenden Dreidimensionalen Tragflächen mit Endlicher Dicke in Inkompressibler Strömung*, DFVLR-AVA-Internal Report, 253-76 J 05, 1976.

[13] Geissler, W., *Nonlinear Unsteady Potential Flow Calculations for Three-Dimensional Oscillating Wings*, AIAA-Journal, Vol.16 no.11, 1978, pp.1168-1174.

[14] Prananta, B.B.and Djojodihardjo, H., *Development of Green Identity Computational Procedure for Three Dimensional Flow Around Finite Thickness Oscillating Wings*, Symposium on Aeroelasticity, Institut Teknologi Bandung, 1990.

[15] Hess, J.L., and Smith, A.M.O., *Calculation of Potential Flow about Arbitrary Bodies*, Douglas Aircraft Company - Aircraft Division : Long Beach, California, Progress in Aeronautical Sciences Vol.8, Pergamon Press, 1976.

[17] Djojodihardjo, H., and Widnall, S.E., *A Numerical Method for The Calculation of Nonlinear Unsteady Lifting Potential Flow Problem*, AIAA 7th Aerospace Sciences Meeting, January 1974.

[18] Djojodihardjo, H. and Bunyamin, S., (in Indonesian), *Aeronotika dan Astronotika Indonesia*, Vol.,No.,1987.

[19] Sloof, J.W., *Numerical Methods in Aircraft Aerodynamics*, Part 2, Lecture Note given at Institut Teknologi Bandung, Division of Aerospace Engineering, 1991.

[20] Katz, Joseph., and Plotkin, Allen., *Low-Speed Arodynamics - from Wing Theory to Panel Methods*, McGraw-Hill, Inc., Singapore, 1991.

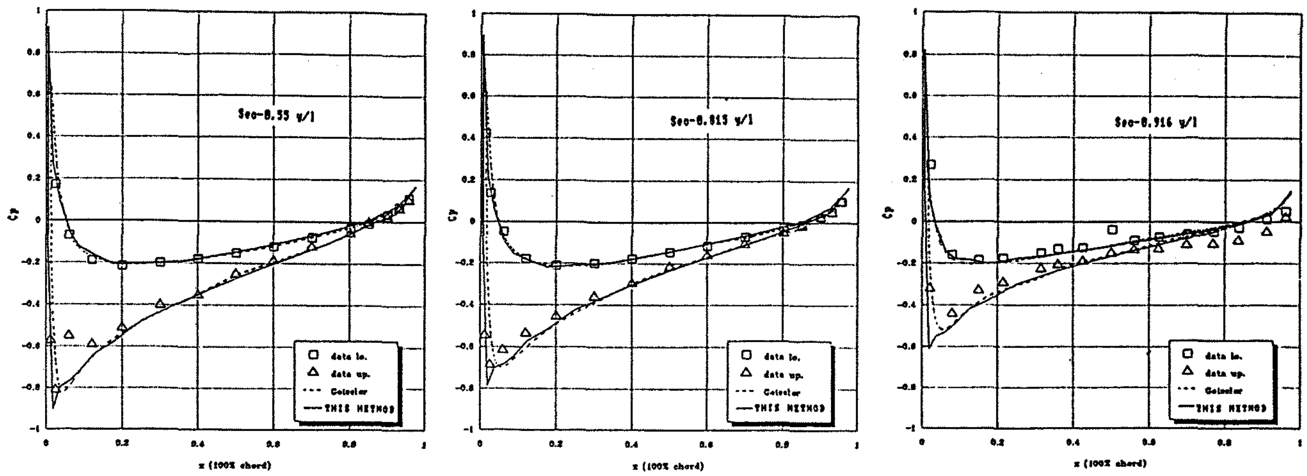

FIGURE 3. Steady pressure distribution, $\alpha=3^{\circ}$

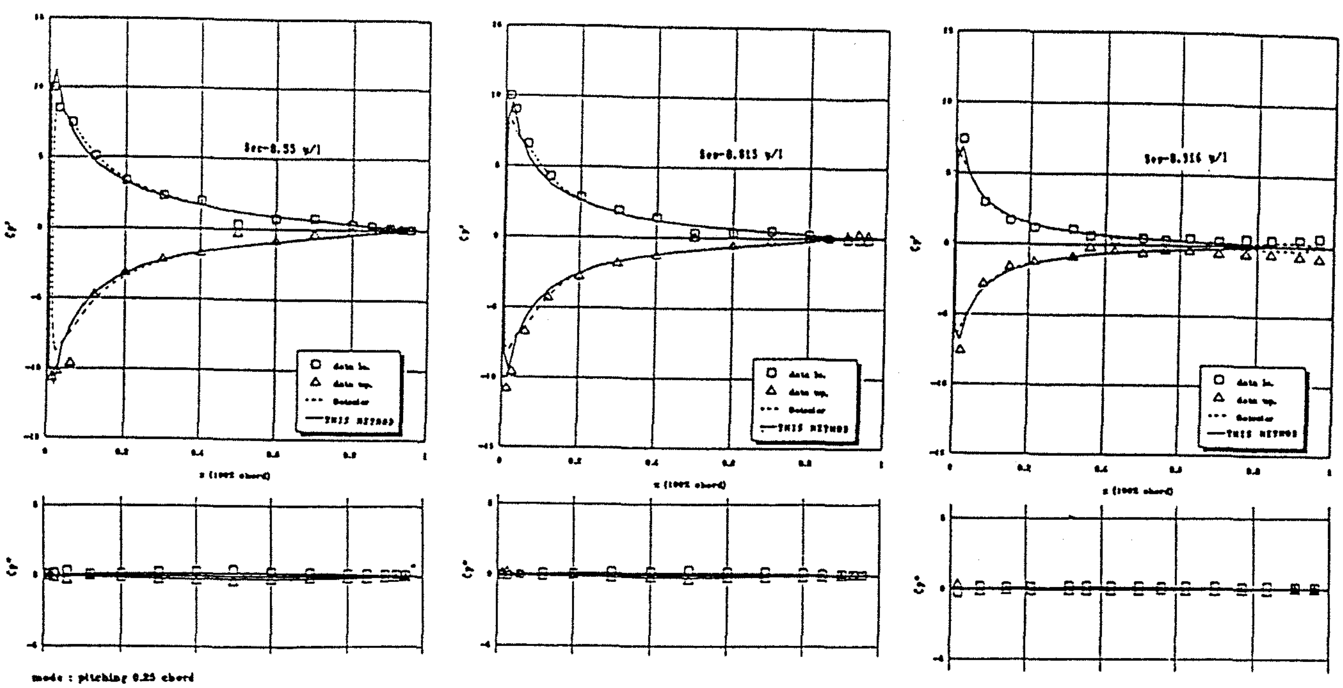

FIGURE 4. Unsteady pressure distribution, $\alpha=0^{\circ}$, $\omega^{*}=0.14$

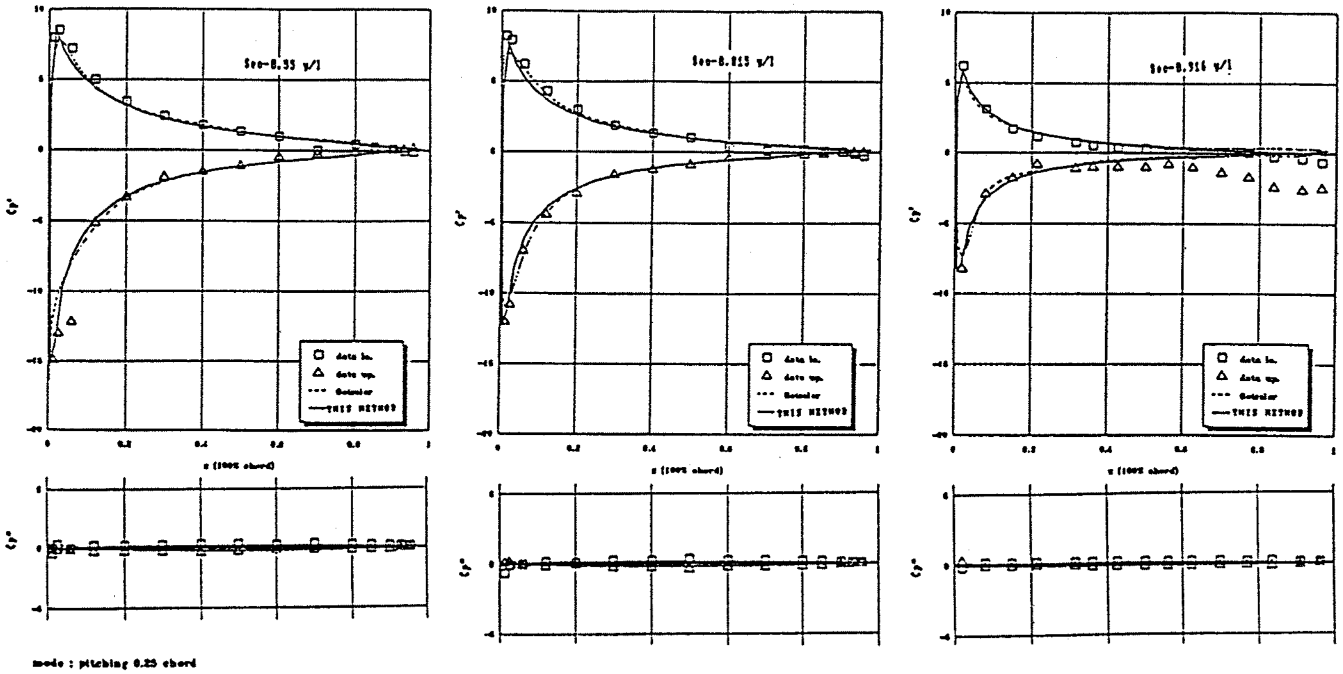

FIGURE 5. Unsteady pressure distribution, $\alpha=3°$, $\omega^{*}=0.14$

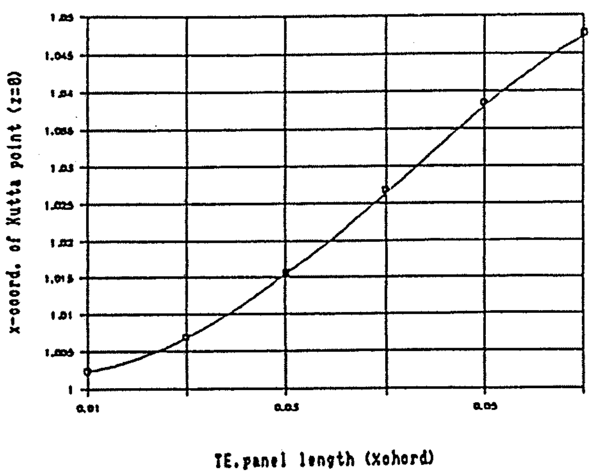

FIGURE 6. Effect of trailing edge panel's chordwise length to Kutta control point coordinate

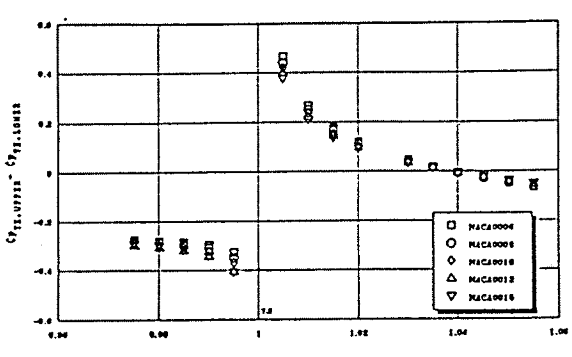

FIGURE 7. Error of calculated pressure distribution due to unsuitable coordinate of Kutta control point

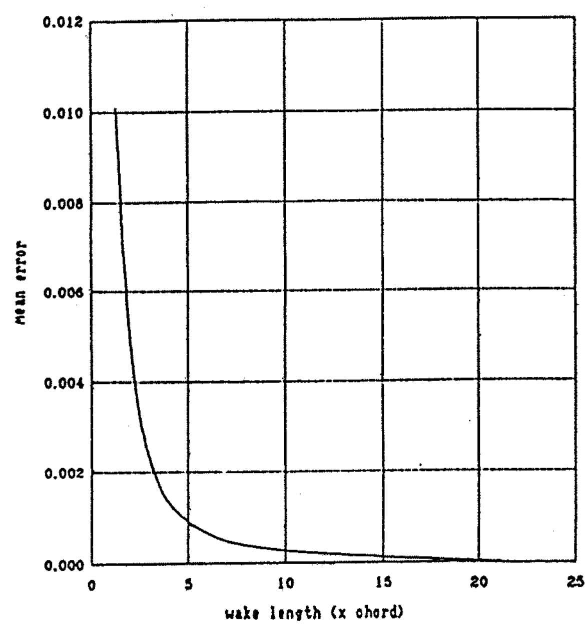

FIGURE 8. Mean error of pressure distribution due to the use of finite discretizised wake