MATHEMATICAL MODELLING OF SELECTED MANEUVERS OF AN AIRCRAFT USING MODIFIED VORTEX LATTICE METHOD

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Abstract

The focus of this paper is an overview of the aircraft's dynamic modelling from three key perspectives: the dynamic stability, panel method approach, and vortex wake dynamics.

The model presented here assumes that the pressure field around the wing can be predicted by the potential flow method.

A method is presented for estimating dynamic stability derivatives in unsteady flow. Ring vortices are employed as singularity elements. As an example the sudden acceleration of the wing is investigated. The stability derivatives of lift, drag and piching moment coefficients with respect to angle of attack and to pich rate are computed.

Results are presented for wing of aspect ratio in the range from 2 up to 50 with sweep angle equal to 15 deg. Differences in steady and unsteady stability derivatives are considerable.

Nomenclature

A	: aspect ratio
c_a	: mean aerodynamic chord
C_D	: drag coefficient
C_L	: lift coefficient
C_m	: moment coefficient
n	: vector normal to wing surface
p	: pressure
S	: wing area
t	: time
U	: wing forward velocity
u, v, w	: velocity components in x,y,z-direction
x, y, z	: body coordinates
X,Y,Z	: inertial coordinates
α	: angle of attack
Γ	: circulation
ϕ	: total velocity potential
φ	: perturbation velocity potential
Λ	: angle of sweep
∞	: uniform flow
BC	: boundary condition
LE	: leading edge
PM	: panel method
RHS	: right-hand side
TE	: trailing edge
VLM	: vortex lattice method

Introduction

From the Flight Dynamics point of wiev the maneuverability of an aircraft is an ability to change its speed, altitude, and direction of flight, i.e., its orientation in space within a given interval of time. Maneuvers may be steady states in which the forces and accelerations are constant (e.g., a steady turn), or they may be transient states (e.g., a sudden acceleration). Aircraft maneuvering in such states is subject to unsteady aerodynamic loads. In addition, when the aircraft operates in the high-angle-of attack regime, nonlinear aerodynamic loads appear as well. Therefore, accurate and efficient prediction of these airloads is of great importance in the analysis of the vehicle's flight motions and in the design of its flight control system.

However, mathematical modelling of an aircraft during maneuvering flight at high angles of attack including the complete nonlinear fluid dynamic equations along time-dependent flight path is still in its initial phases[1]. An alternative approach postulates the use of simplified fluid dynamic models while retaining the complex 3-D nature of both an aircraft geometry and its flight path.

The focus of this paper is an overview of the aircraft's dynamic modelling from three key perspectives: the dynamic stability, panel method approach, and vortex wake dynamics.

The objective of this paper is to examine an ability of a modified Vortex Lattice Method (VLM) for estimating dynamic stability derivatives in unsteady flow.

Dynamic stability of aircraft

Investigations of the transition from equilibrium to a nonequilibrium steady state, or from one maneuvering steady state to another, form part of the subject matter of airplane control [2].

Inasmuch as a maneuverability of an aircraft is related very closely to its controllability, so concept of "stability derivatives" has to be appeared. The calculation of the dynamic stability derivatives has always been an essential as well as a difficult part of the dynamic motion analysis and the aerodynamic design of an aircraft.

Current mathematical models of the unsteady aerodynamic response to an arbitrary motion of an aircraft are usually based on the concept of linear stability derivatives.

It is important to keep in mind that the linearization of the equations of motion and the aerodynamic representation by stability derivatives is only valid provided that the motions of the aircraft are small and that the angles of attack and sideslip are in a range in which the aerodynamic forces and moments are linear.

According to the stability derivatives definition[2] we have

$$C_{x\alpha} = \frac{\partial C_x}{\partial \alpha} = C_L - \frac{\partial C_D}{\partial \alpha} = C_L - C_{D\alpha} \quad (1)$$

$$C_{z\alpha} = \frac{\partial C_z}{\partial \alpha} = -C_D - \frac{\partial C_L}{\partial \alpha} = -C_D - C_{L\alpha} \quad (2)$$

$$C_{m\alpha} = \frac{\partial C_m}{\partial \alpha} \; ; \; C_{mq} = \frac{\partial C_m}{\partial\left(\frac{q c_a}{2U}\right)} \quad (3)$$

$$C_{zq} = \frac{\partial C_z}{\partial\left(\frac{q c_a}{2U}\right)} = -\frac{\partial C_L}{\partial\left(\frac{q c_a}{2U}\right)} = -C_{Lq} \quad (4)$$

$$C_{xq} = \frac{\partial C_x}{\partial\left(\frac{q c_a}{2U}\right)} = -\frac{\partial C_D}{\partial\left(\frac{q c_a}{2U}\right)} = -C_{Dq} \quad (5)$$

where derivatives $C_{D\alpha}$, $C_{L\alpha}$, $C_{m\alpha}$, C_{Dq}, C_{Lq}, and C_{mq} are computed numerically.

Evaluation of aerodynamic loads

The evaluation of the aerodynamic lift, drag, and moment coefficients are all based on the proper integration of the pressure coefficient on the lifting surface. In this paper, the pressure is obtained through an integral representation based on the potential model.

The ability of this concept to predict the pressure field of highly swept wings up to high angles of attack (but no vortex breakdown) under steady-state conditions was demonstrated by the large spectrum of such work as [3]. The extension of these methods into the unsteady aerodynamic regime was done in [4].

The considerations in this paper are restricted to linear, unsteady case.

Panel method approach

Panel methods (PMs) are numerical schemes for solving the model for linear, inviscid and irrotational flow about aircraft flying at subsonic and supersonic speeds.

In the PM approach, the differential equation is converted to an integral one over the configuration surface by means of Green's Theorem. This integral equation is then solved by a discretization process. The configuration under consideration is divided into panels to which a certain distribution of singularities of unknown strength is assigned. Thus, PMs should be more precisely called as surface-singularity methods.

It is assumed that the free vortex sheet remains flat in the plane of the wing and that the free vortices can be visualized as straight lines leaving the wing TEs.

In routine PM codes a combination of source and doublet distributions on the panels are used. Some codes use elementary horseshoe vortices instead of doublets. The strength of the singular elements are determined by satisfying the proper boundary conditions (BCs). Once these are known, the surface velocity components and pressures may be computed.

Basic mathematical flow model

The starting point for the various potential formulations is the *Euler model*. The fundamental assumptions for a physical model of potential flows are that the fluid is inviscid, non-conducting, and isentropic. External forces and heat sources are not taken into account. The governing equations for such model are as follows (cf.(5), chap.7):

-the continuity eq.:

$$\frac{\partial \rho}{\partial t} + div\, \rho V = 0 \quad (6)$$

-the momentum eq.:

$$\rho \frac{dV}{dt} = -grad\, p \quad (7)$$

-the isentropy eq.:

$$p \rho^{-\gamma} = p_\infty \rho_\infty^{-\gamma} \quad (8)$$

where ρ is the density, V is the velocity vector, p is the pressure, and γ is the ratio of specific heats.

Potential models

Panel methods solutions are governed by the second-order linear partial differential equation, called *Prandtl-Glauert equation*

$$(1 - M_\infty^2)\, \varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0 \quad (9)$$

where M_∞ is the Mach number, and φ is the perturbation velocity potential

$$\varphi = \phi - \phi_\infty \qquad (10)$$

$$\nabla\phi = grad\,\phi = V \qquad (11)$$

where ϕ is the total velocity potential.

Starting from the *Navier-Stokes model* , which is the most general model to describe the flow, the following assumptions are necessary to derive Eq. (9):

- no viscosity terms
- steady state
- irrotationality of the flow
- small perturbation approximation.

If one assumes that the flow is incompressible ($M_\infty = 0$), then from Eq.(9) we have the *Laplace equation*

$$\varphi_{xx} + \varphi_{yy} + \varphi_{zz} \equiv \nabla^2\varphi = 0 \qquad (12)$$

It is worth noting that the similar Laplace Equation can be obtained at once from Eq.(6). Namely, taking into account the assumption that the flow is incompressible (ρ=const), and the definition (11) we have by virtue of Eq.(6).

$$\nabla^2\phi = 0 \qquad (13)$$

That why the Laplace Eq.(13) is sometimes called simply the continuity equation, what may seem surprising at the first look.

It is also important to note that so-called *Kelvin's condition*

$$\frac{d\Gamma}{dt} = 0 \quad for\ all\ t \qquad (14)$$

is a form of momentum conservation (and in this sense it replaces Eq.(7)).

In order to complete the problem we need to give proper BCs on the body surface, at the TE (*Kutta-Joukowski condition*), and at infinity.

The first BC requiring zero normal velocity across the body's solid boundaries

$$\nabla\varphi \circ n = 0 \qquad (15)$$

where n is a unit vector normal to the body's surface.

Along the wing's TEs the velocity has to be limited in order to fix the rear stagnation line and therefore

$$\nabla\varphi < \infty \qquad (16)$$

The third BC requires that the flow disturbance, due to the body's motion through the fluid, should diminish far from the aircraft

$$\lim \nabla\varphi = 0 \qquad (17)$$

Modified Vortex Lattice Method (VLM)

It is essential to note that recently there are many versions of the VLM available for solving of Eq. (12): Quasi-VLM [6], Unsteady Quasi-VLM [7], Subsonic Nonlinear VLM [8], Discrete-Vortex Method [9], and even ... Generalized VLM [10]. The proposed method is an extension of the classical VLM [11] for the calculation of the aerodynamic forces on lifting surfaces undergoing complex 3-D unsteady motions.

An essential feature of our proposal is using vortex rings as singular elements (Fig.1). The proposal is based on the equivalence between constant doublet panel and vortex ring (see [12], p.288). By selecting the vortex ring representation, Eq. (12) and condition at infinity (17) are already fulfilled because the vortex is a fundamental solution of the Laplace equation. It should be emphasized that such vortex element ensures the existence of the Eq.(14), and the Kutta-Joukowski condition (16) as well. Thus, to obtain the solution effectively, only boundary condition on the body surface (15) should be fulfill.

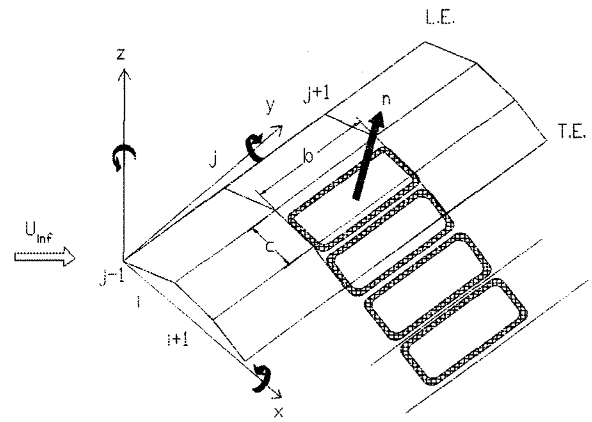

Fig.1. Nomenclature for unsteady motion of a thin lifting surface along a predetermined path.

Time-stepping technique

When we want to treat time-dependent motions of an airplane, we have to keep in mind that the selection of frame systems becomes very essential. Therefore, consider an inertial (X,Y,Z) and body (x,y,z) frame of reference. It is usually useful to describe the unsteady motion of the body on which the "zero normal flow" BC (15) is applied in the body frame of reference. The motion of this frame is then prescribed in the inertial frame and is assumed to be known. However, the BC (15) in this frame becomes

$$\nabla\phi \cdot n - V_f \cdot n = 0 \qquad (18)$$

where V_f is the kinematic velocity, as viewed from the inertial frame. Note that ϕ is the total velocity potential, but as a result of its definition in a frame that is attached to the undisturbed flow, its magnitude is small, like to the perturbation potential.

Then, the solution of the problem, which is time-dependent because of the BC (18), is easier in the body frame. Consequently, the following transformation has to be established

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = f(X_0, Y_0, Z_0, \Phi, \Theta, \Psi) \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (19)$$

where Φ, Θ, Ψ are Eulerian angles.

Similarly, the transormation of the velocities is given by

$$V_f = V_0 + \Omega \times r + v_{ref} \quad (20)$$

where $v_0 = (\dot{X}_0, \dot{Y}_0, \dot{Z}_0)$ is the velocity of the (x,y,z) frame's origin, $\mathbf{r}$=(x,y,z) is the position vector, $\Omega=(\Phi,\theta,\psi)$ is the rate of rotation of the body's frame of reference, and v_{ref}=(x,y,z).

The results presented by Katz & Plotkin (sec.13.1- 13.4 in [12]) indicate that steady-state flow methods can be extended to treat the time dependent problem with only a few modifications. The modification includes the treatment of the condition (14) and the use of the unsteady Bernoulli equation. Therefore, the solution can be reduced to solving an equivalent steady-state flow problem, at each time step. So, this method is called the *time-stepping technique.*

The pressure can be determined from unsteady Bernoulli equation, written either in the body frame of reference or in the inertial one (see sec.13.1 in [12]). In the inertial frame this equation is

$$\frac{p_\infty - p}{\rho} = -\frac{V^2}{2} + \phi_t \quad (21)$$

where by virtue of (11)

$$V^2 = \phi_x^2 + \phi_y^2 + \phi_z^2 \quad (22)$$

Concluding, the staedy-state solution technique can be updated to treat unsteady flows. The typical flow chart is shown in Fig.2.

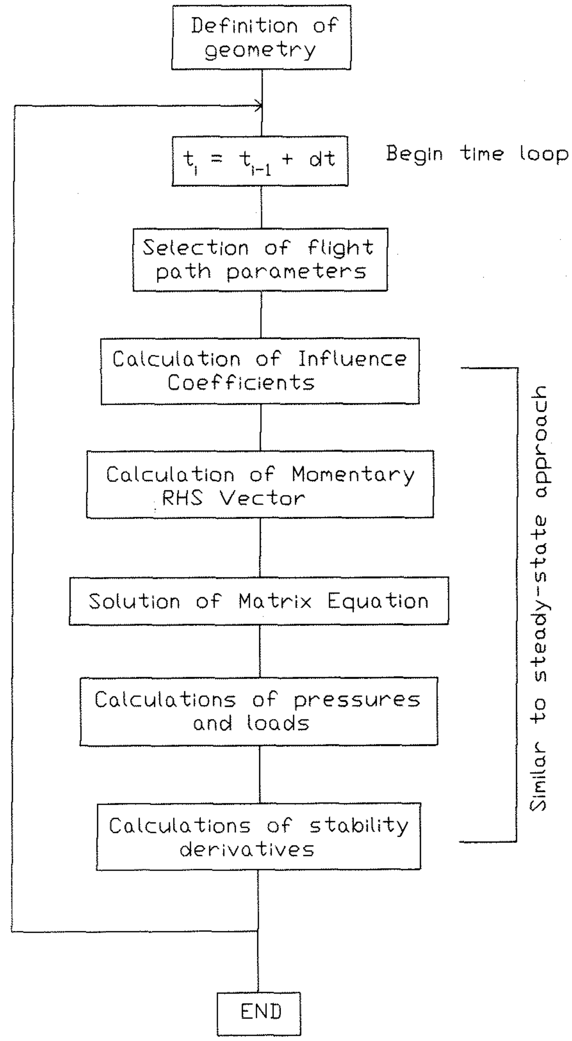

Fig.2. Schematic flow chart for the computation of the unsteady stability derivatives.

Computation of stability derivatives

Once the computations of the influence coefficients $\{a_{KL}\}$ and the righ-hand side velocities $\{RHS_K\}$ are completed (see Fig. 2), the Neumann BC at an arbitrary control point K will result in the following set of linear equations

$$\sum_{L=1}^{M} a_{KL}\Gamma_L = RHS_K \quad (23)$$

When the circulation distribution Γ_{ij} after the solution of Eq.(23) is obtained, the pressure across each panel is determined by using the Bernoulli Eq. (21) for the upper and lower surface streamlines. The pressure difference is defined as

$$\Delta p \equiv p_l - p_u = \rho\left[\left(\frac{V_t^2}{2}\right)_u - \left(\frac{V_t^2}{2}\right)_l + \left(\frac{\partial\phi}{\partial t}\right)_u - \left(\frac{\partial\phi}{\partial t}\right)_l\right] \quad (24)$$

According to[12] tangential velocities due to the wing vortices can be approximated as (see Fig. 1)

$$\frac{\partial\phi}{\partial\tau_i}=\frac{\gamma_y}{2}=\frac{\Gamma_{ij}-\Gamma_{i-1,j}}{2\Delta c_{ij}}\,,\qquad \frac{\partial\phi}{\partial\tau_j}=\frac{\gamma_x}{2}=\frac{\Gamma_{ij}-\Gamma_{i,j-1}}{2\Delta b_{ij}} \tag{25}$$

where Δc_{ij} and Δb_{ij} are the panel lengths in the x-th and y-th directions, respectively (see Fig.1). Similarily, τ_i and τ_j are the panel tangential vectors in the proper directions.

The velocity-potential time derivative is obtained by using the relation

$$\frac{\partial\phi_{ij}}{\partial t}=\frac{\partial}{\partial t}\frac{\Gamma_{ij}}{2} \tag{26}$$

Thus, substituting the terms (25) and (26) into the Eq. (24) results in

$$\Delta p_{ij}=\rho\left[U+u_w,\ V+v_w,\ W+w_w\right]_{ij}\cdot\left(\vec{t}_i\frac{\Gamma_{ij}-\Gamma_{i-1,j}}{\Delta c_{ij}}+\vec{t}_j\frac{\Gamma_{ij}-\Gamma_{i,j-1}}{\Delta b_{ij}}\right)+\rho\cdot\frac{\partial}{\partial t}\Gamma_{ij} \tag{27}$$

where $\{u_w, v_w, w_w\}$ are velocity components induced by the wake vortices and undisturbed flow velocity is $V_\infty = \{U, V, W\}$.

We assume now that the wake is directed along the wing chord and undisturbed velocity V_∞ has only one non-zero component U. In that case Eq.(27) leads to

$$\Delta p_{ij}=\rho\left(U\frac{\Gamma_{ij}-\Gamma_{i-1,j}}{\Delta c_{ij}}+\frac{\partial}{\partial t}\Gamma_{ij}\right) \tag{28}$$

For the simplicity of the calculation of the induced drag, we limit the motion of the lifting surface in such the way that it moves forward along a straight line without sideslip. Then, the induced drag is the force component parallel to the flight direction and each panel contribution is

$$\Delta D_{ij}=-\rho\,(w_{ind}+w_w)_{ij}\,(\Gamma_{ij}-\Gamma_{i-1,j})\,\Delta b_{ij}+\rho\frac{\partial}{\partial t}\Gamma_{ij}\,\Delta S_{ij}\,\sin\alpha_{ij} \tag{29}$$

where α_{ij} is the panel's angle of attack, w_{ind} is downwash induced by the wing's streamwise vortices, and w_w is downwash induced by the wake's streamwise vortices.

The lift, drag and piching moment coefficients can be computed from Eq. (28) (taking into account Eq.(29) for C_D) as follows

$$C_L=\frac{2}{US}\sum_{i,j}S_{ij}\left(\frac{\Gamma_{ij}-\Gamma_{i-1,j}}{\Delta c_{ij}}+\frac{1}{U}\frac{\partial}{\partial t}\Gamma_{ij}\right) \tag{30}$$

$$C_D=\frac{2}{U^2S}\sum_{i,j}S_{ij}\left(-\frac{w_{ind}+w_w}{\Delta c_{ij}}\cdot(\Gamma_{ij}-\Gamma_{i-1,j})+\frac{\partial}{\partial t}\Gamma_{ij}\sin\alpha_{ij}\right) \tag{31}$$

$$C_{m25}=C_L\frac{x_{25}}{c_a}-\frac{2}{USc_a}\cdot\sum_{i,j}S_{ij}x_{ij}\left(\frac{\Gamma_{ij}-\Gamma_{i-1,j}}{\Delta c_{ij}}+\frac{1}{U}\frac{\partial}{\partial t}\Gamma_{ij}\right) \tag{32}$$

where S_{ij} is the panel area, x_{ij} and x_{25} are coordinates of the panel quarter-chord and the wing quarter-mean-aerodynamic-chord, respectively. The difference $\Gamma_{ij}-\Gamma_{i-1,j}$ represents the strength of panel bound vortex, which is placed along the local panel quarter-chords. If the panel is at the LE (i=1) then $\Gamma_{i-1,j} = 0$.

The center of pressure in percent of MAC with respect to the LE can be computed according to the following formula

$$x_{cp}=25+100\cdot\left(\frac{2}{C_Lc_aS}\sum_{i,j}S_{ij}x_{ij}\cdot\left(\frac{\Gamma_{ij}-\Gamma_{i-1,j}}{\Delta c_{ij}}+\frac{1}{U}\frac{\partial}{\partial t}\Gamma_{ij}\right)-\frac{x_{25}}{c_a}\right) \tag{33}$$

One of the simplest but very instructive example of unsteady aerodynamic is the motion after sudden acceleration of the wing. This example was studied in [13, 14], and [4] but without estimating dynamic stability derivatives.

So then, let us consider constant-speed, horizontal flight after the sudden acceleration of an ancambered, tapered, swept wing. Two cases are analyzed. The first one is the flight with constant angle of attack and pitch rate equal to zero. In this case Eq. (23) is solved for the RHS vector as follows

$$RHS_K=-\left((U+u_w)\sin\alpha+w_w\cos\alpha\right)_K \tag{34}$$

After finding the ring vortex circulations Γ_K, the derivatives $C_{L\alpha}$, $C_{D\alpha}$, $C_{m\alpha}$ were computed.

The second case was the flight with pitch rate equal to 1 rad/s around the wing quarter-mean-aerodynamic-chord that results in local angle of attack equal to $q(x_K - x_{25})/U$. Consequently, the k-th component of RHS vector becomes

$$RHS_K=-\left(\frac{q(x_K-x_{25})}{U}+w_w\right)_K \tag{35}$$

After that Γ_K and derivatives C_{Dq}, C_{Lq}, C_{mq} were computed according Eqs. (3)-(5).

Computing process was performed with the relation $U\,\Delta t/c = 0.4$. The wing was divided into M=4 chordwise and N=6 spanwise equally spaced panels. The wing taper ratio $c_T/c_R=0.4$.

Fig.3 shows the nondimensional, transient lift variation with time for swept, tapered wings with various aspect ratios from 2 up to 50. These results are typical[4] in this sense that the initial lift loss and length transient decrease with the reduction in wing aspect ratio. It is also obvious that for wings of small aspect ratios the influence of the wake on the lift coefficient is limited up to about 4 or 5 wing chords behind the TE. Of course, we have to remember that in our model the wake surface is flat.

Fig.4. presents the variation of the position of pressure centre with time. It can be shown that again the steady-state position is reach faster for wings of small aspect ratios.

Fig.5-6 show the stability derivatives $C_{m\alpha}$ and C_{mq}, computed according to Eqs. (3-4), and (34-35). From Fig. 5 it is clear that stability derivative $C_{m\alpha}$ for small aspect ratios is positive what means that these wings have neutral points ahead of the quarter-mean-chord. The steady-state position is again reached faster for small aspect ratios.

Conclusions

In the course of modelling of maneuvers of an aircraft we are facing the simulation of unsteady aerodynamic and the resulting wake dynamics. The modified vortex lattice method with ring elements is shown to be a useful tool for predicting the dynamic stability derivatives. For the numeric examples only the simplest model is presented.

Differences in steady and unsteady stability derivatives are considerable, what may have some meaning during in initial phases of the transition from a steady state of flight to another one.

We want to emphasize that for wings of small aspect ratios and in the case when the wake surface is assumed to be flat, the influence of the wake on the lift coefficient is limited up to a few wing chords behind the trailing edge.

Acknowledgments

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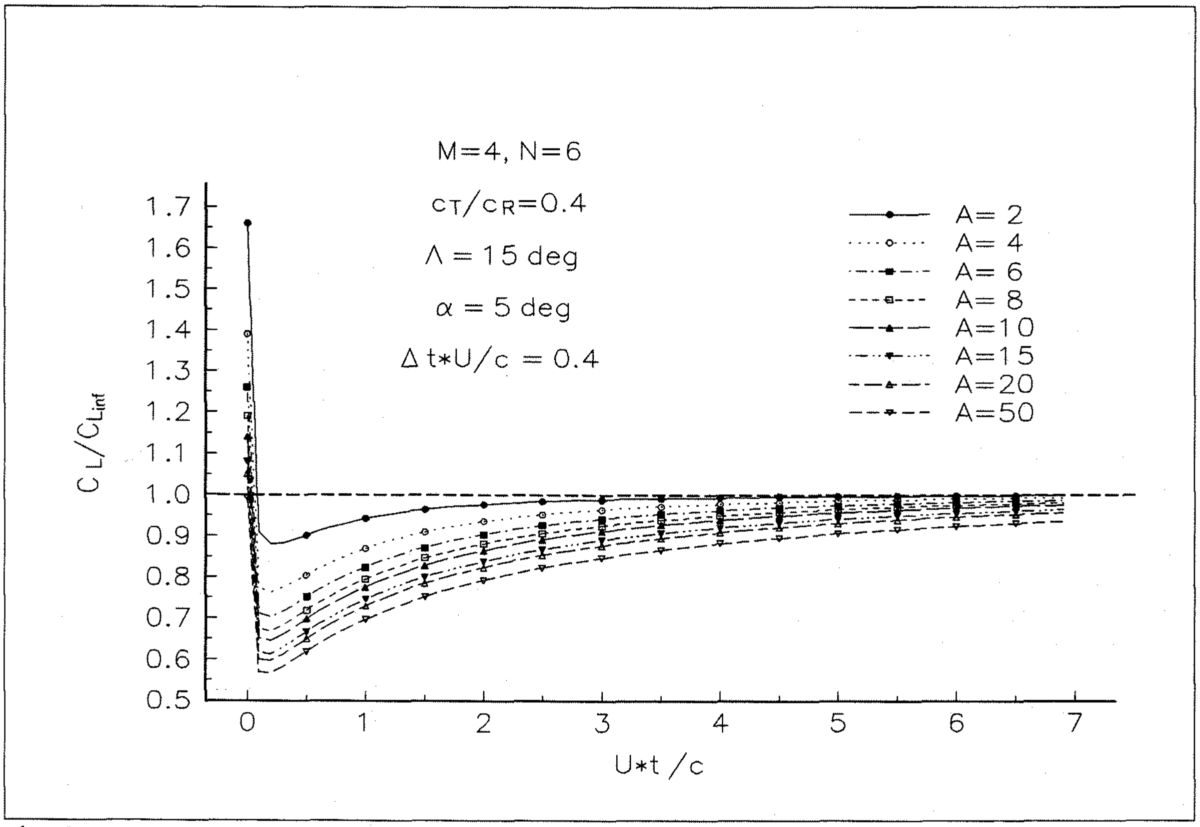

Fig.3. Effect of aspect ratio on the nondimensional transient lift of swept tapered wings that were suddenly set into a constant speed forward flight.

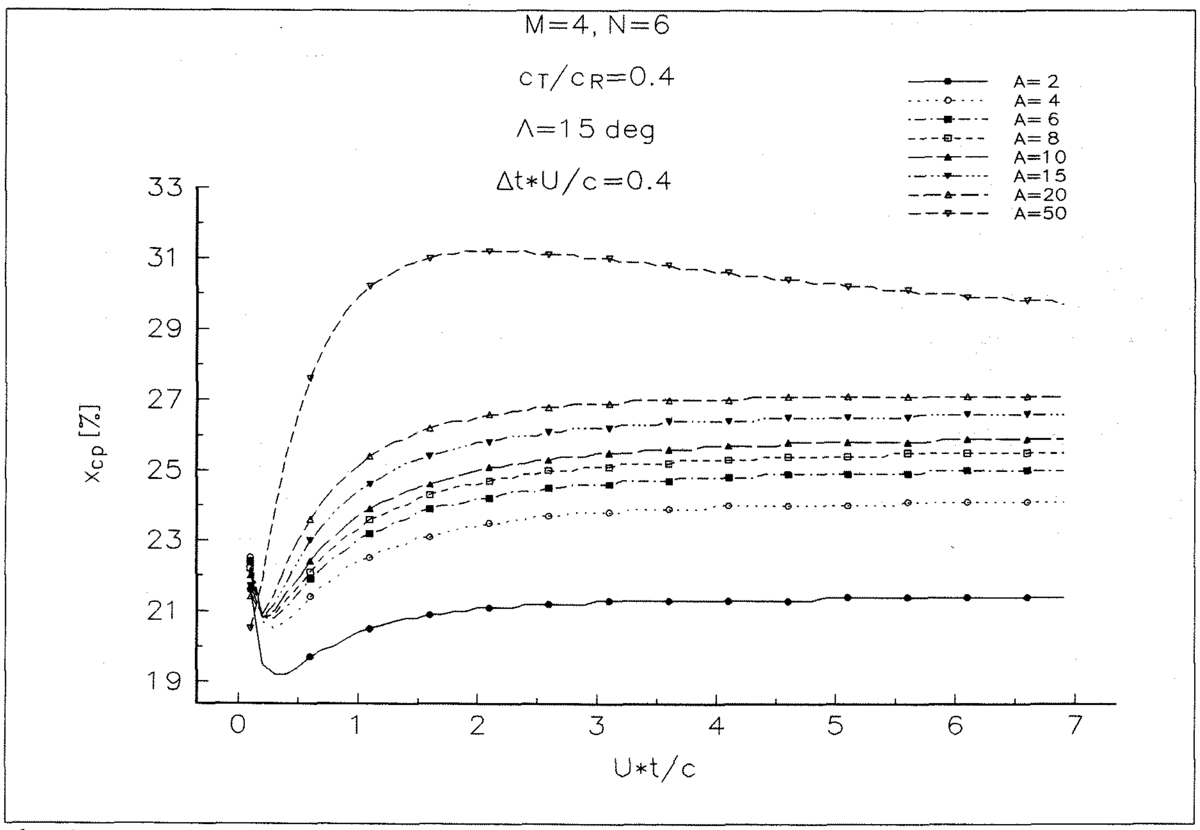

Fig.4. Effect of aspect ratio on the transient centre of pressure position on wings that were suddenly set into motion.

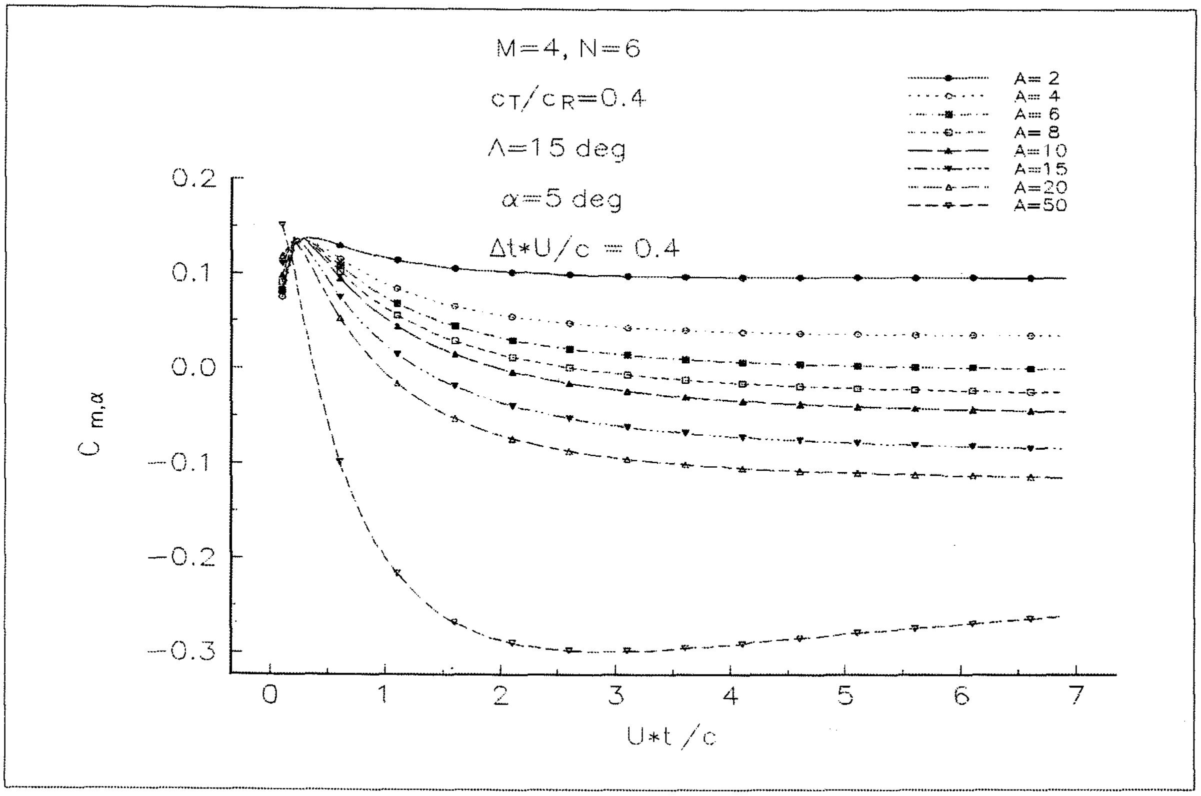

Fig.5. Nondimensional stability derivatives of pitching moment with respect to angle of attack for wings of different aspect ratio that were suddenly set into motion.

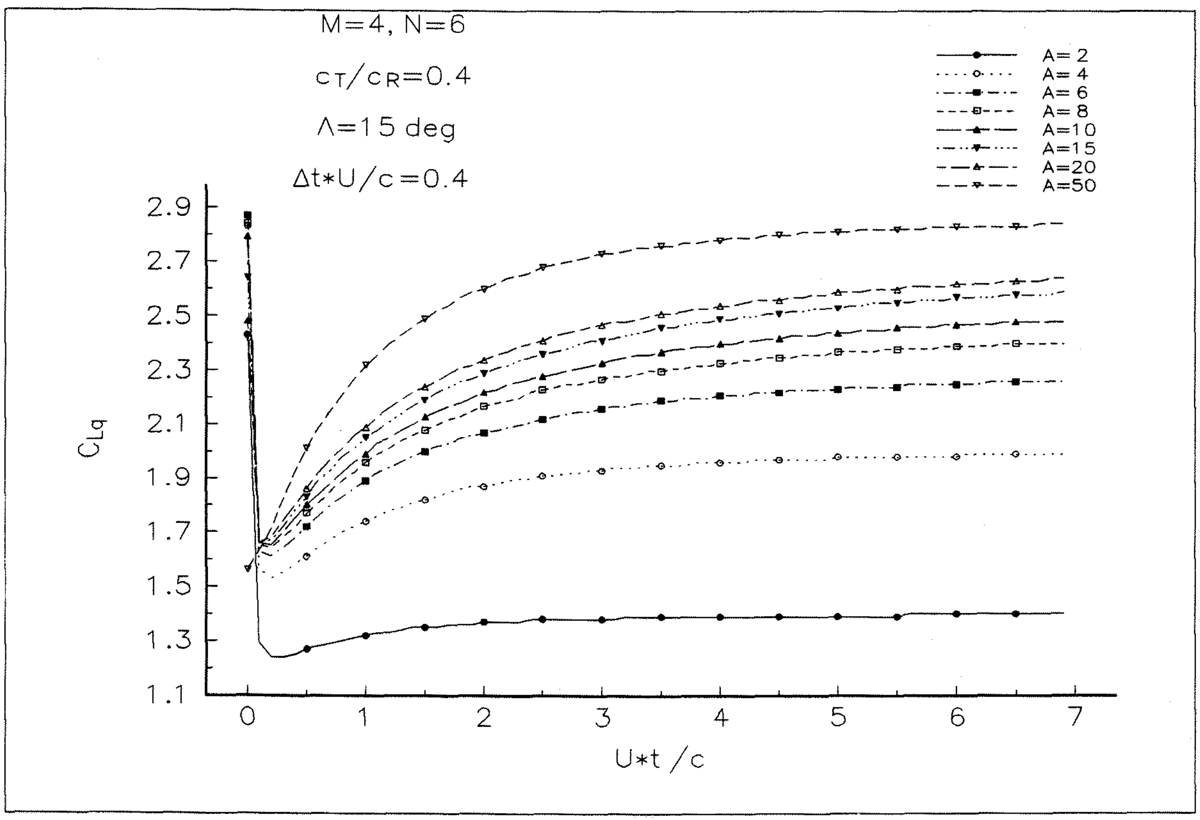

Fig.6. Nondimensional stability derivatives of lifting coefficient with respect to pitch rate for wings of different aspect ratio that were suddenly set into motion.