COMPUTATION OF SUPERSONIC/HYPERSONIC FLOW NEAR COMPLEX CONFIGURATIONS

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Abstract

A numerical method based on the hypersonic small-disturbance theory, the high-incidence slender-body Sychev's theory and Godunov's method have been developed for computing supersonic / hypersonic flows near complex aircrafts at arbitrary angle of attack. This method permits the calculation of the aerodynamic characteristics of the complex airplane and airspace configurations, two or more bodies placed closely and influencing each other.

The basic theory is asymptotic, but in practice it was shown that the applicability range of this method is: $2 < M_\infty < 10, \alpha < 90°$. Method is reliably tested.

The program can be quickly adjusted for many classes of shapes by creating a corresponding file of geometry and is a tool of operative analysis for an automatic engineering design.

Introduction

A new approach to numerical method creation has been realised in this work. This is not direct application of numerical method to equations solving. The preliminary theoretical consideration permits to reduce the problem and simplifies the numerical solution.

Theoretical basis of the method is:

- hypersonic small-disturbance theory;
- hypersonic high-incidence slender-body Sychev's theory;
- new theoretical results, obtained by authors of method:
- extension of Sychev's theory to high-incidence small-sweep wings;
- wing blunt leading edges calculations by Sychev's theory with time-axis, directed along edge.

This theory reduces the 3-D Euler equations to the 2-D unsteady Euler equations (the longitudinal coordinate X is replaced by time t , and is assumed that $u = U_\infty \cos(\alpha)$, which are integrated by Gogunov's numerical method.

Program package is designed on the basis of this theory.

Mathematical statement

Time-independent ideal-gas flow near a three-dimensional (3-D) body is described by differential Euler equations. This equations in the coordinate system moving with the body have the form:

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0,$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0,$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial y} = 0, \qquad (1)$$

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} = 0,$$

$$u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z} = 0, \quad s = \frac{p}{\rho^{æ}}$$

The boundary conditions are given on the surface of the body (kinematic boundary condition):

$$\vec{U} \cdot \vec{n} = 0 \qquad (2)$$

and on the shock wave

$$\rho_\infty U_{\infty_n} = \rho U_n,$$

$$\rho_\infty U_{\infty_n} \vec{U}_\infty + p_\infty \vec{n}_b = \rho U_n \vec{U} + p\vec{n}_b,$$

$$\rho_\infty U_{\infty_n}\left(\frac{U_\infty^2}{2} + e_\infty\right) + p_\infty U_{\infty_n} =$$

$$= \rho U_n\left(\frac{U^2}{2} + e\right) + pU_n, \qquad (3)$$

$$e = \frac{1}{\kappa - 1}\cdot\frac{p}{\rho}$$

Here, n is the normal to the body surface, $\vec{U}$ is the gas particle velocity, $\vec{U}_\infty$ is the undisturbed flow velocity, U_n is the velocity component along the normal to the surface of shock wave, e is the internal energy of unit mass of perfect gas with constant specific heats, and n_b is the normal to the surface of the shock wave.

Relations (1)-(3) formulate the whole mathematical description of 3-D perfect gas flow.

Hypersonic small-disturbance theory (Hayes's theory).

It is assumed that (Fig. 1a):

$$M_\infty >> 1, \quad \tau \sim \delta = d/L << 1, \quad K = M_\infty \tau - 1$$

Here, L is the length of the body, d is the maximum transverse dimension of the body, and τ is the maximum inclination of the body surface to the free-stream direction.

Hayes also assumed that the angle of attack is small. In this case, as shown in [1], it is possible to simplify the equations of the ideal gas motion (1)-(3) in accordance with hypersonic small-disturbance theory provided that the angles of inclination of the body surface to the free-stream direction are small. In fact, introducing, in accordance with an order-of-magnitude estimates, the dimensionless variables

$$x = Lx_0, \quad y = dy_0, \quad z = dz_0$$

$$u = U_\infty\left(1 + \delta^2 u_0\right), \quad v = U_\infty \delta v_0, \quad w = U_\infty \delta w_0$$

$$p = æ p_\infty \delta^2 M_\infty^2 p_0 = \rho_\infty U_\infty^2 \delta^2 p_0,$$

$$\rho = \rho_\infty \rho_0$$

and discarding terms of the order of $o\left(\tau^2 + M_\infty^{-2}\right)$, we can obtain relations identical to 2-D unsteady flow equations, if the variable x_0 is replaced by t_0 $(x_0 = U_\infty t_0)$:

$$\frac{\partial \rho_0}{\partial t_0} + \frac{\partial \rho_0 v_0}{\partial y_0} + \frac{\partial \rho_0 w_0}{\partial z_0} = 0,$$

$$\frac{\partial v_0}{\partial t_0} + v_0 \frac{\partial v_0}{\partial y_0} + w_0 \frac{\partial v_0}{\partial z_0} + \frac{1}{\rho_0}\frac{\partial p_0}{\partial y_0} = 0$$

$$\frac{\partial w_0}{\partial t_0} + v_0 \frac{\partial w_0}{\partial y_0} + w_0 \frac{\partial w_0}{\partial z_0} + \frac{1}{\rho_0}\frac{\partial p_0}{\partial z_0} = 0, \qquad (4)$$

$$\frac{\partial s_0}{\partial t_0} + v_0 \frac{\partial s_0}{\partial y_0} + w_0 \frac{\partial s_0}{\partial z_0} = 0,$$

$$s_0 = \frac{p_0}{\rho_0^{æ}}$$

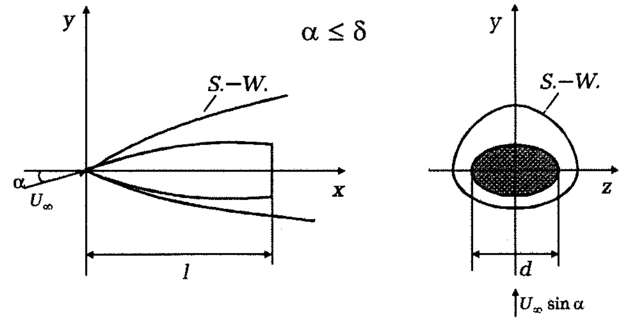

Fig. 1a. Basic theory, Hayes's theory.

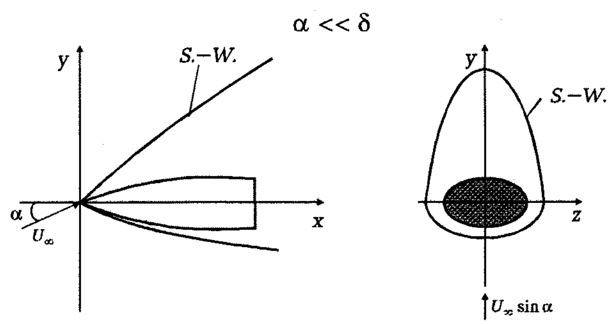

Fig. 1b. Basic theory, Sychev's theory.

The approximate conditions on the surface of the body $f(x_0, y_0, z_0) = 0$ will have the form:

$$\frac{\partial f}{\partial t_0} + v_0 \frac{\partial f}{\partial y_0} + w_0 \frac{\partial f}{\partial z_0} = 0 \qquad (5)$$

The transformation of the conditions at the shock wave $F(x_0, y_0, z_0) = 0$ leads to the following relations:

$$\left[\rho_0 \frac{DF}{Dt_0}\right] = 0, \left[\rho_0 v_0 \frac{DF}{Dt_0} + p_0 \frac{\partial F}{\partial y_0}\right] = 0,$$

$$\left[\rho_0 w_0 \frac{DF}{Dt_0} + p_0 \frac{\partial F}{\partial z_0}\right] = 0$$

$$\left[\rho_0 \frac{DF}{Dt_0}\left(\frac{v_0^2 + w_0^2}{2}\right) + \frac{æ}{æ - 1}\frac{p_0}{\rho_0} + p_0\left(v_0 \frac{\partial F}{\partial y_0} + w_0 \frac{\partial F}{\partial z_0}\right)\right] = 0 \qquad (6)$$

$$\frac{DF}{Dt_0} = \frac{\partial F}{\partial t_0} + v_0 \frac{\partial F}{\partial y_0} + w_0 \frac{\partial F}{\partial z_0}$$

Here, the expressions in square brackets denote the difference in the corresponding quantity ahead and behind the shock, and as the flow velocity ahead of the shock wave we have taken the component of the velocity U_∞ in the plane $x = \text{const}$.

This is the problem of 2-D unsteady motion of gas caused by expansion of 2-D piston.

Sychev's theory

Suchev's theory [2] considered the case of hypersonic flow about an arbitrary slender body at an arbitrary angle of attack. He assumed that (Fig.1b):

$$\delta << 1, \quad M_\infty \delta \geq 1, \quad M_\infty \sin\alpha >> 1 \qquad (7)$$

For small angles of attack theory [2] transforms to theory [1]. The most important parameter, characterizing the precision of theories [1] and [2] is a disturbed flow thickness τ. For small angles of attack the flow disturbances are confined to a region near the body by the highly swept bow shock wave (Fig. 1a). For high angles of attack the disturbance field on the compression side of the body will again be confined to a region near the body (Fig. 1b). But on the leeside the disturbance field will extend a considerable distance. However the leeside field contributes only minimally to the overall loading as long as the compression surface loading is sufficiently high. Hence, the flowfield disturbances of interest are close to the body even for high angles of attack.

Sychev introduced the following dimensionless variables:

$$\begin{aligned} & x = Lx_0,\ y = dy_0,\ z = dz_0 \\ & u = U_\infty \cos\alpha \cdot u_0 \\ & v = U_\infty \sin\alpha \cdot v_0 \\ & w = U_\infty \sin\alpha \cdot w_0 \\ & p = \rho_\infty U_\infty^2 \sin^2\alpha \cdot p_0 \\ & \rho = \rho_\infty \rho_0 \end{aligned} \qquad (8)$$

By substituting (8) into (1)-(3), neglecting values of higher then $o(\delta^2)$-order, and substituting $x_0 = U_\infty \cos\alpha \cdot t_0$ Sychev again obtained equations (4) and corresponding conditions (5)-(6). Axis t with origin at the nose tip is aligned along the body axis but not along free-stream velocity as in theory [1]. And the flat piston not only expands but also moves on the plane with velocity $U_\infty \sin\alpha$.

So, this theory reduces the 3-D steady Euler equations to the 2-D unsteady Euler equations (the longitudinal coordinate x is replaced by time t, and it is assumed that $u = U_\infty \cos\alpha$). This equation transformation simplifier the numerical solution of the problem.

The above theory has been extended to new classes of flows by works [3] and [4]. Theoretical and numerical investigations presented in paper [3] show that theory [1,2] is applicable to large class of body configurations with any-swept wings.

A new approach to the problem of flow near supersonic blunt leading edge has been developed in paper [4]. In this paper flow near the blunt leading edge is considered analogous to the flow on compression side of a slender body which has the form of leading edge. Sychev's theory with time-axis directed along the leading edge is applied in this case in the neighborhood of the leading edge of the wing. The flow parameters in the central part of disturbed region are calculated within the small-disturbance theory. Thus, in both regions of flow over the wing it is possible to use a single universal numerical method based on the use of Hayes's and Sychev's theories(unsteady analogy).

We note that, the combined method make it possible to calculate flows in which subsonic zones are formed near the blunt leading edge of the wing. In fact, relations (4)-(6) are valid when $\delta << \alpha_\tau < \frac{\pi}{2}$, which corresponds , in particular, to large angles between x axis and the leading edge, for which behind the shock wave near the leading edge the local Mach numbers M < 1.

The theoretical and numerical investigations presented in paper [4] show that the above theories are applicable to calculations of supersonic flows near any-swept wings in a large angles-of-attack range. The applicability range of this theories was determined and it was shown that this range is considerable largely than it was assumed in papers [1] and [2]. So, a large class of airframes can be considered using this theory.

Advantages

Advantages of numerical method based on theory [1-4]:

- the numerical method integrating equations (4) is more stable in operation and utilization than methods based on equations (1) (3-D method) because the main difficulty in using 3-D methods is the arising of subsonic domains ($u < a$) where equations change the type. The method under consideration assume that $u = U_\infty \cos\alpha$ always and the equation type does not change.
- this method permits one to solve problems with real subsonic domains such as:
- the problem of the flow on compression side of arbitrary slender airframes at high angles of attack.
- the problem of the flow near blunt leading edge of small-swept wings.

Basic theory is asymptotic, but in practice it was shown that the applicability range of this method is: $2 \le M_\infty \le 10, |\alpha| < 90°$.

Numerical method

Hypersonic slender-body theory equations (4) are integrated by Godunov's first-order approximation method [5]. Bow shock wave is fitted by procedure (6) and assumed to be a boundary of calculation region. The disturbed region is confined to a body surface and bow shock wave. The calculation grid is attached to the body surface and bow shock wave. If disturbed region configuration is complex this region is divided into simple subregions. In each subregion the calculation grid is built so that grid points coincide on the boundaries of subregions. Cross section shape determines subregion number and arrangement.

The program can be quickly adjusted for many classes of shapes by creating of corresponding file of geometry.

Program permits to calculate flow around:

- isolated body;
- body-wing configuration;
- body-wing-stabilizer-tail configuration;
- all mentioned above configurations with engine;
- double-body configurations such as: aircraft-carrier with orbital vehicle and fuel tank with orbital vehicle.

Cross section types of this configurations are shown by fig. 2.

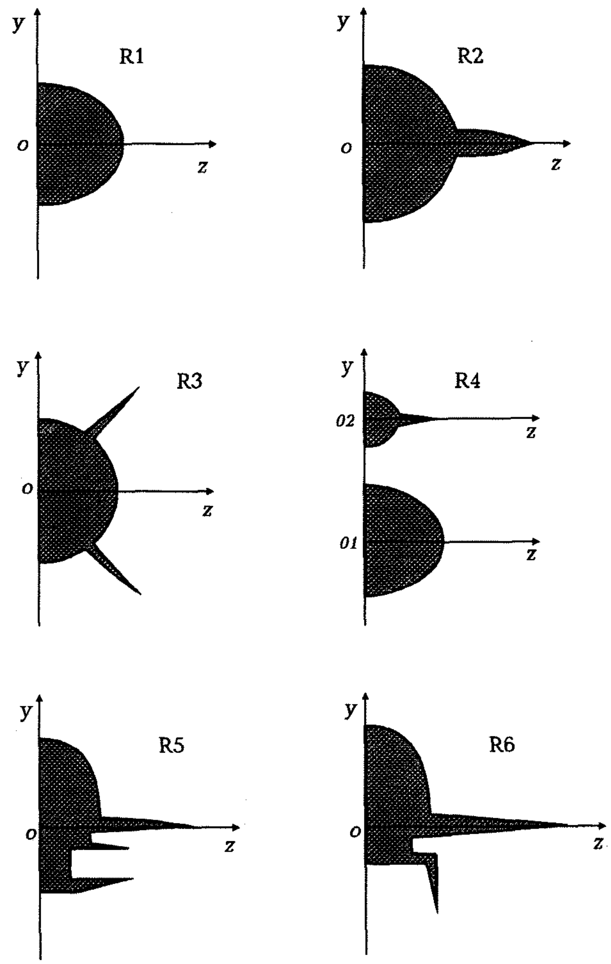

Fig. 2. Cross section types.

Results

This program is a tool to calculate:

- flow fields near one or more closely placed airborne bodies;
- shock wave location;
- flow parameters distribution on the bodies surface;
- summary characteristics (aerodynamic forces and moment coefficients);
- aerodynamic loads on selected elements of designs.

Method is reliably tested. Calculations of many shapes and comparison of results obtained with aerodynamic experiment results and other calculation results shows that the difference in distributed characteristics does not exceed 10%, and in total characteristics - 5%.

Some results of this method (called NINA) are shown on fig. 3-10 in comparison with experimental and other calculation methods results.

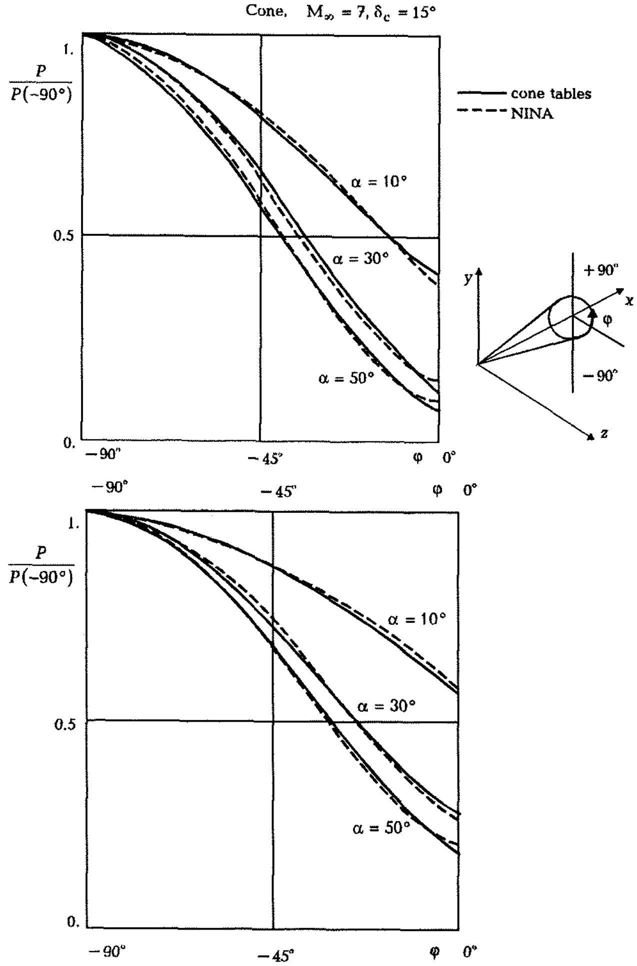

Fig. 3. Cone at high angles of attack

The applicability of this method to slender-body calculations at high angles of attack is demonstrated by fig. 3-5. Pressure and density distribution on cone-surface calculated by NINA-method are compared with 3-D second-order numerical method at $M = 7$, $\alpha = 10°, 30°, 50°$ (fig. 3).

Total characteristics C_x, C_y, M_z and X_g obtained by NINA method (solid lines) are compared with experimental data (points) at $M = 7$ and $\alpha = 0° \div 60°$ on fig. 4.

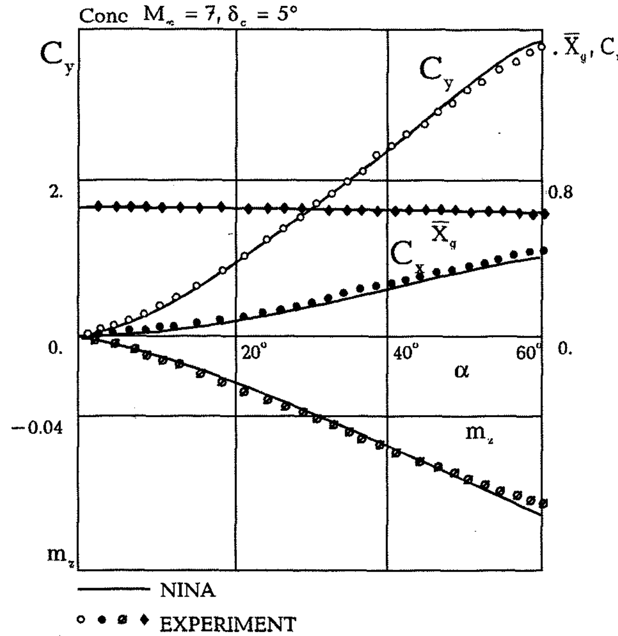

Fig. 4. Total characteristics of cone at $\alpha = 0° \div 60°$

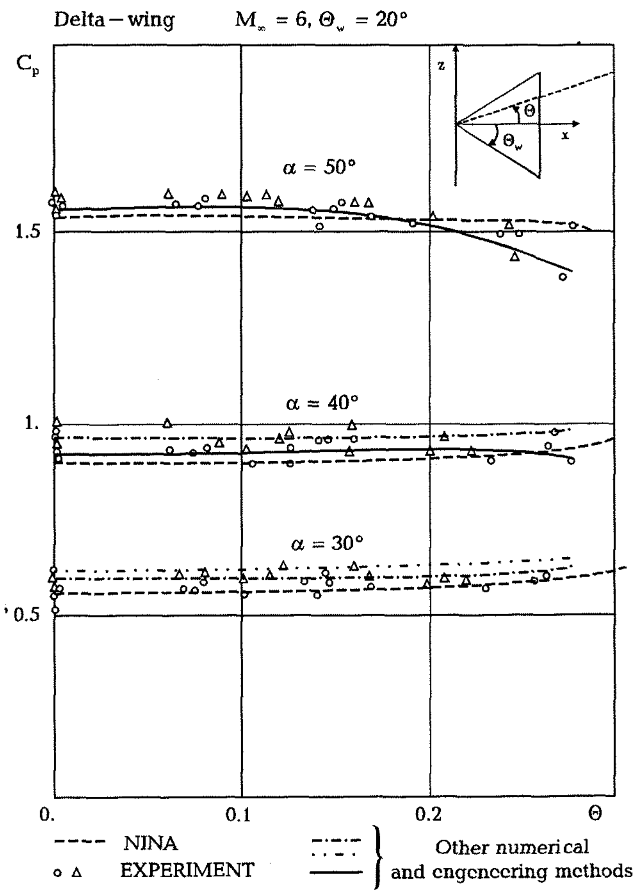

Fig. 5. C_p - distribution on compression side of Delta wing at $\alpha = 30°, 40°, 50°$

Fig. 5 shows C_p distribution on compression side of delta-wing at $M = 6$ $\alpha = 30°, 40°, 50°$. Touch lines (NINA) are this method results. Other lines are obtained by other calculation methods, points are experimental results.

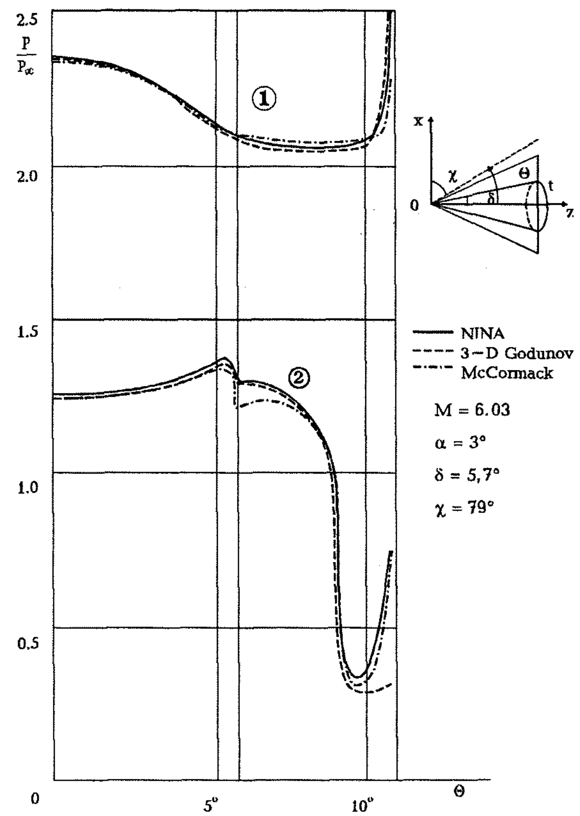

Fig. 6. Pressure distribution on compression side (1) and on leeside (2) cone-delta wing configuration.

Pressure distribution on compression side (1) and leeside (2) of cone-delta wing configuration is presented on fig. 6. Three numerical methods results (NINA, 3-D Godunov and McCormack) are compared on this figure at $M = 6.03$, $\alpha = 3°$. Solid lines are NINA's results, touch lines - 3-D Godunov's method results, touch-point lines - 3-D McCormack's method results.

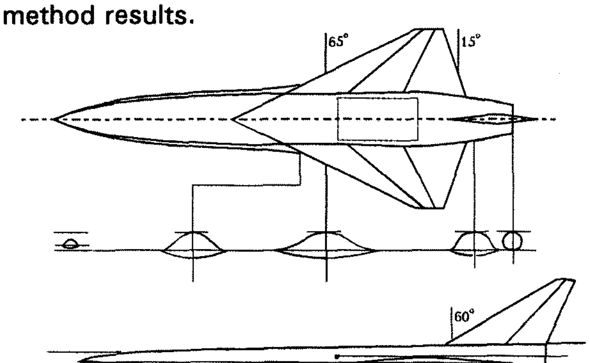

Fig. 7. Hypersonic transport aircraft model.

Hypersonic transport aircraft model [7] shown by fig. 7 has been calculated by NINA-program. Fig. 8 demonstrates the calculation results in comparison with experimental data [7] (solid lines are NINA's results, points - experiment.

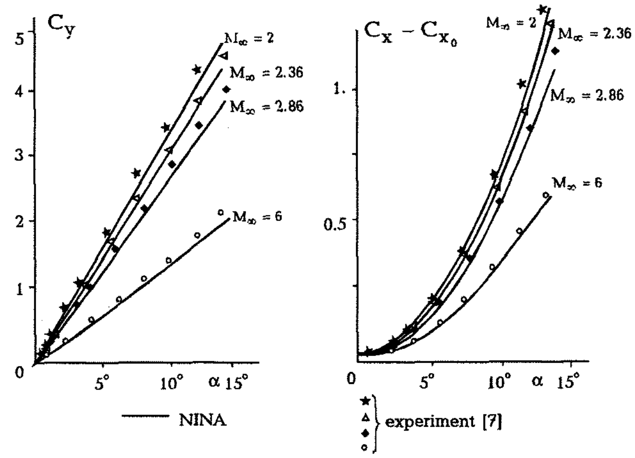

Fig. 8. Total characteristics of hypersonic transport aircraft model.

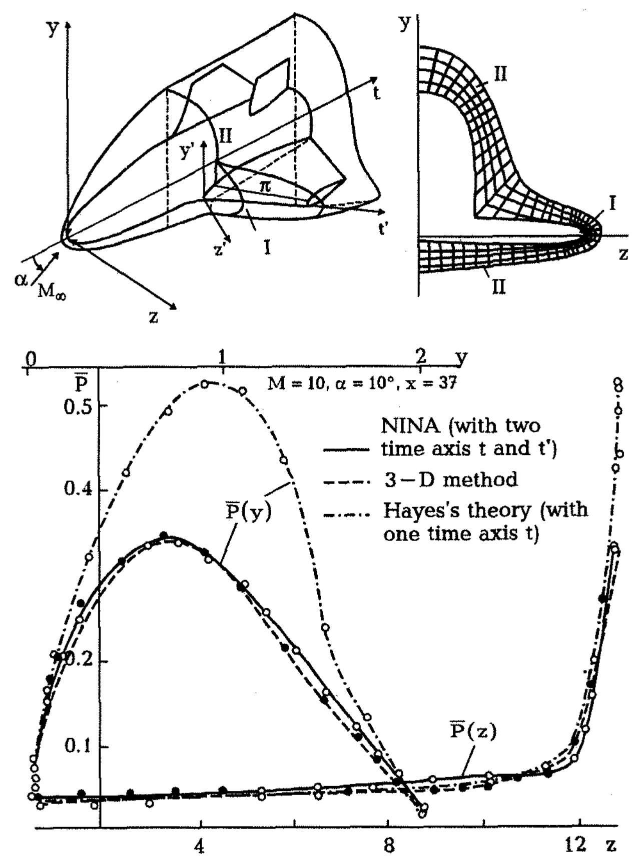

Fig. 9. Aircraft with blunt leading edges.

Calculation results for space aircraft model with blunt leading edges are presented by fig. 9. Aircraft configuration and calculation grid are schematically shown on the same figure.

Pressure distribution on lee-side of the body and on blunt leading edge surface calculated by combined method NINA are compared with the results of 3-D second order calculation method. This investigation had demonstrated that the combined method (with two time-axes: t-along body axes and t' -along leading edge) provides the sufficient accuracy in the neighborhood of the leading edge. But the results obtained within the framework of classical hypersonic small disturbance theory differ from exact solution essentially.

Flow field near two-bodies configuration calculated by NINA-program is presented on fig. 10

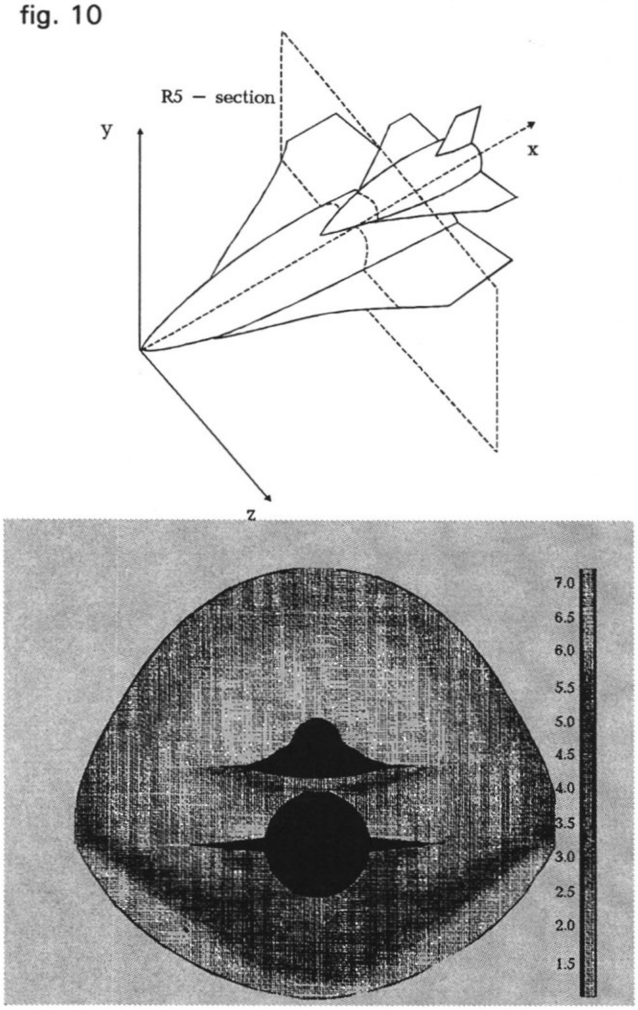

Fig. 10. Double-body configuration: aircraft-carrier-orbital vehicle ($M = 5.0$, $\alpha = 10°$, pressure distribution)

One regime calculation time is 10 - 20 min. for 1 million operations per seconds computer, hence this program can be used for operative analysis for automatic engineering design.

Conclusions

Program package based on the previous theory is of kind that solve the problem of flow near three-dimension bodies by a certain numerical method (for example, McCormack's method or Godunov's method). Such programs integrate three-dimension time-independent Euler's equations.

The most differ of present method from another is that it solves the problem in the limits of hypersonic small-disturbance theory (exactly, in Sychev's generalization of this theory for any incidence angles).

Comparison with other methods results shows that a small loss of precision (10% in distributive characteristics and 5% in summary characteristics in the worst case) gives a great extension of applicability region. Present program gives reliable results at $2 < M < 10$ and arbitrary angles of attack. But 3-D programs applicability region is restricted by small angles of attack because of subsonic domains arising.

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