Guidance System Design by the Linear Exponential Quadratic Gaussian and Dual Control Method

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In this paper the guidance system of a missile is designed by Linear Exponential Quadratic Gaussian and Dual Control (LEQG/DC) method, i.e., both optimal control and estimation problems are considered simultaneously early in the beginning of the design process. In addition to applying traditional Linear Quadratic Gaussian and Dual Control (LQG/DC) approach by taking the state weighting matrix to be negative definite, such that the magnitudes of both state and control input can be raised to increase the observability of the system, the most important key point of the proposed method is to change the performance index into Linear Exponential Quadratic (LEQ) form, by this way the optimal control gain can take both system and measurement noise covariances into consideration. An example of missile guidance system design is also given to show that the performances such as miss distance and state estimation of the proposed method are better.

Introduction

In general, the Linear Quadratic Gaussian (LQG) method [1-7] is extensively used in optimal control and multivariable control systems design, which is based on the regulator concept [8], i.e., to keep the states regulated from system and measurement noises as well as to make the effort of control inputs to a minimum value. The state estimator is usually designed by the robustness recovery process at the final step to retain the robustness stability of the optimal control, e.g., the LQG/LTR (Loop Tansfer Recovery) method. Since the magnitudes of both states and control inputs are kept to the minimum values, that the observability of states is always poor. If there are larger system and measurement noises, then the performance of resulting system would even be decreased. This is the major disadvantage of guidance and control systems obtained by the traditional LQG and LQG/LTR methods [1-9].

Although the balanced realization methods [10-13] are proposed to solve this problem, nevertheless, the compensator, control inputs as well as states trajectory are defined in the first step. Therefore, the improvement of this balanced realization method is limited. The other methods to increase the observability are obtained by either adding the observability weighting factor into the performance index [14] or letting the state weighting matrix to be negative definite [9,15-16], such that the magnitudes of both state and control input can be raised. The disadvantage is that which will reduce the stability margin of the original systems.

The main idea of the proposed method is to take the dual control [9,17-26], i.e., both optimal control and estimation problems, into consideration from the beginning of the design process. Therefore, in addition to applying traditional Linear Quadratic Gaussian and Dual Control (LQG/DC) approch [9] by taking the state weighting matrix to be negative definite, such that the magnitudes of both state and control input can be raised to incease the observability of the system, the most important key point of the proposed method is to change the performance index into Linear Exponential Quadratic (LEQ) form [27-30], by this way the optimal control gain of the control system can take both system and measurement noise covariances into consideration, which can provide better performances than those obtained by LQG based methods.

In this paper, three other different guidance laws of a missile obtained by LQG, LEQG and LQG/DC methods are compared with the proposed LEQG/DC design, it can be seen that the performances such as miss distance and state estimation obtained by LEQG/DC method are better than the others. The organization of this paper is as follows: Section 1 is the introduction. Section 2 derives the general solution of LEQG problem. Section 3 formulates the LEQG/DC problem of a missile guidance system. Section 4 gives the simulation results and discussions. Finally, the conclusion is drawn in Section 5.

General Solution of LEQG Problem

In this section, the optimal control based on Linear Exponential Quadratic Gaussian (LEQG) performance criteria is derived by applying the Hamilton-Jacobi-Bellman(HJB) equation [31]. Let the system be represented by the following time-invariant state equation

$$\dot{x} = Ax + Bu + \Gamma w \tag{1}$$

and the measurement process

$$y = Cx + v \tag{2}$$

where x, u and y are respectively n-, m- and q-dimensional state, control, and measurement vectors, A,B,Γ and C are $n \times n$, $n \times m$,$n \times p$ and $q \times n$ matrices, respectively, w and v are p- and q-dimensional uncorrelated Gaussian white noise processes with zero mean and covariances

$$E\{w(t)w^T(\tau)\} = W\delta(t-\tau) \tag{3}$$
$$E\{v(t)v^T(\tau)\} = V\delta(t-\tau) \tag{4}$$

and

$$E\{v(t)w^T(\tau)\} = 0 \tag{5}$$

The problem is to minimize the following LEQG performance criterion:

$$J(x, t_0) = \sigma E\{exp\{\frac{\sigma}{2}x^T(t_f)S_f x(t_f) + \frac{\sigma}{2}\int_{t_0}^{t_f}[x^T(t)Qx(t) + u^T(t)Ru(t)]dt\}\} \tag{6}$$

where σ is a real number, $E\{\cdot\}$ and $exp\{\cdot\}$ are expection and exponential function operators, S_f is an $n \times n$ positive semi-definite weighting matrix for the terminal states. In order to increase the states observability, Q is set as an $n \times n$ negative definite state weighting matrix [9,15-16], and R is an $m \times m$ postive-definite control weighting matrix.

Applying the separation theorem, a Kalman filter is used to obtain the optimal estimated state from the noisy measurements, and the state estimation equation can be written as

$$\dot{\hat{x}} = A\hat{x} + Bu + K_f(y - C\hat{x}) \tag{7}$$

Since the input to the optimal controller is a linear combination of $\hat{x}$, therefore, the performance criterion of Eq.(6) can be rewritten as

$$\begin{aligned} J(\hat{x}, t_0) &= \sigma E\{exp\{\frac{\sigma}{2}\hat{x}^T(t_f)S_f\hat{x}(t_f) \\ &+\frac{\sigma}{2}\int_{t_0}^{t_f}[\hat{x}^T(t)Q\hat{x}(t) + u^T(t)Ru(t)]dt\}\} \end{aligned} \tag{8}$$

where the original state vector x in Eq. (6) is replaced by the estimated state vector $\hat{x}$, and K_f is the Kalman filter gain defined as

$$K_f = PC^TV^{-1} \tag{9}$$

where the error covariance $P = E\{(x - \hat{x})(x - \hat{x})^T\}$ is propagated forward in time by the Riccati equation

$$\dot{P} = AP + PA^T + \Gamma W\Gamma^T - PC^TV^{-1}CP, \quad P(t_0) = P_0 \tag{10}$$

Since the Kalman filter is an unbiased estimator, that the correction term $(y - C\hat{x})$ of Kalman filter in Eq.(7) may be regarded as an equivalent white noise with zero mean, and the covariance of which is the same as that of v, i.e.,

$$E\{y - C\hat{x}\} = 0 \tag{11}$$

and

$$E\{(y - C\hat{x})(y - C\hat{x})^T\} = R\delta(t) \tag{12}$$

Now the Hamilton-Jacobi-Bellman(HJB) equation, with the reformulated LEQG performance criterion defined by Eqs.(7) and (8), can be derived as follows:

By Eq.(7) the estimator equation of stochastic optimal control can be rewritten as

$$d\hat{x} = f(\hat{x}, u, t)dt + K_f(y - C\hat{x})dt \tag{13}$$

where d is an incremental operator and

$$f(\hat{x}, u, t) = A\hat{x}(t) + Bu(t) \tag{14}$$

By Eq.(8) the performance criterion can be rewritten as

$$J(\hat{x}, t) = \sigma E\{exp\{\sigma\Theta[\hat{x}(t_f), t_f] + \sigma\int_t^{t_f}\Omega[\hat{x}(s), u(s), s]ds\}\} \tag{15}$$

where

$$\Theta[\hat{x}(t_f), t_f] = \frac{1}{2}\hat{x}^T(t_f)S_f\hat{x}(t_f) \tag{16}$$
$$\Omega[\hat{x}(s), u(s), s] = \Phi[\hat{x}(s), s] + \Psi[u(s), s] \tag{17}$$
$$\Phi[\hat{x}(s), s] = \frac{1}{2}\hat{x}^T(s)Q\hat{x}(s) \tag{18}$$

and

$$\Psi[u(s), s] = \frac{1}{2}u^T(s)Ru(s) \tag{19}$$

Let the value function $V(\hat{x}, t)$ be the minimum performance [32] from t to t_f,i.e.,

$$V(\hat{x}, t) = \min_{u(t)} J(\hat{x}, t) \tag{20}$$

then by Eqs.(15) and (20) one has

$$
\begin{aligned}
V(\hat{x},t) &= \min_{u(t)} \sigma E\{exp\{\sigma\Theta[\hat{x}(t_f),t_f] + \sigma\int_t^{t_f} \Omega[\hat{x}(s),u(s),s]ds\}\} \\
&= \min_{u(t)} \sigma E\{exp\{\sigma\Theta[\hat{x}(t_f),t_f] + \sigma\int_t^{t+\Delta t} \Omega[\hat{x}(s),u(s),s]ds \\
&+\sigma\int_{t+\Delta t}^{t_f} \Omega[\hat{x}(s),u(s),s]ds\}\} \qquad (21)
\end{aligned}
$$

where Δt is a very small time interval. Because $u(t_1)$ at $t_1 \geq t$ do not affect $x(s)$ at $s \leq t$, therefore, the minimizing operator may be operated on each term in Eq.(21) and yields

$$
\begin{aligned}
V(\hat{x},t) &= \min_{u(t)} \sigma E\{exp\{\sigma\int_t^{t+\Delta t} \Omega[\hat{x}(s),u(s),s]ds\} \\
&\times \min_{u(t+\Delta t)} E\{exp\{\sigma\Theta[\hat{x}(t_f),t_f] \\
&+\sigma\int_{t+\Delta t}^{t_f} \Omega[\hat{x}(s),u(s),s]ds\}\}\} \\
&= \min_{u(t)} \sigma E\{exp\{\sigma\int_t^{t+\Delta t} \Omega[\hat{x}(s),u(s),s]ds\} \\
&\times \frac{1}{\sigma} V[\hat{x}(t+\Delta t),t+\Delta t]\} \\
&= \min_{u(t)} \sigma E\{exp\{\sigma[\Omega\hat{x}(t),u(t),t]\Delta t\} \\
&\times \frac{1}{\sigma} V[\hat{x}(t+\Delta t),t+\Delta t]\} \qquad (22)
\end{aligned}
$$

In the above derivation, since Δt is very small, the following approximation is applied:

$$
\int_t^{t+\Delta t} \Omega[\hat{x}(s),u(s),s]ds \cong \Omega[\hat{x}(t),u(t),s]\Delta t \qquad (23)
$$

The exponential term and $V[\hat{x}(t+\Delta t),t+\Delta t]$ in Eq.(22) can be expressed in a Taylor series expansion about $\hat{x}$ and t, after neglecting the higher order terms above $(\Delta t)^2$, one has

$$
exp\{\sigma\Omega[\hat{x}(t),u(t),t]\Delta t\} \cong 1 + \sigma\Omega[\hat{x}(t),u(t),t]\Delta t \qquad (24)
$$

and

$$
\begin{aligned}
V[\hat{x}(t+\Delta t),t+\Delta t] &\cong V(\hat{x},t) + V_t(\hat{x},t)\Delta t + V_{\hat{x}}^T(\hat{x},t)\Delta\hat{x} \\
&+V_{t\hat{x}}^T(\hat{x},t)\Delta t\Delta\hat{x} \\
&+\frac{1}{2}(\Delta\hat{x})^T V_{\hat{x}\hat{x}}(\hat{x},t)\Delta\hat{x} \qquad (25)
\end{aligned}
$$

where the subindex denotes the partial derivation with respect to that variable. By Eq.(13)

$$
\begin{aligned}
\Delta\hat{x} &= \hat{x}(t+\Delta t) - \hat{x}(t) \\
&= f(\hat{x},u,t)\Delta t + K_f(y - C\hat{x})\Delta t \qquad (26)
\end{aligned}
$$

then by Eqs.(11) and (12) both mean and variance of $\Delta\hat{x}$ defined by Eq.(26) can be derived as

$$
E\{\Delta\hat{x}\} = f(\hat{x},u,t)\Delta t \qquad (27)
$$

and

$$
E\{\Delta\hat{x}\Delta\hat{x}^T\} = f(\hat{x},u,t)f^T(\hat{x},u,t)(\Delta t)^2 + K_f V K_f^T \Delta t \qquad (28)
$$

Substituting Eqs.(23)-(28) into Eq.(22), one has

$$
\begin{aligned}
V(\hat{x},t) &= \min_{u(t)}\{V(\hat{x},t) + V_t(\hat{x},t)\Delta t + V_{\hat{x}}^T(\hat{x},t)f(\hat{x},u,t)\Delta t \\
&+V_{t\hat{x}}^T(\hat{x},t)f(\hat{x},u,t)(\Delta t)^2 \\
&+\frac{1}{2}t_r[V_{\hat{x}\hat{x}}(\hat{x},t)f(\hat{x},u,t)f^T(\hat{x},u,t)(\Delta t)^2 \\
&+V_{\hat{x}\hat{x}}(\hat{x},t)K_f V K_f^T \Delta t] \\
&+\sigma\Omega(\hat{x},u,t)V(\hat{x},t)\Delta t + \sigma\Omega(\hat{x},u,t)V_t(\hat{x},t)(\Delta t)^2 \\
&+\sigma\Omega(\hat{x},u,t)V_{\hat{x}}^T(\hat{x},t)f(\hat{x},u,t)(\Delta t)^2 \\
&+\sigma\Omega(\hat{x},u,t)V_{t\hat{x}}^T(\hat{x},t)f(\hat{x},u,t)(\Delta t)^3 \\
&+\frac{\sigma}{2}\Omega(\hat{x},u,t)t_r[V\hat{x}\hat{x}(\hat{x},t)f(\hat{x},u,t)f^T(\hat{x},u,t)(\Delta t)^3 \\
&+V_{\hat{x}\hat{x}}(\hat{x},t)K_f V k_f^T(\Delta t)^2]\} \qquad (29)
\end{aligned}
$$

where $t_r[\cdot]$ is a trace operator.

Substituting Eqs.(14) and (17) into Eq.(29) one can obtain the Hamilton-Jacobi Bellman equation of LEQG performance criterion as:

$$
\begin{aligned}
-V_t(\hat{x},t) &= \min_{u(t)}\{\frac{\sigma}{2}[x^T(t)Qx(t) + u^T(t)Ru(t)]V(\hat{x},t) \\
&+\frac{1}{2}t_r[V_{\hat{x}\hat{x}}(\hat{x},t)K_f V K_f^T] \\
&+V_{\hat{x}}^T(\hat{x},t)[A\hat{x} + Bu]\} \qquad (30)
\end{aligned}
$$

Following optimal control theory [32], the Hamiltonian function H is related to the derivation of value function as:

$$
V_t(\hat{x},t) = -\min_{u(t)} H(\hat{x},u,t) \qquad (31)
$$

Comparing Eqs. (30) and (31) one can find

$$
\begin{aligned}
H(\hat{x},u,t) &= \frac{\sigma}{2}[x^T(t)Qx(t) + u^T(t)Ru(t)]V(\hat{x},t) \\
&+V_{\hat{x}}^T(\hat{x},t)[A\hat{x} + Bu] \\
&+\frac{1}{2}t_r[V_{\hat{x}\hat{x}}(\hat{x},t)K_f V K_f^T] \qquad (32)
\end{aligned}
$$

Applying optimal control theorem, the optimal control must satisfy the following equation

$$
\begin{aligned}
0 &= \left.\frac{\partial H}{\partial u}\right|_{u=u^*} \\
&= \sigma R u^* V(\hat{x},t) + B^T V_{\hat{x}}(\hat{x},t) \qquad (33)
\end{aligned}
$$

So from Eq.(33), the optimal control $u^*(t)$ can be obtained as

$$
u^*(t) = -\frac{1}{\sigma}R^{-1}B^T V_{\hat{x}}(\hat{x},t)V^{-1}(\hat{x},t) \qquad (34)
$$

Substituting Eq.(34) into Eq.(30), the HJB equation is reduced to

$$
\begin{aligned}
- &\quad V_t(\hat{x},t) = \frac{\sigma}{2}[\hat{x}^T Q\hat{x} \\
+ &\quad \frac{1}{\sigma^2}V^{-1}(\hat{x},t)V_{\hat{x}}^T(\hat{x},t)BR^{-1}B^T V_{\hat{x}}(\hat{x},t)V^{-1}(\hat{x},t)]V(\hat{x},t) \\
+ &\quad V_{\hat{x}}^T(\hat{x},t)[A\hat{x} - \frac{1}{\sigma}BR^{-1}B^T V_{\hat{x}}(\hat{x},t)V^{-1}(\hat{x},t)] \\
+ &\quad \frac{1}{2}t_r[V_{\hat{x}\hat{x}}(\hat{x},t)K_f V K_f^T] \qquad (35)
\end{aligned}
$$

Suppose optimal value function $V(\hat{x},t)$ to be as the following form

$$V(\hat{x},t) = \sigma D exp(\frac{\sigma}{2}\hat{x}^T S\hat{x}) \tag{36}$$

where D is a scalar function of time and S is a positive-definite symmetric matrix, both of which are to be determined as follows:

By Eq.(36) one has

$$V_t = \sigma \dot{D} exp(\frac{\sigma}{2}\hat{x}^T S\hat{x}) + \frac{\sigma^2}{2} D\hat{x}^T \dot{S}\hat{x} exp(\frac{\sigma}{2}\hat{x}^T S\hat{x}) \tag{37}$$

$$V_{\hat{x}} = \sigma^2 D S\hat{x} exp(\frac{\sigma}{2}\hat{x}^T S\hat{x}) \tag{38}$$

$$V_{\hat{x}\hat{x}} = \sigma^2 D S exp(\frac{\sigma}{2}\hat{x}^T S\hat{x}) + \sigma^3 D S\hat{x}\hat{x}^T S exp(\frac{\sigma}{2}\hat{x}^T S\hat{x}) \tag{39}$$

Substituting the corresponding terms defined by Eqs. (36)-(39) into Eq. (35) one has

$$\begin{aligned} - & \sigma \dot{D} exp(\frac{\sigma}{2}\hat{x}^T S\hat{x}) - \frac{\sigma^2}{2} D\hat{x}^T \dot{S}\hat{x} exp(\frac{\sigma}{2}\hat{x}^T S\hat{x}) \\ = & \frac{\sigma^2}{2}\hat{x}^T Q\hat{x} D exp(\frac{\sigma}{2}\hat{x}^T S\hat{x}) \\ & + \frac{\sigma^2}{2}\hat{x}^T S B R^{-1} B^T D S\hat{x} exp(\frac{\sigma}{2}\hat{x}^T S\hat{x}) \\ & + \sigma^2 D\hat{x}^T S F\hat{x} exp(\frac{\sigma}{2}\hat{x}^T S\hat{x}) \\ & - \sigma^2 D\hat{x}^T S B R^{-1} B^T S\hat{x} exp(\frac{\sigma}{2}\hat{x}^T S\hat{x}) \\ & + \frac{1}{2} tr[\sigma^2 D S K_f V K_f^T] exp(\frac{\sigma}{2}\hat{x}^T S\hat{x}) \\ & + \sigma^3 D S\hat{x}\hat{x}^T S K_f V K_f^T exp(\frac{\sigma}{2}\hat{x}^T S\hat{x}) \end{aligned} \tag{40}$$

Since Eq. (40) must be hold for any $\hat{x}$(t), so equating the corresponding terms one can obtain the following relationships

$$-\dot{D} = \frac{\sigma}{2} D tr[S K_f V K_f^T] \tag{41}$$

and

$$-\dot{S} = Q + SA + A^T S - S(BR^{-1}B^T - \sigma K_f V K_f^T)S \tag{42}$$

with the boundary conditions of D and S to be derived from Eqs. (6), (21) and (36), i.e.,

$$D(t_f) = 1 \tag{43}$$

and

$$S(t_f) = S_f \tag{44}$$

Substituting Eqs.(36) and (38) into Eq.(34), the optimal control can be obtained as a linear combination of the estimated states:

$$u^*(t) = -R^{-1}B^T S\hat{x} \tag{45}$$

where S must satisfy the Riccati equation defined by Eq.(42).

It should be noted that by Eqs. (9), (10), (42) and (45), if the weighting factor $\sigma \neq 0$ in Eq. (42), then the optimal control gains would take both system and measurement noise covariances (W and V) into consideration,which is different from those obtained by LQG method, and this is the reason why certainty equivalence principle cannot be held by LEQG method, but it provides another degree of freedom for the design to get better performances. In the mean while, since the state weighting matrix Q is negative definite, that the observability of the proposed method is better than that obtained by LQG method.

In addition, if σ_{max} is the upper limit of σ in Eq.(42) to make the effective control weighting R_{eff} to be positive definte, i.e., R_{eff} and σ_{max} are defined as

$$BR_{eff}^{-1}B^T = BR^{-1}B^T - \sigma K_f V K_f^T \tag{46}$$

and

$$R_{eff} > 0 \quad \text{for} \quad \sigma < \sigma_{max} \tag{47}$$

Then for $0 < \sigma < \sigma_{max}$, the magnitude of effective control weighting R_{eff} in Eq. (46) is larger than that of the original R. According to optimal control theory it is well known that the bandwidth and overshoot of the system can be decreased, therefore, the system is less sensitive to the environmental noises. However, if $\sigma < 0$, then the above conclusions will be reversed, i.e., the magnitude of R_{eff} is reduced, and the bandwidth as well as the overshoot of the system would be increased. It should be noted that if $\sigma \geq \sigma_{max}$, then R_{eff} would become negative semi-definite, therefore, the gain margin (-6dB to ∞dB) and phase margin ($\pm 60^o$) of the optimal control system cannot be preserved.

LEQG/DC Guidance Law Formulation

In general, the optimal guidance laws are derived by LQG method [32-33], Therefore, both state and control input are always regulated to zero, and the missile is always in constant bearing triangular geometry. However, it is well-known that the observability of such a homing trajectory is poor [9], so that the acceleration of the target maneuver cannot be estimated accurately, which will degrade the performance of LQG optimal guidance law in practical engagement.

It was reported [34] that the observability of homing guidance system is proportional to the time-to-go. Therefore, if the state weighting matrix is taken to be negative definite [9], then the magnitudes of both state and control input can be raised to increase the time-to-go, and the observability of system can also be increased.

For the convenience of presentation, the guidance law development is based on two dimensional analysis as shown in Fig.1. In addition, the target maneuver is assumed to be randomly reversing Poisson square wave [35] as shown in Fig.2. Then both the performance index and the state equation are respectively as:

$$J = \sigma E\{exp[\frac{\sigma c}{2} y_f^2 + \frac{\sigma}{2} \int_{t_0}^{t_f} (u^2(t) - q y_d^2) dt]\} \quad (48)$$

and

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -2\nu \end{bmatrix} \begin{bmatrix} y_d \\ \dot{y_d} \\ A_t \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 2\nu \end{bmatrix} w \quad (49)$$

where u is missile acceleration command in y-axis, ν is the number of times of target maneuver crossing zero per second, w is a gaussian white noise with zero mean and variance to be as

$$E[w(t)w(\tau)] = \sigma_s(t)\delta(t-\tau) \quad (50)$$

It should be noted that in Eq.(49) the autopilot time delay of missile is neglected. Let the measurement equation be as

$$z = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_d \\ \dot{y_d} \\ A_t \end{bmatrix} + v \quad (51)$$

where v is also a gaussian white noise with zero mean and variance as

$$E[v(t)v(\tau)] = \sigma_v(t)\delta(t-\tau) \quad (52)$$

Noted also that w and v are uncorrelated, i.e.,

$$E = [w(t)v^T(\tau)] = 0 \quad (53)$$

By LEQG method developed in Section 2, one has the estimated state equation as

$$\dot{\hat{x}} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -2\nu \end{bmatrix} \begin{bmatrix} \hat{y_d} \\ \hat{\dot{y_d}} \\ \hat{A_t} \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} u + K(z - \hat{x}) \quad (54)$$

where K is the Kalman gain of the filter [32]

$$K = \begin{bmatrix} 2w_o \\ 2w_o^2 \\ w_o^3 \end{bmatrix} \quad (55)$$

and where

$$w_o = (\frac{4\nu^2\sigma_s}{\sigma_v})^{\frac{1}{6}} \quad (56)$$

Then one can obtain the guidance acceleration commands and compare the results by computer simulation in the following section.

Simulation Results

In this section the guidance laws obtained by LQG, LEQG, LQG/DC and the proposed LEQG/DC methods are compared with Monte-Carlo computer simulations. Three variances of measurement noise (i.e., σ_v=0.01^0, 0.1^0 and 1^0) are used for the performance analysis of miss distance and state estimation. Three values of weighitng factor σ (e.g., 0.00001, 0.001 and 0.01) are also used for trade-off study. The related system parameters are listed in Table 1.

Firstly, the average, standard deviation, and maximum values of miss distance are considered. From Tables 2 to 4, it can be seen that if σ_v is small (i.e., 0.01^0), then there are no much differences among the methods. However, if σ_v becomes larger (e.g., 0.1^0), the results obtained by the proposed LEQG/DC method become better. As σ_v is equal to 1^0, there are much more improvement especially for the case with σ=0.001.

Secondly, since the target acceleration estimation is the most important one for optimal guidance, that the estimation error of target acceleration is studied as follows. From Figs. 3 to 11 one can see that if σ_v is smaller (i.e., 0.01^0) then the target acceleration estimation errors are larger for the larger σ of the proposed LEQG/DC method. However, when σ_v becomes larger (e.g., 0.1^0 or 1^0), the target acceleration estimation error becomes smaller. The reason of these effets can be seen from Figs. 12 to 20, that is, if σ_v is small (i.e., 0.01^0) then the acceleration commands of missile is delayed, however, when σ_v becomes larger (e.g., 0.1^0 or 1^0), then the acceleration commands of missile are advanced. Therefore, the proposed LEQG/DC method can increase the observability for the larger measurement noise conditions, and improve the miss distance performance.

Conclusion

(1) From the results of simulation, one can see that if the magnitude of measurement noise is small, there are no much differences among LQG, LEQG, LQG/DC and LEQG/DC methods, However, the performance of the proposed LEQG/DC method would be better for the larger measurement noise conditions, which is consistent with the theoretical prediction.

(2) One should trade off the weighting factor σ of the proposed LEQG/DC method, which should be kept to a small value especially when the magnitude of measurement noise is small.

References

[1] J. C. Doyle, and G. Stein,"Multivariable Feedback Design: Concept for a Classical / Modern Synthesis," *IEEE Trans. Auto. Contr.*, Vol.26, No.1, pp.4-16, 1981.

[2] W. H. Pfeil, M. Athans, and H. A. Spang, "Multivariable Control of the GE T700 Engine Using the LQG/LTR Design Methodology," *Proc. ACC*, pp. 1297-1312, 1986.

[3] R. J. Martin, V. Valavani, and M. Athans, "Multivariable Control of a Submersible Using the LQG/LTR Design Methodology," *Proc. ACC*, pp. 1313-1324, 1986.

[4] A. A. Rodriguez, and M. Athans, "Multivariable Control of a Twin Lift Helicopter System Using the LQG/LTR Design Methodology," *Proc. ACC*, pp. 1325-1332, 1986.

[5] M. Athans, P. Kapasouris, H. Kappos, and H. A. Spang, "Linear -Quadratic-Gaussian with Loop-Transfer-Recovery Methodology for F-100 Engine," *AIAA J. Guidance, Control and Dynamics*, Vol. 9, No.1. pp. 45-52, Jan.-Feb. 1986.

[6] G. Stein, and M. Athans, "The LQG/LTR Procedure for Multivariable Feedback Control Design," *IEEE Trans. Auto. Contr.*, Vol. 32, No.2, pp. 105-114, 1987.

[7] H. Kazerooni, and K. Narayanan, "Loop Shaping Design Related to LQG/LTR for SISO Minimum Phase Plants," *Proc. ACC*, pp. 512-519, 1987.

[8] J. M. Maciejowski, *Multivariable Feedback Design*, Addison-Wesley Publishing Co., Chap 5, pp. 244-258, 1989.

[9] D. G. Hull, J. L. Speyer, and D. B. Burris, "A Linear-Quadratic Guidance Law for Dual Control of Homing Missiles," *AIAA Guidance and Control Conference*, Paper No. 87-2381, pp. 551-559, August, 1987.

[10] A. J. Laub, "Computation of System Balancing Transformation and Other Applications of Simultaneous Diagonalization," *IEEE Trans. Auto. Contr.*, Vol. AC-32, pp. 115-122, 1987.

[11] K. V. Fernando and H. Nicholson, "On the Structure of Balanced and Other Principal Representetion of SISO Systems," *IEEE Trans. Auto. Contr.*, Vol. AC-28, pp. 228-231, 1983.

[12] J. A. De Abren-Garcia and F. W. Fairman, "Balanced Realization of Orthogonally Symmetric Transfer Function Matrices," *IEEE Trans. Circuit Syst.*, Vol. CAS-34, pp. 997-1009, 1987.

[13] A. J. Laub, L. M. Silverman, and M. Verma, "A Note Cross Gramians for Symmetric Realization," *Proc. IEEE*, Vol. 28, pp. 904-905, 1983.

[14] J. R. Sesak and K. M. Strong, "Reduced-Bandwidth Compensator Design via Control and Observation Normalization," *J. of Guidance*, Control, and Dynamics, Vol. 15, No. 2, pp. 457-460, March-April 1992.

[15] H. Ohta, M. Kakinuma and P. N. Nikiforuk, "Use of Negative Weights in Linear Quadratic Regulator Synthesis," *J. of Guidance*, Control, and Dynamics, Vol. 14, No. 4, pp. 791-796, July-August 1991.

[16] M. Hayase, "Linear Constant Coefficient Optimal Control Problems in which the Weighting Matrices Are Not Limited to Be Positive Semi-Definite," *Transactions of the Society of Instrument and Control Engineerings*, Vol. 9, No. 3 pp. 311-318, June 1973.

[17] S. N. Balakrishnan, "A Dual Control Homing Guidance Law," *AIAA Guidance and Control Conference*, Paper No. 87-2448, pp. 836-841, August 1987.

[18] J. Alster and P. R. Belanger, "A Technique for Dual Adaptive Control," *Automatica*, Vol. 10, pp. 627-634, 1974.

[19] Y. Bar-Shalom and E. Tse, "Dual Effect, Certainty Equivalence, and Separation in Stochastic Control", *IEEE Trans. Auto. Contr.*, Vol. AC-19, pp. 494-500, 1974.

[20] A. A. Feldbaum,"Dual Control Theory I-IV," *Automation and Remote Control*, Vol. 21, pp. 874-880, pp. 1033-1039, 1960, and Vol. 22, pp. 1-12, pp. 109-121, 1961.

[21] O. L. R. Jacobs and D. J. Hughes, "Simultaneous Identification and Control Using a Neutral Control Law," *Proc. IFAC 6th World Congress*, Paper No. 30. 2, Boston, U. S. A. 1975.

[22] R. Mehra, "Dual Control of Linear Dynamic Systems with Unknown Parameters Using Bayesian Preposterior Analysis", *Allerton Conf.*, October 1974.

[23] J. Sternby, "A Simple Dual Control Problem with an Analytical Solution," *IEEE Trans. Automatic Control*, Vol. AC-21, pp. 840-844, 1976.

[24] E. Tse and Y. Bar-Shalom, "An Actively Adaptive Control for Linear Systems with Random Parameters via the Dual Control Approach," *IEEE Trans. Auto. Contr.*, Vol. AC-18, pp. 109-117, 1973.

[25] E. Tse and Y. Bar-Shalom and L. Meier, "Wide-sense Adaptive Dual Control for Nonlinear Stochastic Systems," *IEEE Trans. Auto. Contr.*, Vol. AC-18, pp. 98-108, 1973.

[26] B. Wittenmark, "An Active Suboptimal Dual Controller for Systems with Stochastic Parameters," *Automatic Control Theory and Application*, Vol. 3, pp. 98-108, 1973.

[27] D. H. Jacobson, "Optimal Stochastic Linear Systems with Exponential Performance Criteria and Their Relation to Deterministic Differential Games," *IEEE Trans. Auto. Contr*, Vol. 18, No. 2, pp. 124-131, April 1973.

[28] J. L. Speyer, J. J. Deyst, and D. H. Jacobson, "Optimization of Stochastic Linear Systems with Additive Measurement and Process Noise Using Exponential Performance Criteria," *IEEE Tran. Auto. Contr.*,Vol. 19, No. 10, pp. 366-385, August 1974.

[29] J. L. Speyer, "An Adaptive Terminal Guidance Scheme Based on an Exponential Cost Criterion with Application to Homing Missile Guidance," *IEEE Trans. Auto. Contr.*, Vol. 21, pp. 660-665, June 1976.

[30] J. M. Lin, "Bank-To-Turn Optimal Guidance with Linear-Expoential- Gaussian Performance Index and Constant Acceleration Bias," *Proc. ACC*, Boston, June 1991.

[31] R. F. Stengel, *Stochastic Optimal Control, Theory and Application,* A Wiley-Interscience Publication, pp. 203-220, 1986.

[32] F. W. Nesline and P. Zarchan,"A New Look at Classical Versus Modern Homing Missile Guidance," *J. Guidance and Control*, Vol. 4, No. 1, Jan.-Feb., pp. 78-85, 1981.

[33] T. L. Riggs and P. L. Vergez,"Advanced Air-to-Air Missile Guidance Using Optimal Control and Estimation," *AFATL-TR-81-56*, June 1981.

[34] J. L. Speyer, D. G. Hull, and C. Y. Tseng, "Estimation Enhancement by Trajectory Modulation for Homing Missiles," *J. Guidance*, Vol. 7, No. 2, pp. 167-174, March-April 1984.

[35] D. V. Stallard, Classical and Modern Guidance ofHoming Interceptor Missiles, Seminar of Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, Massachusetts, April 1968.

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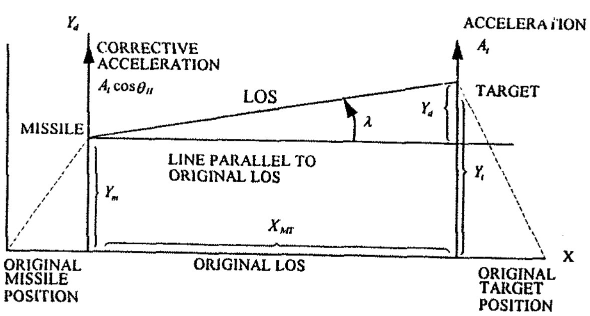


Fig. 1 The intercept geometry of a guided missile system.

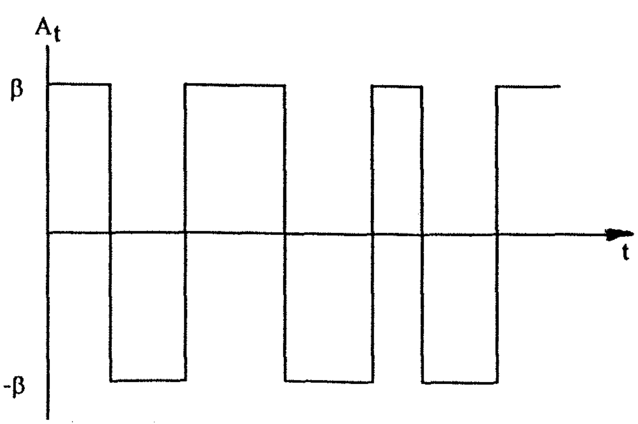


Fig. 2 The poisson square wave model target maneuver.

Table 1 The related system parameters.

Parameters	*Values*
V_M	600 (m/sec)
V_T	400 (m/sec)
β	49 (m/sec^2)
ν	0.1 $(times/sec)$
$R_{MTx}(0)$	5000 (m)
$R_{MTy}(0)$	0 (m)
σ_s	$\frac{\beta^2}{\nu}$
σ_v	$(\frac{R^*_{MT}\sigma_\nu}{57.3})^2$
q	0.004
c	600

Table 2 Miss distance comparisons for σ_v=0.01°.

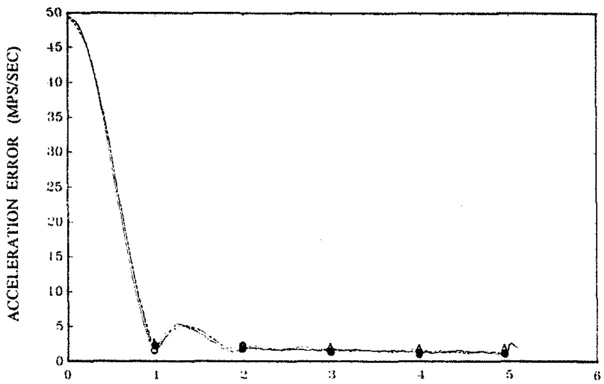
Guidance laws \ Items		Average Miss Distance (m)	Standard Deviation of Miss Distance (m)	Maximum Miss Distance (m)
LQG		0.037633883	0.062401257	0.15024368
LEQG	σ=0.00001	0.023243365	0.065291123	0.22146552
	σ=0.01	0.037834436	0.058527075	0.17002971
LQG/DC		0.11470131	0.12501497	0.16115850
LEQG/DC	σ=0.00001	0.053071676	0.12800397	0.27853720
	σ=0.001	1.0514085	0.28181469	0.49388628
	σ=0.01	4.6976543	1.6154090	0.41237802

Table 3 Miss distance comparisons for σ_v=0.1°.

Guidance laws \ Items		Average Miss Distance (m)	Standard Deviation of Miss Distance (m)	Maximum Miss Distance (m)
LQG		1.8905047	1.5348276	0.91341507
LEQG	σ=0.00001	1.4871212	1.6151485	4.0419690
	σ=0.01	3.3691893	1.8212800	0.023674632
LQG/DC		4.1018886	1.3387010	0.43087086
LEQG/DC	σ=0.00001	0.56192421	0.14283878	0.98649687
	σ=0.001	0.098664293	0.11971943	0.16569720
	σ=0.01	1.0521388	0.29819372	0.47587123

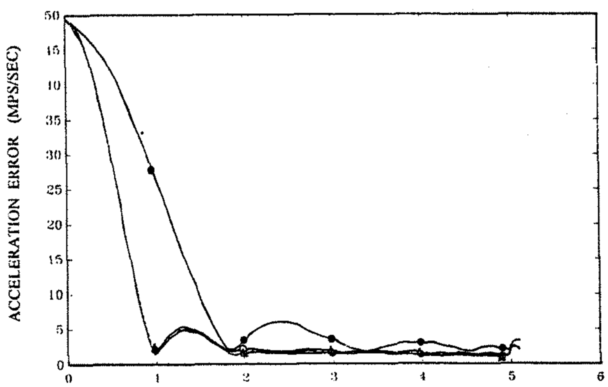
Table 4 Miss distance comparisons for σ_v=1°.

Guidance laws \ Items		Average Miss Distance (m)	Standard Deviation of Miss Distance (m)	Maximum Miss Distance (m)
LQG		0.078427851	0.27275038	0.52427806
LEQG	σ=0.00001	0.094204381	0.23125879	0.57256636
	σ=0.01	0.071076989	0.26709555	0.58610445
LQG/DC		1.0755232	0.21427937	0.63409944
LEQG/DC	σ=0.00001	0.15857727	0.13461492	0.49934753
	σ=0.001	0.78148403	0.16970472	0.39831113
	σ=0.01	2.7924885	0.85715813	0.82697904



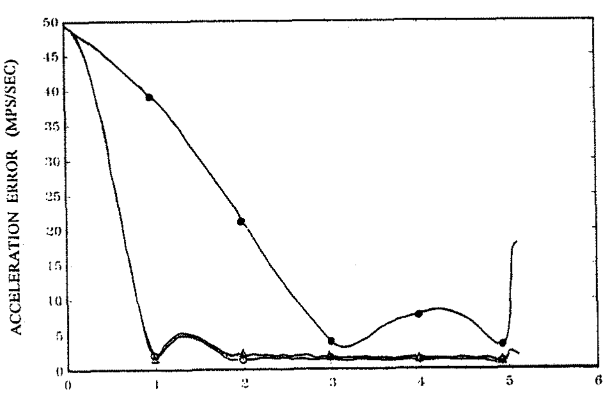
TIME(SEC) LQG(Δ) LEG(+) LQG/DC(o) LEQG/DC(•)

Fig. 3 The estimating error comparison of target acceleration for σ_V=0.01 and σ=0.00001 (LEQG, LEQG/DC).



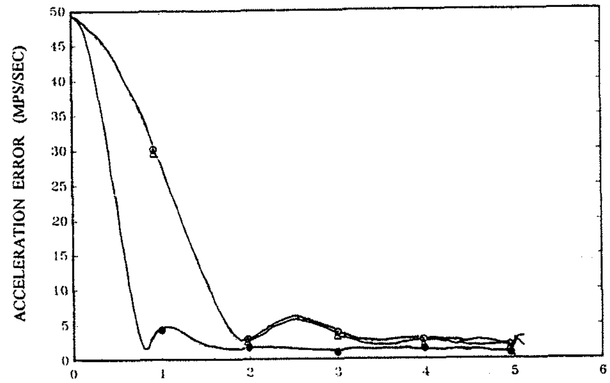
TIME(SEC) LQG(Δ) LEG(+) LQG/DC(o) LEQG/DC(•)

Fig. 4 The estimating error comparison of target acceleration for σ_V=0.01 and σ=0.001 (LEQG, LEQG/DC).



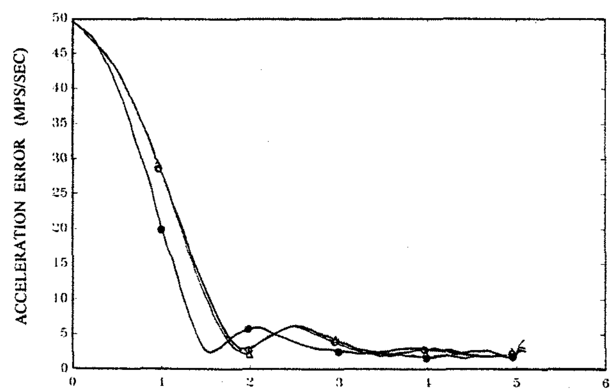
TIME(SEC) LQG(Δ) LEG(+) LQG/DC(o) LEQG/DC(•)

Fig. 5 The estimating error comparison of target acceleration for σ_V=0.01 and σ=0.01 (LEQG, LEQG/DC).



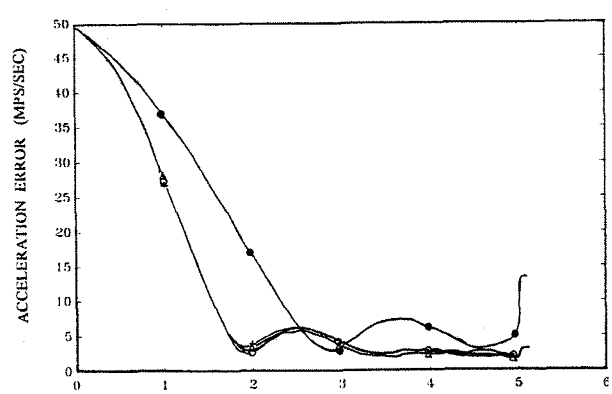
TIME(SEC) LQG(Δ) LEG(+) LQG/DC(o) LEQG/DC(•)

Fig. 6 The estimating error comparison of target acceleration for σ_V=0.1 and σ=0.00001 (LEQG, LEQG/DC).



TIME(SEC) LQG(Δ) LEG(+) LQG/DC(o) LEQG/DC(•)

Fig. 7 The estimating error comparison of target acceleration for σ_V=0.1 and σ=0.001 (LEQG, LEQG/DC).



TIME(SEC) LQG(Δ) LEG(+) LQG/DC(o) LEQG/DC(•)

Fig. 8 The estimating error comparison of target acceleration for σ_V=0.1 and σ=0.01 (LEQG, LEQG/DC).

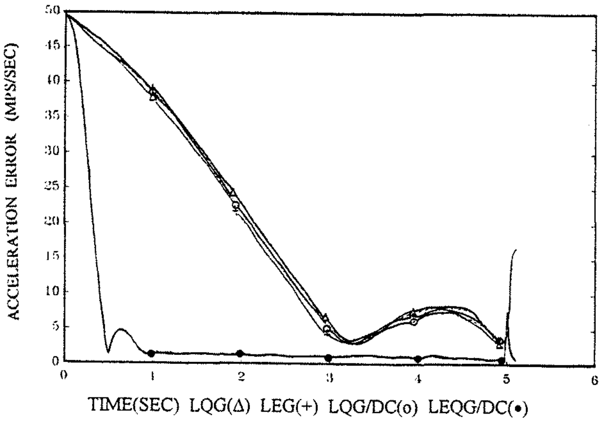


Fig. 9 The estimating error comparison of target acceleration for σ_V=1 and σ=0.00001 (LEQG, LEQG/DC).

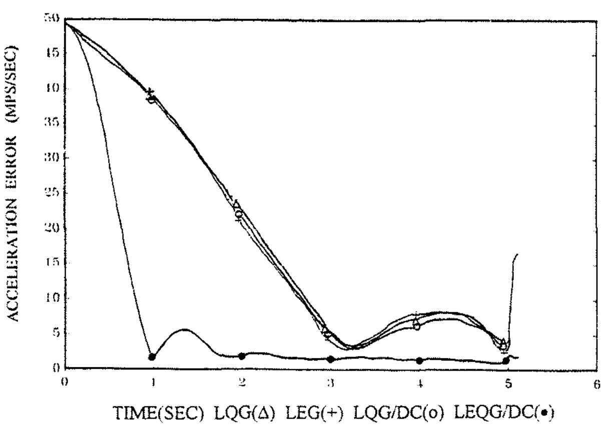


Fig. 10 The estimating error comparison of target acceleration for σ_V=1 and σ=0.001 (LEQG, LEQG/DC).

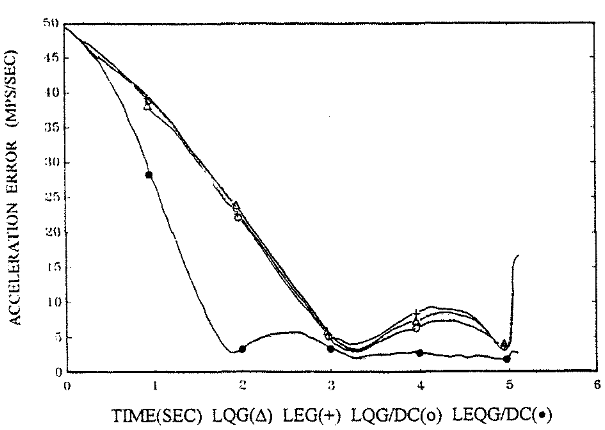


Fig. 11 The estimating error comparison of target acceleration for σ_V=1 and σ=0.01 (LEQG, LEQG/DC).

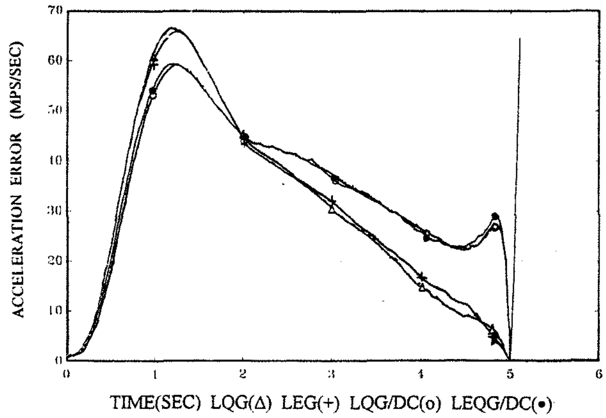


Fig. 12 The comparison of missile acceleration command for σ_V=0.01 and σ=0.00001 (LEQG, LEQG/DC).

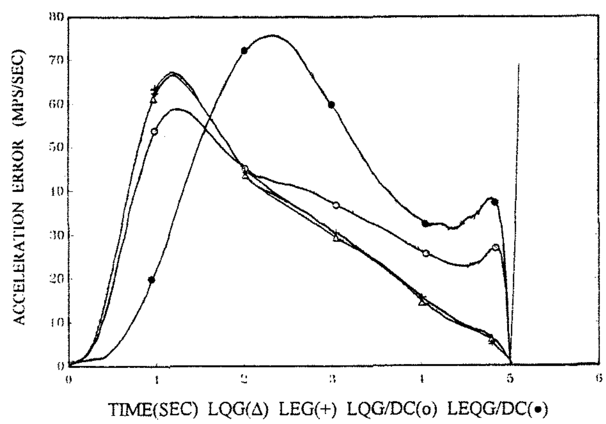


Fig. 13 The comparison of missile acceleration command for σ_V=0.01 and σ=0.001 (LEQG, LEQG/DC).

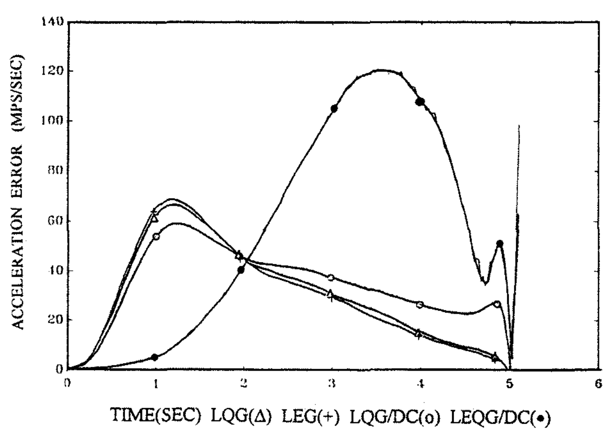
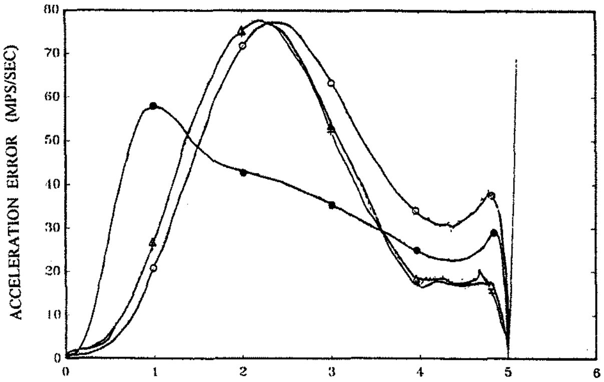
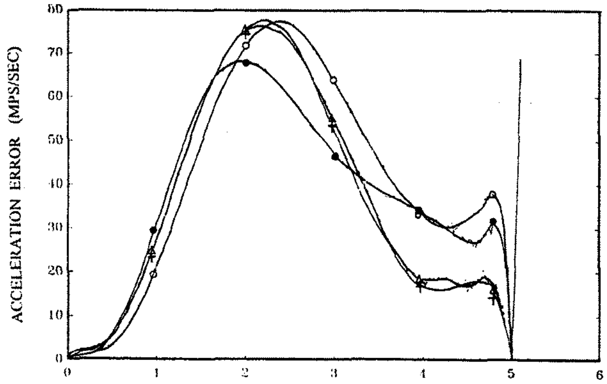


Fig. 14 The comparison of missile acceleration command for σ_V=0.01 and σ=0.01 (LEQG, LEQG/DC).



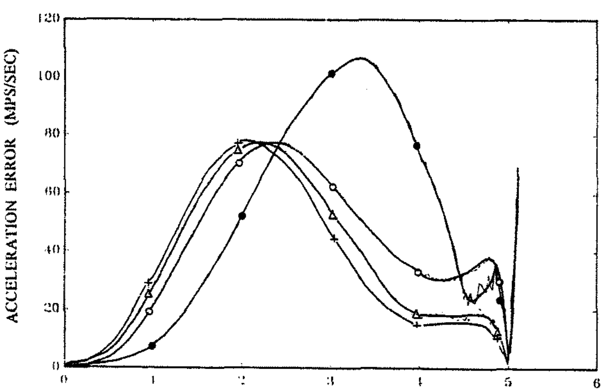
TIME(SEC) LQG(Δ) LEG(+) LQG/DC(o) LEQG/DC(•)

Fig. 15 The comparison of missile acceleration command for σ_V=0.1 and σ=0.00001 (LEQG, LEQG/DC).



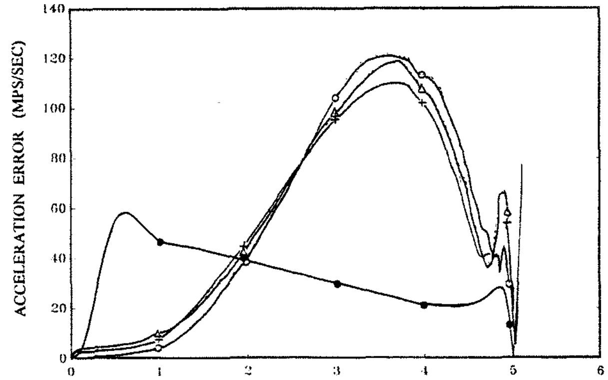
TIME(SEC) LQG(Δ) LEG(+) LQG/DC(o) LEQG/DC(•)

Fig. 16 The comparison of missile acceleration command for σ_V=0.1 and σ=0.001 (LEQG, LEQG/DC).



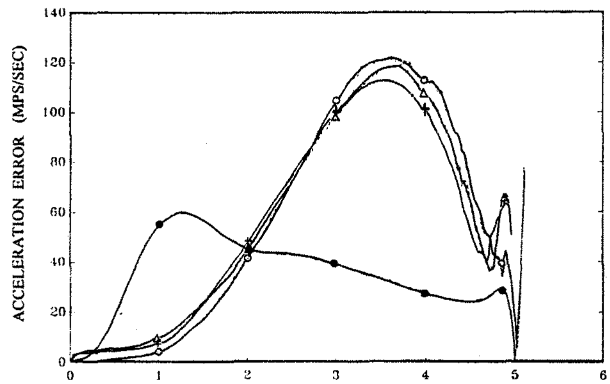
TIME(SEC) LQG(Δ) LEG(+) LQG/DC(o) LEQG/DC(•)

Fig. 17 The comparison of missile acceleration command for σ_V=0.1 and σ=0.01 (LEQG, LEQG/DC).



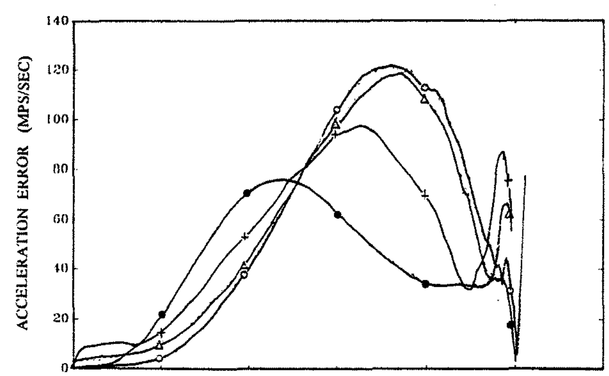
TIME(SEC) LQG(Δ) LEG(+) LQG/DC(o) LEQG/DC(•)

Fig. 18 The comparison of missile acceleration command for σ_V=1 and σ=0.00001 (LEQG, LEQG/DC).



TIME(SEC) LQG(Δ) LEG(+) LQG/DC(o) LEQG/DC(•)

Fig. 19 The comparison of missile acceleration command for σ_V=1 and σ=0.001 (LEQG, LEQG/DC).



TIME(SEC) LQG(Δ) LEG(+) LQG/DC(o) LEQG/DC(•)

Fig. 20 The comparison of missile acceleration command for σ_V=1 and σ=0.01 (LEQG, LEQG/DC).