ON THE EFFECT OF VARYING THICKNESS ON THE FUNDAMENTAL FREQUENCY OF CLAMPED ELLIPTIC PLATES

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Abstract: The effect of the taper parameter on the fundamental frequency of clamped elliptic plates with variable thickness is studied. For this, the thickness is assumed to change both parabolically and exponentially. A five term deflection function, which satisfies the geometric boundary conditions of zero edge deflection and zero slope, is chosen and the collocation method is used to solve the basic equation governing the transverse vibrations of a plate with variable thickness. The fundamental frequencies of elliptic plates with aspect ratios varying from 1.0 to 0.3 , which are of equal volume, have been compared.

INTRODUCTION

Although some literature has recently been available on the transverse vibrations of elliptic plates with variable thickness and different boundary conditions such as Singh's [1] and Prasad's [2] work, there is still need in extending the literature. For instance, the effect of the change in taper parameter of the plates having equal volume on the fundamental frequency has not been investigated. It is therefore aimed in this paper to increase the fundamental frequency of such plates.

BASIC EQUATIONS

The basic equation governing the transverse vibrations of a plate with variable thickness in an open form [3], [4] is as follows :

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + 2\frac{\partial D}{\partial x}\frac{\partial^3 w}{\partial x^3} +$$

$$2\frac{\partial D}{\partial x}\frac{\partial^3 w}{\partial x \partial y^2} + 2\frac{\partial D}{\partial y}\frac{\partial^3 w}{\partial y \partial x^2} + 2\frac{\partial D}{\partial y}\frac{\partial^3 w}{\partial y^3} +$$

$$\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 D}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial y^2} +$$

$$\frac{\partial^2 D}{\partial y^2}\frac{\partial^2 w}{\partial y^2} - (1 - \nu)\Big(\frac{\partial^2 D}{\partial x^2}\frac{\partial^2 w}{\partial y^2} -$$

$$2\frac{\partial^2 D}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 D}{\partial y^2}\frac{\partial^2 w}{\partial x^2}\Big) + \gamma h \frac{\partial^2 w}{\partial t^2} = 0 \qquad (1)$$

where w, ν (taken to be 0.3), γ, h, t denote the displacement, Poisson ratio, density, the variable thickness and time respectively. Also appearing in equation (1) is the flexural rigidity D defined by

$$D = \frac{Eh^3}{12(1 - \nu^2)} \qquad (2)$$

where E is the Young's Modulus.

The boundary of the elliptic plate is expressed as

$$\xi^2 + \eta^2 - 1$$

Here,

$$\xi = \frac{x}{a} \text{ and } \eta = \frac{y}{b}$$

and a and b are the semi-major and the semi minor axes of the ellipse respectively .

Assuming simple harmonic motion one seeks a solution of the form

$$w(\xi, \eta, t) = W(\xi, \eta)e^{i\omega t} \qquad (3)$$

where ω is the circular frequency. Substituting equations (2) and (3) into equation (1) and taking the variable thickness to be of the form

$$h = lh_0 f(\xi, \eta) \qquad (4)$$

,where h_0 denotes the non-dimensional thickness of the plate at the origin and l represents a characteristic length defined by c.a , while c is a constant value (Table 1) depending on the aspect ratio and the taper parameter, one gets:

$$F\left(k^4 \frac{\partial^4 W}{\partial \xi^4} + 2k^2 \frac{\partial^4 W}{\partial \xi^2 \partial \eta^2} + \frac{\partial^4 W}{\partial \eta^4}\right) +$$

$$2k^4 \frac{\partial F}{\partial \xi}\frac{\partial^3 W}{\partial \xi^3} + 2k^2 \frac{\partial F}{\partial \xi}\frac{\partial^3 W}{\partial \xi \partial \eta^2} +$$

$$2k^2 \frac{\partial F}{\partial \eta}\frac{\partial^3 W}{\partial \eta \partial \xi^2} + 2\frac{\partial F}{\partial \eta}\frac{\partial^3 W}{\partial \eta^3} +$$

$$k^4 \frac{\partial^2 F}{\partial \xi^2}\frac{\partial^2 W}{\partial \xi^2} + k^2 \frac{\partial^2 F}{\partial \eta^2}\frac{\partial^2 W}{\partial \xi^2} + k^2 \frac{\partial^2 F}{\partial \xi^2}\frac{\partial^2 W}{\partial \eta^2} +$$

$$\frac{\partial^2 F}{\partial \eta^2}\frac{\partial^2 W}{\partial \eta^2} - k^2(1-\nu)\left(\frac{\partial^2 F}{\partial \xi^2}\frac{\partial^2 W}{\partial \eta^2} - 2\frac{\partial^2 F}{\partial \xi \partial \eta}\frac{\partial^2 W}{\partial \xi \partial \eta} +\right.$$

$$\left.\frac{\partial^2 F}{\partial \eta^2}\frac{\partial^2 W}{\partial \xi^2}\right) - \frac{k^4}{c^4}\lambda^2 f(\xi,\eta) W = 0 \qquad (5)$$

where $F = f^3$, the aspect ratio $k = b / a$, the non-dimensional frequency parameter

$$\lambda^2 = 12(1-\nu^2)\gamma l^2 \omega^2 / E h_0^2$$

and the variation of thickness

$f(\xi,\eta) = c_1\left[1 + \beta\left(\xi^2 + \eta^2\right)\right]$ for parabolically varying thickness,

$f(\xi,\eta) = c_2 e^{\gamma\left(\xi^2+\eta^2\right)}$ for exponentially varying thickness.

Here β and γ are the taper parameters for the parabolically and exponentially varying thickness respectively. In order to be able to compare the fundamental frequencies of the plates, the volume of all the plates is equated to π by using factors c_1 and c_2 . Values of c_2 can be found in Table 2, while c_1 is defined as

$$c_1 = 2/(2+\beta)$$

Table 1. Values of c

β	b/a					
	0.3	0.4	0.5	0.6	0.8	1
0	1.041	1.035	1.029	1.024	1.013	1
0.2	1.041	1.035	1.03	1.025	1.014	1.001
0.4	1.041	1.035	1.03	1.025	1.015	1.002
0.6	1.041	1.036	1.032	1.027	1.019	1.005
0.8	1.044	1.039	1.036	1.033	1.024	1.01
1	1.048	1.047	1.046	1.045	1.031	1.021
1.2	1.048	1.048	1.047	1.047	1.038	1.032

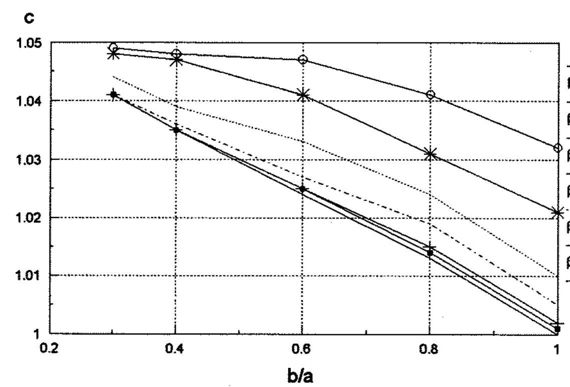

Figure 1. Values of c

Table 2. Values of c_2

γ	c_2
0.0	1.0
0.2	0.9033328
0.4	0.8133005
0.6	0.7298239
0.8	0.6527741
1.0	0.581978

METHOD OF SOLUTION

Since the problem does not have an analytical solution, the collocation method, which is one of the simplest weighted residual methods, has been used. In order to obtain enough accuracy a five-term deflection function satisfying the geometric boundary conditions of zero edge deflection and zero slope is chosen:

$$W = (a_1 + a_2\Phi + a_3\Phi^2 + a_4\Phi^3 + a_5\Phi^4)\Phi^2 \qquad (6)$$

where $\Phi = \xi^2 + \eta^2 - 1$. The residual ε_r obtained by substituting equation (6) into equation (5) is equated to zero in five different points of the plate considered (Fig.2).

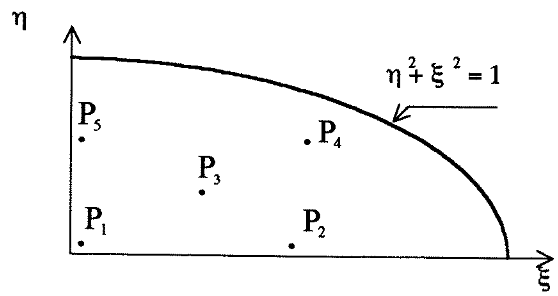

Figure 2.

These points are as follows:

$P_1(0.02,0.02)$, $P_2(0.5,0.02)$, $P_3(0.3,0.3)$, $P_4(0.6,0.6)$, $P_5(0.02,0.6)$

Hence five homogenous equations in a_i are obtained :

$$\left[\mathbf{A} - \lambda^2 \mathbf{B}\right]\{a_i\} = 0. \qquad (7)$$

For a non-trivial solution , the determinant of the coefficient matrix should be equal to zero.

$$\left|\mathbf{A} - \lambda^2 \mathbf{B}\right| = 0. \qquad (8)$$

Solution of equation (8) is an eigenvalue problem leading to a characteristic equation which involves a polynomial of fifth degree in λ^2. The smallest positive real root of these polynomials corresponds to the fundamental frequency. The fundamental frequency values are presented in Tables 3 and 4 and are shown in Figures 3a-b and 4a-b respectively.

Table 3. The values of λ for plates with parabolically varying thickness.

β	b/a					
	1	0.8	0.6	0.5	0.4	0.3
0	10.205	13.226	20.363	27.815	41.588	70.292
0.2	10.796	13.987	21.549	29.426	44.045	75.063
0.4	11.709	15.204	23.424	32.06	48.186	82.944
0.6	12.79	16.694	25.806	35.414	53.463	92.899
0.8	13.658	17.94	27.976	38.528	58.431	102.78
1	14.014	18.178	29.321	40.382	61.391	107.94
1.2	13.549	17.849	28.439	39.328	61.02	106.77

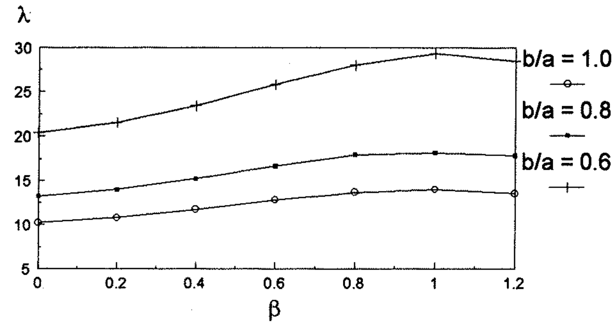

Fig. 3a Frequencies for b/a = 1.0, 0.8 and 0.6

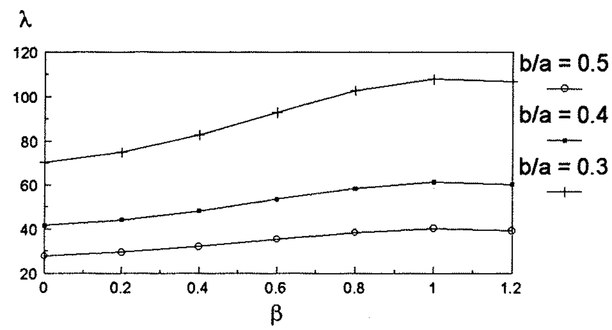

Fig. 3b Frequencies for b/a = 0.5, 0.4 and 0.3

Table 4. The values of λ for plates with exponentially varying thickness.

γ	b/a					
	1	0.8	0.6	0.5	0.4	0.3
0	10.205	13.226	20.363	27.815	41.588	70.292
0.2	11.027	14.295	22.04	30.114	45.121	77.094
0.4	12.905	16.825	26.04	35.764	54.046	94.106
0.6	15.233	20.13	31.547	43.704	66.856	119.07
0.8	15.51	21.013	33.846	47.253	73.391	129.13
1	11.651	16.649	28.964	42.03	67.639	105.48

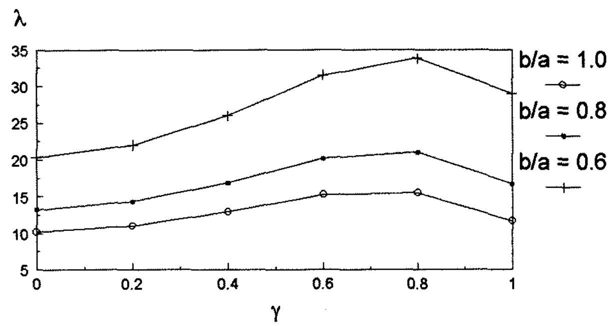

Fig. 4a. Frequencies for b/a = 1.0, 0.8 and 0.6

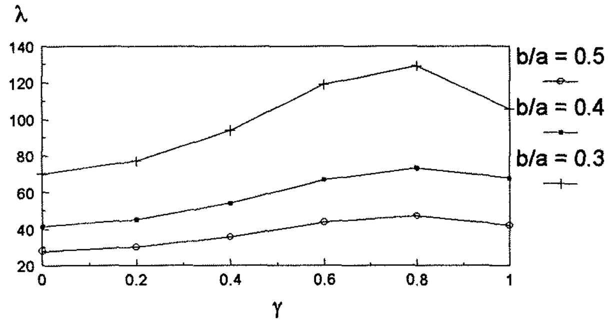

Fig. 4b. Frequencies for b/a = 0.5, 0.4 and 0.3

CONCLUSIONS

The fundamental frequency of an elliptic plate with parabolically varying thickness attains its highest value when the taper parameter β is around 1.0 (see Figure 3a-b), while this is achieved when the taper parameter γ is around 0.8 for plates with exponentially varying thickness (see Figures 4a-b). It is interesting to note that the increase in fundamental frequency is between 37 % and 53 % for parabolically varying thickness, while it is between 52 % and 80 % for exponentially varying thickness.

REFERENCES

(1) B. Singh and D. K. Tyagi 1985 *Journal of Sound and Vibration* **99**, 379-391. Transverse vibrations of an elliptic plate with variable thickness.

(2) K. L. Prasad, A. Venkateshwar and B. Nageswara Rao 1992 *Journal of Sound and Vibration* **158**, 383-386. Free vibration of simply supported and clamped elliptical plates.

(3) S. Timoshenko and K. S. Woinowsky 1959 *Theory of Plates and Shells.* New York: McGraw-Hill.

(4) A. W. Leissa 1969 NASA SP-160 *Vibration of Plates.*