

Yechiel Crispin

Department of Aerospace Engineering
Embry-Riddle Aeronautical University
Daytona Beach, FL 32114
e-mail: crispinj@db.erau.edu

ABSTRACT

The problem of constrained, nonlinear, multiple criteria optimal aircraft synthesis is solved using a genetic algorithm as the optimization module. The nonlinear constraints are incorporated into the genetic search by imposing exact penalties on the objective function in the infeasible domain. The resulting pseudo-objective function is discontinuous and is easily handled by the algorithm. Two different criteria, minimum gross weight and minimum fuel weight are considered. Combinations of the two criteria are treated through the use of scalarization and the set of non-inferior Pareto optimal solutions is then obtained. The method is used to study the sensitivity of the solutions to variations in the required range of the aircraft, using the two criteria.

1. INTRODUCTION

Genetic algorithms (GAs) are a direct simulation of biological evolution processes using simplified first principles of genetics and Darwinian evolution. GAs have been used in optimization, since they mimic adaptation processes believed to play an important role in the causes of evolution. They have been applied to many optimization problems in science and engineering^(1,2), and the number of applications continues to grow. Recently, they have also been applied to aircraft optimization^(3,4). In this paper, a genetic algorithm is used to study the problem of optimum aircraft synthesis using multiple criteria. Specifically, we study the effect of two contradicting criteria, minimum gross weight and minimum fuel weight, subject to performance constraints and requirements.

Genetic algorithms present some advantages for solving complex constrained minimization problems in engineering. Traditional optimization methods, such as gradient and other hill-climbing methods, make use of the properties of continuous functions of several variables and their continuous gradient functions in order to detect local extrema. Usually, the independent variables are assumed to be continuous, and if the problem involves discrete variables, it would need to be reformulated as a continuous problem, which introduces difficulties even in the most simplified problems. The approach in GAs is completely different. GAs belong to the class of combinatorial optimization methods. The optimization problem, whether integer, discrete or continuous, is actually formulated as a combinatorial search problem. This capability is very tempting, since it presents the potential of solving "exotic", practical and complex optimization problems involving discontinuous functions with a combination of integer/discrete/continuous variables, which were otherwise intractable using traditional methods. For example, in aircraft synthesis, integer variables such as the number and type of engines, number of aisles and number of seats abreast might be included as design variables rather than kept constant. In addition to that, in GAs, the search

is conducted from a set of initial designs, the initial population, scattered over a domain of the variables space, which increases the chance of finding the global optimum. Traditional methods, on the other hand, are susceptible of "falling" in local minima, such as happens often in gradient hill climbing methods. Unlike gradient techniques, GAs use values of the objective function and no information on the gradients is required. Therefore, problems involving discontinuous objective functions, discontinuous gradients, and non-smooth data, can still be treated. In aircraft synthesis, possible origins of the discontinuities are the FAR regulations, which are stated in the form of discontinuous ranges in performance requirements, discontinuities in the properties of the standard atmosphere, especially when the flight altitude is used as a design variable, and discontinuities in the values of the maximum lift coefficient C_{Lmax} when it is necessary to switch between various high lift devices (e.g. double slotted versus single slotted trailing edge devices) in order to meet stall speed criteria or takeoff and landing criteria. Other discontinuities result from non-smooth data contained in the knowledge base used for the aircraft synthesis. GAs have also great potential and advantages over other methods in treating the exact nonlinear constraints, especially when the variables are discrete or mixed discrete/continuous variables.

The next section, section 2 is a brief introduction to genetic algorithms. Section 3 describes the aircraft synthesis and sizing method. The aircraft optimization problem is described in section 4, where results of the optimum synthesis of an executive jet aircraft are presented. Finally, a summary and conclusions are presented in section 5.

2. GENETIC ALGORITHMS IN CONSTRAINED OPTIMIZATION

One of the early works on the application of simulated evolution to optimization problems is that of Fogel and co-workers⁽⁵⁾ in the field of cybernetics. The problem was to optimize the function of finite state machines using an evolutionary process. The interested reader may also consult the recent review paper of Forrest⁽¹⁾. Genetic algorithms are based on the paradigm of Darwinian evolution^(6,7,8,9). They are started with many possible points which are randomly and uniformly distributed over the problem space to create an initial population from which the search is started. Then, this initial population undergoes an evolution process during which it adapts to the minimum or maximum value (or values) of the objective function. The search is guided by selecting better adapted solutions that have higher values of the fitness function (objective function). They are especially efficient in locating "interesting" domains within a broad range of the search space and therefore are more suited for finding global minima.

In GAs, since the problem is posed as a discrete combinatorial problem, the treatment of discrete variables is inherent

in the formulation, whereas continuous variables are approximated by discrete variables. Design variables are coded, and in the present work, they are represented by binary numbers of given length (bit strings). This binary coding is a simplified simulation of the genetic information contained in the genotype of each member of the population. Discrete variables are represented exactly by binary numbers whereas continuous variables can be represented to any required degree of approximation by discrete numbers defined between lower and upper bounds. This representation has also the advantage of taking into account side constraints on the design variables in an easy and natural way. The genotype of each member of the population is formed by concatenating the binary strings representing the set of independent variables into a single binary string.

The genetic algorithm consists of a number of relatively simple and well documented operations^(1,2), which upon acting on a population of solutions, improve the value of the fitness function. Each member is assigned a fitness value through the process of evaluation and the population of designs is sorted and ranked from best to worst. The fitness of a member can be defined as the value of the objective function whose extremum is sought, such as aircraft gross weight or system life cycle cost. The genetic operations include coding, evaluation, selection, crossover and mutation. Members in the population are selected as parents in order to produce the next generation. Parents are then replaced by better fit children which are created through a process of recombination (crossover). In the present work, the number of members within the population is kept constant. The genetic operations of this evolution process are repeated until genetic diversity is lost, i.e. all members in the population become identical. A measure of convergence can be obtained from the behavior of the difference between the fitness of the best member and the average fitness of the population. Upon loss of diversity, the average fitness becomes identical to the best fitness.

3. THE AIRCRAFT SYNTHESIS METHOD

The synthesis method consists of an initial sizing module, followed by a detailed estimation of the empty weight components of the aircraft, and an estimation of the fuel weight from the mission profile. This module is then driven by the genetic algorithm in order to attain an optimal aircraft configuration. In order to obtain the weights and size of a new configuration the following parameters need to be specified: the number of engines and engine type (a choice of turbofans having a range of bypass ratios), the payload, the cruise Mach number, the mission range, and the landing and takeoff field lengths.

The main steps used in the synthesis process are described below. The gross weight is given by:

$$W_g = W_e + W_f + W_p \quad (3.1)$$

where W_g , W_e , W_f and W_p are the gross weight, empty weight, fuel weight and payload, respectively. The payload is a known design requirement, the fuel weight is estimated from the mission profile and the empty weight is estimated using correlations for existing aircraft of the same class. The approach used in obtaining these correlations is to gather statistical weights data from a variety of existing aircraft and find best fits using regression analysis. The aircraft empty weight is broken up into several components and systems:

$$W_e = W_w + W_{ht} + W_{vt} + W_{fus} + W_{maing} + W_{noseg} + W_{eng} + W_{fsys} + W_{fc} + W_{hydr} + W_{elec} + W_{avion} + W_{ac} \quad (3.2)$$

Here W_w , W_{ht} , W_{vt} and W_{fus} are the weights of the wing, horizontal tail, vertical tail and fuselage, respectively. W_{maing} , W_{noseg} , W_{eng} and W_{fsys} are the weights of the main landing gear, nose gear, installed engines and fuel system, respectively. W_{fc} , W_{hydr} , W_{elec} , W_{avion} and W_{ac} are the weights of the flight control system, hydraulics system, electrical system, installed avionics and air conditioning system, respectively. There exist detailed correlations for the weights of the various components and systems, see for example the data given by Raymer⁽¹⁰⁾, Roskam⁽¹¹⁾, Torenbeek⁽¹²⁾ and Nicolai⁽¹³⁾. For example, the correlation for the wing weight given by Torenbeek⁽¹²⁾ is:

$$W_w = a_1 W_{mzf} (1 + (6.3 \cos \Lambda_{1/2} / b)^{a_2}) (b / \cos \Lambda_{1/2})^{a_3} (n_{ult})^{a_4} (bS / t_r W_{mzf} \cos \Lambda_{1/2})^{a_5} \quad (3.3)$$

Here $W_{mzf} = W_g - W_f$ is the maximum zero fuel weight, n_{ult} is the ultimate load factor, $\Lambda_{1/2}$ is the wing semi-chord line sweep angle, b is the wing span, S is the wing planform reference area and t_r is the maximum thickness of the wing root chord. The values of the numerical constants a_1 to a_5 are given in Torenbeek⁽¹²⁾. It was found that Raymer's equations⁽¹⁰⁾ for general aviation aircraft give good estimates for executive jets in the 10 passengers class. In the above equations, the weights are in lbs, lengths are in feet, areas in sqf, and angles in degrees. Similar correlations for the other components and systems are given in the above references⁽¹⁰⁻¹³⁾ and are omitted here.

The empty weight given by eq.(3.2) depends on the gross weight W_g , the wing reference area S , the wing span b , the root thickness t_r and the sweep angle of the semi-chord line, which depends on the quarter-chord sweep and taper ratio. Similarly, other components weights depend on the design variables and the gross weight. However, these parameters are not known a priori, i.e., eq.(3.1) is a nonlinear equation for W_g . An initial sizing method is needed in order to obtain estimates of W_g , S , b , t_r and other parameters during the optimization process. Several authors have addressed the problem of initial sizing: a simplified and efficient method is given by Raymer⁽¹⁰⁾, and more detailed methods have been proposed by Loftin⁽¹⁴⁾ and Roskam⁽¹¹⁾. A very brief description of the initial sizing process is given in ref.(3). Several criteria are used in order to obtain the critical values for the wing loading W_g/S and the thrust loading T/W_g : stall speed, landing and missed approach, takeoff and second segment climb gradient, and cruise performance criteria. The landing field length and the stall speed criterion place a direct constraint on the wing loading. The second segment climb gradient and the missed approach criteria place a direct constraint on thrust loading. The takeoff field length and the cruise criteria determine two relations between the thrust loading and the wing loading. The lowest value of W_g/S and its corresponding value of T/W_g are selected as the matching point that satisfies all the performance criteria. For a given combination of the design variables, first estimates of the gross weight and aircraft size are obtained from the relations $T = W_g (T/W_g)$ and $S = W_g / (W_g/S)$, which are then followed by a detailed calculation of the empty weight and the fuel weight. A new value for the gross weight is obtained, which is then used to refine the first estimates of the

empty weight, the fuel weight required for the mission and the size of the wing and engines.

4. THE OPTIMIZATION PROBLEM, RESULTS AND DISCUSSION

The design variables considered are: the wing aspect ratio AR , the wing quarter-chord line sweep angle $\Lambda_{c/4}$, the wing taper ratio λ , the thickness ratio t/c , the thickness taper τ , the fuselage diameter D_f and the fuselage length L_f . We seek the vector of design variables $z = (AR, \Lambda_{c/4}, \lambda, t/c, \tau, D_f, L_f)$ that minimizes a composite function $F(x, z)$ of the gross weight and fuel weight defined by:

$$F(x, z) = x(W_g(z)/W_{g0}) + (1-x)(W_f(z)/W_{f0}) \quad (4.1)$$

where the gross weight $W_g(z)$ and the empty weight $W_f(z)$ are functions of the vector of design variables z . x is a weighting scalar that can vary between 0 and 1, and W_{g0} and W_{f0} are typical values of the gross weight and fuel weight used to normalize $F(x, z)$ such that it is of order one. The solutions are to be selected from the class of admissible aircraft configurations having a fixed number of engines and a fixed payload. They should satisfy prescribed range, cruise speed, stall speed, takeoff and landing performance constraints, and the wing cantilever ratio structural constraint. Side constraints are imposed on the design variables, for example the aspect ratio varies between a prescribed minimum value and a prescribed maximum value: $AR_{min} \leq AR \leq AR_{max}$. Similarly, the other design variables are constrained to vary within certain bounds, so that the genetic search is limited to a finite subset of the design space, prescribed a priori. When $x=1$, the problem is reduced to that of minimum gross weight. When $x=0$, the minimum fuel weight problem is recovered. The other values of x allow the designer to place more emphasis on one of the criteria while still taking the other criterion into account. In the example problem described below, the following requirements were used: $N_p = 10$, $R = 2700$ nm, $E = 45$ min, $M = 0.8$ at best altitude. Here N_p is the number of passengers, R is the range, E is the loiter time and M is the cruise Mach number. The performance constraints of the problem are given by:

$$V_s = (2(W_g/S)/\rho C_{Lmax})^{1/2} \leq V_{s1} \quad (4.2)$$

$$LFL = LFL(W_L/S, C_{Lmax}) \leq LFL_1$$

$$TOFL = TOFL(W_g/S, T/W_g) \leq TOFL_1$$

$$18 \leq CR = b / (2 t_r \cos \Lambda_{c/4}) \leq 22$$

where V_s is the stall speed, ρ the air density and C_{Lmax} is the aircraft maximum lift coefficient. LFL is the landing field length, W_L is the design landing weight, $TOFL$ is the takeoff field length, T is the takeoff thrust, C_{LTO} is the takeoff lift coefficient, and CR is the wing cantilever ratio. The subscripts 1 denote specific values of the constraints. Typical values are: $V_{s1} = 100$ knots, $LFL_1 = 2700$ ft @ S.L. and $TOFL_1 = 5000$ ft @ S.L.

For the results presented below, a uniform crossover was used and the mutation probability was 5 percent. The aspect ratio was allowed to vary between 5 and 15, the wing sweep angle between 20 and 50 degrees, the taper ratio between 0 and 1, the fuselage diameter between 6.2 and 8 ft, and the fuselage length between 55.5 and 70 ft. The thickness ratio

t/c was kept constant at 0.14 and a thickness taper of 1 was used throughout. In determining the size of the population, two requirements have to be considered. Large populations have the advantage of increasing the genetic diversity of the designs, which helps locate the global optimum, but on the other hand, they require a larger number of objective function evaluations. If a very small population is chosen (for instance, 5 members) the algorithm converges prematurely to a false result. In our simulations, it was found that a population of 30 members provides a good compromise between these two requirements.

The aircraft configuration that was studied in this example is an executive jet airplane with a conventional wing and tail arrangement in the 10 to 12 passengers class. Design and performance specifications, as well as three view drawings for this class of aircraft can be found in many editions of Jane's All the World Aircraft and in many other sources. Figs. 1–6 describe the mechanisms of the genetic algorithm as it applies to the specific problem at hand, such as evolution, convergence, constraint propagation and non-inferior solutions using two criteria. Figs. 7–13 describe an example application of the GA to an optimization study using two design criteria, minimum gross weight versus minimum fuel weight.

Fig.1 describes the propagation of the wing cantilever ratio constraint CR as defined in Eq.(4.2) through the population of designs. This ratio is a measure of the bending stress at the wing structural root. For example, increasing the wing span while keeping the root thickness and wing sweep constant, would increase CR , or equivalently the stress at the wing root. Usually, executive jets and transport aircraft have values of CR between 18 and 22.

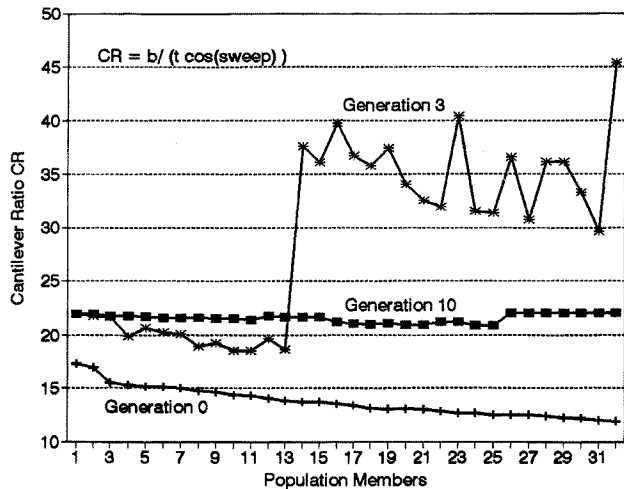


Fig.1: Constraint Propagation Through the Population.

Generation 0 shown in the figure is composed of 32 members, all having an aspect ratio of 5 and values of the wing sweep randomly and uniformly distributed between 20 and 50 degrees. All members in this zeroth generation have a value of CR below the lower limit of 18, and therefore do not satisfy the CR constraint. The zeroth generation is used as a seed for starting the algorithm. These infeasible designs are gradually removed from the population as the genetic search proceeds and new feasible designs are found and introduced into the population. For example it can be seen from the figure, that the first 13 members in the third generation are feasible whereas the rest 19 members are infeasible.

In generation 10, on the other hand all the members are feasible. The designs are sorted from best to worst so that the first member has the minimum value of the objective function (in this case, the gross weight) and the 32nd has the highest value.

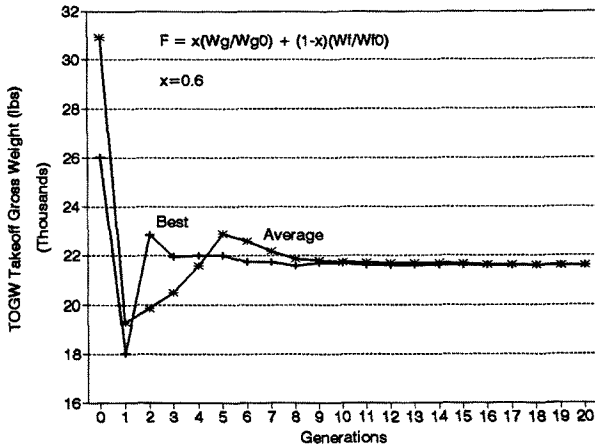


Fig.2: Evolution of the Takeoff Gross Weight.

The evolution of the takeoff gross weight over 20 generations is presented in Fig. 2. For each generation, the value of the best design and the average value taken over all the members in the population are shown. The objective function in this case is given by Eq.(4.1), with $x=0.6$. Since the GA is started with a low aspect ratio value of 5, the initial weight is much higher (26000 lbs) than the optimal converged value of less than 22000 lbs. After generation 9, the average value approaches the best value and they become eventually identical when the diversity of the population is lost after some 14 generations. Another insight into the propagation of the constraint through the population of designs can be obtained from Fig. 3, which shows the evolution of the CR constraint over 20 generations. The best solution satisfies the CR constraint as early as generation 2 whereas the average value is lagging and becomes feasible starting at generation 5.

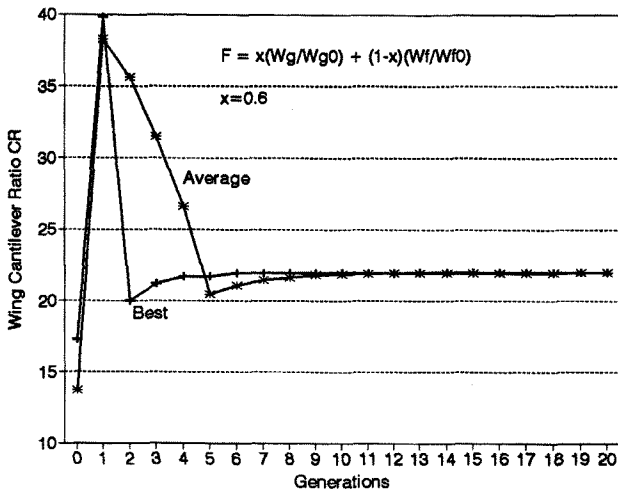


Fig.3: Evolution of the CR Constraint.

The evolution of one of the design variables, the wing quarter chord sweep angle is given in Fig.4 for the same com-

pound objective function F with the scalar $x=0.6$. Strong fluctuations are obtained during the first 8 generations, and the best solution settles on a value of 30° starting at generation 11.

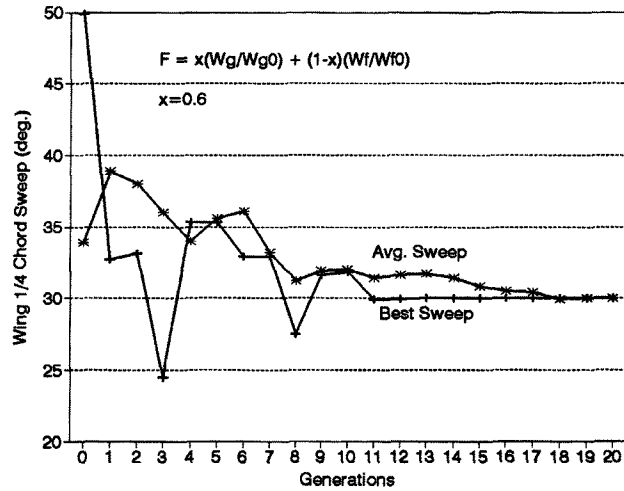


Fig.4: Evolution of the Quarter-Chord Sweep Angle.

The fluctuations are probably due to the present approximate model used in estimating the wave drag when the cruise Mach number is higher than the drag divergence Mach number M_{DD} . This model is based on a USAF DATCOM method. Apparently, some noise is introduced into the calculation of the wave drag coefficient C_{Dw} due to the interpolation of unsmooth empirical data. The evolution of another design variable, the wing aspect ratio, is shown in Fig.5. The fluctuations here are smaller, probably because the wing CR structural constraint forces the wing span and aspect ratio to stay within reasonable bounds.

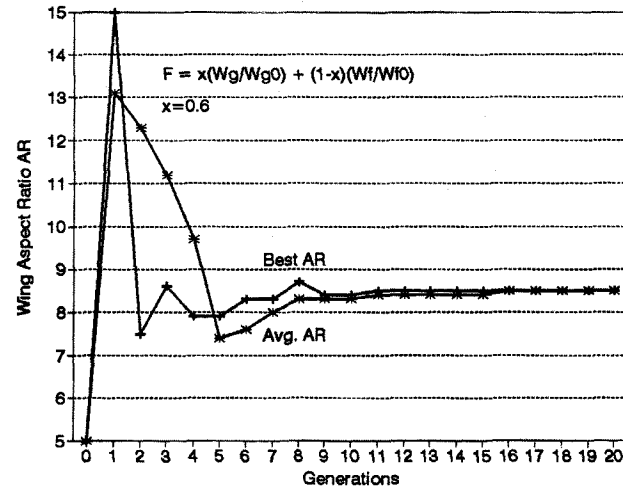


Fig.5: Evolution of the Wing Aspect Ratio.

A more global picture of the multiple criteria optimization using scalarization is presented in Fig.6. The weighting factor x is varied between 0 and 1 so that a set of non-inferior solutions, also known as the Pareto optimal set, is obtained. When $x=0$, the single criterion problem of minimum fuel weight W_f is recovered, and when $x=1$ the problem is reduced to that of obtaining the minimum gross weight W_g . For $x=0$ the fuel weight is minimum and the gross weight is maximum and vice versa, when $x=1$, the gross weight is

minimum and the fuel weight is maximum, because of the contradictory nature of the two criteria.

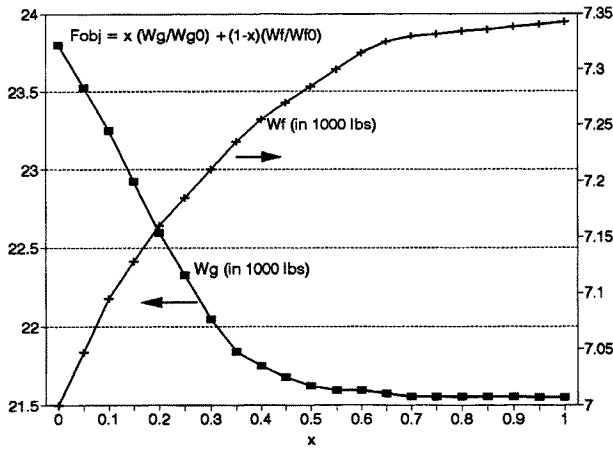


Fig. 6: Pareto Optimal Set of Non-Inferior Solutions.

Minimizing the fuel weight is equivalent to minimizing the wave drag coefficient C_{Dw} , which requires a higher sweep angle and a lower aspect ratio, but not necessarily a lower gross weight. Increasing wing sweep, for example, means an increased wing weight. Also, when the other constraints are taken into account, such as the stall speed constraint, an increased wing reference area is required to maintain the same stall speed V_s because of the adverse effect of increased sweep on C_{Lmax} . When the gross weight is minimized, the present results show that the wave drag coefficient C_{Dw} is approximately equal to the parasite drag coefficient C_{D0} , the fuel weight required is higher, but the combination of design variables gives a lower gross weight. A set of compromised solutions are obtained when the value of the weighting factor x is greater than zero but less than one.

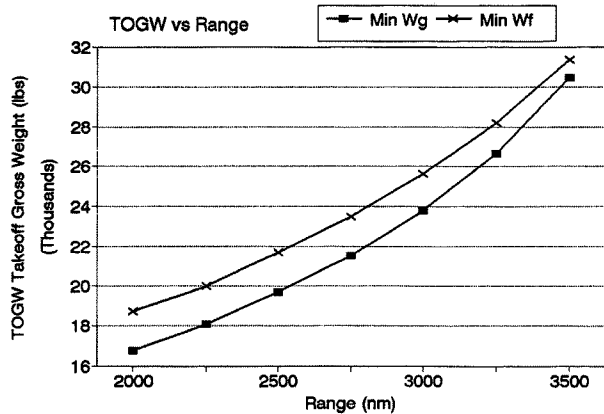


Fig. 7: Takeoff Gross Weight Versus Range Using Two Optimization Criteria.

In Fig. 7, a comparison is made between the gross weight of two configurations, one designed for minimum gross weight and the other for minimum fuel weight for various range requirements (the range varies between 2000 and 3500 nm). The main result is that the configuration designed for mini-

mum fuel has a higher gross weight than the configuration designed for minimum gross weight. A similar result is obtained for the empty weight as displayed in Fig. 8. A comparison between the fuel weight required for the same mission profile is given in Fig. 9. It can be seen that the difference in fuel weight between the two configurations increases with the required cruise range. For larger aircraft this difference might become substantial and might be a major driver of the aircraft synthesis.

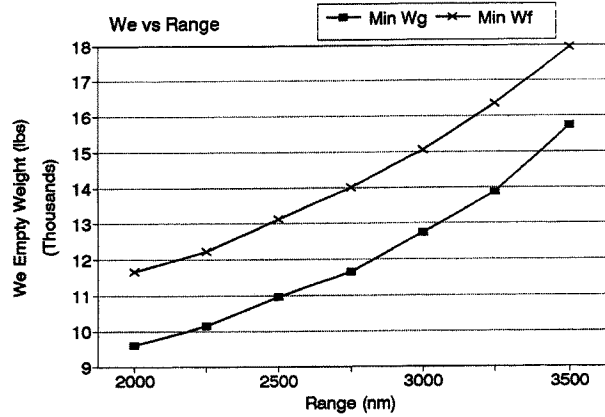


Fig. 8: Empty Weight Versus Range for Two Optimization Criteria.

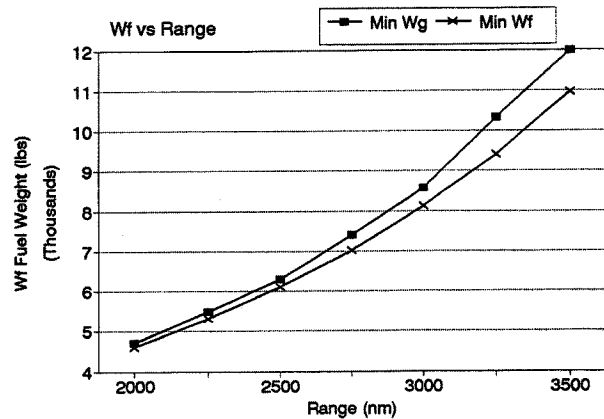


Fig. 9: Fuel Weight Versus Range for the Two Criteria.

Figs. 10, 11, 12 and 13 show that other penalties can be incurred when the criterion is fuel weight rather than gross weight. Excessive wing weight, wing span, wing area and thrust are required if the criterion of minimum fuel is chosen instead of minimum gross weight. When the gross weight is chosen as the objective function, the wing structural constraint imposed on the cantilever ratio drives the feasible designs to moderate values of the wing aspect ratio and sweep angle. The optimum solution is a compromise between aerodynamic efficiency on one hand, which requires a high AR, and on the other hand, wing structural feasibility which requires lower wing spans. High sweep increases the wing weight and decreases the value of C_{Lmax} required for takeoff and landing (which in turn forces the synthesis algorithm to select a low wing loading and therefore a higher wing planform and higher parasite drag). Low wing sweep causes an increase in wave drag which in turn increases fuel consumption and weights. On the other hand, when the

minimum fuel weight criterion is used for this class of executive jet aircraft, a higher gross weight is obtained, with little savings in fuel weight, especially for moderate cruise ranges.

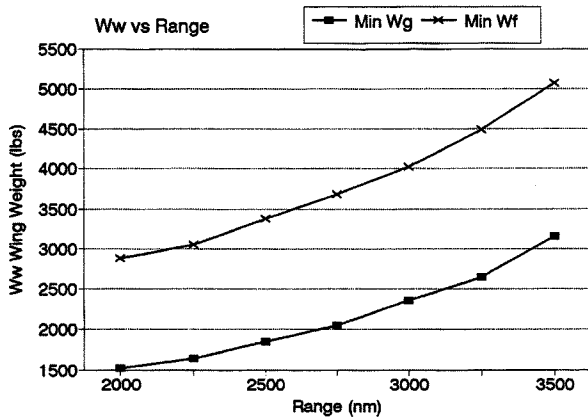


Fig. 10: Wing Weight Versus Range for the Two Criteria.

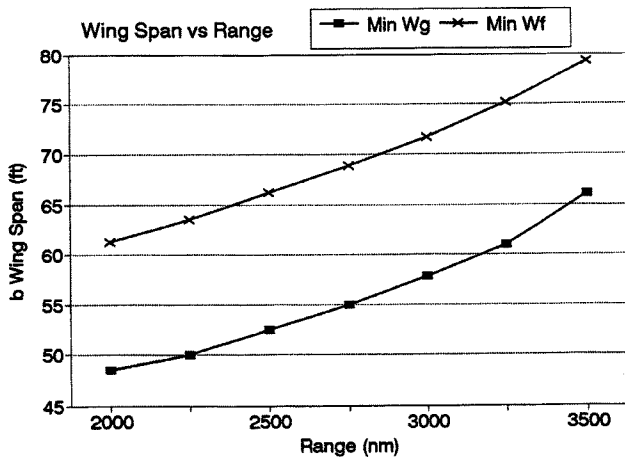


Fig. 11: Wing Span Versus Range for the Two Criteria.

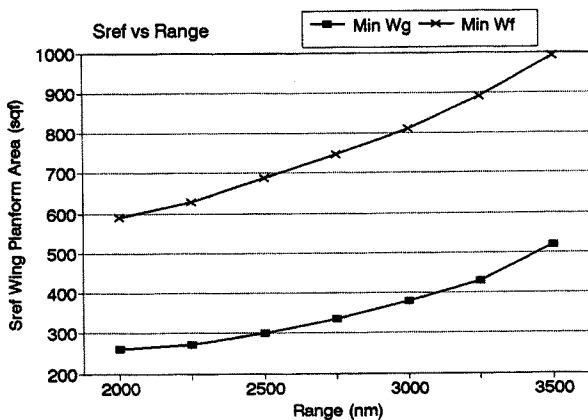


Fig. 12: Wing Planform Area for the Two Criteria.

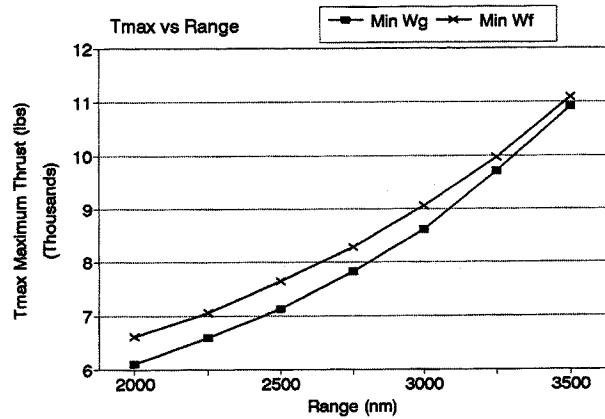


Fig. 13: Maximum Thrust Versus Range for the Two Optimization Criteria.

5. SUMMARY AND CONCLUSIONS

The problem of constrained, nonlinear, multiple criteria optimal aircraft synthesis is solved using a genetic algorithm as the optimization module and a conceptual refined sizing and weight estimation method as the analysis module. The nonlinear constraints are incorporated into the genetic search by imposing exact penalties on the objective function in the infeasible domain, i.e. if the constraints are violated, the fitness function is assigned a (problem-specific) low and unfavorable value, so that the infeasible solutions are eliminated by the selection process. The resulting pseudo-objective function is discontinuous and is easily handled by the algorithm since a genetic search is used rather than a gradient technique. Two different criteria, minimum gross weight and minimum fuel weight are considered. Combinations of the two criteria are treated through the use of scalarization, and the set of non-inferior Pareto optimal solutions is then obtained.

Other methods for obtaining the Pareto-optimal set of non-inferior solutions, such as the min-max method and the breeding of separately evolved populations are currently under investigation. The treatment of problems that involve mixed continuous and discrete variables are also being studied.

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