SCHEME OF ALIGNMENT FOR STRAPDOWN INERTIAL NAVIGATION SYSTEMS

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ABSTRACT

The transfer alignment for strapdown inertial navigation systems (INS) uses the master's and the slave's inertial measurement unit's information to estimate the relative misaligning angle between the master and the slave INS's , so to get slave INS's initial attitude . The master INS is installed in cockpit while the slave INS is located on an aircraft wing . The relative misaligning angle includes fixed misaligning angle and angle of wing flexible deformation .

Transfer alignment methods can be classified into two classes ,one named calculated parameter matching and the other named measured parameter matching . The calculated parameter matching methods have certain accuracy in estimating the fixed misaligning angle , but they are incapable of estimating the angle of wing flexible deformation , and their aligning speeds are usually low . The measured parameter matching methods have high aligning speeds while their precisions are seriously affected by the model errors of wing flexible deformation .

By quantitative analysis , the author realizes that different matching method is suitable for different inertial sensor signal .This paper combines the advantages of calculated parameter matching and measured parameter matching , proposes a new transfer alignment method . The calculated parameter matching is used to process longitudinal acceleration signal , While the measured parameter matching is used to process angular rate signal . A damped vibration model is adopted as the model of flexible deformation . The fixed misaligning angle , the flexible deformation angle and angle rate as well as the constant gyro drifts of slave INS's are taken as the state variables for Kalman filter , and can be estimated fastly and accurately by this method .

I. Introduction

If an inertial navigation guided missile is carried and launched by aircraft , then it's inertial navigation system (INS) is called slave INS in comparison with the aircraft's INS which is called master INS . The master INS is mounted in cockpit . When the missile is to be launched , it's INS should be initialized with it's initial states such as position , velocity and attitude (or directional cosine matrix which is closely related to attitude) . Suppose that the coordinate frame of slave INS is in accordance with the coordinate frame of master INS , then the initial states of slave INS are exactly the same as the master INS's in the very moment the missile is launched . In this ideal condition , what we have to do is to convey the states of master INS's to slave INS by data bus . While , owing to the physical error and flexure deformation of aircraft's wing , the frames of slave and master INS's will never be in accordance with each other , and an angle called relative misaligning angle always exists between them . We designate this angle as β The misaligning angle β makes the motion states of slave system's different from those of master system's . In this case , if we still take the master INS's states as the initial states of slave INS , then the missile will be misguided to a wrong region , and fail to fulfill its task .

The transfer alignment is such a process in which the difference between master's and slave's measured or calculated information is used to estimate the relative misaligning angle , and therefore the initial attitude of slave INS is determined[1].

We designate the master INS's coordinate frame as m , the slave INS's frame as s , the frame of inertial space as i and the navigational coordinate frame as n . The three axes of m frame point to forward , right and down respectively while the axes of n frame point to north , east and down . The coordinate transformation matrix (we also call it direction cosine matrix) between m frame and n frame is C_{mn} and the matrix between s frame and n frame is C_{sn} , while the transformation matrix between m and s frames is C_{sm} , and the following relationship is among them

$$C_{sn} = C_{sm} \cdot C_{mn} \tag{1}$$

When aircraft is in flight , the C_{mn} is always known with a rather high accuracy . hence , if we can find out C_{sm} , then C_{sn} can also be found out , and the slave INS's attitude can therefore be resolved from C_{sn} .

When relative misaligning angle β is small

$$C_{sm} = \begin{bmatrix} 1 & \beta_z & -\beta_y \\ -\beta_z & 1 & \beta_x \\ \beta_y & -\beta_x & 1 \end{bmatrix} = I - B \tag{2}$$

and

$$B = \begin{bmatrix} 0 & -\beta_z & \beta_y \\ \beta_z & 0 & -\beta_x \\ -\beta_y & \beta_x & 0 \end{bmatrix} \tag{3}$$

Where β_x , β_y , β_z are three components of vector β . While as a vector , β consists of constant misaligning angle $\Phi$and wing elastic vibration ρ , that is

$$\beta = \Phi + \rho \tag{4}$$

Where Φ is due to mounting error and constant flexure deformation in flight .

Generally speaking , transfer alignment schemes can be classified into two classes , the one is called calculated parameter matching and the other is called measured parameter matching . The calculated parameter matching schemes take the whole β as state variable , and estimate it by using the difference between positions or velocities reckoned out by master INS and slave INS respectively . Because this type of schemes lack models depicting the wing elastic vibration , so they cannot give accurate estimation of elastic vibration deformation , and their convergence speed of filtering is relatively low , while their accuracy for estimating constant angle Φ is reported high .

The measured parameter matching schemes directly use the differences between inertial signals sensed by master and slave INS's sensing elements respectively , and take Φ and ρ as different state variables . Based upon the vibration model and observation of up to six dimensions , this type of schemes can carry out the alignment fastly , while its accuracy is greatly affected by the error of vibration model .

Having consulting the theory and practice of elastic vibration , this paper selects a damped vibration model as the model of wing elastic vibration . The measured parameter matching is adopted here to settle ρ while the calculated parameter matching is adopted to settle Φ . Hence a new , combined matching transfer alignment scheme is proposed . Cases of alignment simulation show the effectiveness of this new scheme .

Part 2 of this paper will discuss the dynamic model for misaligning angle . Part 3 discusses the angular motion formulation . Part 4 discusses the longitudinal acceleration formulation .Part 5 introduces the so called combined matching transfer alignment scheme . Part 6 shows the simulation results and part 7 is the conclusion of the whole article .

II. Dynamic model

The total misaligning angle β consists of constant misaligning angle Φ and elastic vibration deformation ρ , among which , the modal for Φ is very simple , that is

$$\dot{\Phi} = 0 \tag{5}$$

while the model for ρ is rather complicated . The wing elastic vibration has innumerable modes according to elastic vibration theory[2],[3]. Take the bending of wing for example , it is an angular vibration around the longitudinal axis x_m . the general solution for the displacement is as follow

$$z(y,t) = \sum_{i=1}^{\infty} z_i(y)\sin(\omega_i t + \varphi_i) \tag{6}$$

where $z_i(y)$ is the ingredient mode function of displacement , ω_i is the angular frequency and φ_i is the initial phase . Therefore , the angular vibration becomes

$$\rho_x = tg^{-1}\frac{\partial z(y,t)}{\partial y} \approx \frac{\partial z(y,t)}{\partial y}$$

$$= \sum_{i=1}^{\infty} \frac{\partial z_i(y)}{\partial y} \cdot \sin(\omega_i + \varphi_i) \tag{7}$$

This model doesn't take the damping into consideration , so it is the summation of countless of undamped oscillation modes . However , the damping does exist when the aircraft's wing vibrates . In this case , stimulated by external random force which is assumed to have an uniformly distributed power spectrum , the amplitudes of higher frequency portion of the vibration will be small in comparison with amplitudes of lower frequency portion , and they are so small that they can be omitted altogether . Based on this viewpoint , and for sake of simplicity , this paper makes such an omitting that only the lowest frequency ingredient in the right hand side of equation (7) is retained as the model of angular vibration around x_m axis . Same model reductions are taken for angular vibrations around y_m and z_m axes .Thus the model selected is as follow

$$\ddot{\rho}_j + 2\xi_j\omega_j\dot{\rho}_j + \omega_j^2\rho_j = w_j \qquad j=x,y,z \tag{8}$$

where the w_x,w_y,w_z are generalized random forces . In reference[4], Schneider takes the following model as the angular vibration model

$$\ddot{\rho}_j + 2\omega_j\dot{\rho}_j + \omega_j^2\rho_j = w_j \qquad j=x,y,z \tag{9}$$

The model (9) is similar to model (8) except that it doesn't consider the factor of damping , in this viewpoint , model (8) is closer to reality than model (9) .

III. Formulation for angular motion

The rotation speed of master INS with respect to inertial space is designated as ω_m , which is sensed by the gyros of master INS with little errors . ω_m is superposed of ω_{ie} , ω_{en} and ω_{nm} ,that is

$$\omega_m = \omega_{im}^m = \omega_{ie}^m + \omega_{en}^m + \omega_{nm}^m \tag{10}$$

where the expression " ω_{im}^m " means rotation speed of m frame with respect to i frame , and this speed vector is projected onto the m frame .

The algorithms for ω_{ie}^m and ω_{en}^m can be found in texts on inertial navigation[5],[6],[7] , while ω_{nm}^m should be expressed carefully as bellow

$$\omega_{nm}^{m} = \begin{bmatrix} \dot{\gamma}_m \\ 0 \\ 0 \end{bmatrix} + C_y \begin{bmatrix} 0 \\ \dot{\theta}_m \\ 0 \end{bmatrix} + C_z \begin{bmatrix} 0 \\ 0 \\ \dot{\psi}_m \end{bmatrix} \quad (11)$$

$$C_y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma_m & -\sin\gamma_m \\ 0 & \sin\gamma_m & \cos\gamma_m \end{bmatrix} \quad (12)$$

$$C_z = C_y \cdot \begin{bmatrix} \cos\theta_m & 0 & \sin\theta_m \\ 0 & 1 & 0 \\ -\sin\theta_m & 0 & \cos\theta_m \end{bmatrix} \quad (13)$$

where ψ_m , θ_m , γ_m are heading , pitch , and roll angle of the master INS respectively .

The rotation speed of slave INS with respect to inertial space is designated as ω_s , it contains all the contents of ω_m as well as the rotation speed of s frame with respect to m frame , thus becomes

$$\omega_s = \omega_{is}^{s} = \omega_{im}^{s} + \dot{\rho} \quad (14)$$

ω_s can be sensed by gyros fixed in slave INS , so (14) is expressed in s frame .

By consulting (1) and (2) , equation (14) immediately turns into

$$\omega_s = (I-B)\omega_{im}^{m} + \dot{\rho} = (I-B)\omega_m + \dot{\rho} \quad (15)$$

Since

$$B\,\omega_m = \beta \times \omega_m = -\omega_m \times \beta = -\Omega_m \beta \quad (16)$$

Therefore

$$\omega_s = \omega_m - \Omega_m \beta + \dot{\rho} = \omega_m - \Omega_m(\phi + \rho) + \dot{\rho} \quad (17)$$

and

$$\Omega_m = \begin{bmatrix} 0 & -\omega_{mz} & \omega_{my} \\ \omega_{mz} & 0 & -\omega_{mx} \\ -\omega_{my} & \omega_{mx} & 0 \end{bmatrix} \quad (18)$$

Because the slave INS's gyros always have errors such as constant drifts d_s and random drifts ε_s , so the measured angular rate of slave INS is

$$\omega_s^{*} = \omega_s + d_s + \varepsilon_s$$
$$= \omega_m - \Omega_m(\phi+\rho) + \dot{\rho}\; d_s + \varepsilon_s \quad (19)$$

IV. Formulation for acceleration

We use R_m , R_s to designate the positions of master INS and slave INS with respect to inertial space , and use l to designate displacement of slave INS relative to master INS . Making all these vectors projected on to m frame , then we have

$$R_s = R_m + l \quad (20)$$

and

$$l = (I + P)l_0 = l_0 + \rho \times l_0 \quad (21)$$

$$P = \begin{bmatrix} 0 & -\rho_z & \rho_y \\ \rho_z & 0 & -\rho_x \\ -\rho_y & \rho_x & 0 \end{bmatrix} \quad (22)$$

where l_0 is the primary displacement before vibration occurs .

Performing derivation to (20) with respect to inertial space , we get

$$\left.\frac{dR_s}{dt}\right|_i = \left.\frac{dR_m}{dt}\right|_i + \left.\frac{dl}{dt}\right|_i$$
$$= \left.\frac{dR_m}{dt}\right|_i + \omega_m \times l + \dot{\rho} \times lo \quad (23)$$

once again derivation is performed ,and get

$$\left.\frac{d^2R_s}{dt^2}\right|_i = \left.\frac{d^2R_m}{dt^2}\right|_i + \dot{\omega} \times l + 2\omega \times (\dot{\rho} \times l_0)$$
$$+ \ddot{\rho} \times l_0 + \omega_m \times (\omega_m \times l) \quad (24)$$

Because what can be sensed by accelarometer is not acceleration but specific force , so gravity should be added to both sides of equation (24) , thus get

$$f_s^{m} = \left.\frac{d^2R_s}{dt^2}\right|_i + g \quad (25)$$

$$f_m^{m} = f_m = \left.\frac{d^2R_m}{dt^2}\right|_i + g \quad (26)$$

$$f_s^{m} = f_m^{m} + \dot{\omega}_m \times l + 2\omega_m \times (\dot{\rho} \times l_0)$$
$$+ \ddot{\rho} \times l_0 + \omega_m \times (\omega_m \times l) \quad (27)$$

where f_m^{m} is provided by master INS , and it can be considered to be accurate . While the specific force sensed by slave INS's accelarometer is always mixed with accelarometer's random drift , thus becomes

$$f_s^{*} = f_s^{s} + \nabla_s = (I-B)f_s^{m} + \nabla_s \quad (28)$$

V. New scheme for transfer alignment

This section studies how to use the inertial measurement information to estimate the relative misaligning angle between master and slave INS's . The sensor errors of master INS are assumed to be compensated for by GPS or other techniques , therefore they are not taken into consideration here . Sensor errors of slave INS include gyro's constant and random drifts as well as accelarometer's constant and random drift's . Usually , the accelarometer's constant drift is small in magnitude , and it's influence to the transfer alignment is small , hence , it is also not considered . While the constant drift of gyro exert's a great influence to the result of alignment , so it is taken as a state variable . and it's dynamic model is as follow

$$\dot{d}_s = 0 \tag{29}$$

so ,the total state variable includes $\dot{\rho}$, ρ , ϕ and d_s , and is arranged as follow

$$X=[\dot{\rho}_x\ \rho_x\ \dot{\rho}_y\ \rho_y\ \dot{\rho}_z\ \rho_z\ \phi_x\ \phi_y\ \phi_z\ d_{sx}\ d_{sy}\ d_{sz}]^T \tag{30}$$

State equation for X is written as

$$\dot{X} = \begin{bmatrix} A_\rho & 0 \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} G_\rho \\ 0 \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} \tag{31}$$

where

$$A_\rho = \begin{bmatrix} A_x & 0 & 0 \\ 0 & A_y & 0 \\ 0 & 0 & A_z \end{bmatrix} \tag{32}$$

$$A_i = \begin{bmatrix} -2\xi_i\omega_i & -\omega_i^2 \\ 1 & 0 \end{bmatrix} ,i=x,y,z \tag{33}$$

$$G_\rho = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \tag{34}$$

The discrete form of (31) is as follow

$$X_k = A\ X_{k-1} + \Gamma\ w_{k-1} \tag{35}$$

Now let's shift ω_m in equation (19) from right side to left side of equality , thus (19) turns into

$$d\omega = \omega_s^* - \omega_m = \dot{\rho} - \Omega_m\rho - \Omega_m\phi + d_s + \varepsilon_s$$
$$= H_1\ X + V_1 \tag{36}$$

thus we get the observation equation using information measured by gyros of master and slave INS's . The pair of (31) and (36) offers an excellent observability to $\dot{\rho}$ and ρ ,while it's so stingy to ϕ that it is difficult to estimate ϕ by (36) alone if the maneuver of aircraft is not sufficiently great . So we have to seek for a better observation for ϕ .

If we shift f_m^m in (27) from right to left , then we get

$$f_s^m - f_m^m = \dot{\omega}_m \times 1 + 2\omega_m \times(\dot{\rho} \times 1_0) + \ddot{\rho} \times 1_0$$
$$+ \omega_m \times(\omega_m \times 1) \tag{37}$$

but our problem won't get solved because f_s^m is not a signal sensed by accelarometers . What can be sensed is f_s^* . Upon the assumption that misaligning angle β is small , from (28) we can obtain

$$f_s^m = (I - B)^{-1}(f_s^* - \nabla_s) \approx (I + B)f_s^* - \nabla_s$$
$$= f_s^* - F_s^*(\phi + \rho) - \nabla_s \tag{38}$$

and

$$F_s^* = \begin{bmatrix} 0 & -f_{sz}^* & f_{sy}^* \\ f_{sz}^{*s} & 0 & -f_{sx}^* \\ -f_{sy}^* & f_{sx}^* & 0 \end{bmatrix} \tag{39}$$

Substituting (38) into (37) we get

$$f_s^* - f_m^m = F_m^*(\phi + \rho) + \dot{\omega}_m \times 1_0$$
$$+ 2\omega_m \times(\dot{\rho} \times 1_0) + \ddot{\rho} \times 1_0$$
$$+ \omega_m \times(\omega_m \times 1) + \nabla_s \tag{40}$$

Equation (40) still cannot be used as state observation equation because on the right hand side of it exist $\dot{\omega}_m$ and $\ddot{\rho}$ which are neither state variables nor signals that can be sensed by gyros . Furthermore , the $\ddot{\rho}\times 1_0$ and $F_s^*\rho$ in (40) are great in magnitudes and usually make $F_s^*\phi$ be submerged . So , the observability of (40) is also rather poor . A remedy adopted here is to integrate both sides of equation (37) .

By reviewing (23) through (26) we know that

$$\int(f_s^m - f_m^m)dt = \left.\frac{dR^s}{dt}\right|_i - \left.\frac{dR^m}{dt}\right|_i$$
$$= \omega_m \times 1 + \dot{\rho} \times 1_0 \tag{41}$$

so

$$\int_{t1}^{t2}(f_s^m - f_m^m)dt = \Big[\ \omega_m \times 1 + \dot{\rho} \times 1_0\ \Big]_{t1}^{t2} \tag{42}$$

Substituting (38) into (42) we get

$$\int_{t1}^{t2}\Big[\ f_s^* - f_m^m\ \ F_s^*(\phi + \rho) - \nabla_s\ \Big]dt$$
$$= \Big[\ \omega_m \times 1 + \dot{\rho} \times 1_0\ \Big]_{t1}^{t2} \tag{43}$$

We can see that $\dot{\rho} \times 1_0$ in (43) is much smaller than $\ddot{\rho} \times 1_0$ in (40) since angular frequency ω_j in (7) is usually much greater than 1 . While by virtue of integration , the coefficient matrix of ϕ becomes larger than ever . so the observability of ϕ is improved apparently .

Strictly speaking , since $\dot{\rho}$, ρ and ϕ are all state variables , they should all be put into right hand side of equation (43) like in (36) . Nevertheless , simulations negative this doing by the fact's that both ρ and ϕ diverge rapidly . Having considered that equation (36) has an excellent observability for $\dot{\rho}$ and ρ , this paper adopts another remedy which treats $\dot{\rho}$ and ρ in (43) as known quantities and puts them into left side of equality . In this case , ϕ becomes the only state variable and is put into right hand side of equality . So (43) is changed into

$$\int_{t1}^{t2}\left[f_s^* - f_m^m - F_s^* \cdot \rho \right] dt - \omega_m \times 1 - \dot{\rho} \times 1_0$$

$$= \int_{t1}^{t2} F_s^* \, dt \, \Phi + \int_{t1}^{t2} \nabla_s \, dt \qquad (44)$$

The effect of this trick is shown to be very well via simulations (see next section) .

Because $\dot{\rho}$ and ρ in (44) are assumed to be known , and they can be known only when the observing data of (36) has been used in filtering , so observation (44) cannot keep in pace with observation (36) . That is to say , in the kth cycle of filtering , t_2 , the upper limit of integration should obey the following restriction

$$t_2 \le (k-1)\Delta t \qquad (45)$$

In this paper , the equality is chosen .

In equation (36) , the observation noise is the random drift of gyros , and the RMS of observation noise is the RMS of random drift of gyros . While in (44) , although the RMS of observation noise can be fixed if the length of interval $\left[t_1 , t_2 \right]$ is fixed , the observation noise is no longer a white noise , but rather a strongly coherent colored noise . This is a drawback . Fortunately , this colored noise is small in magnitude , so the result of filtering is intact . The length of interval $\left[t_1 , t_2 \right]$ taken here is 10 seconds . That is to say that every time we add information to the integrals of (44) , we drop from them the old information gathered 10 seconds ago . Hence , as time goes on , the estimated $\dot{\rho}$ and ρ get more and more accurate , and so does the observation for Φ in (44).

From (41) we can see that the background for deriving (44) is a velocity of slave system relative to master system . So equation (44) belongs to the class of calculated parameter matching , but it's different from those calculated matchings in references [8] and [9] owing to it's simple and distinct physical meaning .

Now let's change (44) into a simpler form

$$du = H_2 X + v_2 \qquad (46)$$

The combined matching transfer alignment procedure is summed up as follows

$$X_k = A X_{k-1} + \Gamma w_{k-1} \qquad (35)$$

$$z_k = H X_k + v_k \qquad (47)$$

$$z_k = \begin{bmatrix} d\omega_k \\ du_k \end{bmatrix} \qquad (48)$$

$$H = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} \qquad (49)$$

$$v_k = \begin{bmatrix} v_{1k} \\ v_{2k} \end{bmatrix} \qquad (50)$$

VI. Simulation analysis

When transfer alignment is being carried out , the aircraft's maneuver is necessary and helpful to the scheme proposed by this paper as well as to schemes proposed by other persons . Generally speaking , the attitude maneuver is favorable to measured parameter matching alignment while the longitudinal acceleration is favorable to calculated parameter matching alignment . Because this new scheme is a combination of these two types of matching , so both attitude maneuver and longitudinal acceleration are favorable to it's execution . And because the observability of this scheme is quite well , so just a slight maneuver is enough to help carrying out the alignment , thus makes the pilot comfortable .

To embody the slightness of maneuver , this paper simulates the attitude angular motion in a parabola way and simulates the longitudinal acceleration by a step function . It is assumed that the aircraft's velocity vector is coincide with the x_m axis . Several cases are simulated to demonstrate the feasibility of the scheme developed here . In these cases , the three components of constant misaligning angle are supposed to be 0.007 , -0.0035 , 0.0026 rad respectively . the three constant gyro drifts of slave INS are supposed to be 0.05 , -0.06 , and 0.03 °/h respectively . The magnitudes of ρ_x , ρ_y , ρ_z are 2° , 10′ and 2′ . The RMS's of $\tilde{\varepsilon}_s$ and $\tilde{\nabla}_s$ are 0.01°/h and 10^{-5}g . The random noise is generated by a white noise generator that has a very good zero-mean property . The sampling frequency is 40 Hz and the aligning time is 60 seconds (1 minute) .

Case 1

Condition : Only heading changes in a parabola way , it's values at 0 , 30 , 60 second are

$$\psi_m(0) = 0^\circ \ , \ \psi_m(30) = 10^\circ \ , \ \psi_m(60) = 0^\circ$$

while

$$\theta_m = \gamma_m \equiv 0^\circ \ , \ v_{xm} \equiv 300 \text{ m/s}$$

Result : After a minute's alignment filtering , the final results are

$$\hat{\Phi} = \begin{pmatrix} 0.00704558 \\ -0.00349949 \\ 0.00261982 \end{pmatrix} \text{rad}$$

$$\hat{d}_s = \begin{pmatrix} 0.221570 \\ -0.063229 \\ 0.032301 \end{pmatrix} {}^\circ/\text{h}$$

$$\tilde{\Phi} = \begin{pmatrix} 9.400942 \\ 0.104549 \\ 4.087833 \end{pmatrix}''$$

$$\tilde{\rho} = \begin{pmatrix} -8.083203 \\ 0.205651 \\ -0.153374 \end{pmatrix}''$$

Case 2

Condition : Only roll angle changes in a parabola way , and it's values at 0 , 30 , 60 second are

$\gamma_m(0) = -5^\circ$, $\gamma_m(30) = 5^\circ$, $\gamma_m(60) = -5^\circ$

while

$$\psi_m = \theta_m \equiv 0^\circ \ , \ v_{xm} \equiv 300 \text{ m/s}$$

Result :

$$\hat{\phi} = \begin{pmatrix} 0.00704016 \\ -0.00349816 \\ 0.00260278 \end{pmatrix} \text{rad}$$

$$\hat{d}_s = \begin{pmatrix} -0.190411 \\ -0.064331 \\ 0.026791 \end{pmatrix} {}^\circ/\text{h}$$

$$\tilde{\phi} = \begin{pmatrix} 8.283563 \\ 0.379261 \\ 0.572853 \end{pmatrix}''$$

$$\tilde{\rho} = \begin{pmatrix} 10.026092 \\ 0.133397 \\ 0.224677 \end{pmatrix}''$$

Case 3

Condition : Only pitch changes in a parabola way , and it's values at 0 ,30 ,60 second are

$$\theta_m(0) = -30^\circ, \ \theta_m(30) = 0^\circ, \ \theta_m(60) = 30^\circ$$

while

$$\psi_m = \gamma_m \equiv 0^\circ \ , \ v_{xm} \equiv 300 \text{ m/s}$$

Result :

$$\hat{\phi} = \begin{pmatrix} 0.00700364 \\ -0.00350760 \\ 0.00253257 \end{pmatrix} \text{rad}$$

$$\hat{d}_s = \begin{pmatrix} 0.302329 \\ -0.049013 \\ 0.044836 \end{pmatrix} {}^\circ/\text{h}$$

$$\tilde{\phi} = \begin{pmatrix} 0.750699 \\ -1.568244 \\ -13.825490 \end{pmatrix}''$$

$$\tilde{\rho} = \begin{pmatrix} -0.291821 \\ -0.339542 \\ -0.123485 \end{pmatrix}''$$

Case 4

Condition : A constant longitudinal acceleration is added 40 seconds after the beginning of alignment and all the attitudes keep unchanged , that is

$$v_{xm} = 300 \text{ m/s} \ , \ 0 \le t \le 40 \text{ second}$$

$$a_{xm} = \begin{cases} 0 & , \ 0 \le t \le 40 \text{ second} \\ 2.5 \text{ m/s}^2 & , \ t > 40 \text{ second} \end{cases}$$

$$\psi_m = \theta_m = \gamma_m \equiv 0^\circ$$

Result :

$$\hat{\phi} = \begin{pmatrix} 0.00702578 \\ -0.00350081 \\ 0.00248302 \end{pmatrix} \text{rad}$$

$$\hat{d}_s = \begin{pmatrix} -0.187770 \\ -0.056591 \\ 0.026648 \end{pmatrix} {}^\circ/\text{h}$$

$$\tilde{\phi} = \begin{pmatrix} 5.318236 \\ -0.166410 \\ -24.127959 \end{pmatrix}''$$

$$\tilde{\rho} = \begin{pmatrix} 9.899694 \\ -0.220508 \\ 0.149793 \end{pmatrix}''$$

From all these four cases we can see that the transfer alignment scheme proposed by this paper is feasible and effective , especially when the aircraft turns horizontally or rolls around it's longitudinal axis , for in the first two cases , the errors of $\hat{\phi}$ are small than $10''$. Usually , people take $\tilde{\phi}$ as the criteria to assess the effect of alignment . In this respect , the precision given by this scheme can be said to be in a magnitude of arcsecond . But in our opinion , $\tilde{\rho}$ is also a part of aligning error .So it may be more persuasive to use a comprehensive error that includes $\tilde{\phi}$ and $\tilde{\rho}$ as the criteria . In this sense , the comprehensive precision of this scheme is about 30 arcseconds since some times the $\tilde{\rho}$ may be up to $20''$.

In case 4 we see that the error of $\hat{\phi}_z$ is much greater than $10''$. This is because when the aircraft flies with a constant acceleration , the observability for ϕ is as poor as that when it flies without acceleration . In this case , since the Kalman filter only takes responsibility to recent observations , the best observability appears only in a short period after the maneuver occurs . As time passes on , and if there is no new maneuver's occurring , the observability will descend again . While , as we know , the least square algorithm takes responsibility to a whole process recorded . Therefore , as a remedy for the above mentioned shortcoming of Kalman filter , a least square algorithm is adopted to settle ϕ using observations in equation (44) while the procedure proposed in last section is carrying on . The time interval during which the least square algorithm works is selected from 20 second through 60 second . The least square estimation result for case 4 is

$$\hat{\phi}_{ls} = \begin{pmatrix} 0.00699754 \\ -0.00349811 \\ 0.00260816 \end{pmatrix} \text{rad}$$

And

$$\tilde{\phi}_{ls} = \begin{pmatrix} -0.508402 \\ 0.389387 \\ 1.682162 \end{pmatrix}''$$

Much better than the result provided by Kalman filter in this case . The L.S. results are also very well for case 1 ~ case 3 . But if the interval is too short for L.S. to work , the least square's result will be worse than the filtering's result . So the scheme recommended in last section is still a good choice if a very fast alignment is needed .

In the four cases , the constant gyro drift's around y_s and z_s axes are estimated approximately

. While the estimation for constant drift around x_s axis fails .

VI. Conclusion

In this paper , a combined matching transfer alignment scheme is developed which uses angular rate information sensed by gyros to estimate ρ via measured parameter matching and uses acceleration information sensed by accelarometers to estate Φ via calculated parameter matching . A damped vibration model is used as the model of wing elastic vibration .

The effect of this new , combined matching transfer alignment scheme is very well . The precision of $\hat{\Phi}$ can reach a magnitude of arcsecond in a time of a minute , while the comprehensive precision of $\hat{\beta}$ is about 30 arcseconds .

Aircraft's maneuver is necessary and helpful to this alignment scheme . But the maneuver can be slight and gentle . So the pilot will feel comfortable , and even a larger , heavier aircraft can perform this scheme of alignment easily .

By using this alignment scheme , the constant gyro drift's around y_s and z_s axes can be estimated approximately . This is beneficial to enhance the precision of inertial navigation and guidance . But the estimation result of constant gyro drift around x_s axis is unreliable .

REFERENCES

(1). Yu Jixiang " A study of transfer alignment methods of INS " . ACTA AERONAUTIC ET ASTRONAUTICA SINICA Vol.9, No.5 May, 1988

(2). Shi Shuzhao , Chang Lianfang , Sun Baoli & Peng Lishen « Structure Mechanics and Dynamics »

(3). Yie Fengpei « Foundation of Aero Elastic Dynamics »

(4). Alan M . Schneider " Kalman filter formulations for transfer alignment of strapdown inertial units ". Navigation Vol.30, No.1, Spring, 1983

(5). Chui Zhongxing « Inertial Navigation System »

(6). Chen Zhe « Principle of Strapdown Inertial Navigation System »

(7). Yi Guangqu « Principle of Inertial Navigation »

(8). Itzhack Y. Bar-Itzhack & Eugene F. Mallove " Accurate INS transfer alignment using a monitor gyro and external navigation measurements ". IEEE Trans on AES , Vol.16, No.1

(9). James E. Kain & James R. Cloutier " Rapid transfer alignment for tactical weapon applications ". AIAA , 1989