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INVESTIGATION ON AIR REFUELING SCHEDULING

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Abstract

Air refueling is an effective measure to extend endurance and enhance fighting efficiency for aircraft. As designing the air refueling scheduling, this paper uses the knowledge of navigation, operations research and optimization theory. Refueling positions and scheduling plan are optimized comprehensively. The author obtains a truely optimal solution beyond mathematical programing techniques, and design optimal tracks of assigning tankers to receivers with fixed velocity and height, and computes optimal refueling times and fuel weights with minimum total fuel consumption. This paper takes the maximum number of tankers required, the maximum number of tankers available take-off bases, destination bases and post refueling bases etc. into consideration. Fuel consumption of a single pair of tanker and receiver is minimized with optimization of a constrained enroute method. Total fuel consumption of receivers and tankers is minimized with optimization of Conig efficacy matrix. This paper gives an example. The method of this paper suits not only for single air refueling, but also for military large-scale refueling. It serves the basis for refueling fight, training command or imitation.

I. Introduction

Transport aircraft carry various tasks from these origin bases to those other destination bases. When the distance between these bases is greater than the range of aircraft with its available fuel, it is necessary to air refuel enroute to enlarge their ranges. In the complex multi-aircraft refueling problem, solving refueling scheduling is a more complex problem that deals with the knowledge of navigation, operations research and optimization theory. This problem can be divided into two sections. First, refueling position, refueling fuel and refueling time must be determined. Second, refueling scheduling plan must be selected(1).

The first task of refueling scheduling is to locate the optimal refueling positions. Refueling positions are mainly determined by onload capabilities of receivers and offload capabilities of tankers. The offload or onload capabilities are functions of take-off gross weights, take-off fuel loads, take-off bases, destination bases and post refueling bases etc. If all other factors are held constant, each receiver's onload capability increases with flight time, and each tanker's offload capability decreases with flight time(2, 3).

In order to study easily, some appropriate assumptions are required with the different refueling demands. Ref. 2 and Ref. 3 use the method that the receiver's onload capability is equal to the tanker's offload capability to determine refueling locations. This method suits for buddy-buddy approaching or flighting to the same destination of tanker and receiver. For some other refueling cases, it is difficult to obtain an optimal solution. Optimization of a constrained spherical location problem is studied in Ref.4. The objective function is non-linear, and some of the constraints are a function of the decision variables. This method is complex in mathematics. Ref.5 researches the multi-aircraft refuelings, using the method of Network and Greedy.

In this paper, the following assumptions and notations are used:

1. The receivers' take-off original states (take-off weights, fuels, take-off times, take-off bases and destination bases) are fixed.
2. The tankers' take-off original states are determined by the refueling scheduling plan.

3. The earth is a perfect sphere, and receivers and tankers follow the great circle route in flighting from one point to another.

4. High weather (such as temperature) is neglected. Only constant wind is considered.

5. Refueling is a period process, not a point.

6. Flight weights and their fuel consumpion rates of Tankers and receivers are allowed to vary.

7. Refueling times of one receiver in flight course is no more than twice.

8. After once refueling, the tanker lands at the appropriate post refueling base.

9. The numbers of receivers and tankers are not fixed, but their respective bases positions are fixed.

This paper's reference parameter is time or distance, according to the original states. Using optimization of a constraind enroute determines the optimal refueling positions and refueling times. Conig efficacy matrix is used to solve the tanker and receiver scheduling.

Ⅱ. Theory and Method

Optimal Refueling Position

Determination Receiver's Required Fuel. The difference between the fuel of receiver flighting in its whole enroute range and that of its take–off available fuel is its required fuel. The fuel is determined by iteration. The reason is that aircraft's weight will be changed while refueling occured, and its fuel consumption will be changed.

In level cruise, the aircraft weight W equals left Y, and thrust T equals drag Q, so

$$\frac{dW}{dt} = \frac{dW}{dx} \cdot \frac{dx}{dt} = -c_e T = -\frac{c_e W}{(Y/Q)}$$

where, Ce is aircraft's fuel consumption rate. x is distance between take–off base and flight position. The combined parameter is assumped as:

$$K = \frac{V(Y/Q)}{c_e}$$

we have

$$\frac{dW}{dx} = -\frac{W}{K} \tag{1}$$

For the receiver, the general relationship between weight and distance is

$$W_R(x) = W_{R0}e^{(-\frac{x}{K_R})} \tag{2}$$

or

$$W_{Re}(y) = W_{Re}e^{(\frac{y}{K_R})} \tag{3}$$

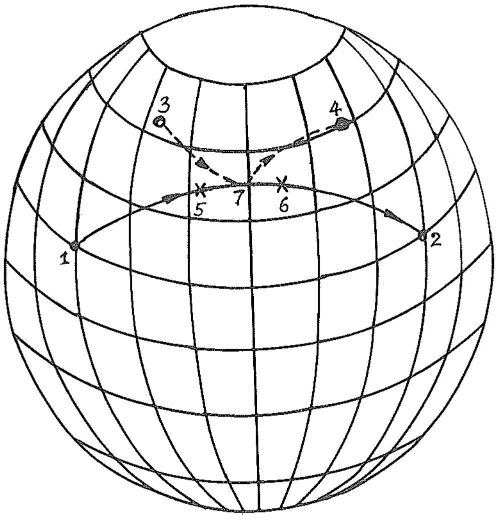
where, the receiver's take–off gross weight W_{R0} is known. W_{Re} is the basic weight that receiver reaches its destination base. It includes aircraft's empty weight, cargo weight and prepared fuels. It is also known.

If the optimal refueling point is at the distance r, the required refueling fuels of receiver is obtained as follows,

$$\begin{aligned} \Delta F_b &= W_{Re}(L-r) - W_R(r) \\ &= W_{Re}e^{(\frac{L-r}{K_R})}) - W_{R0}e^{(-\frac{r}{K_R})}) \end{aligned} \tag{4}$$

There, S is the distance of the receiver's whole range.

Determination the Possible Refueling Area. The possible refueling area is determined by the aircraft's properties and its surroundings. For example, as shown in Fig.1, the possible refueling point is between the earlest possible refueling point and the latest possi-



1 take–off base of receiver
2 destination base of receiver
3 take–off base of tanker
4 post refueling base of tanker
5 the earliest possible refueling point
6 the latest possible refueling point
7 refueling position

Fig.1 Determination the possible refueling area ble refueling point on receiver's flight route. The earlest possible refueling point is the point that receiver with full fuels can reach the destination base. The latest possible refueling point is the point that the receiver with its take–off available fuels can flight the farthest point. Before this area, receiver with refueling can't carry its task. After this area, receiver can't be refueled. In general, the earlest possible refueling point will not fall into aircraft's climb part, and the latest possible refueling point will not fall into aircraft's down part.

Determination the Optimal Refueling Position. Optimization refueling position is to minimize the whole fuel consumption according to the optimal function. It relates to tankers and receivers original states. $W_R(r)$ is the function of the optimal refueling position r. So it is optimized by Eq. (4). Different tankers serve the same receiver will have the different results, optimal refueling positions and required fuels. It is important for the whole refueling schedeling.

For once refueling, the general relationship between weight and flight distance of tanker is as Eq. (2) and Eq. (3)

$$W_T(x) = W_{T0}e^{(-\frac{x}{K_T})} \tag{5}$$

or

$$W_{Te}(y) = W_{Te}e^{(\frac{y}{K_T})} \tag{6}$$

It's assumpted that the offload fuel that the tanker can provide to the receiver is ΔF_a . l is the distance of the tanker reaching the refueling point. So

$$\Delta F_a = W_{T0}e^{(-\frac{l}{K_T})} - W_{Te}e^{(\frac{S-l}{K_T})} \tag{7}$$

where, W_{Te} is the basic weight that the tanker reaches its post refueling base, and is known. S is the distance of the tanker's whole range.

The original optimal refueling position is defined, according to the equivalence of onload and offload capability, i.e.

$$\Delta F_a = \Delta F_b$$

Then, W_{T0} can be defined by formular from Eq. (4) and Eq. (7), then the tanker's take–off time can be defined.

In this paper, in the area of possible refueling, method of iteration is used for searching the optimal result,refueling position r and refueling time T. The error between the lasted two iterations is enough small, then ΔF_b can be defined and the accurate refueling position can be defined.

Twice Refueling

If the required fuels of receiver is greater than the offload fuels of tanker once, twice refueling is considered. In this paper, the first refueling position should be early, and the receiver will be fulled with the tanker's offload fuel. The method of determining the second refueling position is similar to the above one.

Scheduling of Tankers and Receivers

If the number of receivers is n, the possible refueling points is 2n. The number of tankers is m. Mathematically, if K = max(m, 2n), the efficacy matrix is a square matrix. It's optimal subjective function is

$$GK = \sum_{i=1}^{k}\sum_{j=1}^{k} c_{ij}x_{ij} = Min \tag{8}$$

Where, $C_{ij}$is the efficacy (fuel cost) coefficient of the tanker i assigning to the receiver j. The constraint conditions is

$$\sum_{j=1}^{k} x_{ij} = 1 \qquad (i = 1,2,\cdots,k)$$

$$\sum_{i=1}^{k} x_{ij} = 1 \qquad (j = 1,2,\cdots,k)$$

$x_{ij} = 0$ (tanker i not assigning to receiver j)

$x_{ij} = 1$ (tanker i assigning to receiver j)

This conditions must be satisfied by the refueling point. We can use Conig efficacy matrix to solve the problem. Its theory basis is: the optimal assignment will not be changed while a line or a row of the efficacy matrix adds or reduces a contant. The optimal assignment is not completed until at least a zero element in one line or one row is reached.

1. m> 2n The available tanker number is greater than the required number of tanker. So, all refueling tasks will be finished.
2. m < 2n The available tanker number is less than the required number of tanker. Not all refueling tasks will be finished. We think that the available tankers is a kind of source. This problem is how to use the source fully to finish the task.

Ⅲ. Example

We have a assumption that the tanker have a waitline flight of 15 minites when it reaches the refueling position early. The computation results of refueling positions, refueling times, refueliing fuels and the scheduling assignment are shown in the following table 1 and table 2.

Table 1 Refueling Positions and Refueling Times of Receivers

Receiver No.	Required Fuel (kg)	First position	Refueling time	Point No.	Second position	Refueling time	Point No.
1	6280.12	(27.0,111.0)	09:24:14	2	(16.4,115.6)	11:23:13	10
2	5392.16	(16.0,111.8)	10:06:47	7	(4.4,109.4)	12:05:43	15
3	5444.32	(26.8,114.7)	09:38:07	4	(11.1,113.0)	12:06:59	16
4	3378.65	(13.5,107.5)	10:07:50	8	(0.0, 0.0)		
5	7023.54	(10.0,109.5)	10:24:05	9	(12.7,111.2)	11:31:11	12
6	7166.86	(24.0,114.2)	09:37:49	3	(15.3,109.6)	12:14:06	17
7	6934.18	(24.7,113.4)	09:17:10	1	(15.4,108.2)	11:59:30	14
8	3889.86	(14.0,113.4)	09:50:29	5	(12.0,113.9)	11:24:56	11
9	* * *	(0.0, 0.0)			(0.0, 0.0)		
10	4714.49	(16.1,110.3)	09:56:50	6	(14.3,113.5)	11:57:07	13

Table 2 Refueling Scheduling of Tankers

Tanker No.	Take–offf weight(kg)	time	Refueling point No.	fuel(kg)	Post Refueling–Point
1	66730.9	08:38:41	8	3378.65	(30.1,104.0)
2	57571.6	11:18:11	14	3467.09	(23.0,113.0)
3	54221.7	09:08:05	3	3583.43	(25.0,121.1)
4	64368.8	07:16:31	7	2696.08	(20.0,110.1)
5	63459.9	09:51:08	11	1944.93	(20.0,120.0)
6	67544.6	10:06:59	16	2233.03	(25.0,105.0)
7	63054.7	10:37:22	13	2357.25	(19.0,115.0)
8	66914.1	09:10:27	6	2357.25	(24.0,120.0)
9	61135.7	08:15:10	5	1944.93	(20.0,110.0)
10	54425.1	08:40:41	2	3140.06	(33.0,115.0)
11	70078.1	10:11:13	15	2696.08	(20.0,120.0)
12	55300.8	08:45:48	1	3467.09	(25.0,105.0)
13	58439.7	08:13:39	4	3163.10	(19.0,115.0)
14	57063.0	10:20:58	10	3140.06	(24.0,120.0)
15	62162.1	10:26:13	17	3583.43	(20.0,110.0)

Ⅳ. Conclusions

This paper studies the determination of the optimal refueling position and the assignment on air refueling scheduling. Optimal refueling position is determined by optimization of a constrained enroute method. This method suits for all kinds of respective positions of tankers and receivers. Optimal refueling scheduling is determined by optimization of Conig efficacy matrix. This method is suitable for arbitrary number of tankers and receivers. The results are reasonable and practical, and can serve for air force directly.

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