An Adaptive Region Method for Computation of Vortex Sheet Behind Wing in Compressible Flow

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Abstract

An adaptive region method is presented for the computation of vortex sheet behind wing in compressible flow, by adjusting automatically computation region in size on every trefftz's plane. The method can keep the boundary of computation region sufficiently away from the region of vorticity to ensure the accuracy with less computation time. In addition,the first derivatives of the potential on branch cuts is redefined and a second derivatives of potential function is obtained for improving calculation of the cross velocity on branch cuts. Computation shows that the "hook"at vortex tip which appeared by using Stremeland Murman's method [1] for elliptically loaded wing is eliminated in the present method and the computation results are quite agreement with exact solution of Betz theory for elliptically loaded wing.

I. Introduction

Vortex flow arises from flow Separation. Vortex sheets shed from a wing gradually roll up and concentrate into two or more vortex cores. Generally, the details of the wake structure have only small influence on the pressure distribution of the airplane surface. However, the strong and persistent trailing vortex system caused by large transport aircraft has proven that it may bring about hazardous for air traffic, which has resulted the present stringent rules imposed on air traffic involving large planes. Also the evolution and position of these vortex sheets must be properly accounted for if it is necessary to determine the aerodynamic forces correctely.

A fundamental computation method of vortex sheets is that the vortices are representated by a finite number of discrete vortex filaments and the motion of these filaments under their mutual influence is tracked by lagrangian, rather than Eulerian method.

Baker[2] have introduced the "cloud in cell" technique in plasma physics to the calculation of vortex flows, In this approach, the velocities of the vortices are obtained by solving the equation of streamfunction on the Eulerian finite difference grid. The velocities are then interpolated to the vortex positions, and the vortices are tracked in a Lagrangian reference frame. Using a fast Poisson solver for the streamfunction, these calculations require $O(M\log_2 M)$ operations for every time step, where M is the number of grid points. The grid introduces fine scale structures of the flow, which are amalgamated into larger structures independent of the grid. Large numbers of vortices can be efficiently represented by this approach.

This approach was modified by stremel and Murman[1] who solved the equation of velocity potential rather than the streamfunction. The method has been applied to computation of the flow behind a conventional wing and flapped wing. By tracking the vortices in the lagrangian frame and redistributing it with "cloud in cell" methed in the Eulerian frame for the potential calculation. In their result a small "hook" appears at the vortex tip. When the computation is conducted downstream, this irregularity becomes larger "hook" shown in the spiral vortex core.

This paper presents an adaptive region method to improve the results given by reference[1]. In the reference[1] the size of computation region is constant on every trefftz's plane, In fact, the computation error is importantly affected by the region size, when the computation station goes downstream, the region size should be adjusted in order to keep the boundary of the computation region sufficiently away from the region of vorticity. Assuming H is the maximum distance between adjacent local centroids of vorticity in the given trefftz's plane, the computation region is defined by H as well as the distance between the computation boundary and the nearest vortex marker. The first derivatives of the potential on branch cuts is redefined and a second derivatives of potential function is obtained for improving the calculation of the cross velocity on branch cuts.

With these improvement, The computation results for elliptically loaded wing is quite agreement with exact solution of Betz theory on the horizontal position of vortex centroid and on descend speed of vortex centroid. The "hook" inside the spiral appeared by using Stremel and Murman's algorithm is eliminated in the present method.

The method has been applied to calculation of the vortex sheets behind an aircraft. The computation results has been used to conduct of formation flight of the aircraft.

II. Potential Flow computation of the vorex sheets

The flow considered in the present paper is symmetrical. All the vortices in flow field is considered concentrated on a finite number of vortex filaments. The vortices move under their mutual induction. The flow everywhere outside the cores of the vortices is irrotational. Figure 1 gives illustration about the problem. Time t represents substantially the distance from computing station to the trailing edge of the wing. Coordinate y

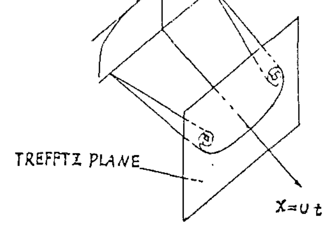

Fig. 1 Schematic of vortex wake problem

and z are in the spanwise and vertical direction. The initial position of the wake corresponds to the wing trailing edge and its circulation distribution is determined from the slope of spanwise circulation distribution $\Gamma_w(y)$, the strength of the vorticity at $t=0$ is given by

$$\Gamma_i(0,y,0) = -\int_{y_1}^{y_2} \frac{d\Gamma_w}{dy} \qquad 0 \leqslant y \leqslant 1$$

$$= -(\Gamma_w(y_2) - \Gamma_w(y_1)) \tag{1}$$

where y_1, y_2 are spanwise coordinates. Defining the i^{th}vortex marker and let $i=1,\cdots,N$, the initial position of each vortex marker ($t=0$) is given by

$$y_i(0) = \frac{y_1 + y_2}{2} \quad ; \quad z_i(0) = 0 \tag{2}$$

the spline smooth interpolation is used to distribute the initial circulation to the marker points subsequently, the location of the vortex markers at the next station downstream are determined from the trajectory equations.

$$\begin{cases} \dfrac{dy_i}{dt} = v_i = v(y_i, z_i) \\ \dfrac{dz_i}{dt} = w_i = w(y_i, z_i) \end{cases} \tag{3}$$

where v_i and w_i are the flow velocities at the location of the vortex marker. They can be obtained by solving the full potential equation. For the velocity field, the vorticity is a given by

$$\omega = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \tag{4}$$

It is assumed that the velocity is consisted of potential component and a non potential component as

$$\begin{cases} v = \varphi_y - g_1(y,z) \\ w = \varphi_z - g_2(y,z) \end{cases} \tag{5}$$

we obtain from the vorticity definition,

$$g_{2y} - g_{1z} = -\omega \tag{6}$$

Substituting equation (5) into the continuity equation

$$v_y + w_z = 0 \tag{7}$$

Hence we get

$$\varphi_{yy} + \varphi_{zz} = g_{2z} + g_{1y} \tag{8}$$

An outer boundary condition away from the wake is given by superposition of the potential φ_i induced by each vortex and its image vortex.

$$\varphi = \sum_{i=1}^{N} \Gamma_i (\theta - \theta_r)_i \tag{9}$$

where

$$\theta = tan^{-1}\left[\frac{z_B - z_i}{y_B - y_i}\right] \quad \theta_r = tan^{-1}\left[\frac{z_B - z_i}{y_B + y_i}\right] \tag{10}$$

lower subscript B denotes outer boundary. Except the symmetry boundary, equation (10) is used on the outside boundaries. On the symmetry boundary with $y=0$ the condition $\varphi_y=0$ should be satisfied.

The branch cut is introduced to produce a single valued function. Consideration of the fact that there is an image for each vortex leads to the condition,

$$\delta\varphi = \varphi^+ - \varphi^- = 2\pi\Gamma_i, \quad -y_i \leqslant y \leqslant y_i, \quad z = z_i \tag{11}$$

where $\varphi^{\pm} = \varphi(y, z_i \pm)$. This jump condition also can be imposed on the symmetry boundary. Thus the derivatives of φ are continuous and finite everywhere except at the location of the markers, where they are singular. On branch cuts, the first derivatives with respect to z at point j,k

$$(\varphi_z)_{j,k} = \frac{1}{2}[(\varphi_z)^-_{j,k} + (\varphi_z)^+_{j,k}] \tag{12}$$

where the upper superscript minus denotes the lower side of the branch cut and upper superscript plus denotes the upper side of the branch cut.

Since the motion of each vortex is tracked through space, the method is lagrangian in nature. However, the velocities are found by solving the equation of the velocity potential on a finite computation region surrounding the vortices at each time station and then interpolated to each vortex location, this solution of the potential equation at each time station is an Eulerian description of the flow, A redistribution method is for transferring vortex strength at the vortex markers from the lagrangian coordinate system to the Eulerian coordinate system. A area weighting method for the transference scheme is used as follows.

$$\begin{cases} \Gamma_1 = (A_1/A)\Gamma_i \\ \Gamma_2 = (A_2/A)\Gamma_i \\ \Gamma_3 = (A_3/A)\Gamma_i \\ \Gamma_4 = (A_4/A)\Gamma_i \end{cases} \tag{13}$$

where A and $A_1,\cdots,A_4$ are the areas defined in Fig 2. This is just a bilinear interpolation approximation that conserves the total circulation. Values of v and w are determined on the Eulerian mesh from the solution of potential equation and then they can be interpolated to the marker locations (y_i, z_i) by using the bilinear interpolation again. The velocities at the marker location y_i, z_i are then given by

$$q_i = \frac{A_1 q_1 + A_2 q_2 + A_3 q_3 + A_4 q_4}{A} \tag{14}$$

where q represents v or w.

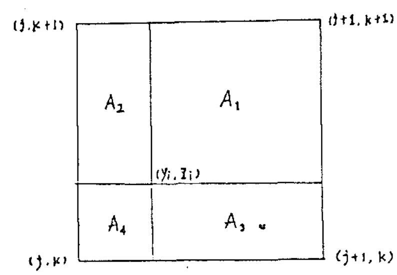

Fig. 2 Redistribution scheme

III. Computation region

The problem considered in the present paper is unsteady, The marker location (y_i, z_i) can be obtained from equation (3)

$$\begin{cases} y_i^{n+1} = y_i^n + \triangle t v_i^n \\ z_i^{n+1} = z_i^n + \triangle t w_i^n \end{cases} \tag{15}$$

The stability condition of the equation was suggested by Baker

$$\triangle t \leqslant \min_{j,k}\left[\frac{\triangle y}{v}, \frac{\triangle z}{w}\right] \tag{16}$$

As the calculation of marker location (y_i, z_i) is an integrate process. the errors due to improper size of the computation region on each trefftz's plane will be accumulated while the calculation is done downstream. The accumulated errors may cause incorrect computation results of the location of vortex marker. So adaption of region size on every trefftz's plane is very important to the wake computation.

The size of computation region is adjusted automatically as following. The maximum distance H between the adjacent local vortex centroids is used as a basic parameter to determine the first approximation of the region, then the truncation error of the derivatives of the potential on the boundary is used to adjusted the size of computation region. The calcalation of the local centroids of vorticity is

$$\begin{cases} \Gamma_m = \sum_{i=(m-1)p+1}^{mp} \Gamma_i \\ \Gamma_m \bar{x}_m = \sum_{i=(m-1)p+1}^{mp} \Gamma_i x_i \\ \Gamma_m \bar{y}_m = \sum_{i=(m-1)p+1}^{mp} \Gamma_i y_i \end{cases} \tag{17}$$

where m = 1, 2,···, M = N / P; P is the number of the vortex markers which composes of a local vortex centroid. The resulting error is small when the boundary is $A_0 \cdot H$ away from the nearest vortex marker (x_i, y_i). $A_0$is a control parameter determined by the truncation error of the velocity on the boundary, The error can be expressed as

$$E_B = \left[\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right]_B \tag{18}$$

where subscript B denotes outer boundary. Given the number of local centroids, the minimum region can be determined without loss of accuracy.

Ⅳ. Computed Examples

Elliptic Span Load

The spanwise circulation distribution of an elliptically loaded wing is

$$\Gamma_w = (1 - y^2)^{1/2} \tag{19}$$

the horizontal and vertical position of the vortex centroid defined by

$$\bar{y} = \sum_{i=1}^{N} y_i \Gamma_i \qquad \bar{z} = \sum_{i=1}^{N} z_i \Gamma_i \tag{20}$$

The results are compared with the results of Stremel and Murman algorithm in Figure 3. the horizontal position of the vortex centroid is 0.78537. It agrees with the classical theory of Betz, The descent speed of the vortex centroid nearly a constant downstream. The velocity is −0.639

Span load for a swept wing

The computation of the vortex sheets for a swept wing is another example. The spanwise lift coefficient distribution of the swept wing is acquired by solving transonic full potential equation. The circulation of the swept wing obtained

$$\Gamma_w = \frac{C_l \cdot U_\infty \cdot C}{2} \tag{21}$$

where C_l is lift coefficient distribution of the wing. U_∞ is the free stream velocity. C is the local chord length. There are two cores for the problem (fig.4) due to the effect of wing root and wing tip.

Ⅴ. Conclusions

An improvement computation method has been developed to compute vortex sheets for time dependent potential flows by "cloud in cell" method. An adaptive region is used and a new expression of second derivatives of potential funtion is introduced. The computation result show that the "hook" at the very tip of the wake for elliptically loaded wing which appeared in other references is eliminated,and the positions of the vortex centroid is of good agreement with classical theory. Finally, it should be pointed that the method has been applied to giving data for conducting formation flight of aircraft.

Ⅵ. Reference

〔1〕 Murman. E.M and Stremel.P.M, "A Vortex Wake Capturing Method for Potential Flow Calculations", AIAA 82 0947

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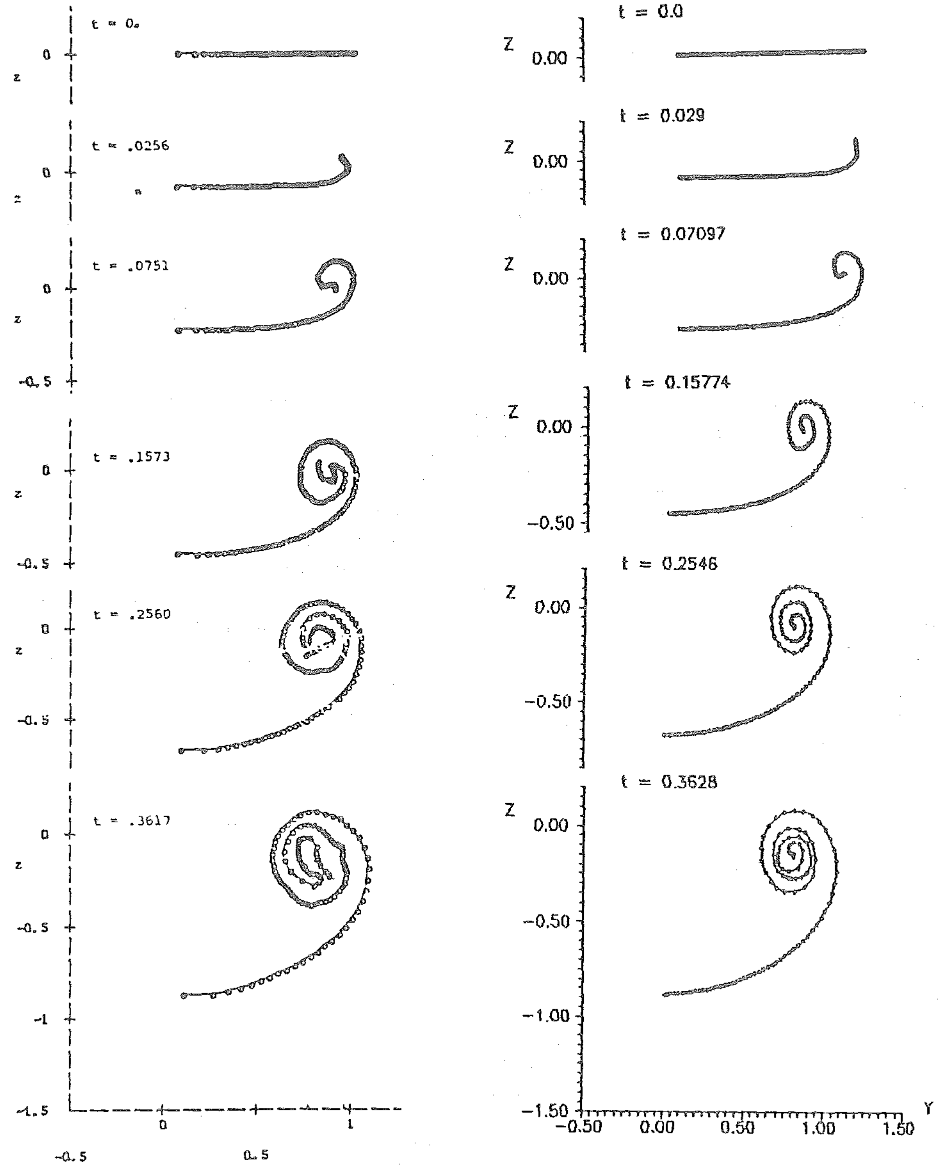

Results of reference (1)　　　　Present result

Fig. 3 Vortex wake geometry for elliptically loaded wing.

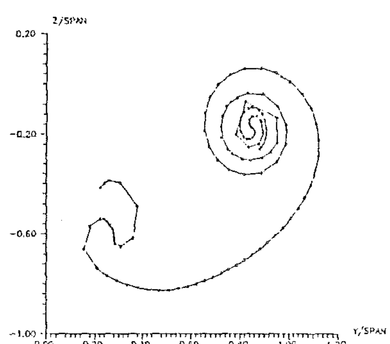

Fig. 4 Vortex wake geometry for a swept wing