COMPARISON AND SELECTION OF SIMPLE MODELS FOR COMPOSITE BEAMS REPRESENTING LIFTING SURFACES

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Abstract

A wide range of technical papers have discussed the subject of analytical models for thin walled composite beams, especially composite box beams. Many of these articles have dealt with the modeling of composite lifting surfaces in various analyses, and present a variety of methods for representing wings and rotor blades as composite beams. These different models provide several different qualitative values for beam properties and responses to loading. A comparison of these models showed these differences and allowed selection of the "best" composite beam model to represent a particular lifting surface application. Additionally, this process of selection demonstrates the relative advantages and disadvantages of the models being examined.

Nomenclature

A	laminate extensional stiffness matrix
c	beam chord
d	beam depth
D	laminate flexural stiffness matrix element
$\mathcal{D}$	effective beam stiffness matrix elements for Austin model
EI	beam flexural stiffness coefficient
F	inverse beam stiffness matrix element
F	applied load vector
G	effective shear modulus
GJ	beam torsional stiffness coefficient
K	beam bend-twist coupling stiffness coefficient
K	stiffness matrix
L	beam length
M	spanwise bending moment
S'	effective shear stiffness coefficient of shear webs for Austin model
t	beam wall thickness
T	chordwise bending torsion
v	horizontal displacement in y-axis direction
w	vertical displacement in z-axis direction

Greek Letters

α	angular displacement about x-axis
α	warping parameter
β	warping parameter
δ	denominator term
ε_{xx}	axial strain
γ_{xy}	shear strain

Subscripts

$(\,)_h$	horizontal box beam panel
$(\,)_v$	vertical box beam panel

Introduction

In view of the many different models presented in existing literature for representing lifting surfaces, selecting a small number of these models for closer examination proves to be prudent. Because of this, four different published models were chosen for comparison; one of these models was simplified by this author for a total of five models to be examined. These models, in general, are from the literature dealing specifically with aerospace lifting surface applications.

To examine these models, a typical box beam needs to be defined. The box beam for this comparison is considered to have a rectangular cross-section, and has vertical and horizontal walls of equal thickness. Figure 1 shows the general box beam dimensions, displacements and coordinate system used.

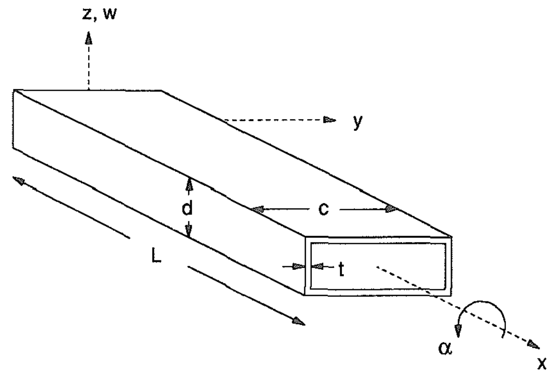

Figure 1. Box Beam with Dimensions and Coordinate Axes Labeled

In any model for a beam-like structure, some form of the force-displacement matrix equation describes the beam response to loading:

$$\{\mathbf{F}\} = [\mathbf{K}]\{x\} \tag{1}$$

The way that this equation is expressed and the stiffness matrix generated provide the distinguishing features between the models. Also, the methods for calculation of strain from the displacements vary slightly between some of the models.

Description of Models for Composite Lifting Surfaces

The five beam models used in this comparison were all selected to represent a lifting surface which has a symmetric laminate construction. The box beam representation is considered to have equal thickness of both the vertical and horizontal components. Four of these beam models are presented in published articles or texts; the fifth beam model is a simplification of one of the published models.

Published Beam Models

The first beam model is taken from Weisshaar and Foist[1]. This model is considered as a High-Aspect-Ratio Plate model, and laminated plate theory directly provides the basis for the model. Following the development in the article for a lifting surface built from symmetric laminates, with the assumption that the chordwise (or lag) bending moment is zero, the relationships between applied moments and curvatures is:

$$\begin{Bmatrix} M \\ T \end{Bmatrix} = \begin{bmatrix} EI & K \\ K & GJ \end{bmatrix} \begin{Bmatrix} w'' \\ \alpha' \end{Bmatrix} \tag{2}$$

In this equation the stiffness matrix elements are found by evaluating the expressions:

$$EI = 2c\left[D_{11_h} + A_{11_h}\left(\frac{d}{2}\right)^2 - \frac{\left\{ D_{12_h} + A_{12_h}\left(\frac{d}{2}\right)^2 \right\}^2}{\left\{ D_{22_h} + A_{22_h}\left(\frac{d}{2}\right)^2 \right\}} \right] \tag{3}$$

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$$K = -4c\left[D_{16_h} + A_{16_h}\left(\frac{d}{2}\right)^2 - \frac{\left\{D_{12_h} + A_{12_h}\left(\frac{d}{2}\right)^2\right\}\left\{D_{26_h} + A_{26_h}\left(\frac{d}{2}\right)^2\right\}}{\left\{D_{22_h} + A_{22_h}\left(\frac{d}{2}\right)^2\right\}}\right] \tag{4}$$

$$GJ = 8c\left[D_{66_h} + A_{66_h}\left(\frac{d}{2}\right)^2 - \frac{\left\{D_{26_h} + A_{26_h}\left(\frac{d}{2}\right)^2\right\}^2}{\left\{D_{22_h} + A_{22_h}\left(\frac{d}{2}\right)^2\right\}}\right] \tag{5}$$

Strains in the horizontal beam panels are examined using the equations:

$$\varepsilon_{xx} = -z\,w'' \tag{6}$$

$$\gamma_{xy} = -2\,\alpha' \tag{7}$$

Unfortunately, the development of this High-Aspect-Ratio Plate model as described in this article, does not account for the presence of shear webs, or vertical walls, that exist in box beams.

The Vinson and Sierakowski[2] text discusses a very simplistic model of a box beam which includes the effects of shear webs and the "parallel axis" effect of the panels of a box beam, but this model lacks bend-twist coupling terms. The displacement-curvature equations for this model are decoupled and are quite simply:

$$M = EI\,w'' \tag{8}$$

$$T = GJ\,\alpha' \tag{9}$$

The stiffness matrix elements for this model are expressed as:

$$EI = 2D_{11_h}c + 2A_{11_h}c\left(\frac{d}{2}\right)^2 + \frac{2A_{11_v}d^3}{12} \tag{10}$$

$$GJ = \frac{c\,d}{2}\left[A_{66_h}d + A_{66_v}c\right] \tag{11}$$

These relatively crude calculations provide for a very computationally simple model for a box beam. Strain is still calculated by equations (6) and (7). This model is the only one of the models not from literature specifically dealing with topics in aerospace.

A recent article by Smith and Chopra[3] offers a more fully developed model for a composite box-beam with a force-displacement matrix equation in all six degrees of freedom. It also includes the effects of warping. For symmetric laminate beam walls, the six equations uncouple to provide two three-by-three matrix equations. In representing lifting surfaces, only the equations relating bending moments to curvatures need to be examined:

$$\begin{Bmatrix} T \\ M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} K_{44} & K_{45} & K_{46} \\ K_{45} & K_{55} & 0 \\ K_{46} & 0 & K_{66} \end{bmatrix} \begin{Bmatrix} \alpha' \\ w''-\gamma_{xz}' \\ v''-\gamma_{xy}' \end{Bmatrix} \tag{12}$$

To develop the stiffness matrix elements for this model, Smith and Chopra introduce the parameters α and β to include warping effects:

$$\beta = -\frac{(1-\alpha)}{(1+\alpha)} \tag{13}$$

$$\text{and } \alpha = \left(\frac{c}{d}\right)\left(\frac{t_v}{t_h}\right)\left(\frac{G_v}{G_h}\right) \tag{14}$$

Using these warping parameters, the stiffness matrix elements are expressed as:

$$\begin{aligned} K_{44} = {} & (1+\beta)^2\left\{2c\,D_{66_h} + 2c\,A_{66_h}\left(\frac{d}{2}\right)^2\right\} + (1-\beta)^2\left\{2d\,D_{66_v} + 2d\,A_{66_v}\left(\frac{c}{2}\right)^2\right\} \\ & - \frac{(1-\beta)^2\left\{2d\,D_{26_v} + 2d\,A_{26_v}\left(\frac{c}{2}\right)^2\right\}^2}{\left\{\frac{4}{3}\left(\frac{c}{2}\right)^3 A_{22_h} + 2d\,D_{22_v} + 2d\,A_{22_v}\left(\frac{c}{2}\right)^2\right\}} \\ & - \frac{(1+\beta)^2\left\{2c\,D_{26_h} + 2c\,A_{26_h}\left(\frac{d}{2}\right)^2\right\}^2}{\left\{\frac{4}{3}\left(\frac{d}{2}\right)^3 A_{22_v} + 2c\,D_{22_h} + 2c\,A_{22_h}\left(\frac{d}{2}\right)^2\right\}} \end{aligned} \tag{15}$$

$$\begin{aligned} K_{45} = {} & (1+\beta)\left\{\frac{4}{3}\left(\frac{d}{2}\right)^3 A_{16_v}\right\} - (1+\beta)\left\{2c\,D_{26_h} + 2c\,A_{26_h}\left(\frac{d}{2}\right)^2\right\} \\ & * \frac{\left\{2c\,D_{12_h} + 2c\,A_{12_h}\left(\frac{d}{2}\right)^2 + \frac{4}{3}\left(\frac{d}{2}\right)^3 A_{12_v}\right\}}{\left\{\frac{4}{3}\left(\frac{d}{2}\right)^3 A_{22_v} + 2c\,D_{22_h} + 2c\,A_{22_h}\left(\frac{d}{2}\right)^2\right\}} \end{aligned} \tag{16}$$

$$\begin{aligned} K_{46} = {} & -(1-\beta)\left\{2d\,D_{16_v} + 2d\,A_{16_v}\left(\frac{c}{2}\right)^2\right\} + (1-\beta)\left\{2d\,D_{26_v} + 2d\,A_{26_v}\left(\frac{c}{2}\right)^2\right\} \\ & * \frac{\left\{2d\,D_{12_v} + 2d\,A_{12_v}\left(\frac{c}{2}\right)^2 + \frac{4}{3}\left(\frac{c}{2}\right)^3 A_{12_h}\right\}}{\left\{\frac{4}{3}\left(\frac{c}{2}\right)^3 A_{22_h} + 2d\,D_{22_v} + 2d\,A_{22_v}\left(\frac{c}{2}\right)^2\right\}} \end{aligned} \tag{17}$$

$$K_{55} = \left\{2c\,D_{11_h} + 2c\,A_{11_h}\left(\frac{d}{2}\right)^2 + \frac{4}{3}\left(\frac{d}{2}\right)^3 A_{11_v}\right\} - \frac{\left\{2c\,D_{12_h} + 2c\,A_{12_h}\left(\frac{d}{2}\right)^2 + \frac{4}{3}\left(\frac{d}{2}\right)^3 A_{12_v}\right\}^2}{\left\{2c\,D_{22_h} + 2c\,A_{22_h}\left(\frac{d}{2}\right)^2 + \frac{4}{3}\left(\frac{d}{2}\right)^3 A_{22_v}\right\}} \tag{18}$$

$$K_{66} = \left\{2d\,D_{11_v} + 2d\,A_{11_v}\left(\frac{c}{2}\right)^2 + \frac{4}{3}\left(\frac{d}{2}\right)^3 A_{11_h}\right\} - \frac{\left\{2d\,D_{12_v} + 2d\,A_{12_v}\left(\frac{c}{2}\right)^2 + \frac{4}{3}\left(\frac{d}{2}\right)^3 A_{12_h}\right\}^2}{\left\{2d\,D_{22_v} + 2d\,A_{22_v}\left(\frac{c}{2}\right)^2 + \frac{4}{3}\left(\frac{d}{2}\right)^3 A_{22_h}\right\}} \tag{19}$$

Evaluating the integral equations given in Reference 3 provides the above expressions.

Calculation of strains in this model also includes the warping parameters:

$$\varepsilon_{xx} = - y\,(v''-\gamma_{xy}') - z\,(w''-\gamma_{xz}') \tag{20}$$

$$\gamma_{xy} = - (1 + \beta)\, z\, \alpha' \tag{21}$$

The next model for examination is a model presented by Austin, et al.[4] This model approximates vertical panels in the box beam as orthotropic shear webs. The equation representing beam response for this model was presented in an inverse form:

$$\begin{Bmatrix} w'' \\ \alpha' \end{Bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{12} & F_{22} \end{bmatrix} \begin{Bmatrix} M \\ T \end{Bmatrix} \tag{22}$$

This inverse form is derived from the three-by-three system:

$$\begin{Bmatrix} M \\ 0 \\ T \end{Bmatrix} = \begin{bmatrix} \mathcal{D}_{11} & \mathcal{D}_{12} & 2\mathcal{D}_{16} \\ \mathcal{D}_{12} & \mathcal{D}_{22} & 2\,\mathcal{D}_{26} \\ 2\,\mathcal{D}_{16} & 2\,\mathcal{D}_{26} & 4\,\mathcal{D}_{66} \end{bmatrix} \begin{Bmatrix} \frac{d^2w}{dx^2} \\ \frac{d^2w}{dy^2} \\ \frac{d^2w}{dxdy} \end{Bmatrix} \tag{23}$$

where the coefficients are:

$$\mathcal{D}_{11} = \frac{c\,d^2}{2} A_{11} \left\{ 1 - \frac{A_{16}^2}{A_{11}\,(S' + A_{66})} \right\} \tag{24}$$

$$\mathcal{D}_{12} = \frac{c\,d^2}{2} A_{12} \left\{ 1 - \frac{A_{16}\,A_{26}}{A_{12}\,(S' + A_{66})} \right\} \tag{25}$$

$$\mathcal{D}_{22} = \frac{c\,d^2}{2} A_{22} \left\{ 1 - \frac{A_{26}^2}{A_{22}\,(S' + A_{66})} \right\} \tag{26}$$

$$\mathcal{D}_{16} = \frac{c\,d^2}{2} A_{16} \left\{ \frac{S'}{S' + A_{66}} \right\} \tag{27}$$

$$\mathcal{D}_{26} = \frac{c\,d^2}{2} A_{26} \left\{ \frac{S'}{S' + A_{66}} \right\} \tag{28}$$

$$\mathcal{D}_{66} = \frac{c\,d^2}{2} A_{66} \left\{ \frac{S'}{S' + A_{66}} \right\} \tag{29}$$

$$\text{with, } S' = \frac{1}{t_w} A_{66_w} \frac{c}{d} \tag{30}$$

Then, inverting and discarding the equation for curvature in the y direction provides equation (22). The stiffness terms in equation (22) are given as:

$$F_{11} = \frac{\mathcal{D}_{22}\,\mathcal{D}_{66} - \mathcal{D}_{26}^2}{\delta} \tag{31}$$

$$F_{12} = \frac{\mathcal{D}_{12}\,\mathcal{D}_{26} - \mathcal{D}_{22}\,\mathcal{D}_{16}}{2\,\delta} \tag{32}$$

$$F_{22} = \frac{\mathcal{D}_{11}\,\mathcal{D}_{22} - \mathcal{D}_{12}^2}{4\,\delta} \tag{33}$$

$$\delta = \mathcal{D}_{11}\,(\mathcal{D}_{22}\,\mathcal{D}_{66} - \mathcal{D}_{26}^2) + \mathcal{D}_{12}\,(\mathcal{D}_{16}\,\mathcal{D}_{26} - \mathcal{D}_{12}\,\mathcal{D}_{66}) + \mathcal{D}_{16}\,(\mathcal{D}_{12}\,\mathcal{D}_{26} - \mathcal{D}_{16}\,\mathcal{D}_{22}) \tag{34}$$

For comparison to the other models, equation (22) can be inverted once again to the form:

$$\begin{Bmatrix} M \\ T \end{Bmatrix} = \begin{bmatrix} EI & K \\ K & GJ \end{bmatrix} \begin{Bmatrix} w'' \\ \alpha' \end{Bmatrix} \tag{35}$$

The common stiffness matrix elements are then represented as:

$$EI = \frac{F_{22}}{(F_{11}\,F_{22} - F_{12}^2)} \tag{36}$$

$$K = \frac{-F_{12}}{(F_{11}\,F_{22} - F_{12}^2)} \tag{37}$$

$$GJ = \frac{F_{11}}{(F_{11}\,F_{22} - F_{12}^2)} \tag{38}$$

These terms will include influence of the three-by-three system given in equation (23).

Simplified Model

Because the Smith and Chopra model is rather complex in its development and implementation, an attempt was made by this author to simplify the six by six matrix equation given by Smith and Chopra. Small quantities are ignored, resulting in a two by two stiffness matrix equation model, which is simple to work with and includes some representation of cross-sectional warping. This simplification produces the following matrix equation which represents the discarding of $(v'' - \gamma_{xy}')$, γ_{xz}', and M_z, from equation (12):

$$\begin{Bmatrix} M \\ T \end{Bmatrix} = \begin{bmatrix} EI & K \\ K & GJ \end{bmatrix} \begin{Bmatrix} w'' \\ \alpha' \end{Bmatrix} \tag{39}$$

The stiffness matrix elements in this model are taken from the Smith and Chopra article, using the same warping parameters:

$$EI = \left\{ 2\,c\,D_{11_h} + 2\,c\,A_{11_h} \left(\frac{d}{2}\right)^2 + \frac{4}{3}\left(\frac{d}{2}\right)^3 A_{11_v} \right\} - \frac{\left\{ 2\,c\,D_{12_h} + 2\,c\,A_{12_h} \left(\frac{d}{2}\right)^2 + \frac{4}{3}\left(\frac{d}{2}\right)^3 A_{12_v} \right\}^2}{\left\{ 2\,c\,D_{22_h} + 2\,c\,A_{22_h} \left(\frac{d}{2}\right)^2 + \frac{4}{3}\left(\frac{d}{2}\right)^3 A_{22_v} \right\}} \tag{40}$$

$$K = (1 + \beta) \left\{ \frac{4}{3}\left(\frac{d}{2}\right)^3 A_{16_v} \right\} - (1 + \beta) \left\{ 2\,c\,D_{26_h} + 2\,c\,A_{26_h} \left(\frac{d}{2}\right)^2 \right\} * \frac{\left\{ 2\,c\,D_{12_h} + 2\,c\,A_{12_h} \left(\frac{d}{2}\right)^2 + \frac{4}{3}\left(\frac{d}{2}\right)^3 A_{12_v} \right\}}{\left\{ \frac{4}{3}\left(\frac{d}{2}\right)^3 A_{22_v} + 2\,c\,D_{22_h} + 2\,c\,A_{22_h} \left(\frac{d}{2}\right)^2 \right\}} \tag{41}$$

$$GJ = (1+\beta)^2\left\{2\,c\,D_{66_h} + 2\,c\,A_{66_h}\left(\frac{d}{2}\right)^2\right\}$$

$$+ (1-\beta)^2\left\{2\,d\,D_{66_v} + 2\,d\,A_{66_v}\left(\frac{c}{2}\right)^2\right\}$$

$$- \frac{(1-\beta)^2\left\{2\,d\,D_{26_v} + 2\,d\,A_{26_v}\left(\frac{c}{2}\right)^2\right\}^2}{\left\{\frac{4}{3}\left(\frac{c}{2}\right)^3 A_{22_h} + 2\,d\,D_{22_v} + 2\,d\,A_{22_v}\left(\frac{c}{2}\right)^2\right\}}$$

$$- \frac{(1+\beta)^2\left\{2\,c\,D_{26_h} + 2\,c\,A_{26_h}\left(\frac{d}{2}\right)^2\right\}^2}{\left\{\frac{4}{3}\left(\frac{d}{2}\right)^3 A_{22_v} + 2\,c\,D_{22_h} + 2\,c\,A_{22_h}\left(\frac{d}{2}\right)^2\right\}} \qquad (42)$$

These values are the same as K_{55}, K_{45}, and K_{44} from the Smith and Chopra article, including the warping terms. The warping terms are then neglected for calculating the strains in the component walls, so equations (6) and (7) are used to calculate strains.

Evaluation of Models

The models were evaluated by examining a series of box beams with varying thickness to chord ratios and aspect ratios. Symmetric laminate stacking patterns were used to examine the different beam model responses to bending and/or torsional loading. Using the properties for an AS/3501 graphite-epoxy system given in Reference 2, the stiffness matrix was generated for each beam model, and the displacement response was found for a lift-type loading condition.

Box Beams for Evaluation

To provide a wide range of comparison, 18 different box beam dimensions were evaluated. All of these beams were created with the idea that each must have a "wing area" of 3000 square inches, and carry a lift load equivalent to 1000 pounds. A line load applied along the quarter chord of the beam represents this loading condition. Aspect ratios of the beams vary from 3 to 18, and thickness-to-chord ratios range from 0.10 to 0.20. The wide range of beam dimensions reflects the range from typical wing dimensions to typical rotor blade dimensions. Table 1 provides a summary of these beam dimensions.

Table 1. Beam Dimensions Used for Comparison

AR = *L/c*	*t/c*	*L (in.)*	*c (in.)*	*d (in.)*
3	0.10	94.86832	31.62277	3.162277
3	0.15	94.86832	31.62277	4.743416
3	0.20	94.86832	31.62277	6.324555
6	0.10	134.1640	22.36067	2.236067
6	0.15	134.1640	22.36067	3.354101
6	0.20	134.1640	22.36067	4.472135
9	0.10	164.3167	18.25741	1.825741
9	0.15	164.3167	18.25741	2.738612
9	0.20	164.3167	18.25741	3.651483
12	0.10	189.7366	15.81138	1.581138
12	0.15	189.7366	15.81138	2.371708
12	0.20	189.7366	15.81138	3.162277
15	0.10	212.1320	14.14213	1.414213
15	0.15	212.1320	14.14213	2.121320
15	0.20	212.1320	14.14213	2.828427
18	0.10	232.3790	12.90994	1.290994
18	0.15	232.3790	12.90994	1.936491
18	0.20	232.3790	12.90994	2.581988

Computer Code

To evaluate these beams, a FORTRAN code was used. This code calculates the stiffness matrix elements, root curvatures and strain in the upper surface of the beam at the root from the beam dimensions.

Comparison and Selection

Comparison of the beams was carried out using a specific decision making process. This process accounts for conservatism, realism and simplicity. All of the beam models begin the process; a model is eliminated from the process when it does not meet a decision criterion. The flow chart for this process is shown in Figure 2.

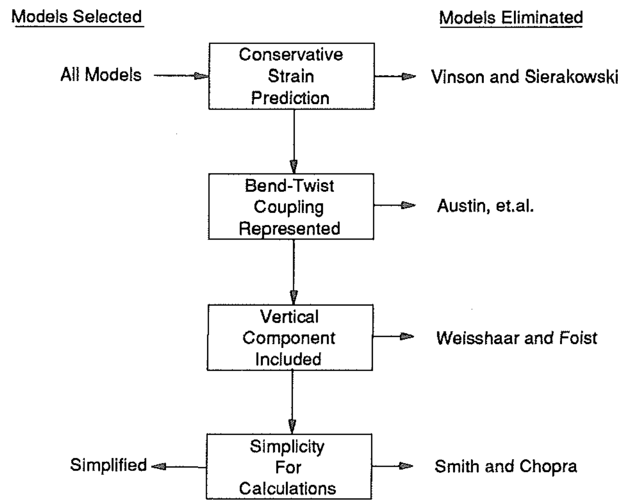

Figure 2. Process for Selecting a Box Beam Model

Conservative Strain Prediction

A reasonable beam model would preferably predict a slightly higher value of strain than would actually exist in the beam under the loading condition. This provides a safe, conservative estimate of loading; therefore, a beam designed using a conservative model would not fail.

All five models were evaluated for each of the different beam dimensions for maximum strain in the top of the beam at the root. Both the in-plane axial strain ε_{xx} and the in-plane shear strain γ_{xy} are considered. Comparison of these values shows that the beam model of Vinson and Sierakowski predicts significantly lower values of strain than the other four models, for all of the cases examined. This lower strain value prediction becomes especially noticeable in the strain for beams of lower thickness to chord ratios, regardless of the aspect ratio. Figure 3 shows a composite of four plots which display a sample of the lower strain predictions given by the Vinson and Sierakowski model. The other four beam models give reasonably similar results, therefore the Vinson and Sierakowski model is deemed inappropriate for modeling a lifting surface due to this conservatism criterion.

Bend-Twist Coupling

Most anisotropic material structures demonstrate the presence of bending-torsion coupling, and this coupling provides the first realism measure for comparison. This coupling contribution is generally small, but for this comparison, it becomes important because a lift load on the beam produces both a spanwise bending moment and a chordwise torsion. All of the remaining four box beam models, with the exception of the Austin model, exhibit bend-twist coupling. In the Austin model, bend-twist coupling is represented in the stiffness matrix by the element K. Looking to equations (37), (32), (28), and (27), it can be seen that to represent bend-twist coupling in the Austin model extension-shear coupling in the laminate extension matrix needs to exist. Because the parameters of this comparison specify the beam to be made of balanced, symmetric laminates, no laminate extension-shear

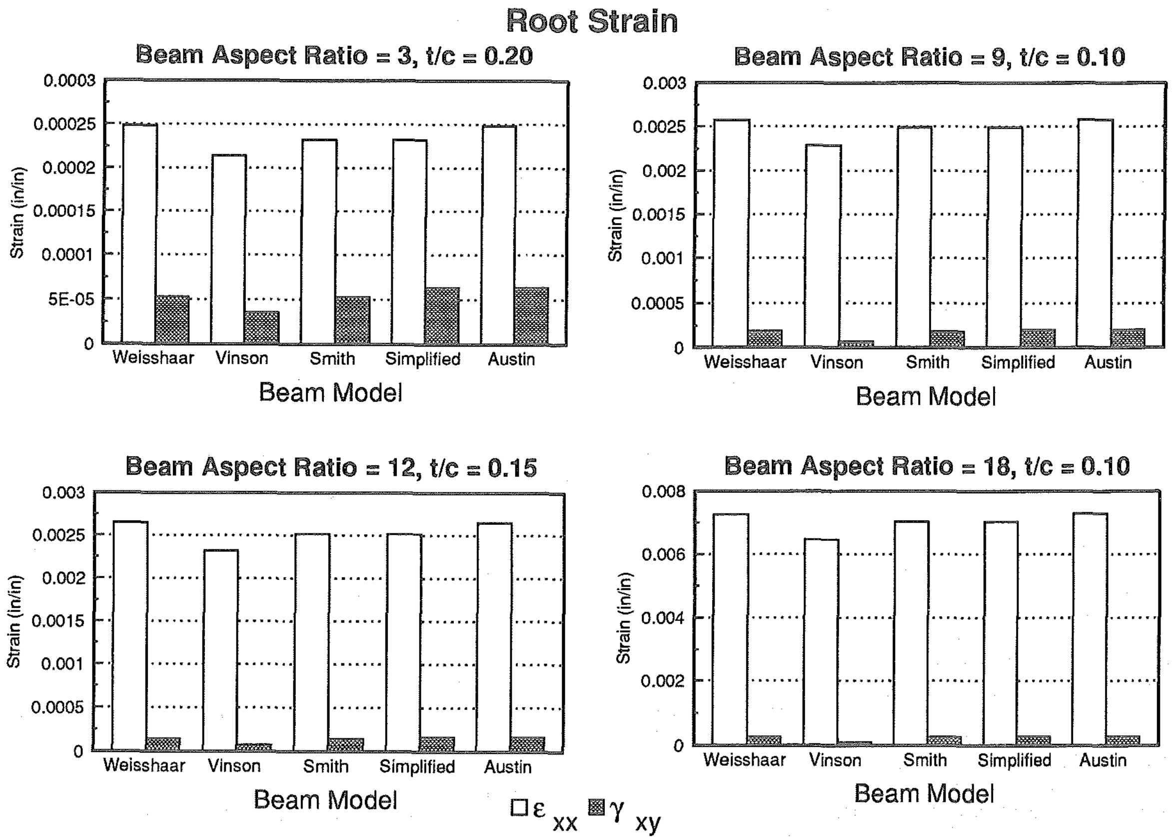

Figure 3. Values of Strain Calculated at the Beam Root for Various Aspect Ratio and Thickness-to-Chord Ratio Beams

coupling exists; so in the Austin model, no bending-torsion coupling exists. This eliminates the Austin model from the comparison process.

Vertical Components

Another realism criterion and an important contributor to the stiffness of a box beam is the inclusion of the vertical components, which affects flexural and torsional stiffness. The range of thickness to chord ratios examined in this study vary from 0.10 to 0.20. At the low end of this range, vertical components will be small and have little effect on stiffnesses; yet at the upper end of the range, the vertical components become more significant. The article by Weisshaar and Foist[1] states about the High-Aspect-Ratio Plate model: "(It) has deficiencies because (it cannot) account for flexible transverse shear webs as are present in box beam design. This is a potentially important deficiency because of the wide-spread use of thin-wall, single- and multicell torque boxes in airplane design." This deficiency is enough to disqualify this model from further consideration in this comparison. Figure 4 displays the difference in the values of torsional and flexural stiffnesses between the Weisshaar and Foist model and the Smith and Chopra model. The stiffness differences are found by:

$$\Delta EI = | EI_{Weisshaar} - EI_{Smith}| \tag{43}$$

$$\Delta GJ = | GJ_{Weisshaar} - GJ_{Smith}| \tag{44}$$

This figure shows that for certain cases, this difference may not be very significant, but at high aspect ratios and high thickness to chord ratios the difference between models is quite significant. It is important to note that in Figure 4, only two models are compared, this is because the flexural and torsional stiffnesses of the simplified model are exactly those of the Smith and Chopra model.

Simplicity

The last criteria for comparison in this case was simplicity. The idea of simplicity drove this author to modify the Smith and Chopra model, and it would follow that this simplified model would be the logical choice to represent the lifting surface. The effect that this simplification has on the beam model deserves discussion. As previously stated, both of these remaining models have basically the same stiffness matrix elements, with the exception that the Smith and Chopra model includes all six stiffness terms. The more complex system of equations does not provide for a more accurate solution for strains in the beams examined in this comparison. The v" term provides a negligible contribution in calculation of the strain in the upper panel of the box beam. The most significant difference here comes from ignoring the warping parameter β in the calculation of the shear strains in the simplified model. The simple expressions of equations (6) and (7) are used to evaluate strains. The axial strain, ε_{xx}, does not appreciably vary between the two models. While the numerical variation of shear strain is noticeable, this difference is also insignificant when comparing the magnitude of the axial and shear strains. It can be seen in Figure 5 that the effect of simplifying the model has little overall effect on the beam response. The plots of shear strain for both remaining models are essentially the same; in fact, the simplified model is slightly more conservative.

Additionally, this model reduces computational time for the curvature calculations due to its size. Solution of a two-by-two system of equations requires less than half as many operations as needed to solve the three-by-three system of equations. For cases where many solutions of this matrix are necessary, as in highly iterative design processes, computational time can be saved.

Discussion and Conclusions

For the specific case examined in this paper, a box beam of symmetric laminates and equal wall thicknesses subject to a lift-like loading, the simplification of the Smith and Chopra model is the best choice to represent the lifting surface for the range of beam dimensions examined. This model appears to be the best overall

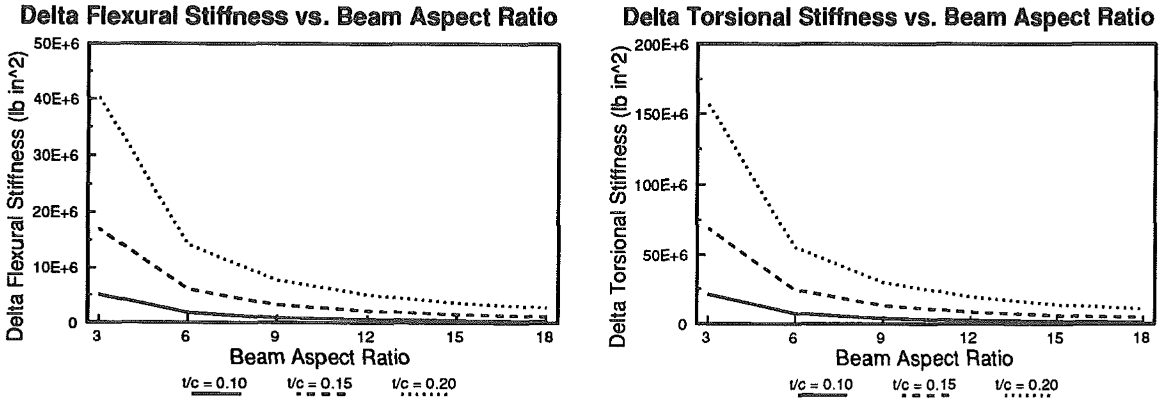

Figure 4. Difference in Flexural and Torsional Stiffnesses Between the Weisshaar and Foist[1] Model and the Smith and Chopra[3] Model

model, but it is evident from the comparison that other models can be used for other cases and may in fact be better choices for those other cases.

For cases where thin wings are being examined, the Weisshaar and Foist model can be used, as the effects of vertical components become relatively unimportant. This would especially be true for thin wings with relatively high aspect ratios, which is implied by the model's label as the "High-Aspect-Ratio Plate Model".

When a highly accurate representation of the beam response is needed, the Smith and Chopra model supplies the most complete representation of a composite box-beam. This model includes bend-twist coupling in two directions, and has a reasonable representation of the effects of beam cross-section warping. If only a few solutions of the moment-curvature relationship are required, this becomes a desirable model.

In this paper, asymmetric laminates were not examined. If a beam of asymmetric laminate construction were to be analyzed, the Austin model would then include terms representing bend-twist coupling, and could prove to be a viable model for use. The introduction of asymmetric laminates also adds additional coupling terms in the laminate stiffness matrices; this will effect all of the models, and a further examination may be required to select a model for beams of asymmetric laminate construction.

Finally, if only a rough order-of-magnitude analysis is required, the simple Vinson and Sierakowski model proves to be the easiest model to develop and implement. It is the epitome of a simple composite box beam model.

This study reveals that for most general lifting surfaces constructed from symmetric laminate panels, the simplification of the Smith and Chopra model is the appropriate model to use. The other models examined have their own advantages, and they may be used when appropriate.

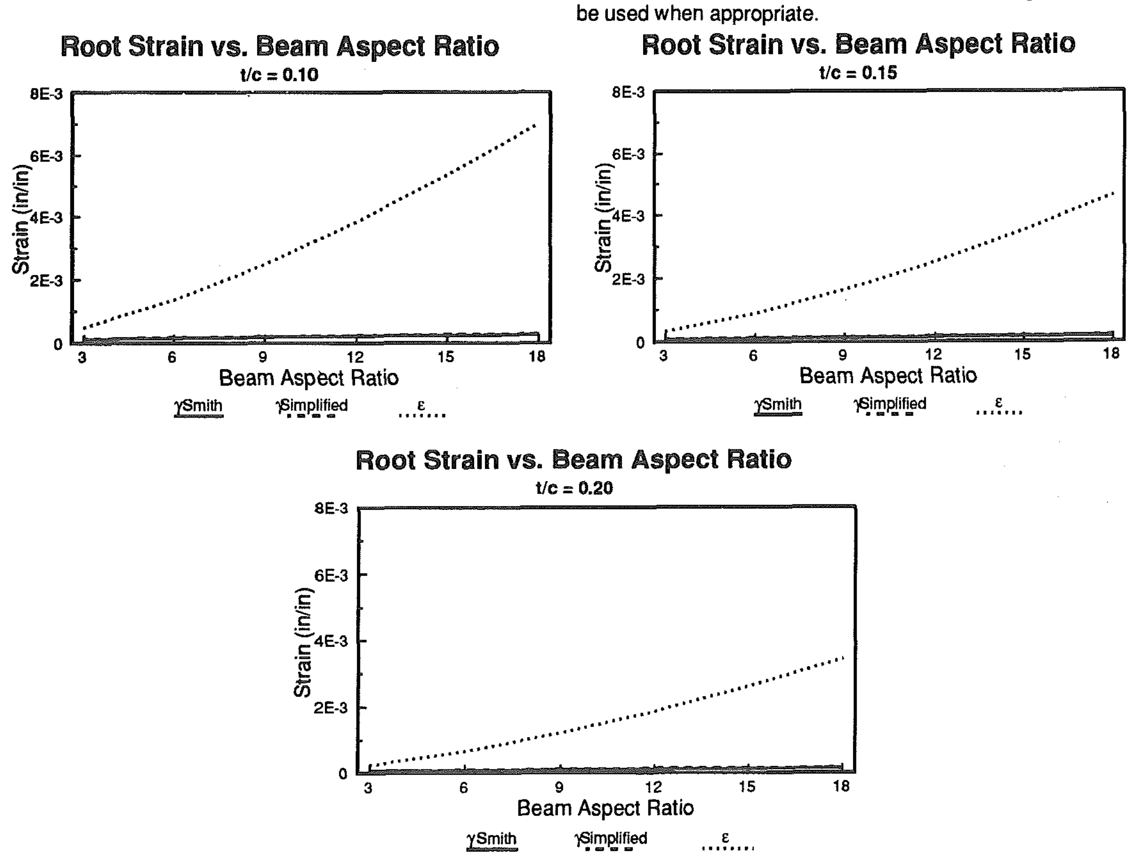

Figure 5. Comparison of Strain Predictions at the Blade Root

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