ICAS-92-6.5.1

Main Landing Gear Plastic Limit Analysis for Fighter Aircraft

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Abstract

This thesis is an analysis on plastic limit of the outer tube of some fighter aircraft main landing gear structures. The outer tube is simplified as a cylinder shell here, upon which the combined action of a bending moment and an internal pressure is imposed. By analysing the relationship between $M_x$and $N_x$in the yield surface, the yield condition is derived, and the static solutions to the plastic limit load and two final damage forms of the shell are obtained. This method of design can greatly reduce the structural weight of the landing gear, and increase its reliability.

I. Introduction

In using engineering beam method to design a structure, the safety rule is $\sigma \leqslant \sigma_s / n$($\sigma_s$is the yield condition, n is the safety factor). It is inadequate, however, to use this rule to determine the strength of a structure, because when a structure develops a certain yield state somewhere, it does not necessarily mean the total loss of its load–bearing capability; a structure will fail only when yield region expands beyond a certain scope. This means that the load–bearing capacity of a plastic material structure far surpasses the scope as limited by the elastic limit. To make full use of the strength potential, further study is needed about the influence of the material plastic property upon the load–bearing capacity of a landing gear structure.

The outer tube of an aircraft landing gear is the main load–bearing structural member, and is the key part of a landing gear structure. Therefore, it is of significant practicability to conduct a study of the outer tube. This thesis presents a study of plastic limit load of the outer tube with the plastic limit analysis method, which can truly describe the safety degree of structures, make full use of the material load–bearing potential and reduce the structural weight by a large margin, thus enhancing aircraft technical performances and combat capability.

II. Mechanical Model

The mechanical model of the outer tube can be simplified as a cylinder shell, upon which the combined action of a non–axisymmetric moment and an axisymmetric internal pressue is imposed. Then the part of the outer tube filled with gas is simplified as a model with one side free and the other joint–supported; the free side is sealed rigidly, and a concentrated moment M_1acts on this side, a distributing moment $\overline{m}_2$ acts on the outer surface and a uniform pressure p acts on the inside(See Fig.1).

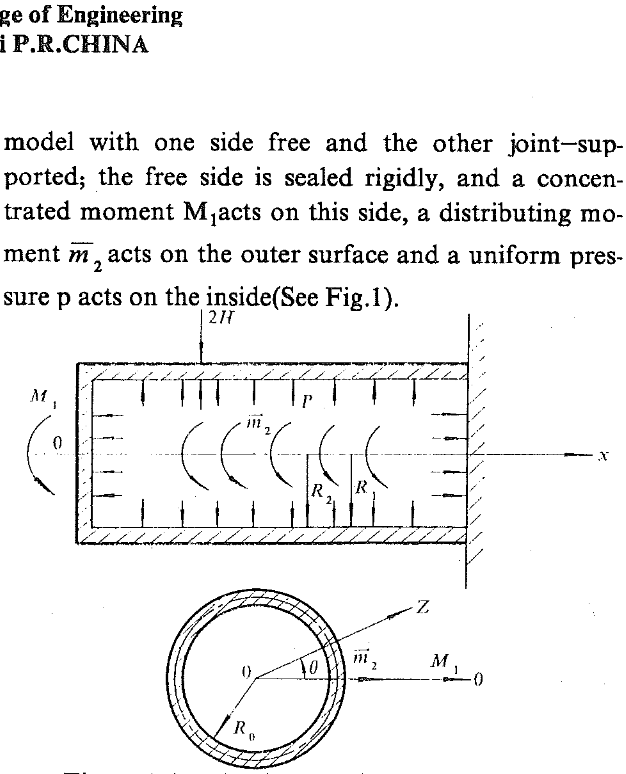

Figure 1. Mechanical Model of Outer Tube

The length of the shell is L and the thickness is 2H, the inside and outside radius are $R_1$and R_2respectively, the distance from the shell axis to the middle surface is R_0.

A cylindrical coordinate system is used: x for horizontal ordinate, z for radial ordinate and $\theta$for ring ordinate(see Fig.1), the origin is at the free side, θ is positive anti–clockwise.

The displacement rates about θ、 x、 z are $\dot{U}$、 $\dot{V}$、 $\dot{W}$respectively.

III. Basic Hypotheses

Basic hypotheses in the limit analysis are :

III.1 Materials are ideally rigid–plastic.

III.2 The deforming of a structure is so small that there is no need to consider the change of the geometric dimension.

III.3 A structure will not lose its stability before it reaches its load limit value.

III.4 The load is increased at a constant proportion.

III.5 The stress along the shell's thick direction is very small and can be ignored (that is $\sigma_\theta \approx 0$).

III.6 Materials cannot be compressed, the poisson rate μ is 0.5.

Ⅳ. Basic Equation

The stress state of the axisymmetric shell is determined by 9 generalized stresses; the film forces are N_θ and N_x; the moments are M_θ and M_x. The generalized strain rates and the curvature rates corresponding to the generalized stresses are $\dot{\varepsilon}_\theta$、$\dot{\varepsilon}_x$ and $\dot{K}_\theta$、$\dot{K}_x$ respectively.

A micro–element is taken from the shell as shown in Fig.2. Assume that the limit film stress is N_p, the limit moment is M_p. $N_p=2\sigma_s H$, $M_p=\sigma_s H^2$.

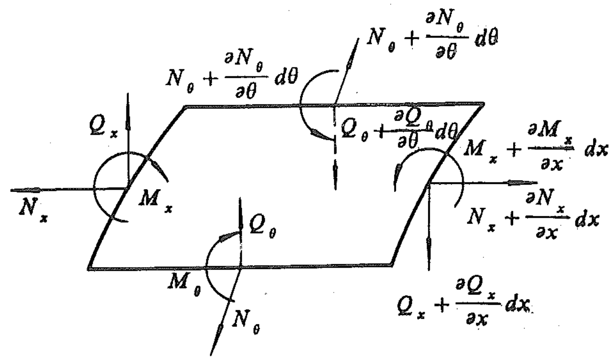

Figure 2. A Micro–element

Now take some non–dimensional values as follows (the characteristic length is R_0):

$$n_\theta=\frac{N_\theta}{N_p},\quad n_x=\frac{N_x}{N_p},\quad m_\theta=\frac{M_\theta}{M_p},\quad m_x=\frac{M_x}{M_p}$$

$$\dot{k}_\theta=\frac{M_p}{N_p}\dot{K}_\theta,\quad \dot{k}_x=\frac{M_x}{N_p}\dot{K}_x,\quad x=\frac{z}{L},\quad \dot{v}=\frac{\dot{V}}{R_0}$$

$$\dot{w}=\frac{\dot{W}}{R_0},\quad p=\frac{PR_0}{N_p},\quad m_1=\frac{M_1}{4N_pR_0^2},\quad m_2=\frac{\vec{m}_2}{4N_pR_0}$$

Ⅳ.1 Balance Equation

Let $ds=R_0d\theta$, the balance equations are simplified and non–dimensionalized:

$$\begin{cases}\dfrac{\partial n_x}{\partial x}=0\\[2mm] \dfrac{\partial n_\theta}{\partial\theta}-\dfrac{Q_\theta}{N_p}=0\\[2mm] \dfrac{R_0}{LN_p}\dfrac{\partial Q_x}{\partial x}+\dfrac{1}{N_p}\dfrac{\partial Q_\theta}{\partial\theta}+n_\theta-p=0\\[2mm] \dfrac{H}{2R_0}\dfrac{\partial m_\theta}{\partial\theta}-\dfrac{Q_\theta}{N_p}=0\\[2mm] \dfrac{H}{2L}\dfrac{\partial m_x}{\partial x}-\dfrac{Q_x}{N_p}=0\end{cases}\qquad(1)$$

Ⅳ.2 Geometric Equation

The relationship [4] of the shell middle surface is as follows:

$$\begin{cases}\dot{\varepsilon}_x=\dfrac{R_0}{L}\dfrac{\partial\dot{v}}{\partial x}\\[2mm] \varepsilon_\theta=\dot{w}\\[2mm] \dot{k}_x=-\dfrac{HR_0}{2L^2}\dfrac{\partial^2\dot{w}}{\partial x^2}\\[2mm] \dot{k}_\theta=-\dfrac{H}{2R_0}\dfrac{\partial^2\dot{w}}{\partial\theta^2}\end{cases}\qquad(2)$$

Ⅳ.3 Flow Law

By Drucker's Hardening postulate,the flow law can be written as follows:

$$\begin{cases}\dot{\varepsilon}_x=\lambda\dfrac{\partial f}{\partial n_x}\\[2mm] \dot{\varepsilon}_\theta=\lambda\dfrac{\partial f}{\partial n_\theta}\\[2mm] \dot{k}_x=\lambda\dfrac{\partial f}{\partial m_x}\\[2mm] \dot{k}_\theta=\lambda\dfrac{\partial f}{\partial m_\theta}\end{cases}\qquad(3)$$

where f is the yeild function of the shell, λ is the proportion factor.

Ⅴ. Yield Condition

If both sides of the uniform shell under internal pressure are not closed, then the yield condition can be

$$n_\theta=1\qquad(4)$$

obtained as :Assuming that a plastic joint is formed at section x_0 of the shell, the joint can move along the x axis with the variations of M_1、$\vec{m}_2$ and p, and the yield condition of the yield section x_0 will be determined by m_x and n_x together.

If the section of the micro–element in x direction yields, its stress distribution can be described as in Fig.3, where t is the length factor, and $-H\leqslant t\leqslant H$, then :

$$\begin{cases}n_x=-\dfrac{t}{H}\\[2mm] m_x=1-\dfrac{t^2}{H^2}\end{cases}\qquad(5)$$

from(5): $m_x=1-n_x^2$

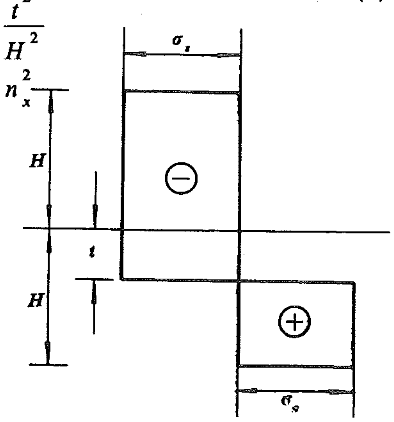

Figure3.Stress Distribution in x Direction Section

When the outer–layer is pulled and the in–layer is pressed, m_x is taken as a negative value. Then the yield condition at the yield section x_0 is:

$$\pm m_x = 1 - n_x^2 \qquad (6)$$

Ⅵ. Static Solution

For the stress state of the cylinder shell studied in this thesis, the load influences on the general stresses are that the internal pressure p will cause moment m_x、m_θ and flim force n_θ、n_x; the single force couple M_1 and the distribution moment $\bar{m}_2$ will cause the axial force n_x. Then the balance equation along x direction discussed in this thesis can be obtained as the sum of the balance equation of a cylinder shell and the axial force n_x as caused by $\bar{m}_2$. Now, two cases will be discussed as follows.

Ⅵ.1 The internal pressure p takes the main effect, and M_1、$\bar{m}_2$ have little influence.

Assuming that the shell yields at the section $x = x_0$, and n_x as caused by $\bar{m}_2$ varies uniformly along axial x, the distribution of n_x is irrelative to θ, and n_x is a constant at limit state (see Fig.4).

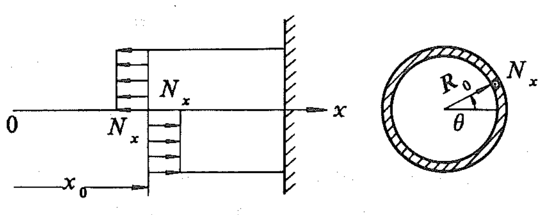

Figure4.N_x Distribution in x_0 Section at Limit State

When $0<\theta<\pi$, the general force is $+N_x$;

$\pi<\theta<2\pi$, the general force is $-N_x$;

From the moment balance, we obtain:

$$M_x + \bar{m}_2 x = 4\int_0^\pi N_x R_0 \sin\theta R_0 d\theta$$

Non–dimensionalize the formula, then we have:

$$n_x = m_1 + m_2 \frac{L}{R} x \qquad (7)$$

The balance eqation in x direction is

$$\frac{\partial n_x}{\partial x} \pm m_2 \frac{L}{R_0} = 0 \qquad (8)$$

When $0<\theta<\pi$, equation (8) is adopted as a positive sign; when $\pi<\theta<2\pi$, it is adopted as a negative sign.

From balance equation (1), we obtain:

$$\frac{R_0 H}{2L^2}\frac{\partial^2 m_x}{\partial x^2} + \frac{H}{2R_0}\frac{\partial^2 m_\theta}{\partial x^2} + n_\theta - p = 0 \qquad (9)$$

$$\frac{H}{2R_0}\frac{\partial m_\theta}{\partial \theta} - \frac{\partial n_\theta}{\partial \theta} = 0 \qquad (10)$$

Substiute (4)、(10) into (9), thus

$$m_x = -\frac{L^2}{R_0 H}(1-p)x^2 + c_1 x + c_2 \qquad (11)$$

where c_1、c_2 are undetermined coefficients. By boundary condition, the solution of equation (11) can be written as

$$m_x = -\frac{L^2}{R_0 H}(1-p)x^2 + [\frac{L^2}{R_0 H}(1-p) - 2m_1 m_2 \frac{L}{R_0} - m_2^2 \frac{L^2}{R_0^2}]x + 1 - m_1^2 \qquad (12)$$

Since the internal pressure takes the main effect, the main cause for lending yield is that the m_x reaches its maximum value.

From $\frac{\partial m_x}{\partial x} = 0$, we can obtain

$$x_0 = \frac{1}{2} - \frac{Hm_1 m_2}{L(1-p)} - \frac{Hm_2^2}{2R_0(1-p)} \qquad (13)$$

Substitute (7)、(12)、(13) into (6), and simplify the formula, then the static force solution is expressed as

$$[-\frac{L^2}{R_0 H}(1-p) + m_2^2\frac{L^2}{R_0^2}][\frac{1}{2} - \frac{Hm_1 m_2}{L(1-p)} - \frac{Hm_2^2}{2R_0(1-p)}] + \frac{L^2}{R_0 H}(1-p) - m_2^2\frac{L^2}{R_0^2} = 0 \qquad (14)$$

Ⅵ.2 The internal pressure is small and the moment M_1 and $\bar{m}_2$ are large

If the internal pressure is small and the moment M_1 and $\bar{m}_2$ are large, then the film force N_x as caused by M_1 and $\bar{m}_2$ will be the main cause for lending yield.

Clearly, moment $M_1 + \bar{m}_2 L$ reaches the maximum value at x = 1, so the shell will yield here first. According to the yield condition, n_x can reach the maximum value only when $m_x = 0$ and $N_x = N_p$, then, yield will occur at x = 1, and a plastic joint be formed there. From (7),we obtain the static force solution:

$$m_1 + m_2 \frac{L}{R_0} = 1 \qquad (15)$$

When x = 1, $m_x = 0$ and when $0 < x < 1$, $m_x + n_x^2 < 1$, then

$$\begin{cases} p\leqslant 1-m_2^2\dfrac{H}{R_0} \\ p\leqslant 1-\dfrac{HR_0}{L^2}(1-m_1^2) \end{cases} \quad (16)$$

This is the applicabe range of the static solution (15). The static solution that cannot satisfy the above formulae is just like (14).

Ⅶ. Shell Limit Damage

The static solution of formula (14) is affected mainly by the internal pressure, then the yield condition is $n_\theta=1$. From the third formula of the flow law (3), we can obtain $\dot{R}_x=0$, then substitute it into the third formula of the equation (2) : $\dot{w}=D_1x+D_2$

where D_1 and D_2 are undetermined coefficients.

According to boundary conditions of the displacement, it goes that:

when $x=0$, $\dot{w}=0$, then $D_2=0$

$x=x_0$, $\dot{w}=\dot{w}_0$, then $D_1=\dfrac{\dot{w}_0}{x_0}$

$\therefore$ when $0\leqslant x\leqslant x_0$, $\dot{w}=\dfrac{\dot{w}_0}{x_0}x$

and $x=1$, $\dot{w}=0$, then $D_2=-D_1$

$x=x_0$, $\dot{w}=\dot{w}_0$, then $D_1=\dfrac{\dot{w}_0}{1-x_0}$

$\therefore$ when $x_0\leqslant x\leqslant 1$, $\dot{w}=\dfrac{\dot{w}_0}{1-x_0}(1-x)$

Now we obtain the damage form of the static solution of equation (14) as shown in Fig.5.

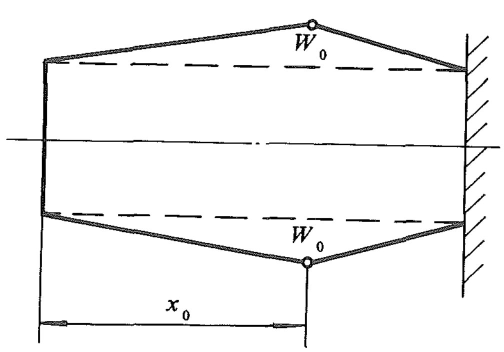

Figure 5. Damage Form I

For the static solution of the equation (15), yielding is mainly effected by $M_1$and $\overline{m}_2$, so it occurs at $x=1$, where $M_1+\overline{m}_2L$ reaches the maximum value, and plastic joint is formed there. The damage form is shown in Fig.6.

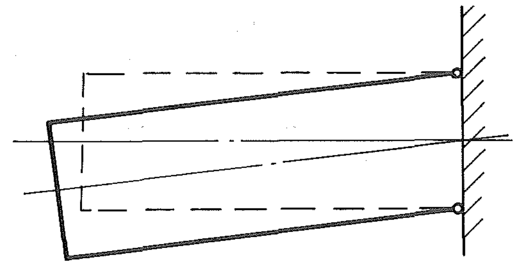

Figure 6. Damage Form II

Ⅷ. Example

In this thesis, plastic limit is calculated for the outer tube of some fighter aircraft main landing gear under operating mode (E_R+G_R).

The loading factor of the calculated result is $\lambda=1.38$. λ is the ratio of the plastic limit load to the using load, which indicates that the using load can be increased by as much as 38% before the plastic limit is reached.

Ⅸ. Conclusion

Ⅸ.1 The plastic limit load is the real failure load for the main landing gear, which is similar to the designed load. The safety factor f (f=design load / using load) as described in a strength report for some fighter aircraft is 1.65, which can be reduced to 1.38 on the basis of limit analysis as presented in this thesis.

Under definite safety conditions (with the same using load), the smaller the safety factor is, the lighter the structural weight will be.The structural weight as determined through the plastic limit analysis is much lighter than that of the original structure, so that economy is achieved for raw material consumption, and aircraft maneuverability can be improved.

If the designed load is a fixed one, then the using load can be raised properly, for the limit load is larger than the action load, thus improving the aircraft landing property.

Ⅸ .2 This thesis obtains two kinds of damage forms under limit state condition, both of which satisfy the law of plastic deformation, and provide theoretical basis for engineering design and aircraft maintenance.

Ⅸ .3 The plastic limit analysis is a study of real damage load for a structure, which is different from the strength limit of materials.It can describe more exactly the real safety condition of an aircraft landing gear, and is helpful in improving the structural reliability. Clearly, it is an advanced design idea, which can be used not only for aircraft landing gear design, but also for designs of other structures.

Ⅸ.4 This thesis is the first example of applying plastic limit analysis to aviation engineering in China. It offers a new method for the strength analysis of aircraft structure, and makes up for the weakness of the design based on elastic analysis only. It presents a new approach for the application of plastic limit analysis to aviation engineering.

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