DYNAMIC STABILITY,COUPLING AND ACTIVE CONTROL OF ELASTIC VEHICLES WITH UNSTEADY AERODYNAMIC FORCES MODELING**

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Abstract

In this paper,longitudinal equations of disturbance motions acting with unsteady aerodynamic forces are derived. A method of analysis of the effect of aeroelasticity on the dynamic stability of elastic vehicles by using a simplified mathematical model of unsteady aerodynamic forces is proposed. Synthesis of optimal design of the active control system is studied from the point of view for coordinations of design of active feedback control,choose of positions of control surfaces and sensors(gyros) locations from the stand point for the efficient suppresion of elastic vibration of the vehicle. Moreover,method of quantitative analysis of the coupling characteristics of elastic vehicles is developed.

I. Introduction

For studying the stability problems of elastic vehicles, some literatures (1),(2) consider the elastic vehicle as a combination of a rigid vehicle and elastic modes link. This kind of model neglects the aerody-namic coupling effect between the rigid vehicle and elastic modes. Other literatures such as (3),(4) consider the aerodynamic coupling, but the mathematical model is based on steady motion with consideration of steady aerodynamic forces only.

Maneuvering flight, transient motion during actuation of control surfaces,and flight in gust wind of vehicles all belong to unsteady flight conditions. In these cases, aerodynamic forces acting on vehicles should be considered as unsteady aerodynamic forces,which depend not only on the instantaneous values of motion parameters,but also on their time history of variations. For elastic vehicles,aerodynamic forces due to elastic vibrations of vehicles are also unsteady in characters, therefore unsteady aerodynamic forces must be well considered therein.

So far didn't exist very appropriate mathematical model for the consideration of unsteady aerodynamic forces for the analysis of dynamic characteristics of the unsteady motion of elastic vehicles. In this paper,we utilize the concept of the "indicial function" to establish practical mathematical model for the consideration of unsteady aerodynamic forces. Thus, by using Duhamel's integral,the incremental lift,pitching moment,and generalized force can be obtained with the help of indicial functions corresponding to unit step increases of angle of attack, angular pitching velocity and modes of elastic vibrations of vehicles. Then the longitudinal equations of disturbance motions with unsteady aerodynamic forces and n-orders of elastic modes considered can be expressed in the form of the integral-differential equations.

Thus far very few literatures concerned with the coordinative design of active feedback control and choose of positions of control actuators and sensors(gyros). In this paper, synthesis of the optimal design of active control system is discussed from the point of view for coordinations of design of the active feedback control,choose of positions of control surfaces and of sensors(gyros) from the stand point for efficient suppression of elastic vibrations of the vehicle. Furthermore, sensitivity factor is introduced in the index function, so that robustness of the active control system to variations of flight parameters well be improved.

In order to investigate the dynamic behavior of vehicle in view of coupling effects, a method is developed for quantitative analysis of the coupling characteristics of elastic vehicles.

II. Equations of Unsteady Disturbance Motion of Elastic Vehicles

In general,the elastic vibrations of flight vehicles consist of bending of wing, torsion of wing, and bending of vehicle body along the longitudinal axis.

The equation for elastic bending vibration of wing may be expressed as:

$$L_1(y,z,\varphi,t)=\frac{\partial^2}{\partial z^2}\left(EJ\frac{\partial^2 y(z,t)}{\partial z^2}\right)+m(z)\frac{\partial^2 y(z,t)}{\partial t^2}$$
$$-m(z)\sigma(z)\frac{\partial^2\varphi(z,t)}{\partial t^2}-Y(z,t)-F_g(z,t)=0 \tag{2.1}$$

For elastic torsion of wing:

$$L_2(y,z,\varphi,t)=-\frac{\partial}{\partial z}\left(GJ_p\frac{\partial\varphi(z,t)}{\partial z}\right)+J(z)\frac{\partial^2\varphi(z,t)}{\partial t^2}$$
$$-m(z)\sigma(z)\frac{\partial^2 y(z,t)}{\partial t^2}-M(z,t)-M_g(z,t)=0 \tag{2.2}$$

For elastic bending vibration of vehicle body:

$$L_3(\xi,x,t)=\frac{\partial^2}{\partial x^2}\left(EJ_\phi\frac{\partial^2\xi(x,t)}{\partial x^2}\right)+m_\phi(x)\frac{\partial^2\xi(x,t)}{\partial t^2}-Y(x,t)-F_g(x,t)$$
$$-2\delta(x-x_{CT})Y_{CT}(t)-\delta(x-x_\delta)\cdot Y_\delta(t)=0 \tag{2.3}$$

In these equations,$y(z,t)$, $\varphi(z,t)$, $\xi(x,t)$ show bending deflection, torsion of wing,and elastic bending deflection of vehicle body along the longitudinal axis; $Y(z,t)$,$M(z,t)$,$Y(x,t)$- aerodynamic loading i.e. unsteady aerodynamic force on wing,

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moment about elastic axis of wing, and unsteady aerodynamic force acting on vehicle body; $F_g(z,t)$, $M_g(z,t)$, $F_g(x,t)$- their corresponding aerodynamic loadings due to external disturbance; $Y_{CT}(t)$-force acting on half stabilizer at $x=x_{CT}$; Y_δ- centralized force acting on control surface at $x=x_\delta$; $\sigma(z)$- distance from c.g. of wing profile section to the elastic axis of wing; $EJ(Z)$, $GJ_p(z)$, $EJ_\phi(x)$-bending rigidity, torsion rigidity of wing along z- axis, and bending rigidity of vehicle body along x-axis respectively; $m(z)$-mass of wing, $m_\phi(x)$- mass of vehicle body. All the above distributed forces and masses are referred to units "per unit length". δ- Dirac function.

For most of space vehicle, the body is of slender shape, with small aspect ratio. The predominent elastic effect for this type of vehicle is elastic bending of vehicle body along the longitudinal axis. In the following, we deal with this type of vehicle mainly for longitudinal motion.

Let the elastic bending deflection of the vehicle be expressed by:

$$\zeta(x,t)=\sum_{i=1}^{N}\Phi_i(x)\cdot\eta_i(t) \qquad (2.4)$$

where $\phi_i(x)$ are functions of elastic modes, and $\eta_i(t)$ are the generalized coordinates.

The equations of unsteady disturbance motions of elastic vehicles with unsteady aerodynamic forces and elastic modes considered may be expressed in the form of Duhamel's integral:

$$\dot{X}(t)+\int_0^t I_1(t-\tau)\dot{X}(\tau)d\tau=AX(t)+\int_0^t I_2(t-\tau)\dot{u}(\tau)d\tau+Bu(t) \qquad (2.5)$$

Here state vector $X(t)$ includes state variables for motions of vehicle— velocity U, angle of attack α, pitching velocity q, and generalized coordinates $\eta_i(t)$, and their derivatives, $u(t)$ is control variable, e.g. control surface deflection $\delta(t)$, A- coefficient matrix, B- control matrix, I_1, I_2- unsteady parts of aerodynamic forces and moments expressed as indicial functions.

Longitudinal equations of disturbance motions with unsteady aerodynamic forces and n- orders of elastic modes considered are expressed as follows:

$$\begin{aligned}M_0(Vq-\dot{W})&=\int_0^t\dot{W}(\tau)H_Y^W(t-\tau)d\tau+\int_0^t\dot{q}(\tau)H_Y^q(t-\tau)d\tau\\&+\sum_{i=1}^{N}\int_0^t\dot{\eta}_i(\tau)H_Y^{\dot{\eta}_i}(t-\tau)d\tau+\sum_{i=1}^{N}\int_0^t\ddot{\eta}_i(\tau)H_Y^{\ddot{\eta}_i}(t-\tau)d\tau\\&+\int_0^t\dot{W}_y(\tau)H_{Y_g}^{W_y}(t-\tau)d\tau+\int_0^t\dot{W}_{yz}(\tau)H_{Y_g}^{W_{yz}}(t-\tau)d\tau\\&+\int_0^t\dot{\delta}(\tau)H_Y^\delta(t-\tau)d\tau\end{aligned} \qquad (2.6)$$

$$\begin{aligned}J_0\dot{q}&=\int_0^t\dot{W}(\tau)H_M^W(t-\tau)d\tau+\int_0^t\dot{q}(\tau)H_M^q(t-\tau)d\tau\\&+\sum_{i=1}^{N}\int_0^t\dot{\eta}_i(\tau)H_M^{\dot{\eta}_i}(t-\tau)d\tau+\sum_{i=1}^{N}\int_0^t\ddot{\eta}_i(\tau)H_M^{\ddot{\eta}_i}(t-\tau)d\tau\\&+\int_0^t\dot{W}_y(\tau)H_{Mg}^{W_y}(t-\tau)d\tau+\int_0^t\dot{W}_{yz}(\tau)H_{Mg}^{W_{yz}}(t-\tau)d\tau\\&+\int_0^t\dot{\delta}(\tau)H_M^\delta(t-\tau)d\tau\end{aligned} \qquad (2.7)$$

$$\begin{aligned}\ddot{\eta}_i+2\zeta_i\omega_i\dot{\eta}_i+\omega_i^2\eta_i&=\frac{1}{m_i}[\int_0^t\dot{W}(\tau)H_{Y\Phi_j}^W(t-\tau)d\tau\\&+\int_0^t\dot{q}(\tau)H_{Y\Phi_j}^q(t-\tau)d\tau+\sum_{i=1}^{N}\int_0^t\dot{\eta}_i(\tau)H_{Y\Phi_j}^{\dot{\eta}_i}(t-\tau)d\tau\\&+\sum_{i=1}^{N}\int_0^t\ddot{\eta}_i(\tau)H_{Y\Phi_j}^{\ddot{\eta}_i}(t-\tau)d\tau+\int_0^t\dot{W}_y(\tau)H_{Yg\Phi_i}^{W_y}(t-\tau)d\tau\\&+\int_0^t\dot{W}_{yz}(\tau)H_{Yg\Phi_i}^{W_{yz}}(t-\tau)d\tau+\int_0^t\dot{\delta}(\tau)H_{Y\Phi_i}^\delta(t-\tau)d\tau]\\&(i=1,2,\cdots,N)\end{aligned} \qquad (2.8)$$

where W- punching velocity ($W=U\alpha$), $H(t)$-indicial functions of the whole unsteady aerodynamic forces or moments for unit step increases of angle of attack, pitching velocity, generalized coordinates etc., with their corresponding unsteady part $I(t)$; e.g. $H_Y^{\eta_i}(t)$- unsteady aerodynamic force for unit step increase of η_i, and $I_Y^{\eta_i}(t)$- its corresponding unsteady part.

III. Simplified Mathematical Model

For the convenience of calculation, we put the longitudinal equations of disturbance motions in Laplace transform in standard matrix form in order to be easy to use the standard computer programs. In doing this, we use polynomial exponential function expressions to represent the above indicial functions in the time domain, then the corresponding Laplace transforms are rational fractions.

In general, we may take e.g.

$$H_Y^W(t)=Y_W\left(1+\sum_{i=0}^{n}a_ie^{-b_it}\right)$$

When only one term of exponential function is introduced, i.e.

$$H_Y^W=Y_W(1+a_0e^{-bt}) \qquad I_Y^W(t)=a_0Y_We^{-bt}$$

take Laplace transform

$$L\int_0^t H_Y^W(t-\tau)\dot{W}(\tau)d\tau=\int_0^t Y_W[1+a_0e^{-b(t-\tau)}]\dot{W}(\tau)d\tau$$

$$=Y_W\left(\frac{1}{S}+\frac{a_0}{S+b}\right)SW(S)=Y_W\frac{aS+b}{S+b}W(S)$$

$$(a=1+a_0)$$

$$L\int_0^t I_Y^W(t-\tau)\dot{W}(t)d\tau=Y_W\frac{a_0S}{S+b}W(S)$$

then we have the relation between the unsteady aerodynamic coefficient H_Y^W and the corresponding steady aerodynamic coefficient Y_W in Laplace transform:

$$H_Y^W(s) = Y_W \frac{aS + b}{s + b}$$

which corresponds to consider the unsteady transient process as nonperiodic character.

If two terms of exponential expression are considered, then

$$H_Y^W(s) = Y_W \frac{eS^2+fS+g}{S^2+cS+d}$$

As a preliminary calculation, we choose one term of exponential expression for the unsteady aerodynamic coefficients, and take $b_i = b$, then the corresponding Laplace transforms for equations (2.5) (2.6), (2.7) will be:

$$U_0 q(S) - SW(S) = Y_W \frac{a_1 S + b}{S + b} W(S) + Y_q \frac{a_2 S + b}{S + b} q(S) + Y_{\dot\eta_1} \frac{a_3 S + b}{S + b} \dot\eta_1 + Y_{\eta_1} \frac{a_4 S + b}{S + b} \eta_1 + Y_{\dot\eta_2} \frac{a_5 S + b}{S + b} \dot\eta_2 + Y_{\eta_2} \frac{a_6 S + b}{S + b} \eta_2 + Y_{\dot\eta_3} \frac{a_7 S + b}{S + b} \dot\eta_3 + Y_{\eta_3} \frac{a_8 S + b}{S + b} \eta_3 + Y_\delta \frac{a_9 S + b}{S + b} \delta(S) \quad (3.1)$$

$$Sq = M_W \frac{C_1 S + b}{S + b} W(S) + M_q \frac{C_2 S + b}{S + b} q(S) + M_{\dot\eta_1} \frac{C_3 S + b}{S + b} \dot\eta_1 + M_{\eta_1} \frac{C_4 S + b}{S + b} \eta_1 + M_{\dot\eta_2} \frac{C_5 S + b}{S + b} \dot\eta_1 + M_{\eta_2} \frac{C_6 S + b}{S + b} \eta_2 + M_{\dot\eta_3} \frac{C_7 S + b}{S + b} \dot\eta_3 + M_{\eta_3} \frac{C_8 S + b}{S + b} + M_\delta \frac{C_9 S + b}{S + b} \quad (3.2)$$

$$\ddot\eta_i + 2\zeta_i \omega_i \dot\eta_i + \omega_i^2 \eta_i = F_{iW} \frac{d_{i1} S + b}{S + b} W(S) + F_{iq} \frac{d_{i2} S + b}{S + b} q + F_{i\dot\eta_1} \frac{d_{i3} S + b}{S + b} \dot\eta_1 + F_{i\eta_1} \frac{d_{i4} S + b}{S + b} \eta_1 + F_{i\dot\eta_2} \frac{d_{i5} S + b}{S + b} \dot\eta_2 + F_{i\eta_2} \frac{d_{i6} S + b}{S + b} \eta_2 + F_{i\dot\eta_3} \frac{d_{i7} S + b}{S + b} \dot\eta_3 + F_{i\eta_3} \frac{d_{i8} S + b}{S + b} \eta_3 + F_{i\delta} \frac{d_{i9} S + b}{S + b} \delta \quad (i = 1,2,3) \quad (3.3)$$

In equations (3.1), (3.2), (3.3), only the first three elastic modes of the vehicle are considered.

Finally we may obtain the linear system of equations of states in matrix form:

$$\Delta(s) \left[\dot W(s), W(s), \ddot\theta(s), \dot\theta(s), \theta(s), \ddot\eta_1(s), \dot\eta_1(s), \cdots \ddot\eta_3(s), \dot\eta_3(s), \eta_3(s) \right]^T = E_1 \delta(s) + E_2 \dot\delta(s). \quad (3.4)$$

where $\Delta(s)$ is the coefficient matrix for the left side.

IV. Stability Analysis

In order to analyse the stability of the elastic vehicle, we must solve the characteristic roots of the matrix determinant:

$$|\Delta(s)| = 0 \quad (4.1)$$

In equation (3.4), if we choose $b = \infty$, $a_i = c_i = d_i = e_i = f_i = 1$, it corresponds to case with steady aerodynamic forces.

For an investigated vehicle, the characteristic roots with 3-modes of elastic vibrations considered have been calculated respectively for case with steady aerodynamic forces, and for case with unsteady aerodynamic forces considered.

A (unsteady)	B (steady)
$-3.012 \pm 34.960i$	$-3.546 \pm 34.777i$
$-4.929 \pm 321.10i$	$-4.584 \pm 321.192i$
$-32.24 \pm 760.2i$	$-31.47 \pm 761.22i$
$-20.58 \pm 1178.0i$	$-20.646 \pm 1177.87i$
$(-598.2,\ 0,\ 0,\ 0.1627\times10^{-31} \pm 0.1627\times10^{-71}i)$	$(-\infty,\ 0,\ 0,\ 0,\ 0,\ 0)$

For the complex roots of these two cases, the the first pair corresponds to natural short period motion, 2,3,4-th pair roots correspond respectively to first, second, third modes of elastic vibrations of the vehicle. From data, we see for the investigated vehicle the consideration of unsteady aerodynamic forces has some effects on the magnitudes of amplitudes and frequencies of the elastic vibrations of the elastic vehicle. More calculations show the effect is more serious in transonic flight.

V. Synthesis of Optimal Active Control System

We now consider the optimal design of active control system.

Let the equations of state of the elastic vehicle in closed loop system be expressed by:

$$\dot X = AX + BU$$
$$Y = CX \quad (5.1)$$
$$U = KY$$

If there are m- active control surfaces situated along the longitudinal axis of the elastic vehicle at $x = s_1, s_2, s_3, \ldots s_m$, then the corresponding deviations of the control matrix B:

$$B = (b_1, b_2, b_3 \ldots b_m)$$

due to deviations of the positions of active control surfaces are

$$dB = \left(\frac{\partial b_1}{\partial x}(s_1)\ \frac{\partial b_2}{\partial x}(s_2)\ \frac{\partial b_3}{\partial x}(s_3) \ldots \frac{\partial b_m}{\partial x}(s_m) \right) \cdot \mathrm{diag}\left(ds_1 ds_2 ds_3 ds_4 \ldots ds_m \right) \triangleq B_x . ds \quad (5.2)$$

Furthermore, if there are L-sensors (rate gyros) situated at $x = r_1, r_2, r_3, \ldots r_l$, the corresponding variations of the observation matrix

$$C = \left(C_1^T C_2^T C_3^T \ldots C_l^T \right)^T \tag{5.3}$$

will then be

$$dC = \begin{bmatrix} dr_1 & & & \\ & dr_2 & & \\ & & \ddots & \\ & & & dr_l \end{bmatrix} \begin{bmatrix} \frac{C}{x}l(r_1) \\ \vdots \\ \frac{C}{x}l(r_l) \end{bmatrix} = dr.C_x \tag{5.4}$$

We treat the optimal active control synthesis for suppressing elastic vibrations as follows: i.e. we need to find factor K of the feedback control, positions of control surfaces $s = (s_1, s_2, s_3 \ldots s_m)$ and positions of sensors (gyros) $r = (r_1, r_2, r_3 \ldots r_l)$, so that index function

$$J = \frac{1}{2}\int^{\infty} (X^T Q\, X + U^T R\, U)dt \tag{5.5}$$

to be minimum.

Since X contains state variables η, $\dot{\eta}$, relating to elastic vibrations, therefore the above criterion is in some sense related to the suppression of the elastic vibrations of the vehicle. (see fig.1,2)

The following relations can be obtained finally after some tedious derivations:

$$\frac{\partial J}{\partial K} = R^T K C T_{11} C^T + B^T P_{11} T_{11} C^T \tag{5.6}$$

$$\frac{\partial J}{\partial s} = V\mathrm{diag}\left(B_x^T P_{11} T_{11} C^T K^T \right) \tag{5.7}$$

$$\frac{\partial J}{\partial r} = V\mathrm{diag}\left(K^T R K C T_{11} C_x^T + K^T B^T P_{11} T_{11} C_x^T \right) \tag{5.8}$$

in which T_{11}, P_{11} satisfy Lyapunov equations:

$$(A+BKC)^T P_{11} + P_{11}(A+BKC) + Q = 0 \tag{5.9}$$

$$(A+BKC)\, T_{11} + T_{11}(A+BKC)^T + I = 0 \tag{5.10}$$

and

$$V(O) = \begin{pmatrix} I & O \\ O & O \end{pmatrix}$$

With $\frac{\partial J}{\partial K}$, $\frac{\partial J}{\partial s}$, $\frac{\partial J}{\partial r}$, by applying the usual optimization method with constraints, we can find optimal values K^*, s^*, r^*, corresponding to J=min. For our investigated vehicle, we have after 80-th optimization search:

$$K = \begin{pmatrix} 0.2776 & 0.2452 & 0.2334 \\ -0.2069 & -0.0914 & -0.0492 \end{pmatrix}$$

If sensitivity factor $e = dX/df$ is introduced in the index function, where f is parameter considered, then

$$J = \frac{1}{2}\int_0^{\infty} (X^T A\, X + U^T R\, U + e^T S\, e)dt \tag{5.11}$$

Let it be called LQS (linear quadratic with sensitivity) feedback design as compared with the former case LQR (linear quadratic regulator). The resulting LQS control system will be less sensitive to external parameters f (e.g. flight Maxa No., height), i.e. the robustness of the active control system for case LQS is improved as compared with case LQR as calculations indicated (see fig.2) for different M.

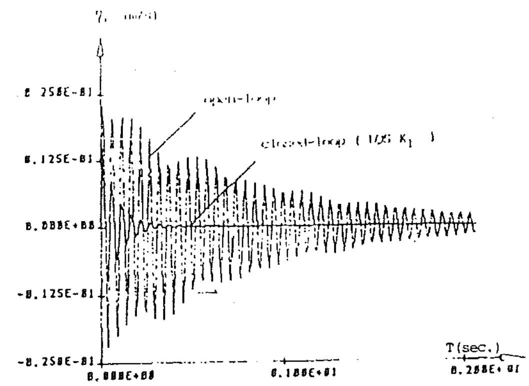

Fig.1 η_1 curve $M = 2.0$

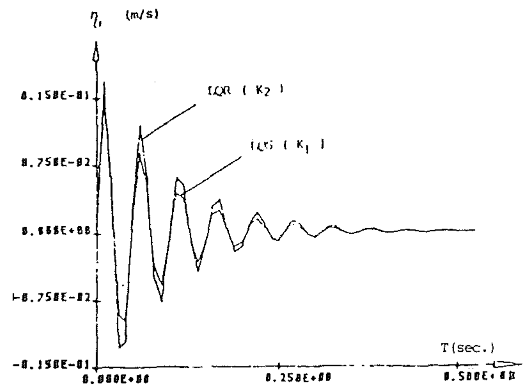

Fig.2 $\dot{\eta}_1$ curve $M = 2.0$

VI. Dynamic Coupling of Elastic Structure with the Control System

We now turn to consider the dynamic coupling of elastic body with the control system.

For Simplicity, we consider the quasi-steady case, and the equations of state of elastic vehicle in closed loop system are expressed by the superposition of vehicle body motion plus elastic vibration of the vehicle:

$$\dot{X}_1 = A_1 X_1 + B_1 \delta \tag{6.1}$$

$$Y_1 = C_1 X_1 \tag{6.2}$$

$$X_1 = |w\ \vartheta\ \dot{\vartheta}|^T \tag{6.3}$$

$$M_V \ddot{\eta}_i + D_V \dot{\eta}_i + K_V \eta_i = -\frac{1}{2}\rho V_0^2 Q(t) + H_\delta \delta(t) \tag{6.4}$$

$$Y_2 = C_2 X_2 \tag{6.5}$$

$$X_2 = [\eta_1\ \eta_2 \ldots \eta_n\ \dot{\eta}_1\ \dot{\eta}_2 \ldots \dot{\eta}_n]^T \triangleq [\eta_i\ \dot{\eta}_i]^T$$

$$U=-KY \tag{6.6}$$

Output of sensors: $Y = Y_1+Y_2 = C_1X_1+C_2X_2$

$$=\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} w \\ \vartheta \\ \dot{\vartheta} \end{pmatrix} - \begin{pmatrix} \phi'(X_{sp}) & 0 \\ 0 & \phi'(X_{sr}) \end{pmatrix}\begin{pmatrix} \eta \\ \dot{\eta} \end{pmatrix} \tag{6.7}$$

where X_{sp} - position of angular sensor,

X_{sr} - position of angular velocity sensor,

and
$$\delta = u + \delta_{com} \tag{6.8}$$

in which δ_{com} - command signal.

Then
$$\delta = -KY+\delta_{com}$$
$$= -[k_1 \;\; k_2]C_1X_1+k_1\phi'(X_{sp})\eta+k_2\phi'(X_{sr})\dot{\eta} + \delta_{com} \tag{6.9}$$

Substituting expression δ into equations of state of the elastic vehicle in closed loop system, we can get finally:

$$\bar{A}(s)X(s) = \bar{B}\delta_{com}(s) \tag{6.10}$$

where

$$\bar{A}(s)=\begin{pmatrix} A_{11}(s) & A_{12}(s) \\ A_{21}(s) & A_{22}(s) \end{pmatrix}, \quad X(s)=\begin{pmatrix} X_1(s) \\ \eta(s) \end{pmatrix}, \quad \bar{B}=\begin{pmatrix} B_1 \\ H_\delta \end{pmatrix}$$

in which

$$A_{11}(s)= sI-A_1+B_1[k_1 \;\; k_2]C_1$$
$$A_{12}(s)= -[B_1k_1\phi'(X_{sp})+B_1k_2\phi'(X_{sr})s]$$
$$A_{22}(s)= M_V s^2+[D_V-H_\delta k_2\phi'(X_{sr})]s + \bar{Q}(s) + [K_V-H_\delta k_1\phi'(X_{sp})]$$
$$A_{21}(s)= H_\delta[k_1 \;\; k_2]C_1$$

Then we can get the following conclusion:

The stability of the whole elastic vehicle in closed loop system will be determined by matrix $\bar{A}$ instead of matrix A_1. Therefore, even if the rigid vehicle system is stable, the whole elastic vehicle in closed system may be unstable due to coupling of the elastic body with control system. Only under suitable choises of positions of sensors (X_{sp}, X_{sr}) and actuator (H_δ), and feedback coefficients k_1, k_2, the whole elastic vehicle system will be stable.

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