

## DRAG/THRUST ESTIMATION VIA AIRCRAFT PERFORMANCE FLIGHT TESTING

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### ABSTRACT

This paper presents a procedure for drag/thrust estimation via aircraft performance flight testing using Weighted Least Square (WLS) estimation method. This procedure was applied to estimate the drag of certain F-16 configurations and the thrust of certain modified engines. The estimated drag/thrust were in good agreement with their prediction. The statistical evaluation of the estimate confidence intervals proved to be a reliable tool for assessing the quality of the results. The estimated drag/thrust were later used to evaluate aircraft performance with greater accuracy.

### I. INTRODUCTION

#### Drag and thrust

The estimation of external store drag and thrust characteristics is an important part of combat aircraft performance evaluation. In general, drag evaluation is composed of the following three phases. At first drag is estimated using analytical methods which are based mainly on computer analyses, handbook formulas and in some cases, when one doesn't have any better alternative, similarity analyses. The second phase is comprised, usually, of wind tunnel testing which is used to refine the drag estimate. But problems like wind tunnel model scale (Reynolds effect), wind tunnel non-linearity and impurity affects the results, mainly the zero lift drag ( $C_{D0}$ ). The third phase, flight testing, is carried out to finally determine the store drag characteristics.

When evaluating the thrust level for a given aircraft/engine combination the procedure is similar. At first thrust is evaluated by the engine manufacturer using a semi-analytical computer program simulation, called "steady state engine deck". The second phase is ground evaluation of installed engine. Analyses and ground testing are performed to evaluate the installed engine performance, mainly the inlet and nozzle losses. The last phase is flight testing where thrust is finally evaluated.

Though being two separate entities, drag and thrust are closely related. When referring to aircraft drag or thrust one has to define the relation between them, or what is known as - "thrust/drag book keeping"<sup>(3)</sup>. The main issues that are usually in dispute are spillage drag and tail boat drag which are sometimes regarded as drag and sometimes as "negative" thrust.

#### Estimation

Flight test data analysis is involved with the estimation of system parameters and their subsequent transformation to aero/thrust data-bases. This may be used to achieve a higher quality evaluation of the aircraft performance.

From the viewpoint of estimation theory, the task of drag/thrust estimation from aircraft flight testing, falls within the category of stochastic non-linear dynamic system identification. There are several estimation methods suited for this wide and quite explored field. Each of those methods has its own advantages and disadvantages, and for each application one has to choose the most promising and suited method.

The method used in this work is based on output error identification algorithm. The optimal estimates of the dynamic model parameters are found by minimizing a cost function which is squared difference between the model output and the actual measurement acquired via flight testing. Belonging to the class of least squares (LS) algorithms, the method does not utilize any statistical information regarding the process and/or measurement noises. This fact renders its structure relatively simple, when compared to statistical algorithms like the Extended Kalman Filter (EKF) which is commonly used for parameter identification problems. Moreover, the output error algorithm becomes rather attractive when considering the highly non-linear nature of the aircraft dynamic model.

The outline of this paper is as follows. In Section II the pre-estimation procedure which includes flight tests definition, data acquisition and data handling is described. In Section III we present the aircraft dynamic model, the estimator structure, the parameters modeling and the statistical tools used for evaluation of the estimates confidence intervals. In Section IV the post-estimation procedure is described. In Section V results of flight testing based estimation are presented and discussed. Section VI contains our conclusions.

## II PRE-ESTIMATION

Pre-estimation is an important stage prior to estimation process. It consists of three phases: (a) Flight test - planning and conducting philosophy, (b) Aircraft instrumentation and data acquisition and (c) raw data (time histories) pre-processing.

### Flight test planning and philosophy

The flight tests objective is to supply reliable and sufficient data for the estimation process. Since the basis for the entire estimation process is the data acquired in the flight tests, much consideration should be taken when planning and conducting the flight tests.

The first stage of the test planning consists of a definition of the maneuvers required given the data required. Let us examine the physical meaning of common maneuvers used in performance flight test. We shall first cover aero performance tests and then thrust performance tests.

A typical drag polar of an aircraft in a given Mach/altitude point is shown in figure 1. A good test planning will provide us with  $C_L/C_D$  test points that will cover required regions in the drag polar. Following, is a description of five, commonly used, maneuvers that covers the drag polar:

(a) Level acceleration - conducted in MIL and MAX A/B power: This untrimmed maneuver covers moderate to low  $C_L$ 's over a range of mach numbers. One should remember that for a given Mach number only one  $C_L/C_D$  data point can be obtained.

(b) Sustained Turn - Conducted in MIL and A/B MAX power at a given Mach number: In MIL power it covers moderate  $C_L$ 's, while in MAX A/B it covers higher  $C_L$ 's. One can extract many repeated data points from this maneuver, and thus increase data reliability at a given Mach/ $C_L$  point.

(c) Trim Point - stabilizing the aircraft at a specific Mach number and altitude gives 1g sustained and trimmed maneuver.

(d) Roller Coaster Maneuver (RCM) - covers moderate to low  $C_L$ 's. This maneuver is usually used to help define  $C_{D0}$ .

(e) Split-S - covers moderate to high  $C_L$ 's. Conducting this maneuver allows us to increase load factor without losing airspeed and with a relatively small change in altitude.

The thrust flight test maneuvers are designed to provide sufficient data so as to cover a given grid of Mach numbers, altitudes and throttle positions. Therefore, some of the maneuvers listed above can be used for thrust and fuel flow evaluation.

A difference between thrust estimation and drag estimation is that in contrast to drag, thrust is strongly dependent on altitude and ambient temperature. This will result in a large amount of flight tests at different altitudes and temperatures. Besides choosing the appropriate maneuvers one should take into account some other considerations. For example, when testing the drag of several stores that can make up many loadings, it has to be decided which loadings are to be tested so that enough data for prediction of other stores' combinations could be obtained. In performance thrust testing, a so called "base-line" sortie is needed to obtain enough aerodynamic data of the specific flight test aircraft.

### Pre-processing

A mid phase between flight tests and the estimation process is called - pre processing. This stage consists of two parts.

At first, raw data is overviewed and 'erroneous data' (outliers, or data that does not conform with known physical properties) is discarded. This is done usually, by graphing the raw data and selecting the, so called, 'erroneous data'. For example, when performing transonic acceleration a measuring problem in some of the parameters arises. Therefore, the transonic Mach range (0.95-1.1) can be cut out and two different estimations can be performed (until 0.95 and from 1.1).

At the second part, immeasurable quantities are calculated from the measurable data and some filtering and/or skewing are applied to the data if needed. An example of "calculated" state is the path angle. When a level acceleration is performed it is almost impossible to maintain a constant altitude, therefore, there is a small path angle. Assuming that the path angle is zero during the acceleration maneuver might lead to errors in the estimated parameters. It is impossible to measure the path angle (without a ground radar), but it can be calculated from the measured altitude, airspeed and ambient temperature as follows:

$$\gamma = \arcsin(\dot{h}/v) * \frac{T_{amb}}{T_{isa}} \quad (2.1)$$

### III. THE ESTIMATOR

The problem of aerodynamic/thrust parameters identification can be posed as: given a set of flight test time histories of the aircraft responses find the values of the unknown parameters in the dynamic model, such that the model's output will be as close as possible to the actual measurements, in a predefined sense. This task is carried out via the following stages : definition of a cost function, selection of a dynamic model to represent the physical system, modeling of the parameters and establishment of an analytical or computational tool to evaluate the estimates quality.

#### The cost function

The most common cost function is the square of the difference between the

actual measurements and the predicted measurements based on the estimated parameters. If the covariance of certain measurement noise is relatively small or big or, if scaling is needed or, if there is any other good reason, a weighting matrix could be added to the cost function to magnify or to reduce this measurement contribution to the cost function. This is called - Weighted Least Squares (WLS). The cost function used in this work is :

$$J = \sum_{i=1}^P \left[ \frac{1}{N} \sum_{k=1}^N (Z_{ki} - \hat{Z}_{ki})^T W (Z_{ki} - \hat{Z}_{ki}) \right] \quad (3.1)$$

where : P is the number of different measurements (Alt., M, etc.), and N is the number of measured data points.

If one has a high confidence in the a-priori parameters, (often called initial "guess") the following term, which means that one would like the estimated parameters to stay "close" to their initial values, can be added to the cost function.

$$(\theta - \theta_0)^T W_1 (\theta - \theta_0) \quad (3.2)$$

where :  $\theta$  is the parameter's vector and  $\theta_0$  is the parameter's initial guess.

#### The dynamic model

Choosing the proper dynamic model has to be based upon deep and profound understanding of the physical system to be estimated. Moreover, that several considerations have to be taken into account when defining the dynamic model. Computation complexity in conjunction with the type of computer available as well as the type of flight test maneuvers and quality of available data are some examples of those considerations. For example, when estimating drag/thrust from level acceleration one can neglect the aircraft lateral equation of motion provided that the bank angle was kept close to zero during the maneuver. In this paper the following two Degrees Of Freedom (2DOF) equations of motion were chosen as the dynamic model for the acceleration maneuver :

$$\dot{\gamma} = g(T \cos \alpha - 0.7PM^2SC_D - W \sin \gamma) \quad (3.3)$$

$$\dot{M} = \frac{(T \sin \alpha + 0.7PM^2SC_L - g)}{WMa} \quad (3.4)$$

#### Parameters modeling

After the dynamic model has been

defined, one has to model the unknown parameters. There are several ways of doing this. For instance, in stability and control parameter estimation it is common to set the parameters as constants whose values should be estimated. On the other hand, when drag coefficient is estimated from level acceleration maneuver it makes sense to model the varying  $C_D$  through the acceleration as a function of, say, Mach number (M). Since the following relations :

$$C_L = F\{M, H\} \quad (3.5)$$

$$C_D = F\{M, C_L\} \quad (3.6)$$

$$C_T = F\{M, H, \Pi\} \quad (3.7)$$

exist, it is quite straightforward, in case of constant throttle/constant altitude maneuver, to choose a model in which the aero/thrust coefficients are functions of M. There are two basic ways to model the aero/thrust coefficients, a continuous form and a discrete form. In this work the following polynomial form was chosen to represent the continuous form :

$$C = \theta_0 + \theta_1 M + \theta_2 M^2 + \dots + \theta_k M^k = \bar{\theta}^T \bar{M} \quad (3.8)$$

where :  $\bar{\theta} = \text{col}(\theta_0, \theta_1, \dots, \theta_k)$   
 $\bar{M} = \text{col}(1, M, M^2, \dots, M^k)$

and k is chosen empirically as a compromise between model flexibility and complexity. (experience shows that values of 4-8 are adequate).

The discrete form is :

$$C = [C_1, C_2 \dots, C_k] \quad (3.9)$$

$$M = [M_1, M_2 \dots, M_k] \quad (3.10)$$

where each  $C_i$  corresponds to each  $M_i$ .

### Confidence intervals

The most interesting question concerning the estimated values of the unknown parameters is how "close" are the estimates to their "true" values. The well known Cramer-Rao (CR) inequality gives a theoretical bound to the accuracy that is achievable, regardless of the estimator used. The CR inequality is as follows :

$$E[(\hat{\theta} - \theta)(\hat{\theta} - \theta)^T] \geq [I + \nabla b(\theta)] Q^{-1}(\theta) [I + \nabla b(\theta)]^T \quad (3.11)$$

where : Q is the Fisher information matrix.  $b(\theta)$  is estimator bias.  $\nabla$  is the

gradient operator.

Since the CR inequality gives just a lower bound for the parameters' error covariance there is a need to estimate the actual parameters' error covariance. Ref. (1) gives the following formula as an estimate for this covariance:

$$E[(\hat{\theta} - \theta)(\hat{\theta} - \theta)^T] = (X^T X)^{-1} X^T v X (X^T X)^{-1} \quad (3.12)$$

where :

X is measurement sensitivity matrix  
 $v = E\{\epsilon \epsilon^T\}$   
 $\epsilon$  is the measurement noise

An estimate for  $v$  is given by :

$$v = \frac{(\hat{Z} - Z)(\hat{Z} - Z)^T}{n - p}$$

where :

n is the number of data points, and  
 $p = \text{dim}(\hat{\theta})$

Since :

$$C = \theta^T \bar{M} \quad (3.13)$$

Then :

$$P_C = \text{VAR}(C - C) = E\{\bar{M}^T [\hat{\theta} - \theta][\hat{\theta} - \theta]^T \bar{M}\} = \text{trace}\{\bar{M} \bar{M}^T P_\theta\} \quad (3.14)$$

Where :

$$P_\theta = \text{COV}(\hat{\theta} - \theta) \quad (3.15)$$

A 95% confidence interval for the estimated aero/thrust coefficient is given by :

$$C - \Delta C < C < C + \Delta C \quad (3.16)$$

where :

$$\Delta C = (n - p) * t_{0.975} * \sqrt{P_C} \quad (3.17)$$

where  $(n - p)t_{0.975}$  is the t-statistic for  $(n - p)$  degrees of freedom.

This concludes the definitions needed in order to carry out the estimation process and to obtain flight test based estimation of aero/thrust coefficients. Now those coefficients need to be transformed into aero/thrust data-bases grid and structure. In the following section the transformation procedure is discussed.

#### IV. POST-ESTIMATION

The task that remains after the aerodynamic/thrust coefficients has been estimated from specific flight test maneuvers, is to evaluate the impact of estimated coefficients over the entire performance envelope. In order to carry it out a profound knowledge of the structure of aerodynamic and thrust data-bases is needed. Since the structure of the aerodynamic data-base and the thrust data-base is quite different the discussion was divided into two parts: drag and thrust post-estimation.

##### Drag post-estimation

Aerodynamic data-base is a discrete representation of the aircraft's aerodynamic data needed for performance evaluation. It consists of two major parts: clean aircraft aero data and stores/-loadings aero data. The clean aircraft aero data is defined by the following tables :

(a)  $C_L$  vs. Angle of attack at various Mach numbers.

(b)  $C_D$  vs. Mach number and  $C_L$  at various aircraft stability margin (= various aircraft center of gravity (c.g.))

(c) Altitude effect on  $C_{d0}$  at various Mach numbers.

Since the post-estimation of the estimated drag based on the different maneuvers is too wide to be included here, we will restrict ourselves to level acceleration post-estimation. During the acceleration,  $C_L$ ,  $C_D$  and Mach number changes each data point, so at a specific Mach number an estimation of only a specific pair of  $C_L$  and  $C_D$  which generally does not coincide with the  $C_L$ 's grid in the aerodynamic data base is obtained. So, for translation of estimated drag to the  $C_L$ 's grid interpolation is carried out as can be seen from the following formula :

$$C_D|_{C_L=C_{L2}} = C_D|_{C_L=C_{L1}} + \frac{\Delta C_D}{\Delta C_L} * (C_{L2} - C_{L1}) \quad (4.1)$$

where the ratio  $\frac{\Delta C_D}{\Delta C_L}$  is numerically calculated from the aerodynamic data-base.

Things get complicated when dealing with MAX A/B acceleration, where significant changes in aircraft center of gravity might occur. This was taken into account in the same manner as above.

After delta drag coefficient due to the loadings has been determined, it is converted to units called Drag Index (DI) which are used in the aircraft performance flight manual.

##### Thrust post-estimation

Thrust post-estimation is generally less complicated than drag post-estimation. In performance calculation it is common to represent the engine thrust in what is called "thrust tables". Actually those tables are a discrete representation of the thrust as a function of : Mach number, altitude, and throttle position at a given ambient temperature profile.

So, if the estimated maneuver was MIL or MAX A/B level acceleration and the ambient temperature was standard, it is straightforward to obtain discrete thrust values at given Mach number and insert them directly to the thrust data-base. If the ambient temperature was non standard we applied the following correction to the estimated thrust :

$$T_{CORR} = T_{TEST} + \frac{\Delta T}{\Delta temp} * (temp_{AMB} - temp_{ISA}) \quad (4.2)$$

where the ratio  $\frac{\Delta T}{\Delta temp}$  is numerically calculated from engine data-base.

The same is done with other fix throttle maneuvers, but when analyzing trim point maneuver things get complicated since interpolation between throttles position is needed too.

#### V. RESULTS AND DISCUSSION

The results of the estimation process can be separated into two categories: The estimated aerodynamic coefficients and thrust and estimation confidence intervals (also called estimation performance analyses). Here the results of level acceleration estimation and the associated confidence intervals are presented.

##### Estimated aerodynamic coefficients

Measured and estimated Mach number time histories and altitude Mach histories are presented in figures 2.& 4., respectively. Figures 3.& 5., present a time history of the error between measured and estimated Mach number and altitude, respectively. This error is almost zero

mean and lightly colored which implies that the quality of the estimation is good.  $C_L$  and  $C_D$  Mach history are shown in figure 6. and 7., respectively. Each figure consists of two lines - the initial guess based on a-priori knowledge and the final estimation. Figure 10. summarizes the estimated drag indexes (DI) versus the predicted, as can be seen there is a good agreement between both.

Table 1 summarize Cramer-Rao Bounds (CRB) obtained from the estimator in certain accelerations. As shown, those bounds are quite low.

As we learned from this work there are several parameters that influence the quality of the results. Many of our aerodynamic coefficients estimated from MAX A/B acceleration had high confidence intervals and some of them gave unrealistic drag estimation. That was contributed to the fact that the greater the acceleration, the fewer the data points per given range of Mach number. Due to cost function minimization algorithm (first order and second order gradient) the scheme is sensitive to the parameters' initial "guess", which should be "close" enough to the parameters' "true" values, so it would prevent the algorithm from wrongly converge to a local minimum which might be far from the "true" minimum. The former implies that if evaluation of aircraft's aerodynamic coefficients using this procedure is needed, a good a-priori knowledge of the coefficients is mandatory. Last but not least is the effect of the maneuver quality, that is how well it was performed, on the estimation results.

#### Estimated thrust

Estimated thrust and its confidence intervals are shown in figure 8.& 9. respectively, for level acceleration case.

In figure 9. estimated thrust and the boundaries of its confidence intervals are shown. Again, as can be seen, this interval is relatively low which implies a high accuracy of the results.

## VI. CONCLUSIONS

In this paper a method which involves three phases :pre-estimation, estimation and post-estimation, was presented as a tool for thrust/drag evaluation via aircraft performance flight testing. The paper

reviews those phases using the example of drag and thrust estimation from level acceleration. The results of the estimation were used later for a more accurate aircraft performance evaluation. Based on our experience, this method provides a good tool for drag and thrust estimation even in cases where flight test measurements are not optimal. The statistical evaluation of the estimate confidence interval proved to be a reliable tool for assessing the quality of the results.

## NOMENCLATURE

### Symbols

$\bar{C}$	- Coefficient's vector
H	- Altitude [m]
J	- Cost function
M	- Mach number [-]
P	- Pressure [Newton/m ]
S	- Ref. area [m ]
T	- Thrust [Newton]
W	- Weight [Newton]
Z	- Measurement
$\hat{Z}$	- Predicted measurement
$\alpha$	- Angle of attack [Rad]
$C_D$	- Drag coefficient [-]
$C_L$	- Lift coefficient [-]
$C_T$	- Thrust coefficient [-]
$\gamma$	- Path angle [Rad]
a	- Speed of sound (m/sec)
$\pi$	- Throttle position
g	- Acceleration of gravity (m/sec <sup>2</sup> )

$E \{ \}$  - Expected value  
 $\theta$  - Parameter's vector

Abbreviations and acronyms

VAR - Variance  
 COV - Covariance  
 EKF - Extended Kalman Filter  
 LS - Least Squares  
 WLS - Weighted Least Squares  
 CRB - Cramer-Rao Bound  
 MIL - Military power  
 MAX A/B - Maximum afterburner

Acc No.	CRB (%)
1	0.7
2	0.2
3	0.1
4	0.5
5	0.3
6	0.2
7	0.5

Table 1 - CRB values  
 (MIL ACCELERATIONS)

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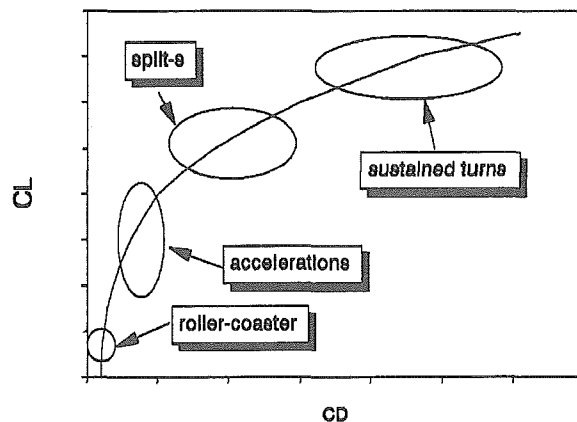


Figure 1 - DRAG POLAR

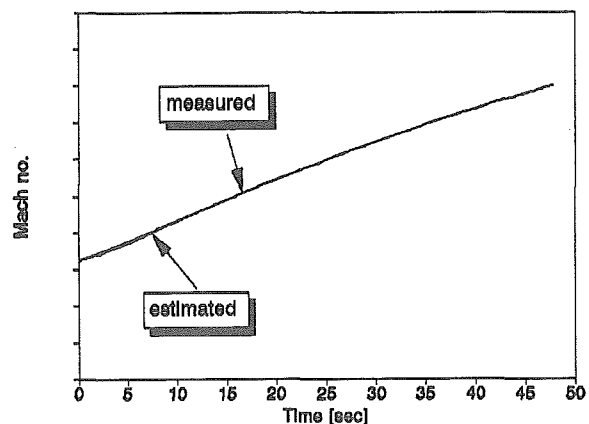


Figure 2 - Mach vs. Time

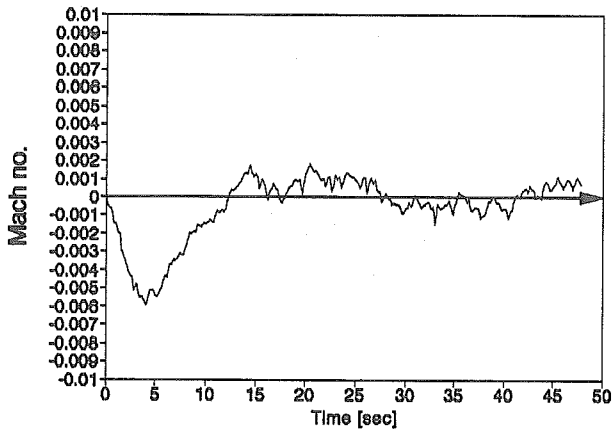


Figure 3 - Mach residual vs. Time

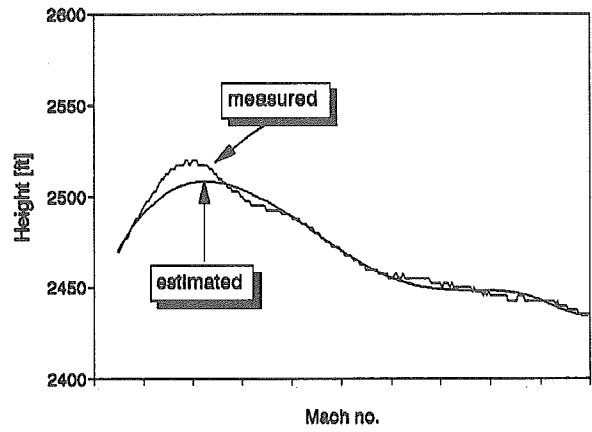


Figure 4 - Height vs. Mach

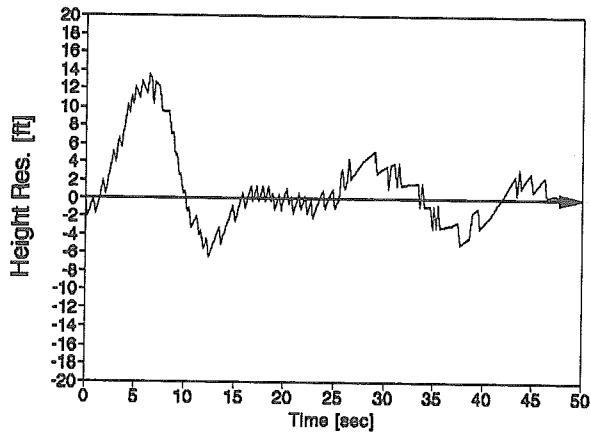


Figure 5 - Height residual vs. Time

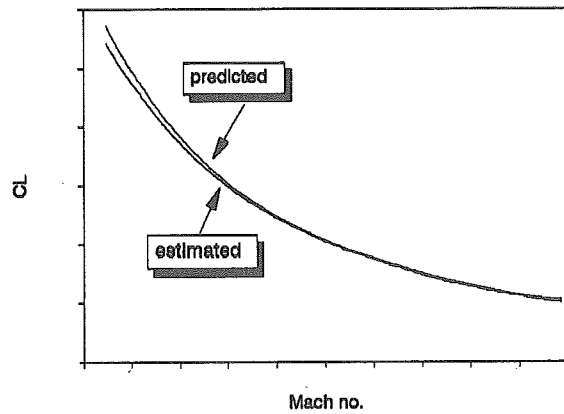


Figure 6 - CL vs. Mach No.

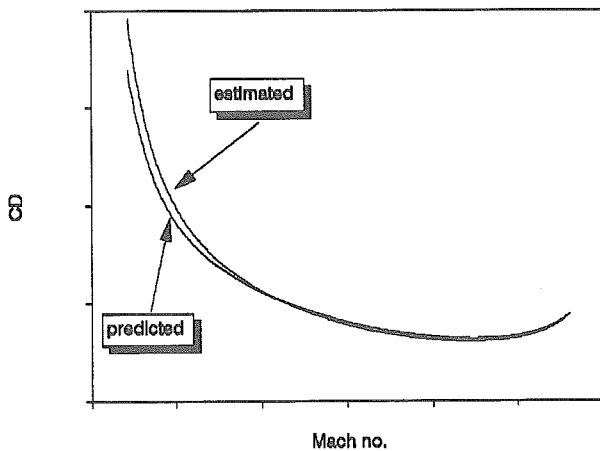


Figure 7 - CD vs. Mach no.

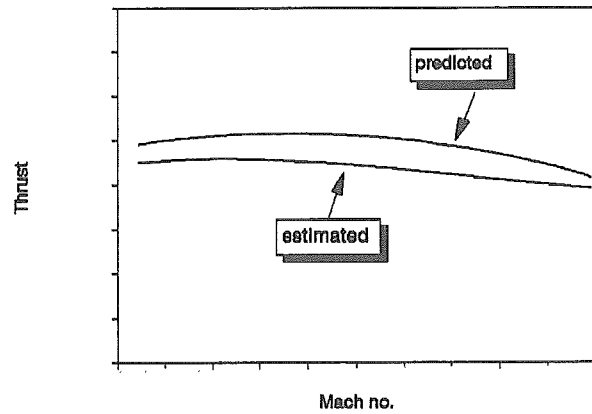


Figure 8 - Thrust vs. Mach No.



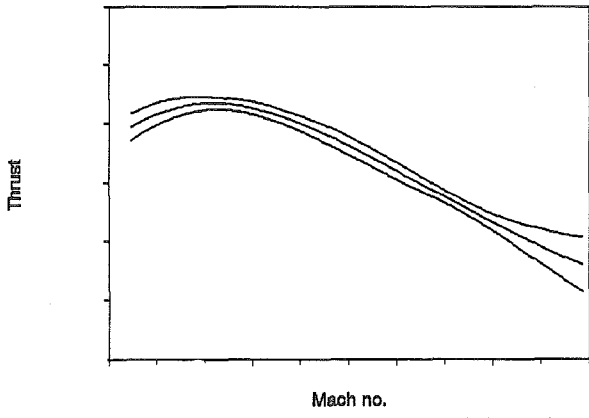


Figure 9 - Thrust Confidence Intervals

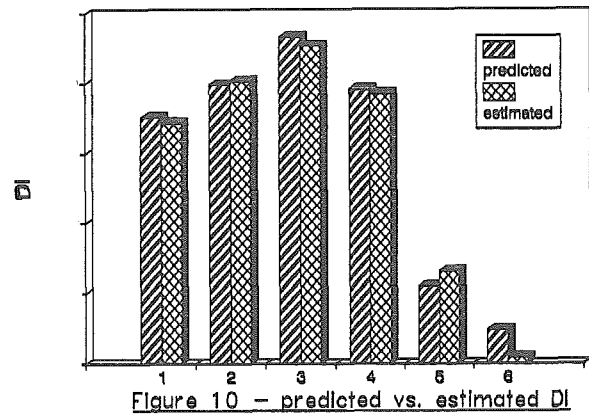


Figure 10 - predicted vs. estimated DI