

FAULT TOLERANT SCHEME FOR MULTISENSOR NAVIGATION SYSTEM

H.G. Zhang and H.Y. Zhang
 Department of Automatic Control
 Beijing University of Aeronautics & Astronautics
 Beijing 100083, P.R. China

Abstract

The problem of failure tolerant estimation in multisensor navigation systems is addressed. A fault tolerant scheme is given for multisensor navigation systems. The local estimates of different subsystems are fed into a master filter to give a global optimal estimate of the system state in the absence of failures. The failed subsystem can be detected and isolated by using chi-square test or GLT (generalized likelihood test). The failed subsystem is then switched off from the master filter, the remained local estimates of unfailed subsystems are recombined in the master filter to give a most accurate estimate. The results are applied to a SINS/GPS/Doppler integrated navigation system for illustration.

Introduction

The current approach to cost-effective achievement of high-accuracy navigation in aircraft is a multisensor navigation system that includes one or more inertial navigation systems (INSs) and one or more navigation reference sensors such as a global positioning system (GPS) receiver, Doppler radar, terrain aided system, air-data system, TACON, VOR/DME etc. The data from these subsystems are processed by an integration filter (master filter) that computes the minimum mean square error (MMSE) estimation of aircraft position, velocity, acceleration, and attitude. The integration filter typically resides in a mission computer.

In multisensor system, different sensors are combined into several subsystems. When all sensor outputs are blended using one Kalman filter, the accuracy of the estimate of navigation states is superior to that of local filters of its individual subsystems. However, this one Kalman filter scheme does not have the capability of fault tolerance. In the presence of sensor failures, the estimate conditioned on all sensor measurements will not be correct,

then the performance of the integrated system will degrade considerably. Kerr⁽¹⁾ first proposed an idea that the above problem can be solved by using decentralized filtering techniques.

Brumback and Srinath⁽²⁾ considered the design of an integrated multisensor navigation system for which fault tolerance in the presence of soft failures is desired. A fault tolerant system will be presented in this paper. The objective is to develop an estimation algorithm that eliminates the effects of a failed sensor, so that reliable system performance is achieved. In this paper, a FDI algorithm is proposed, which is used to determine the validity of the local estimates, and a failed sensor can be identified from analysis of the invalid local estimates. The combining algorithm for decentralized estimation in multisensor system is used to estimate the error states of the integrated navigation system by processing the measurements from unfailed sensors.

The above results are applied to the design of a fault tolerant multisensor navigation system which is composed of three sensors: a SINS, a GPS receiver and a Doppler radar forming SINS/GPS and SINS/Doppler subsystems. The two subsystems provide two measurements for two local estimators, the local estimates are fed into a master filter to construct the MMSE estimate of the aircraft navigation state. When one of the subsystems has failed, the correspondent local estimate will be switched off from the master filter. After the reconfiguration, the master filter can give a most accurate estimate of navigation state in the presence of the subsystem failure. Simulation results are presented to evaluate the performance of the system.

Combining Algorithm for Decentralized Estimation

In most applications of Kalman filtering to multisensor navigation

system, only one Kalman filter is used to compute a single, optimal estimate of the aircraft navigation states based on all available sensor data. However, if one sensor fails, its output is no more correct and the "centralized" estimation given by the single Kalman filter is not reliable anymore. A fault tolerant estimation can be obtained if we use decentralized estimation. The decentralized estimation scheme is based on different subsets of the available sensor data. The multiple local estimates can be compared to determine if they are in agreement within their expected uncertainties. If all estimates are in agreement, no failures are declared, and it would seem reasonable to combine these local estimates into a global estimate. The global estimate is the most accurate estimate which can be computed by a single Kalman filter which processes all sensor data.

The problem of combining several local estimates into a global estimate has been considered by several authors^(3,4,5). For the navigation problem considered herein, Tylee's algorithm⁽⁵⁾ seems to be most appropriate. Tylee considered a system in which there are N processors, each of them having its own local measurements. The measurements may consist of all system states and not just the states of local subsystems, so that each local processor computes its own optimal estimation of the system state. The objective is that if any sensor undergoes a hard failure, the estimates of all other processors are not affected by the failed sensor, and the failed sensor outputs can be estimated by using unfailed sensor outputs. But Tylee's results are derived based on ignoring the correlations between the local estimates that arise from the system process noise. Zhang⁽⁶⁾ have given a sufficient condition for using unfailed sensor outputs to generate optimal estimates of failed sensor outputs, and extended Tylee's results by using the upper bound technique proposed by Carlson⁽⁷⁾.

In the navigation problem it is desired that the optimal global estimation be obtained from the unfailed sensor outputs. When a failure is detected, and the faulty sensor is identified, the estimate that includes the information from this failed sensor would be declared invalid. The estimate that includes information from all other sensors then becomes new global estimate. The estimate after failure is the same as if the failed sensor had never been part of the system.

Consider the discrete-time model

$$x(k+1) = \phi(k)x(k) + G(k)w(k) \quad (1)$$

Assume that there are N sets measurements from m sensors, then we can design N local filters for N local models described by the following equations

$$x_i(k+1) = \phi_i(k)x_i(k) + G_i(k)w(k) \quad (2)$$

$$y_i(k) = H_i(k)x_i(k) + v_i(k) \quad (3)$$

$$i=1,2,\dots,N$$

where $x_i(k)$ is the system state vector which will be estimated through the measurements $y_i(k)$ from the i th sensor subset. The noise $w(k)$ and $v_i(k)$ are independent, zero-mean, white Gaussian sequences having covariances of intensity $Q(k)$ and $R_i(k)$ respectively. The initial state $x_i(0)$ is a Gaussian random vector with zero-mean and covariance $P_i(0)$, and is independent of the noise.

Each local filter is a Kalman filter, which can be used to generate a local estimate of the system state vector. Then all estimates of the local filter are put into a master filter to generate global optimal estimate of the system state vector. If a failure occurs in the i th sensor set, the estimate of the i th local filter will not be correct, it should not be put into master filter so that the global estimate of the system state vector is always correct. We can also use unfailed sensor outputs to generate optimal estimate of the output measured by failed sensor.

The optimal estimates of the local states are given by the local Kalman filter

$$\hat{x}_i(k+1/k+1) = \hat{x}_i(k+1/k) + K_i(k+1)[y_i(k+1) - H_i(k+1)\hat{x}_i(k+1/k)] \quad (4)$$

$$\hat{x}_i(k+1/k) = \phi_i(k)\hat{x}_i(k/k) \quad (5)$$

$$K_i(k+1) = P_i(k+1/k)H_i^T(k+1) \times [H_i(k+1)P_i(k+1/k)H_i^T(k+1) + R_i(k+1)]^{-1} \quad (6)$$

$$P_i(k+1/k) = \phi_i(k)P_i(k/k)\phi_i^T(k) + G_i(k)Q(k)G_i^T(k) \quad (7)$$

$$P_i(k+1/k+1) = [I - K_i(k+1)H_i(k+1)] \times P_i(k+1/k) \quad (8)$$

For generating the global state estimate \hat{x}_g using N local state estimates \hat{x}_i ($i=1,\dots,N$), we have the following theorem⁽⁶⁾.

Theorem 1: In a linear discrete-time system, assume \hat{x}_i ($i=1,\dots,N$) are N local

state estimates with the error covariances P_{ii} , if $P_{ij}=0$ for all $i \neq j$, then the optimal global state estimate is given by

$$\hat{x}_g = P_g \sum_{i=1}^N P_{ii}^{-1} \hat{x}_i \quad (9)$$

where

$$P_g = \left(\sum_{i=1}^N P_{ii}^{-1} \right)^{-1} \quad (10)$$

Corollary 1: In a linear discrete-time system, if there is no process noise, i.e. $Q(k)=0$, and initial value $P_{ij}(0)=0$ for all $i \neq j$, then (9) and (10) hold.

If the j th sensor failed, the local estimate \hat{x}_j should be eliminated from \hat{x}_g . Then (9) and (10) become as

$$\hat{x}_g = P_g \sum_{\substack{i=1 \\ i \neq j}}^N P_{ii}^{-1} \hat{x}_i \quad (11)$$

$$P_g = \left(\sum_{\substack{i=1 \\ i \neq j}}^N P_{ii}^{-1} \right)^{-1} \quad (12)$$

The estimate of the output measured by the j th failed sensor can be obtained by

$$\hat{y}_j = H_j \hat{x}_g \quad (13)$$

In general, $P_{ij} \neq 0$ for all $i \neq j$, thus Theorem 1 can not be applied directly in this case. This problem can be solved by using the upper bound technique proposed by Carlson⁽⁷⁾.

A composite state vector and corresponding covariance matrix are defined as follows:

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} \quad P = \begin{bmatrix} P_{11} & \cdots & P_{1N} \\ \vdots & \ddots & \vdots \\ P_{N1} & \cdots & P_{NN} \end{bmatrix}$$

Then we have following composite state equation

$$\begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}_{k+1} = \begin{bmatrix} \phi_1 & & \\ & \ddots & \\ & & \phi_N \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}_k + \begin{bmatrix} G_1 \\ \vdots \\ G_N \end{bmatrix} w(k) \quad (14)$$

Processing a local measurement from i th sensor subset in a global optimal sense, we obtain

$$y_i = H_i x_i + v_i = Hx + v_i \quad (15)$$

$$H = [0 \cdots 0 \quad H_i \quad 0 \cdots 0] \quad (16)$$

$$A = HPH^T + R_i = H_i P_{ii} H_i^T + R_i \quad (17)$$

Measurement Update

$$\hat{x}^+ = \hat{x} + PH^T A^{-1} (y_i - H\hat{x}) \quad (18)$$

$$P^+ = P - KHP = P - PH^T A^{-1} HP \quad (19)$$

The j th element of (18) is

$$\hat{x}_j^+ = \hat{x}_j + P_{ji} H_i^T A^{-1} (y_i - H_i \hat{x}_i) \quad (20)$$

The jk element of (19) is

$$P_{jk}^+ = P_{jk} - P_{ji} H_i^T A^{-1} H_i P_{ki}^T \quad (21)$$

Note that:

(a) When $j=i$, measurement y_i affects only local state x_i , i.e.

$$\hat{x}_i^+ = \hat{x}_i + P_{ii} H_i^T A^{-1} (y_i - H_i \hat{x}_i) \quad (22)$$

(b) For $j \neq i$, measurement y_i dose not affect \hat{x}_j , leaving $\hat{x}_j^+ = \hat{x}_j$.

(c) For $j \neq i$, if $P_{ij}(0)=0$, $P_{ij}(k)$ remains zero afterwards, then

$$P_{ii}^+ = P_{ii} - P_{ii} H_i^T A^{-1} H_i P_{ii}^T \quad (23)$$

Therefore, if the local estimates are initially uncorrelated, then the local measurement sets can be processed independently, and they remain uncorrelated forever.

Time Update

$$\begin{bmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_N \end{bmatrix}_{k+1} = \begin{bmatrix} \phi_1 & & \\ & \ddots & \\ & & \phi_N \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \vdots \\ \hat{x}_N \end{bmatrix}_k \quad (24)$$

$$\begin{bmatrix} P_{11} & \cdots & P_{1N} \\ \vdots & \ddots & \vdots \\ P_{N1} & \cdots & P_{NN} \end{bmatrix}_{k+1} =$$

$$\begin{bmatrix} \phi_1 & & \\ & \ddots & \\ & & \phi_N \end{bmatrix} \begin{bmatrix} P_{11} & \cdots & P_{1N} \\ \vdots & \ddots & \vdots \\ P_{N1} & \cdots & P_{NN} \end{bmatrix}_k \begin{bmatrix} \phi_1^T & & \\ & \ddots & \\ & & \phi_N^T \end{bmatrix} +$$

$$\begin{bmatrix} G_1 \\ \vdots \\ G_N \end{bmatrix} Q [G_1^T \cdots G_N^T] \quad (25)$$

From (24) and (25), we know that state update can be separated, if there is no process noise, i.e. $Q=0$, and $P_{ij}(0)=0$ for all $i \neq j$, then covariance update can also be separated. Thus Corollary 1 is proved.

If $Q \neq 0$ in (25), then P_{ij} can not remain zero even though $P_{ij}(0)=0$ for all $i \neq j$. Note that

$$\begin{bmatrix} G_1 \\ \vdots \\ G_N \end{bmatrix} Q [G_1^T \cdots G_N^T] = \begin{bmatrix} G_1 & & \\ & \ddots & \\ & & G_N \end{bmatrix} \begin{bmatrix} Q & \cdots & Q \\ \vdots & \ddots & \vdots \\ Q & \cdots & Q \end{bmatrix} \begin{bmatrix} G_1^T & & \\ & \ddots & \\ & & G_N^T \end{bmatrix} \quad (26)$$

From matrix theory, we know that

$$\begin{bmatrix} Q & \cdots & Q \\ \vdots & \ddots & \vdots \\ Q & \cdots & Q \end{bmatrix} \leq \begin{bmatrix} r_1 Q & & \\ & \ddots & \\ & & r_N Q \end{bmatrix} \quad (27)$$

$$1/r_1 + \cdots + 1/r_N = 1, \quad 0 \leq 1/r_i \leq 1 \quad (28)$$

The right hand side is an upper bound of the left hand side.

Substituting this result in (26) then yields the following upper bound on the covariance P

$$\begin{bmatrix} P_{11} & \cdots & P_{1N} \\ \vdots & \ddots & \vdots \\ P_{N1} & \cdots & P_{NN} \end{bmatrix}_{k+1} \leq$$

$$\begin{bmatrix} \phi_1 & & \\ & \ddots & \\ & & \phi_N \end{bmatrix} \begin{bmatrix} P_{11} & \cdots & P_{1N} \\ \vdots & \ddots & \vdots \\ P_{N1} & \cdots & P_{NN} \end{bmatrix}_k \begin{bmatrix} \phi_1^T & & \\ & \ddots & \\ & & \phi_N^T \end{bmatrix} +$$

$$\begin{bmatrix} G_1 & & \\ & \ddots & \\ & & G_N \end{bmatrix} \begin{bmatrix} r_1 Q & & \\ & \ddots & \\ & & r_N Q \end{bmatrix} \begin{bmatrix} G_1^T & & \\ & \ddots & \\ & & G_N^T \end{bmatrix} \quad (29)$$

Thus we can obtain the following partition results

$$P_{ii} = \phi_i P_{ii}^- \phi_i^T + G_i r_i Q G_i^T \quad (30)$$

$$P_{ji} = \phi_j P_{ji}^- \phi_i^T = 0, \quad \text{if } P_{ji}^- = 0 \quad (31)$$

A similar upper bound to that of (27) can be placed on the initial value of the state covariance.

Failure Detection and Isolation for Subsystems

In multisensor systems, multiple local estimates can be computed with each estimate being dependent on a subset of the available sensors. These local estimates can then be combined to obtain a global estimate of the system state. If subtle sensor failures occur that cannot be detected by sensor self-test, local filter performance will be degraded, and overall system performance will be affected. An approach is therefore needed to determine the validity of a local filter estimate computed from sensors that are subject to subtle failures.

Chi-Square Test

We model the problem of failure detection as that of detecting a signal of unknown magnitude that occurs at an unknown time, and assume the following model for system state $x(k)$ and observation $y(k)$:

$$x(k+1) = \phi(k)x(k) + G(k)w(k) \quad (32)$$

$$y(k) = H(k)x(k) + \gamma\sigma(k, \phi) + v(k) \quad (33)$$

where $w(k)$ and $v(k)$ are independent, zero-mean, Gaussian white noise sequences having covariances of intensity $Q(k)$ and $R(k)$ respectively. The initial state $x(0)$ is a Gaussian random vector independent of $w(k)$ and $v(k)$ and has mean x_0 and covariance P_0 . The failure model is represented as a random vector γ . The failure event is represented by a step function $\sigma(k, \phi)$ which is unity for $k \geq \phi$, where ϕ is the time at which the failure occurs, and zero elsewhere. The dimension of vector $x(k)$, $y(k)$ are n and m respectively.

A two-ellipsoid test has been applied to the detection of specific failure modes by Kerr^(8,9). Based on Kerr's results, Brumback and Srinath⁽¹⁰⁾ proposed a chi-square test which is simpler to implement than Kerr's test. But they did not explain how to obtain the scalar test statistics. Using detection theory we will derive the scalar test statistic in the following.

The two-ellipsoid test used two estimates: $\hat{x}_1(k)$ which is the estimate obtained using the measurement $y(k)$ via a

Kalman filter, and the estimate $\hat{x}_2(k)$ which is computed from the appriori information only. The two estimates are obtained from the following sets of equations

$$\hat{x}_1(k+1)=[I-K(k+1)H(k+1)]\phi(k)\hat{x}_1(k)+K(k+1)y(k+1) \quad (34)$$

$$\begin{aligned} \hat{x}_1(0) &= x_0 \\ P_1(k+1/k) &= \phi(k)P_1(k/k)\phi^T(k)+G(k)Q(k)G^T(k) \end{aligned} \quad (35)$$

$$\begin{aligned} P_1(0) &= P_0 \\ K(k+1) &= P_1(k+1/k)H^T(k+1) \times \\ & [H(k+1)P_1(k+1/k)H^T(k+1)+R(k+1)]^{-1} \quad (36) \\ P_1(k+1/k+1) &= [I-K(k+1)H(k+1)]P_1(k+1/k) \quad (37) \end{aligned}$$

and
$$\hat{x}_2(k+1) = \phi(k)\hat{x}_2(k) \quad (38)$$

$$\begin{aligned} \hat{x}_2(0) &= x_0 \\ P_2(k+1) &= \phi(k)P_2(k)\phi^T(k)+G(k)Q(k)G^T(k) \quad (39) \\ P_2(0) &= P_0 \end{aligned}$$

Define the estimation errors $e_1(k)$ and $e_2(k)$ as

$$e_1(k) = \hat{x}_1(k) - x(k) \quad (40)$$

$$e_2(k) = \hat{x}_2(k) - x(k) \quad (41)$$

Define

$$\beta(k) = e_1(k) - e_2(k) \quad (42)$$

Since each filter is linear, and each estimate is unbiased, so that

$$E\{\beta(k)\} = E\{e_1(k) - e_2(k)\} = 0 \quad (43)$$

The covariance of $\beta(k)$

$$\begin{aligned} W(k) &= E\{\beta(k)\beta^T(k)\} \\ &= P_1(k) + P_2(k) - P_{12}(k) - P_{12}^T(k) \end{aligned} \quad (44)$$

where

$$P_{12}(k) = E\{e_1(k)e_2^T(k)\} = P_{21}^T(k) \quad (45)$$

Since $\beta(k)$ is Gaussian with zero mean and covariance $W(k)$ of (44), its distribution is completely defined.

When a failure occurs in the subsystem, the estimate $\hat{x}_1(k)$ will be biased. However, the estimate $\hat{x}_2(k)$ is still unbiased since it is independent of the faulty measurement $y(k)$. Therefore, $\beta(k)$ is biased according to (42). By detecting the difference in the mean of $\beta(k)$, we can determine if failure has occurred.

For the vector $\beta(k)$, the two

hypotheses to be tested are identified as H_0 , the normal mode, and H_1 the failure mode. Under H_1 hypothesis the bias failure is assumed, with the bias failure magnitude and sign being unknown completely. The statistics of $\beta(k)$ under the two hypotheses are:

$$\begin{aligned} H_0: E\{\beta(k)\} &= 0 \quad E\{\beta(k)\beta^T(k)\} = W(k) \\ H_1: E\{\beta(k)\} &= \mu \quad E\{[\beta(k)-\mu][\beta(k)-\mu]^T\} = W(k) \end{aligned}$$

where μ , the mean in the failure mode, can take both negative and positive values.

Since $\beta(k)$ is a Gaussian random vector, the log likelihood ratio $\Lambda(k)$ for the two hypotheses is given by⁽¹¹⁾

$$\begin{aligned} \Lambda(k) &= \{\beta^T(k)W^{-1}(k)\beta(k) - \\ & [\beta(k)-\mu]^T W^{-1}(k)[\beta(k)-\mu]\} / 2 \end{aligned} \quad (46)$$

The maximum likelihood estimate $\hat{\mu}$ of μ is the value which maximizes the expression (46). Clearly, it gives

$$\hat{\mu} = \beta(k) \quad (47)$$

Substituting this result into the expression for $\Lambda(k)$ yields the detection decision function (the test statistic) $\lambda(k)$

$$\lambda(k) = \beta^T(k)W^{-1}(k)\beta(k) \quad (48)$$

The test statistic $\lambda(k)$ is chi-square distributed with n degrees of freedom, n is the dimension of x . The test for failure detection is

$$\begin{cases} \lambda(k) \geq T_D & \text{failure} \\ \lambda(k) < T_D & \text{no failure} \end{cases} \quad (49)$$

where the threshold T_D is determined from the table of chi-square distribution and

$$\Pr[\lambda(k) > T_D | H_0] = P_{fa}$$

where P_{fa} is the probability of false alarm, which can be obtained by integrating the chi-square density function of $\lambda(k)$.

(44) can be simplified to⁽²⁾

$$W(k) = P_2(k) - P_1(k) \quad (50)$$

However, direct computation of $W^{-1}(k)$ can be avoided by computing the Cholesky decomposition,

$$W(k) = LL^T \quad (51)$$

solving a triangular system of equations, and computing $\lambda(k)$ as an inner product of a vector with itself, we obtain

$$\lambda(k) = (L^{-1}\beta)^T(L^{-1}\beta) \quad (52)$$

Residual Test

As the chi-square test uses two estimates, the computation or storage requirements on the board computer will be increased. We will introduce a residual test, which only uses the residual of Kalman filter to detect the possible failure, to avoid the addition computations or storages required in the chi-square test.

The residual of Kalman filter is given by

$$r(k) = y(k) - H(k)\hat{x}(k+1/k) \quad (53)$$

where the prediction $\hat{x}(k+1/k)$ is given by

$$\hat{x}(k+1/k) = \Phi(k)\hat{x}(k) \quad (54)$$

In the absence of failures, the residual $r(k)$ is zero-mean, white Gaussian sequence with the following covariance

$$V(k) = H(k)P(k/k-1)H^T(k) + R(k) \quad (55)$$

In the presence of a failure, the residual $r(k)$ will not be zero-mean, white noise sequence any more. It is the difference in the means of the residual $r(k)$ in the absence and presence of failure which provides a basis for failure detection. Therefore, we have a binary hypotheses as the following

H_0 : no failure

$$E\{r(k)\} = 0 \quad E\{r(k)r^T(k)\} = V(k)$$

H_1 : failure occurred

$$E\{r(k)\} = \mu \quad E\{[r(k) - \mu][r(k) - \mu]^T\} = V(k)$$

Similarly, we can derive the scalar test statistic $\lambda(k)$ as

$$\lambda(k) = r^T(k)V^{-1}(k)r(k) \quad (56)$$

The test statistic $\lambda(k)$ is also chi-square distributed with m degrees of freedom, m is the dimension of y . The test for failure detection is

$$\begin{cases} \lambda(k) \geq T_D & \text{failure} \\ \lambda(k) < T_D & \text{no failure} \end{cases} \quad (57)$$

The threshold T_D can also be determined from the table of the chi-square distribution.

Application

The results of the previous three sections provide the necessary elements from which a fault tolerant multisensor

navigation system can be developed. In this section we apply these results to a system composed of a SINS, a GPS receiver and a Doppler radar. This SINS/GPS/Doppler integrated navigation system consists of two subsystems: SINS/GPS and SINS/Doppler navigation systems which are constructed by integrating a SINS with a GPS receiver and a Doppler radar respectively. The SINS provides outputs of the aircraft navigation state (position, velocity, acceleration and attitude) coordinated in a local-level reference frame. The GPS receiver provides pseudorange and delta pseudorange measurements which are used to provide the best estimate of the aircraft position, velocity and system time. The Doppler radar provides aircraft velocity in an aircraft body reference frame. The integrated system performance objectives are to compute the MMSE estimate of the aircraft navigation state conditioned on all data since system initialization. If a failure is detected, the system must compute the most accurate estimate of the aircraft navigation state conditioned on data from the unfailed sensors, as though the failed sensor has never been part of the system.

The error state equation of the integrated navigation system is

$$\dot{x}(t) = Fx(t) + \Gamma W(t) \quad (58)$$

where $x \in R^{21}$ is defined as $x = [\Delta\lambda \ \Delta L \ \Delta h \ \Delta v_x \ \Delta v_y \ \Delta v_z \ \Delta T_x \ \Delta T_y \ \Delta T_z \ \epsilon_x \ \epsilon_y \ \epsilon_z \ \nabla_x \ \nabla_y \ \nabla_z \ E_{po} \ E_{hsf} \ d_k \ d_\Delta \ d_{tu} \ d_{tru}]^T$

$\Delta\lambda$ — longitude error

ΔL — latitude error

Δh — altitude error

Δv_x — east velocity error

Δv_y — north velocity error

Δv_z — vertical velocity error

T_x — east attitude error

T_y — north attitude error

T_z — azimuth error

ϵ_x — x gyro drift

ϵ_y — y gyro drift

ϵ_z — z gyro drift

∇_x — x accelerometer bias

∇_y — y accelerometer bias

∇_z — z accelerometer bias

E_{po} — altimeter bias

E_{hsf} — altimeter scale factor error

d_k — doppler scale factor error

d_Δ — doppler drift angle

d_{tu} — GPS clock phase error

d_{tru} — GPS clock frequency error

$W(t)$ is system noise, F , Γ are given by Zhang⁽⁶⁾.

The two measurement equations of two subsystems SINS/GPS and SINS/Doppler are

$$y_1 = H_1 x + v_1 \quad (59)$$

$$y_2 = H_2 x + v_2 \quad (60)$$

where v_1 and v_2 are independent, zero-mean, white Gaussian noises with covariances R_1 and R_2 respectively. y_1 , the measurement from SINS/GPS, is given by

$$y_1 = \begin{bmatrix} \rho_G - \rho_I \\ \dot{\rho}_G - \dot{\rho}_I \end{bmatrix} \quad (61)$$

where $\rho_G = \rho + c\Delta t_u$ is the pseudorange vector given by GPS receiver. ρ is the range vector from the GPS user to the satellites, c is the speed of light, Δt_u is the time difference between the user's clock and the clock of satellites. ρ_I is the pseudorange vector calculated from SINS. $\dot{\rho}_G$, $\dot{\rho}_I$ are delta pseudorange vectors, which are given by GPS and SINS respectively.

$$y_2 = [v_I - v_D] \quad (62)$$

where v_I is the aircraft velocity vector obtained from SINS, v_D is the aircraft velocity vector measured by Doppler radar. A detail description of the model is given by Zhang⁽⁶⁾.

Based on equations (58)-(62), two local Kalman filters can be constructed to give two local estimates x_1 , x_2 of system state and their estimate error covariance matrices P_1 , P_2 .

The SINS/GPS/Doppler fault tolerant integrated navigation system is shown in Figure 1. In this system, SINS outputs are measurements of angular rate vector w and specific force vector f . The reliability of SINS can be ensured by using fault tolerant schemes described in second section. Then the SINS/GPS navigation system failure means that GPS has failed. Similarly, the SINS/Doppler navigation system failure means that Doppler radar has failed. Two local filters each processes one of the two sets of measurements, and estimate the states of the integrated system. The global estimate is computed by combining the estimates of the SINS/GPS and

SINS/Doppler filter using the combining algorithm presented in the third section. Each filter implements the chi-square or residual test and reports whether or not it has detected a failure. Since there are only two subsystems, the global estimate after a sensor failure is the estimate which is obtained from the unfailed sensor outputs. After a GPS or Doppler failure the reconfiguration or repair should be done if it is possible, so that the accurate global estimate can be obtained and the much high reliability can be achieved.

The system of Figure 1 was simulated via Monte Carlo techniques. The mission scenario is flight due East at 300m/s. The integrate step is 0.025s, the filter computation cycles are all 1s, and the number of Monte Carlo samples is 100. A $1\mu g$ accelerometer bias failure occurred at $t=30s$, and 100m pseudorange measurement bias failure occurred in GPS at $t=50s$.

Figure 2 and Figure 3 show the position and velocity errors, respectively, of the SINS/GPS subsystem. These errors are not affected by the accelerometer failure since the fault tolerant schemes for SINS are applied. However, after a GPS failure, the altitude error jumped from about 16m to about 30m, and other errors became divergent. It is shown that a GPS failure will result in invalid estimate of the SINS/GPS subsystem.

Figure 4 and Figure 5 show the position and velocity errors, respectively, of the SINS/GPS/Doppler fault tolerant integrated navigation system. From these figures, we know that the errors of the integrated system are not affected by both the accelerometer failure and the GPS failure.

In one word, the proposed fault tolerant system can be used to provide reliable and accurate estimate of the aircraft navigation state in the possible presence of sensor failures. The obtained estimate of error state is applied to the compensation of the integrated system so that the most accurate navigation solution can be obtained.

Conclusions

In this chapter, we have presented an approach for fault tolerant estimation in a multisensor navigation system which includes one or more SINS and one or more navigation reference sensors such as GPS receiver, Doppler radar, terrain aided system, air-data system, TACAN and VOR/DME. The performance objective is to compute the most accurate estimate of the aircraft navigation state, based on only unfailed sensors. The approach is 1)

improve the reliability of SINS using redundant sensor configurations with corresponding fault tolerant schemes, 2) compute multiple local estimates of the aircraft navigation state, 3) determine which estimates are valid, 4) based on the validity pattern decide which sensor has failed, and 5) generate the global estimate by combining those estimates which do not use data from the failed sensor. In the absence of failures, this approach can provide a MMSE estimate of the aircraft navigation state. In the presence of failure, it can provide a most accurate estimate of the aircraft navigation state. A simulated example of an aircraft navigation system consisting of a SINS, a GPS receiver and a Doppler radar was presented to illustrate the design approach and the performance of the resulting system.

References

- [1] Kerr, T.H., Decentralized Filtering and Redundancy Management for Multisensor Navigation, *IEEE Trans. Aerospace and Electronic Systems*, Vol. AES-23, 83-119, 1987
- [2] Brumback, B.D. and Srinath, M.D., A Fault-Tolerant Multisensor Navigation System Design, *IEEE Trans. Aerospace and Electronic Systems*, Vol. AES-23, 738-755, 1987a
- [3] Speyer, J.L., Computation and Transmission Requirements for a Decentralized Linear-Quadratic-Gaussian Control Problem, *IEEE Trans. Automatic Control*, Vol. AC-24, 266-269, 1979
- [4] Willsky, A.S., et al, Combining and Updating of Local Estimation and Regional Maps along Sets of One-Dimensional Tracks, *IEEE Trans. Automatic Control*, Vol. AC-27, 799-813, 1982
- [5] Tylee, J.L., Estimation of Failed Sensor Outputs, *Nuclear Science and Engineering*, Vol.96, 145-152, 1987
- [6] Zhang, H.G., *Decentralized Estimation Method with Application to Fault Tolerant Multisensor Navigation System Design*, Ph.D dissertation, Beijing University of Aeronautics & Astronautics, Sept., 1991
- [7] Carlson, N.A., Federated Square Root Filter for Decentralized Parallel Processes, *Proc. of IEEE NEACON*, 1448-1456, 1987
- [8] Kerr, T.H., Real-Time Failure Detection: a Nonlinear Optimization Problem That Yields a Two-Ellipsoid Test, *Journal Optimization Theory and Application*, Vol. 22, 509-536, 1977
- [9] Kerr, T.H., Statistical Analysis of a Two-Ellipsoid Overlap Test for Real-Time Failure Detection, *IEEE Trans. Automatic Control*, Vol. AC-25, 762-773, 1980
- [10] Brumback, B.D. and Srinath, M.D., A Chi-Square Test for Fault-Detection in Kalman Filter, *IEEE Trans. Automatic Control*, Vol. AC-32, 552-554, 1987
- [11] Van Trees, H.L., *Detection, Estimation, and Modulation Theory*, John Wiley and Sons, Inc., 1968
- [12] Zhang, H.G. and Zhang, H.Y., Fault Management for Redundant Sensor Configurations, *Journal of Beijing University of Aeronautics & Astronautics*, No.3, 21-28, 1990

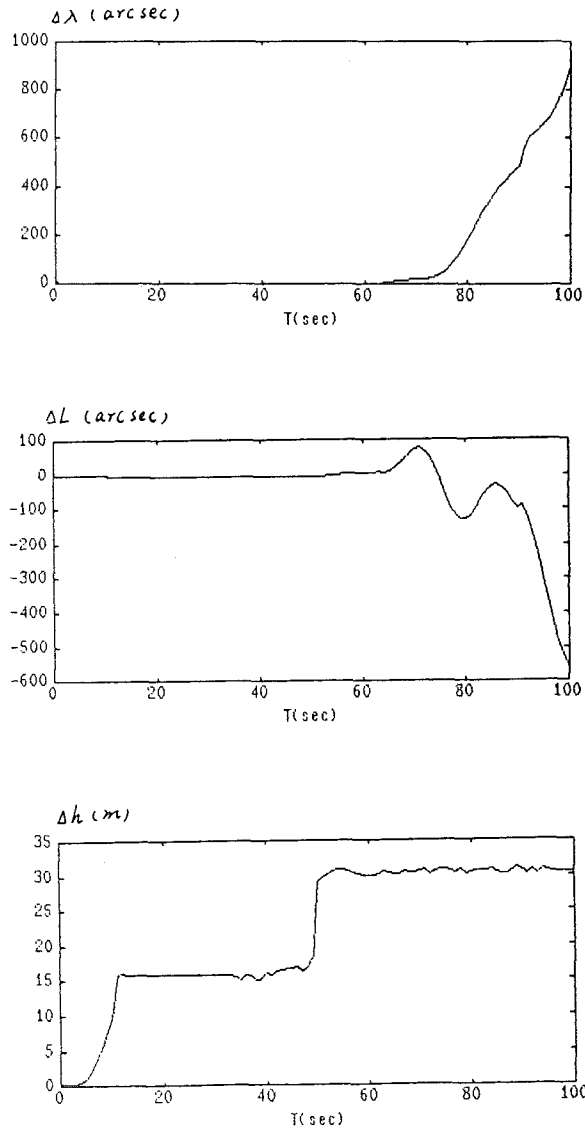


Figure 2. SINS/GPS Subsystem Position Errors

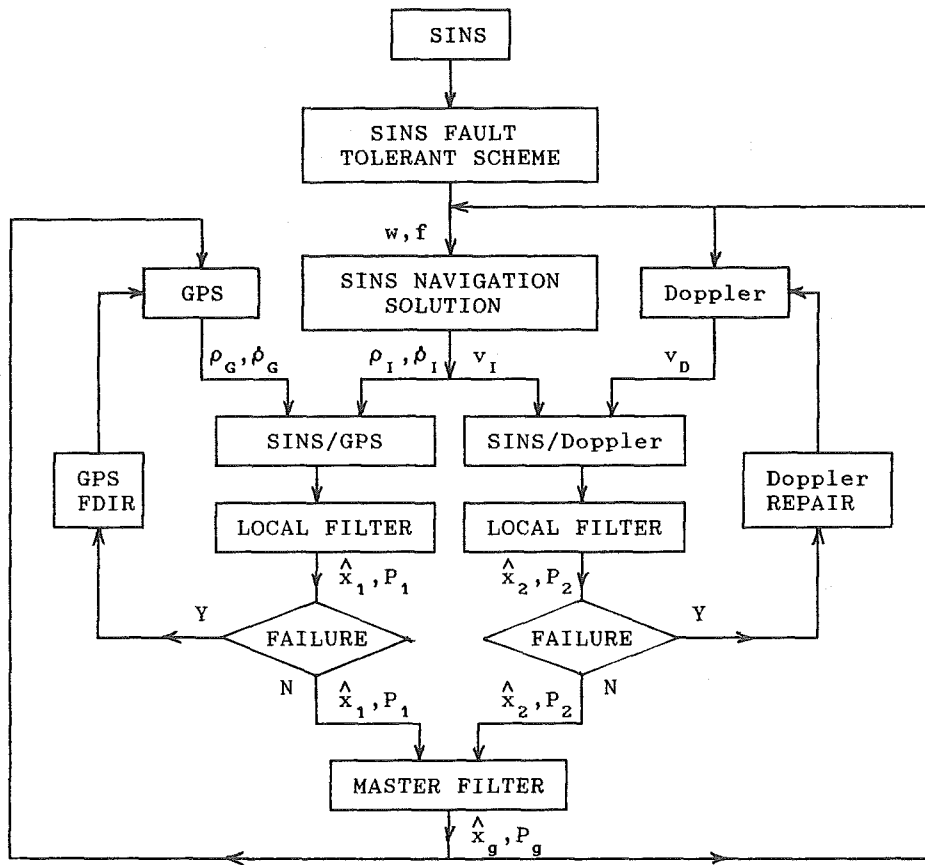


Figure 1. The SINS/GPS/Doppler Fault Tolerant Integrated Navigation System

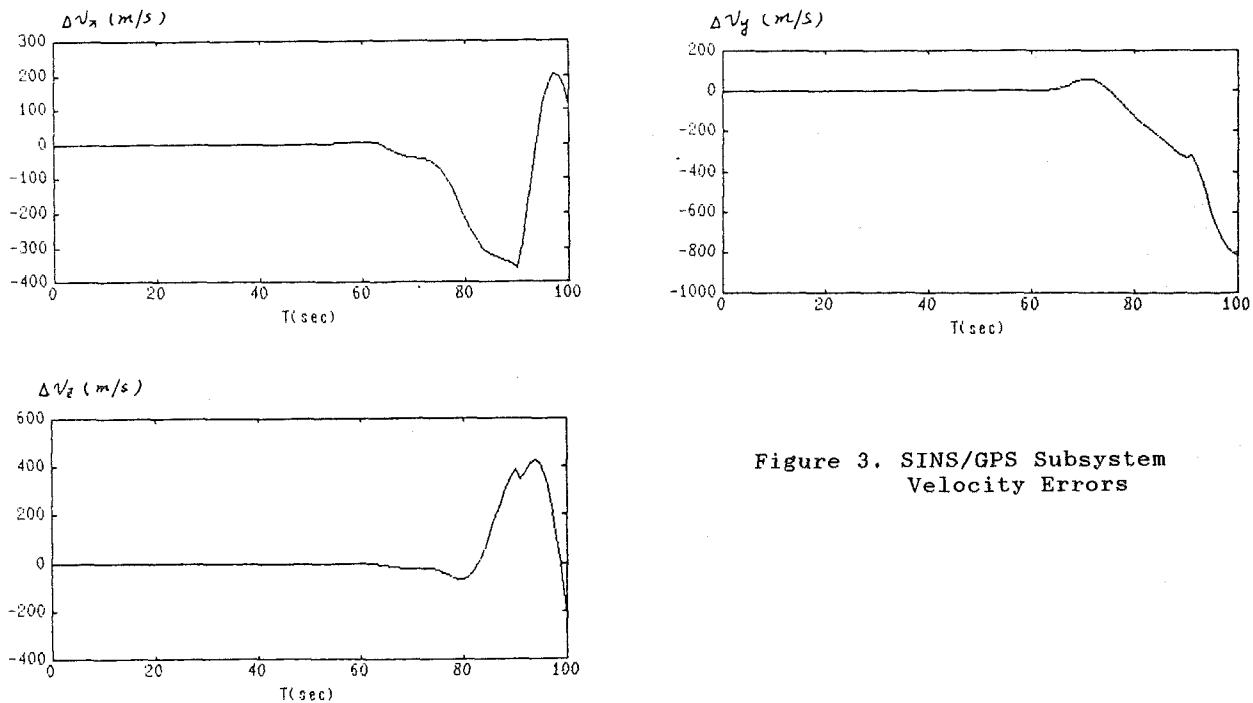


Figure 3. SINS/GPS Subsystem Velocity Errors

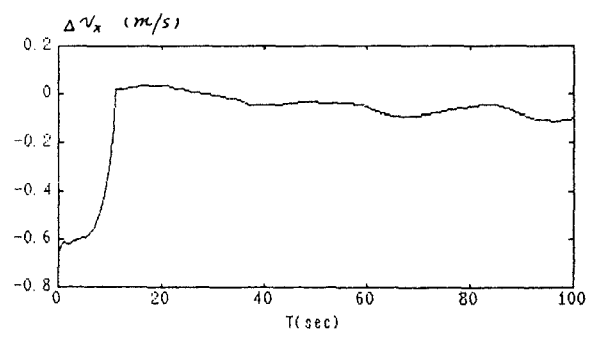
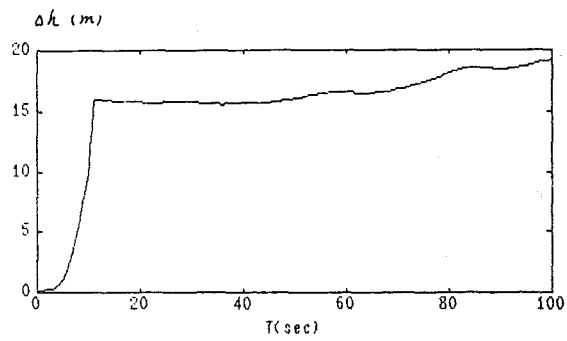
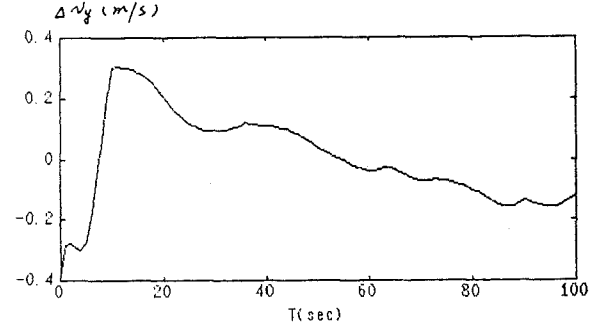
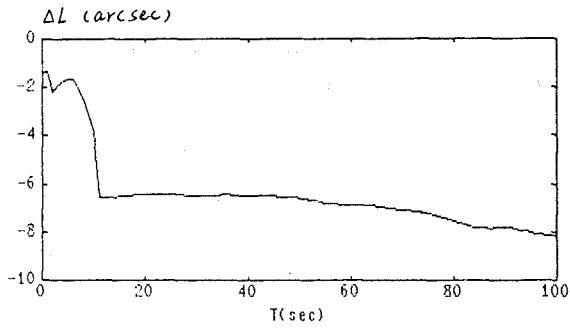
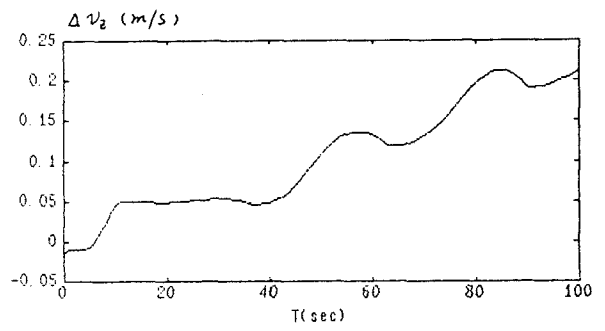
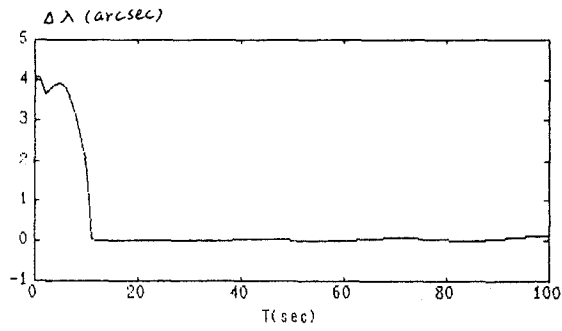


Figure 4. Fault Tolerant System Position Errors

Figure 5. Fault Tolerant System Velocity Errors