FLIGHT PATH OPTIMIZATION AND SUBOPTIMAL CONTROL LAWS SYNTHESIS FOR TRANSPORT MISSION OF MANEUVERABLE AIRCRAFT

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Abstract

The flight path optimization for transport mission of maneuverable aircraft as well as suboptimal control synthesis for the optimal flight path implementation are considered. The calculation of the extremal fields for some examples are carried out by the second order method, which presents the extension of previously known one, and by the maximum principle. The approximations of optimum solutions are proposed for the schedule design. The synthesis for a normal load factor is performed through tracking this approximations. The adaptive extremal law for the engine thrust control is proposed. Some numerical results are presented.

I Introduction

The possibilities of modern onboard computers, as is well known, reveal the ways of optimal solutions usage for aircraft trajectory control. The transport missions for maneuverable aircraft have peculiarity related to the wide range of initial and final conditions which may belong to all range of possible flight regimes. Many papers are devoted to trajectory optimization problems for non maneuverable and maneuverable aircraft including the consideration of the optimal control solution expressed in a feedback form. One of attractive approach to the solution of such problems is related to the so-called singular perturbation method which was successfully employed for practically important tasks[1,2,3,4]. A very useful approach to the solution of usual transportation task in the energy approximation was proposed in Ref.[5]. One needs to point out that the last approach to the solution of transportation task may be extended for two cruise-dash portion on trajectory for maneuverable aircraft. Another approach may be based on computation of extremals field. i.e. on flooding the state-space with extremals and control synthesis on the basis of tracking the approximations to extremals, which satisfy initial and final conditions. This approach is considered in Ref.[6] on the basis of energy formulation.

The similar approach is proposed for some problems in this paper on the basis of the second order method and on the basis of the maximum principle, and is related apparently to the approach in Ref.[7,8]. The second order method, which was developed by one of the authors[9], provides the optimization of control for discrete nonlinear dynamic system (autonomous and non-autonomous) taking account of control/state constraints at fixed and free final time. This method may be considered as a modification or extension of the second order method[10]. There are several ways of engine thrust control. The adaptive extremal control law, which contains Lagrange multipliers, is considered for this purpose in this paper.

This paper does not pretend to describe the transportation control problem solution in full volume, but only to demonstrate some approaches and techniques that may be useful for application.

II Transport Mission

The main transport missions or tasks involve the following functionals: time of flight (t_f), range (l_f) and fuel consumption (w). All the transportation tasks have the initial and final conditions in terms of altitude, speed, flight path angle etc. Some of them may be free.

It is known that cost function for transport tasks to be minimized may be presented in the form

$$J = \lambda_w w + \lambda_t t_f - \lambda_l l_f \qquad (1)$$

where λ_w, λ_t and λ_l are constant non negative Lagrange multipliers.

Taking $\lambda_t = 1$, $\lambda_w = 0$, $\lambda_l = 0$, we have the problem of final time minimization. Taking $\lambda_w = 1$ and $\lambda_w \neq 0$, $\lambda_l \neq 0$, we have the problem of fuel consumption minimization for given t_f and l_f, etc. A similar problem statement may be considered for mission when t_f and l_f correspond to desirable point of interception and it is necessary to minimize fuel consumption. Additional constraints may be imposed on the altitude and speed on some portions of trajectory.

The solution of such problems with the usage of full motion equations for arbitrary initial and final conditions is difficult. It is the reason for considering the simplified models of motion side by side with more exact ones.

III The Main Features of the Second Order Method

The above mentioned transport tasks may be formulated as follows. Let's consider a usual discrete model of nonlinear dynamic system motion

$$x_{k+1} = g^k(x_k, u_k, \Delta t_k),$$
$$\Delta t_{k+1} = \Delta t_k, \qquad (2)$$
$$k = 0, \dots, N-1$$

that corresponds to aircraft continuous motion equations. The equation for Δt is introduced when final time is free.

State variables vector x_k that has n components and control vector u_k that has m components satisfy inequality constraints

$$\varphi^k(x_k, u_k, \Delta t_k) \leq 0, \qquad (3)$$

where φ is vector with p components.

It is necessary to determine the succession $u_0, \dots, u_{N-1}$ and value of Δt so that

$$\psi(x_N) = 0 \tag{4}$$

and

$$J = F(x_N) + \sum_{k=0}^{N-1} L^k(x_k, u_k, \Delta t_k)$$

reaches the minimum, when initial state vector x_0 is given. Vector ψ has q components ($q \le n-1$, if $L \equiv 0$ and $q \le n$, if $L \ne 0$).

Such a problem is similar to that considered in Ref.[10]. The difference is related to introduction of constraints (3) and the use of Δt as state variable. The last permits us to consider the problems with free final time.

Optimality conditions

The first variation of Lagrange function gives

$$\delta J = F_x \delta x_N + \sum_{k=0}^{N-1} (L_x^k \delta x_k + L_u^k \delta u_k + L_{\Delta t}^k \delta \Delta t_k) +$$

$$+ \sum_{k=0}^{N-1} \lambda'_{k+1} (g_x^k \delta x_k + g_u^k \delta u_k + g_{\Delta t}^k \delta \Delta t_k - \delta x_{k+1}) +$$

$$+ \sum_{k=0}^{N-1} \mu_{k+1} (\delta \Delta t_k - \delta \Delta t_{k+1}) + \nu' \psi_x \delta x_N +$$

$$+ \sum_{k=0}^{N-1} \omega'_k (\varphi_x^k \delta x_k + \varphi_u^k \delta u_k + \varphi_{\Delta t}^k \delta \Delta t_k - \delta \varphi^k),$$

where λ, μ, ω, ν are Lagrange multipliers and $()'$ notes transposition. Let's introduce Hamiltonian

$$H^k = L^k(x_k, u_k, \Delta t_k) + \lambda'_{k+1} g^k(x_k, u_k, \Delta t_k) +$$

$$+ \mu_{k+1} \Delta t_k + \omega'_k \varphi^k(x_k, u_k, \Delta t_k)$$

and adopt

$$\lambda'_k = H_x^k, \;\; \mu_k = H_{\Delta t}^k, \;\; k = 0, \dots, N\text{-}1,$$

$$\lambda'_N = (F_x + \nu' \psi_x)', \;\; \mu_N = 0, \tag{5}$$

then

$$\delta J = \lambda'_0 \delta x_0 + \mu'_0 \Delta \delta t_0 + \sum_{k=0}^{N-1} H_u^k \delta u_k - \omega'_k \delta \varphi^k.$$

It may be shown[10] that for optimal control at $\delta x_0 = 0$ the following expressions are valid:

$$H_u^k = 0,$$

$$\omega_k \ge 0, \;\; \varphi^k = 0,$$

$$\omega_k = 0, \;\; \varphi^k < 0, \tag{6}$$

$$k = 0, \dots, N\text{-}1,$$

$$\mu_0 = 0.$$

The determination of optimal solution demands the solution of two-point-boundary-value-problem (TPBVP), described by equations (2), (4), (5) and (6).

The variation of solution

Suppose that we have nominal optimal solution that satisfy (2), (4), (5) and (6). Let's consider small deviations from this solution that arise from desired small disturbances in δx_0, $\delta \psi_0$, $\delta \varphi$, δH_u^k and $\delta \mu_0$. These disturbances lead to the variations of δx_k, $\delta \lambda_k$, $\delta \mu_k$, $\delta \omega_k$, $\delta \nu$ and δu_k that satisfy linearized equations (2), (4), (5) and (6):

$$\begin{aligned}
&\delta x_{k+1} = g_x^k \delta x_k + g_u^k \delta u_k + g_{\Delta t}^k \delta \Delta t_k, \\
&\delta \Delta t_{k+1} = \delta \Delta t_k, \\
&\delta \varphi^k = \varphi_x^k \delta x_k + \varphi_u^k \delta u_k + \varphi_{\Delta t}^k \delta \Delta t_k, \\
&\delta \lambda_k = g_x^{k\prime} \delta \lambda_{k+1} + H_{xx}^k \delta x_k + \\
&+ H_{xu}^k \delta u_k + H_{x\Delta t}^k \delta \Delta t_k + \varphi_x^{k\prime} \omega_k, \\
&\delta \mu_k = \delta \mu_{k+1} + g_{\Delta t}^{k\prime} \delta \lambda_{k+1} + H_{\Delta t x}^k \delta x_k + \\
&+ H_{\Delta t u}^k \delta u_k + H_{\Delta t \Delta t}^k \delta \Delta t_k + \varphi_{\Delta t}^{k\prime} \omega_k, \\
&\delta H_u^k = g_u^{k\prime} \delta \lambda_{k+1} + H_{ux}^k \delta x_k + \\
&+ H_{uu}^k \delta u_k + H_{u\Delta t}^k \delta \Delta t_k + \varphi_u^{k\prime} \omega_k, \\
&k = 0, \dots, N\text{-}1, \\
&\delta \psi = \psi_x \delta x_N, \\
&\delta \lambda_N = (F_{xx} + \nu' \psi_{xx})' \delta x_N + \psi'_x \delta \nu, \;\; \delta \mu_N = 0.
\end{aligned} \tag{7}$$

The equations (7) may be rewritten in more compact form:

$$\delta y_{k+1} = A^k \delta y_k + a^k \delta u_k, \tag{8}$$

$$\delta \pi_k = A^{k\prime} \delta \pi_{k+1} + B^k \delta y_k + b^k \delta u_k + d^{k\prime} \delta \omega_k, \tag{9}$$

$$\delta H_u^k = a^{k\prime} \delta \pi_{k+1} + b^k \delta y_k + H_{uu}^k \delta u_k + \varphi_u^{k\prime} \delta \omega_k, \tag{10}$$

$$\delta \varphi^k = \varphi_u^k \delta u_k + d^k \delta y_k, \tag{11}$$

$$\delta \pi_N = S_N \delta y_N, \tag{12}$$

where

$$\delta y_k = \begin{pmatrix} \delta x_k \\ \delta \Delta t_k \\ \delta \nu \end{pmatrix}, \;\; \delta \pi_k = \begin{pmatrix} \delta \lambda_k \\ \delta \mu_k \\ \delta \psi \end{pmatrix},$$

$$A^k = \begin{pmatrix} g_x^k & g_{\Delta t}^k & Q_{nq} \\ Q_{1n} & 1 & Q_{1q} \\ Q_{qn} & Q_{q1} & I_{qq} \end{pmatrix}, \quad B^k = \begin{pmatrix} H_{xx}^k & H_{x\Delta t}^k & Q_{nq} \\ H_{\Delta tx}^k & H_{\Delta t\Delta t}^k & Q_{1q} \\ Q_{qn} & Q_{q1} & Q_{qq} \end{pmatrix},$$

$$a^k = \begin{pmatrix} g_u^k \\ Q_{1m} \\ Q_{qm} \end{pmatrix}, \quad b^k = \begin{pmatrix} H_{xu}^k \\ H_{\Delta tu}^k \\ Q_{qm} \end{pmatrix}, \quad d^k = (\varphi_x^k \; \varphi_{\Delta t}^k \; Q_{pq}), \tag{13}$$

$$S_N = \begin{pmatrix} F_{xx} + \nu'\psi_{xx} & Q_{n1} & \psi'_x \\ Q_{1n} & 0 & Q_{1q} \\ \psi_x & Q_{q1} & Q_{qq} \end{pmatrix},$$

I_{mm} is the unit matrix, $m \times m$; Q_{ml} is zero matrix, $m \times l$.

The expressions (8)-(12) describe linear TPBVP concerning δy_k, $\delta\pi_k$, δu_k, $\delta\omega_k$. At initial point, the component δx_0 of δy_0 and the components $\delta\mu_0$, $\delta\psi$ of $\delta\pi_0$ are given, at final point, the boundary condition (12) is given. For solving this problem this condition is transferred to the initial point by the help of reverse run method[10]. Assuming that

$$\delta\pi_k = S_k \delta y_k + h_k \tag{14}$$

and substituting (8)-(12) in (14) we'll have

$$\delta\pi_{k+1} = S_{k+1}A^k\delta y_k + S_{k+1}a^k\delta u_k + h_{k+1}.$$

Substitution of this expression in (9), (10) gives

$$\delta\pi_k = Z_{yy}^k \delta y_k + Z_{yu}^k \delta u_k + d^{k\prime}\delta\omega_k + A^{k\prime}h_{k+1}, \tag{15}$$

$$\delta H_u^k = Z_{uy}^k \delta y_k + Z_{uu}^k \delta u_k + \varphi_u^{k\prime}\delta\omega_k + a^{k\prime}h_{k+1}, \tag{16}$$

where

$$Z_{yy}^k = B^k + A^{k\prime}S_{k+1}A^k,$$
$$Z_{yu}^k = b^k + A^{k\prime}S_{k+1}a^k,$$
$$Z_{uu}^k = H_{uu}^k + a^{k\prime}S_{k+1}a^k.$$

If matrix $\begin{pmatrix} Z_{uu}^k & \varphi_u^k \\ \varphi_u^{k\prime} & Q_{pp} \end{pmatrix}$ is not singular, then follows from (11), (16) that

$$\begin{pmatrix} \delta u_k \\ \delta\omega_k \end{pmatrix} = -\begin{pmatrix} Z_{uu}^k & \varphi_u^k \\ \varphi_u^{k\prime} & Q_{pp} \end{pmatrix}^{-1} \begin{pmatrix} Z_{uy}^k \\ d^k \end{pmatrix} \delta y_k - \begin{pmatrix} Z_{uu}^k & \varphi_u^k \\ \varphi_u^{k\prime} & Q_{pp} \end{pmatrix}^{-1} \begin{pmatrix} a^{k\prime}h_{k+1} - \delta H_u^k \\ -\delta\varphi^k \end{pmatrix}. \tag{17}$$

By means of (14), we'll get from (17)

$$\delta\pi_k = \left(Z_{yy}^k - (Z_{yu}^k \; d^k) \begin{pmatrix} Z_{uu}^k & \varphi_u^k \\ \varphi_u^{k\prime} & Q_{pp} \end{pmatrix}^{-1} \begin{pmatrix} Z_{uy}^k \\ d^k \end{pmatrix} \right) \delta y_k + A^{k\prime}h_{k+1} - (Z_{yu}^k \; d^k) \begin{pmatrix} Z_{uu}^k & \varphi_u^k \\ \varphi_u^{k\prime} & Q_{pp} \end{pmatrix}^{-1} \begin{pmatrix} a^{k\prime}h_{k+1} - \delta H_u^k \\ -\delta\varphi^k \end{pmatrix}.$$

Therefore the presentation (14) is valid if

$$S_k = Z_{yy}^k - (Z_{yu}^k \; d^k) \begin{pmatrix} Z_{uu}^k & \varphi_u^k \\ \varphi_u^{k\prime} & Q_{pp} \end{pmatrix}^{-1} \begin{pmatrix} Z_{uy}^k \\ d^k \end{pmatrix},$$

$$h_k = A^{k\prime}h_{k+1} - (Z_{yu}^k \; d^k) \begin{pmatrix} Z_{uu}^k & \varphi_u^k \\ \varphi_u^{k\prime} & Q_{pp} \end{pmatrix}^{-1} \begin{pmatrix} a^{k\prime}h_{k+1} - \delta H_u^k \\ -\delta\varphi^k \end{pmatrix}.$$

It is possible now to compute S_0, h_0 by means of (13) taking into account that $h_N = 0$. The condition (14) at $k = 0$ gives the possibility to compute $\delta\lambda_0$, $\delta\Delta t_0$, $\delta\nu$ that are components of δy_0, $\delta\pi_0$. Thereafter, we'll get the solution of linear problem (8)-(12) using recurrent expressions (8) and (17).

Iterative procedure

Here we'll give a description of some peculiarities of iterative procedure organization, but it is brief because of restricted volume of the paper. Let's assume that we have succession u_k, $k = 0, \ldots, N-1$, t and succession x_k, $k = 0, \ldots, N$, which satisfy (2) and they are near to optimal solution in a sense that initial condition and (3), (4) are satisfied with small disturbances Δx_0, $\Delta\psi$, $\Delta\varphi_k$, ΔH_u^k, $\Delta\mu_0$. Then adopting $\delta x_0 = -\Delta x_0$, $\delta\psi = -\Delta\psi$, $\delta\varphi_k = -\Delta\varphi_k$, $\delta H_u^k = -\Delta H_u^k$, $\delta\mu_0 = -\Delta\mu_0$ it is possible to get the improving variation of solution. But the received in accordance with (8) variations of δx_k after adding to initial succession x_k will not satisfy equations of motion (2) exactly. That is the reason for the next proposal: to compute δx_k (necessary to substitute in (17)) not by use of the first formula in (7), but as the difference

$$\delta x_k^* = g^{k-1}(x_{k-1} + \delta x_{k-1}^*, u_{k-1} + \delta u_{k-1}, \Delta t + \delta\Delta t) - x_k,$$
$$k = 1, \ldots, N,$$

where $\delta x_0^* = \delta x_0$ and δu_{k-1} is determined by (17).

Short description of computer program peculiarities.

The rather complicated computer program was developed on the basis of this problem statement for three-dimensional motion of aircraft.

The list of the main points of the developed program for aircraft path optimization looks as follows:

- smooth approximation of the aerodynamic, thrust-consumption characteristics, and the usage of analytical expressions for the first and the second derivatives, that are necessary in the second order method like Ref.[10],

- the employment of variation of the first order necessary conditions of optimality and solution of TPBVP for linearized direct and conjugate systems by the help of reverse run method in manner like Ref.[10],
- the procedure of calculation of Lagrange multipliers from the minimization of the norm of Hamiltonian derivative vector for the beginning of iteration procedure and their correction by means of neighboring-optimal technique in iterative procedure,
- the employment of the short initial trajectories with fulfillment of convexity condition for iteration procedure, which through the neighboring-optimal technique provides given of vector x_0 and given boundary condition $\psi(x_N) = 0$.
- the correction or improvement of the second derivatives matrix (in the neighboring-optimal technique), which must be reversed on the basis of transition to the principal axes of the appropriate quadratic form.

IV Flight Path Optimization

The results presented in this section are obtained by the neighboring-optimal technique on the basis of the second order method optimization program described in section III. It will be shown that the optimal schedules in space "altitude - specific energy" ((h,E)-space) may by sufficiently simple for approximation. The approximation of throttle control is more complicated.

Equations of motion

The usual equations of motions were used for solving some problems of optimal control in vertical plane by the help of the second order method

$$\dot{V} = g(n_x - \sin\gamma),\ \dot{\gamma} = \frac{g}{V}(n_y - \cos\gamma), \tag{18}$$

$$\dot{h} = V\sin\gamma,\ \dot{l} = V\cos\gamma,$$

$$\dot{W} = -Q_s,$$

where the tangential and the normal load factors

$$n_x = \frac{T_{max}(M,h)\eta}{W}\cos\alpha - \frac{qS}{W}C_D,$$

$$n_y = \frac{T_{max}(M,h)\eta}{W}\sin\alpha + \frac{qS}{W}C_L,$$

the fuel consumption per second

$$Q_s = Q_s(\eta, M, h),$$

the other notation is usual.

All the dependencies such as $T_{max}(M, h)$, $C_D(C_L, M)$, $Q_s(\eta, h, M)$ are approximated by polynomials and fraction-rational functions. The control variables are η and C_L in the presence of inequality constraints:

$$\eta_{min} \le \eta \le 1,\ C_L \le C_{Lmax}(M),\ \frac{qS}{W}C_L \le n_{ymax}.$$

The inequality constraints on state variables are determined by permissible flight domain:

$$V_{min}(h) \le V \le V_{max}(h),\ h \ge h_{min}.$$

The additional constraints on flight path angle may be imposed

$$\gamma_{min} \le \gamma \le \gamma_{max},$$

where γ_{min}, γ_{max} depend on h, V and, maybe, on n_x.

The final conditions for state coordinates are given for each problem.

The transition from take-off to final value of specific energy.

The results of optimization for transition from initial condition corresponding to take-off to the given final value of specific energy ($E = h + V^2/2g$), which correspond to the minimum fuel consumption, are presented in Figures 1-3 for various values of initial weight.

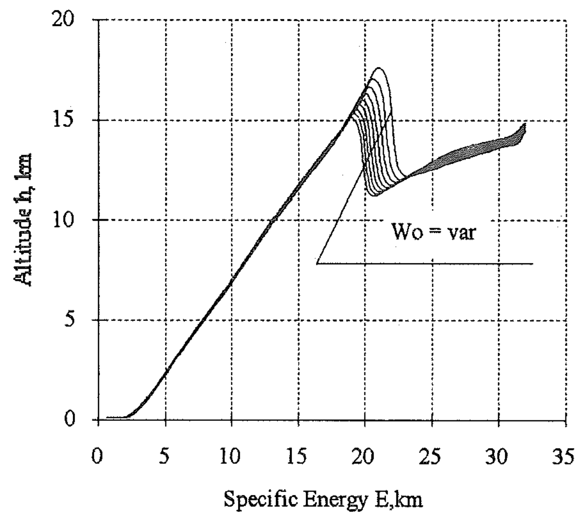

Figure 1. Extremals in (h,E)-space.

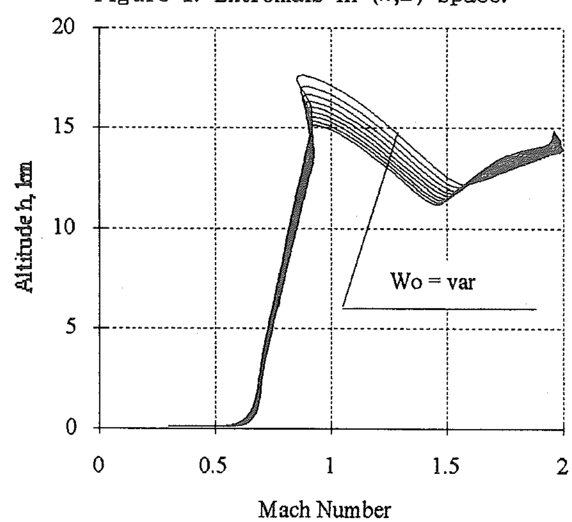

Figure 2. Extremals in (h,M)-space.

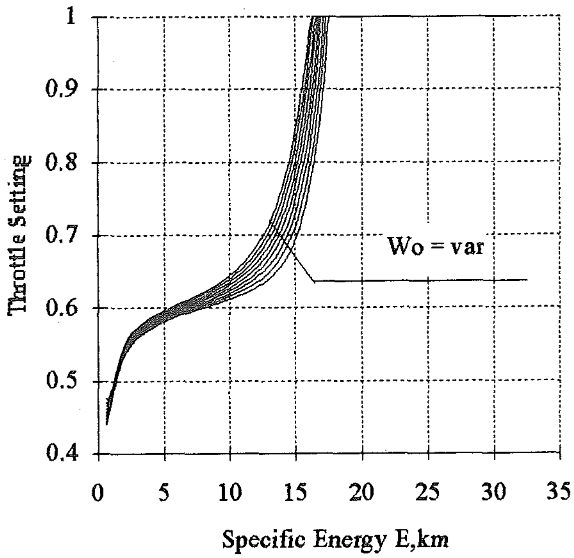

Figure 3. Throttle setting vs specific energy.

It is known that such so-called basic schedules may be considered as the portions of the appropriate more common schedules.

Each of the computed schedules has the portion of descending the shape of which sufficiently depends on initial weight. The constructed family of schedules may be presented in form of two-dimensional function of altitude $h^D(E, W_0)$.** This function after approximation may be used for on-board implementation in control system.

Figures 4-6 show how fuel consumption Δw, time Δt and range Δl for these schedules depend on E for various values of initial weight W_0.

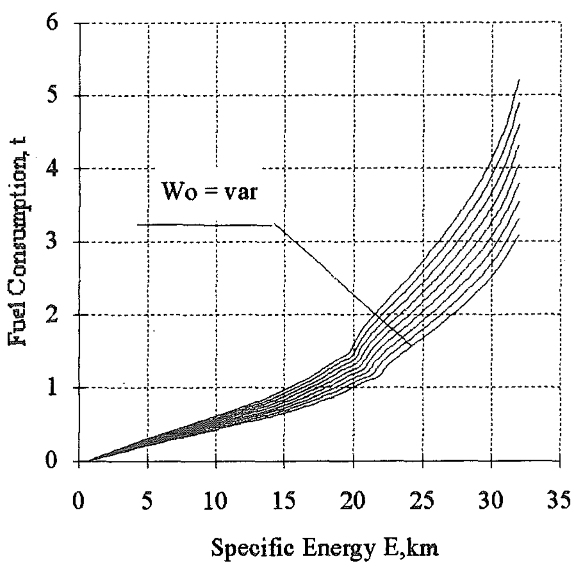

Figure 4. Fuel consumption vs specific energy.

** the superscript D corresponds to desirable value

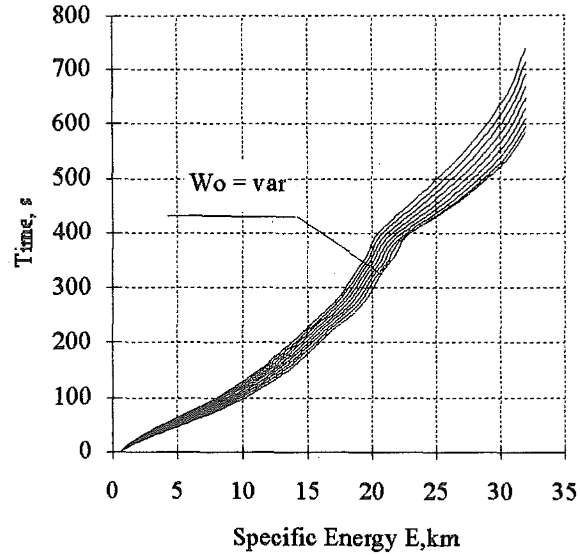

Figure 5. Time of flight vs specific energy.

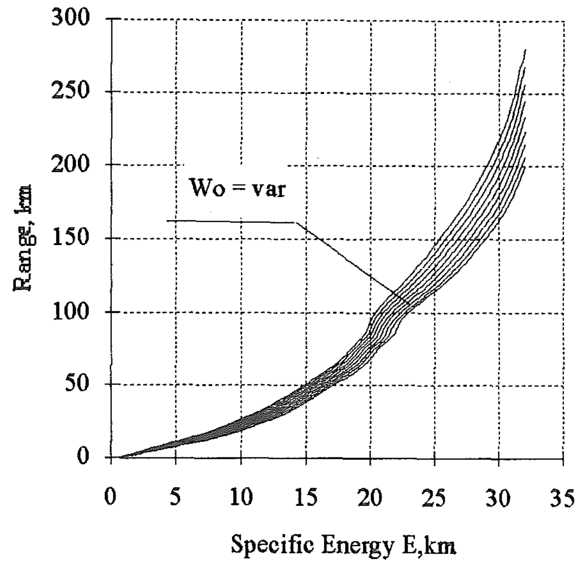

Figure 6. Range vs specific energy.

These dependencies are the functions of two arguments: $\Delta w(E, W_0)$, $\Delta t(E, W_0)$, $\Delta l(E, W_0)$ that have simpler form than basic schedules for approximation. These functions allow us to compute the fuel consumption, time and range for the flight from the point 1 to the point 2 on the schedule:

$$\Delta w_{1,2} = \Delta w(E_2, W_0) - \Delta w(E_1, W_0),$$

$$\Delta t_{1,2} = \Delta t(E_2, W_0) - \Delta t(E_1, W_0),$$

$$\Delta l_{1,2} = \Delta l(E_2, W_0) - \Delta l(E_1, W_0).$$

It may be necessary to go to basic schedule, when instant position is not on basic one corresponding to instant value of aircraft weight.

The dependencies $W_0(E, W)$ allow us to find for instant value of E the weight W_0 that corresponds to initial point of basic schedule and to find the basic schedule $h^D(E, W_0)$ such that it may be desirable to go to the schedule and go along it further. For this purpose a smooth additional schedule of short duration may be constructed for transition from instant point in the vicinity of desirable schedule to this one.

Minimum time transitions in vertical plane

The optimization for minimum time transition in vertical plane from initial condition corresponding to take-off to the final condition h_f = const, $\gamma_f = 0$ for various values of final speed V_f was fulfilled. The appropriate extremals field in (h,E)-space and (h,M)-space are presented in Figures 7,8.

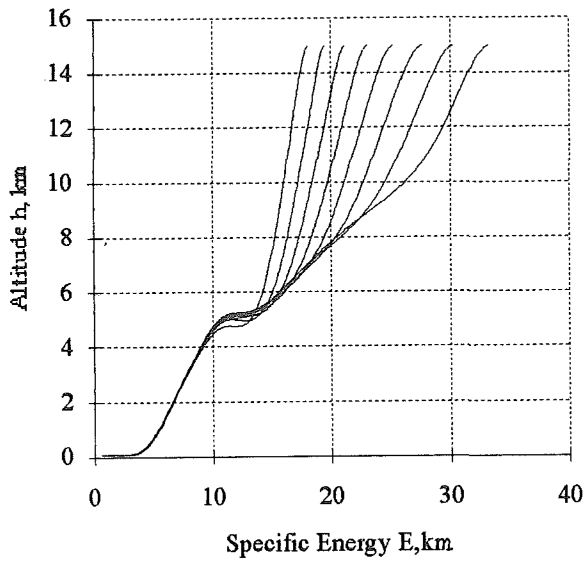

Figure 7. Extremals in the (E,h)-space.

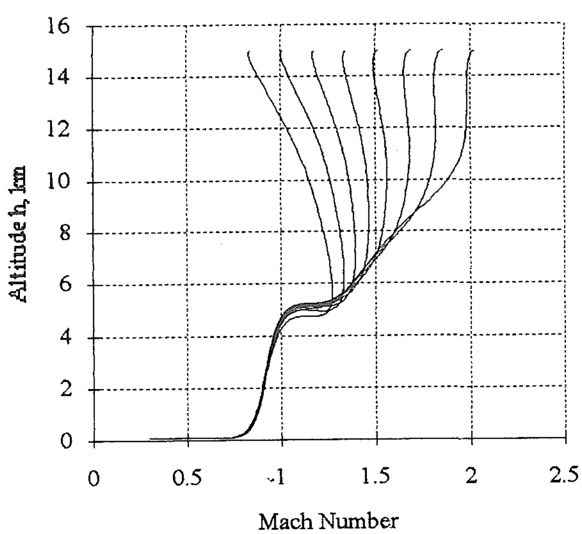

Figure 8. Extremals in the (M,h)-space.

It is seen that schedules in (h,E)-space are simple and more convenient for approximation. The thrust is equal to maximum thrust for all trajectories. These results are analogous to those obtained in Ref.[11,12], but have more regular character.

The flight to cruise point with fixed range

The flight to cruise point with the minimum fuel consumption is the first portion of flight for sufficiently large duration. The final conditions on h, V at $\gamma = 0$ correspond to minimum of the fuel consumption per unit of range (Q_{km}) and depend on the final weight. The approximation of cruise regime in form

$$h_f - a_{11} - a_{12}W_f = 0,$$

$$V_f - a_{21} - a_{22}W_f - a_{23}W_f^2 = 0,$$

$$Q_{km} = a_{31} - a_{32}W,$$

gives good result. Positive coefficients a_{ij} depend on aircraft characteristics, its configuration and atmospheric conditions.

The calculations for several values of range show that, after some value of range, the schedules vary insignificantly and may be easily approximated.

V Minimum Time Descent

We'll consider in this Section the minimum time descent in vertical plane with the usage of maximum principle in simplified problem statement when control variables are η and $\sin\gamma$. This approach is sufficiently simple for generation of extremals field in (h,V)-space for given initial conditions because the problem has only one parameter.

The application of previously described second order method is not successful for such a problem because the regular iteration procedure brings to trajectories with the control and state variables on boundaries of the admissible region. It requires the employment of complicated procedure for removing control and state variables from boundaries.

The equation of motion for this problem can be rewritten as follows

$$\dot{V} = g(n_x^0 - \sin\gamma), \quad \dot{H} = V\sin\gamma,$$

where

$$n_x^0 = \frac{T_{max}(M, h)\eta}{W}\cos\alpha - \frac{qS}{W}C_D,$$

$$C_D = C_D(C_L, M), \quad C_L = \frac{W}{qS}, \quad W = \text{const}.$$

All the assumption in motion model are contained in these expressions.

It is necessary to transit from the given initial condition h_0, V_0 to any other for minimum time by choosing optimal schedules in (h,V)-space and appropriate throttle control with account of inequality constraints in the form

$$\sin\gamma_{min} \le \sin\gamma \le \sin\gamma_{max},$$

where γ_{min} and γ_{max} are functions of h, V and n_x^0. These constraints are related to flight envelope limitations.

We'll give the common scheme of this problem solution omitting some details. The Hamiltonian for this problem may be written in form

$$H = \lambda_V g n_x^0 + \Omega \sin\gamma ,$$

where $\Omega = \lambda_H V - \lambda_V g$. The necessary conditions for optimality include the ajoint equations

$$\dot{\lambda}_V = -\frac{\partial H}{\partial V}, \dot{\lambda}_h = -\frac{\partial H}{\partial h}.$$

The solution of these equations linearly depend on initial values $\lambda_V^0 = \lambda_V(0)$ and $\lambda_h^0 = \lambda_h(0)$. If the final values h_f, V_f would be given then it will be necessary to choose the values of λ_V^0 and λ_h^0 appropriately. But in our case by varying both parameters we can generate the extremals originating from point h_0, V_0 in (h,V)-space. It may be shown however that there is no need in varying these parameters and it is sufficient to vary only one parameter φ choosing the values of λ_V^0 and λ_h^0 in the following manner

$$\lambda_V^0 = \cos\varphi , \Omega^0 = \sin\varphi,$$

$$\lambda_h^0 = \frac{\Omega^0 + \lambda_V^0 g}{V}.$$

It follows from linear dependence of ajoint variables on λ_V^0 and λ_h^0 and of Hamiltonian on λ_V and λ_h. The optimal control then is determined from condition

$$\min_{\eta, \sin\gamma} H < 0. \quad (19)$$

The condition (19) at initial time moment gives the admissible subrange of φ value within 0 to 360° range. The singular control for $\sin\gamma$ arises when $\Omega \equiv 0$. From this condition, it follows that

$$n_x^0 = \frac{V}{g}\left(V\frac{\partial n_x^0}{\partial h} - g\frac{\partial n_x^0}{\partial V}\right).$$

This expression gives the appropriate schedule in (h,V)-space which depends on optimal value of η. It is interesting to note that at η = 1 this schedule corresponds to minimum time energy climb. This follows from the condition

$$\frac{d}{dV}\left[n_x(\eta, V, h(E, V)).V\right] = 0$$

at E = const.

The example of the extremals field in (h,V)-space for initial point A is given in Figure 9. Several regions are shown in this field. For each region the throttle control has some peculiarities. The results may be approximated for their usage for control law synthesis when descending.

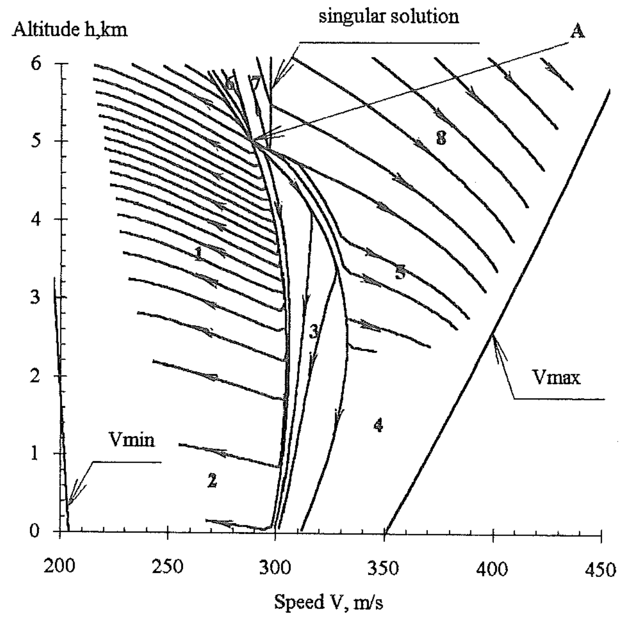

Figure 10. Extremals in (h,V)-space.

VI Control Law for Normal Load Factor

Let's consider the tracking of a given desirable schedule $h^D = h^D(E)$ at $\dot{E} \neq 0$ on the basis of linear equivalent models, that specify the law of compensation of the difference between desirable and current values of the flight parameters. We'll take such models in the form

$$\frac{d}{dt}\Delta h + k_h \Delta h = 0, \frac{d}{dt}\Delta\sin\gamma + k_\gamma \Delta\sin\gamma = 0 , \quad (20)$$

where

$$\Delta h = h^D - h,\ \Delta\sin\gamma = \sin\gamma^D - \sin\gamma$$

and k_h , k_γ - parameters.

These models correspond to smooth transition processes and it is possible to agree the long period motion quality and short one by choosing k_h, k_γ. It is follows from (20) and (18) that

$$\sin\gamma^D = n_x^D \frac{dh^D}{dt} + k_h \Delta h ,$$

$$n_y^D = \frac{V}{g\cos\gamma}\left(\frac{d}{dE}\sin\gamma^D + k_\gamma \Delta\sin\gamma\right) + \cos\gamma . \quad (21)$$

The tangential load factor n_x^D depends on n_y^D. The system of two equations (21) contains two unknown parameters $\sin\gamma^D$ and n_y^D . It is possible to find the solution of this system presenting the dependence of n_x^D upon n_y^D by means of appropriate rank in the vicinity of instant value of n_y. It is important for on-board implementation. For usual trajectory computation it is sufficient to use (21) adopting that $n_x^D = n_x$.

VII Adaptive Control Law for Throttle

If the specific energy varies monotonously ($\dot{E} > 0$ or $\dot{E} < 0$) functional (1) may be rewritten in form

$$J = \int_{\Gamma} f dE,$$
$$f = \frac{\lambda_W Q_s + \lambda_t - \lambda_l V \cos\gamma}{n_x V}, \tag{22}$$

where Γ is the given realized schedule in (h,E)-space, and λ_W, λ_t, λ_l are constant non negative Lagrange multipliers that may be computed when the full optimization problem is solving. Assuming that dependence $T = T(h, V, Q_s)$ is known and monotonous on Q_s, we'll find the optimal thrust control from minimum (at $\dot{E} > 0$) or maximum (at $\dot{E} < 0$) on Q_s of function f in (22).

The function f in (22) depends on fast Q_s, n_y and slow h, V, γ variables. Therefore it is possible to search desirable value of Q_s on-line by means of adaptive procedure like

$$\dot{T} \sim \dot{Q}_s \sim -(\,\mathrm{sign}\dot{E}\,)\frac{\partial f}{\partial Q_s}. \tag{23}$$

In such approach we neglect the interrelation of Q_s and n_y related to the necessity of tracking of the given schedule Γ.

Assuming that the motion proceeds exactly along the given schedule and $n_x = (T - D)/W$ it follows from (23) that the adaptive control law synthesis for throttle may have, for example, the form

$$\dot{T} = -k_T(\mathrm{sign}\dot{E}\,)\left(\lambda_W n_x - \frac{1}{W}(\lambda_W Q_s + \lambda_t - \lambda_l V(\cos\gamma)^{-1})\frac{\partial T}{\partial Q_s}\right), \tag{24}$$

where k_T - positive control law parameter. It is necessary to introduce additionally to (24) the limitations on $\dot{T}$ and T.

The usage of such control law demands the information about instant values of n_x, Q_s, W, V, γ and estimation of the derivative $\partial T/\partial Q_s$ that may be introduced by means of its approximation. If the schedule in (h,E)-space for $\dot{E} > 0$ corresponds to minimum fuel consumption then $\lambda_W = 1$, $\lambda_t = 0$, $\lambda_l = 0$, the expression (24) may be written as follows

$$\dot{T} = -k_T\left(n_x - \frac{Q_s}{W}\frac{\partial T}{\partial Q_s}\right).$$

For minimum time we have $\lambda_t = 1$, $\lambda_W = 0$, $\lambda_l = 0$,

$$\dot{T} = k_T \frac{1}{W}\frac{\partial T}{\partial Q_s} > 0$$

and for this reason $T = T_{max}$. For minimum fuel consumption when time and range are given or free this control law provides acceptable results. The results of computations show that for climb up the usage of such adaptive control law gives the fuel consumption that does not exceed the optimal one more then 0.2% when all information is exact. In more complicated examples this excess reaches 2%.

Conclusions

The examples described in this paper indicate that considered technique of flight path optimization gives the acceptable results for the extremals fields generation. Since the regularities of such fields may be understood there are possibilities for approximation of extremals and appropriate control synthesis. There are ways of simplification of the computed schedules without significant change of results for the worse. But this topic is not discussed in this paper.

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