

## VORTEX CONTROL TECHNOLOGY

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#### Abstract

The basic physical mechanism of vortex formation, development and interaction of vortices is described first. Next various possibilities to modify vortex flows including those employing geometric variation, acoustic excitation, blowing and other artificial devices to produce desirable interference effect are studied. Specific emphasis is placed on the unsteady excitations and some phenomena such as dynamic stall delay, steady streaming, like resonance synchronization are recognized. The purpose is always to get superlift, less drag and better control performance. The paper mainly deals with low speed external flows, however compressibility effects and internal flows are also mentioned. There exists immense potential in using vortex control technology to improve the performance of future aircraft and finally the problems need further study are identified.

#### I. Introduction

It is a great honor for me to present the Guggenheim Lecture at the 18th ICAS congress. It was forty five years ago that I began my graduate study at Guggenheim Aeronautical Laboratory of California Institute of Technology. The Daniel Guggenheim Fund for the promotion of Aeronautics exerted a remarkable influence on the aeronautics not only in the United States but in the world as a whole. The really far sighted Guggenheim family saw the great importance of advancing the art and science of aeronautics and aviation, and one would certainly agree to the above point of view, just look at what a fantastic achievement in aeronautics has been made in about the last seven decades since the establishment of Guggenheim Fund. The flight speed now covers from low subsonic, transonic, supersonic and to hypersonic. Despite of all the successes, some fundamental problems still remain to be solved. In 1965 K uchemann described vortices as "the sinews and muscles of fluid motion." His statement was quoted a good many times in the literature, though this statement was a little descriptive. Recent advances in nonlinear mechanics have shown that the deterministic dynamic equations can lead to chaos and the seemingly random chaos possess internal structures which are not

random in the ordinary interpretation. In fact Brown and Roshko found experimentally the organized vortex structure now called coherent structure in the free shear layer. Different coherent structures may be found in the turbulent flows e.g. hairpin vortices in a turbulent boundary layer. The ideal potential flow theory played an important role in the early stage of development of aeronautics, but even then the lift or circulation could not be explained without a starting vortex and the bound vorticity distribution. K arm n-Sears nonstationary airfoil theory rested upon continuous shedding of vortices from the trailing edge. With the rapid advances in the flow visualization technique, successes in computational fluid dynamics and computer graphics, many details of the flow field may be displayed and sometimes even the real or animated time dependent three dimensional movie flow pictures can be obtained, thus this would give us an intimate perception of the flow phenomenon. Therefore no matter how complex the flow looks like, it does possess a definite structure which might be called the sinews and muscles of fluid motion. As there is no unanimous definition of vortex, we may call this a vorticity structure and vortex is a vorticity concentrated region. Vortices occur in nature almost everywhere such as the giant red spot in the atmosphere of Jupiter etc., but with such a large scale, actually we can do nothing with it. A good summary of vortex flows was given by J.J. Cornish at the AIAA Applied Aerodynamics Conference in 1983. In any case there certainly exists a definite vorticity structure which may involve different time and space scales in the sense of hierarchy for a given flows, and the main purpose of flow research is therefore to understand the related structures and how they vary or response to external excitations. The design philosophy of keeping a streamline body and avoiding separation in the early stage of development of aeronautics was not always adhered to since the discovery of vortex lift due to leading edge separation and also the nonlinear lift increment for a slender body at large angle of attack. Indeed the vortex control technology has become an important theme in the research and development work in aeronautics, and in essence this is to organize or reorganize the vorticity structure to meet our technical or performance requirement. The first part of our paper will deal with the basic physical mechanism of how the vorticity is produced and the vortex is formed. Then we study various means that can be used to modify vortex flow either direct or indirect. Specific emphasis is placed on the unsteady excitation, such that

with a given excitation frequency, some "resonance" effects may be obtained i.e. a relatively small amount of energy input can yield a well organized vortex structure which derives its energy from the main flow itself and yields the desired benefits in aerodynamic performance. In depth and intensive research on vortex control technology is still going on and this is manifested in many papers presented at the present congress. It is evident the improvement of future aircraft design depends to a large extent on tapping the potential use of this technology. The final part of this paper will be devoted to a discussion of future problems that should be studied from both theoretical and practical aspects.

## II . Mechanism of Vorticity Generation and Interaction

In principle the equations governing the motion of fluid are the Navier–Stokes equations. In order to explore the details of the flow field, the introduction of vorticity is very useful. The vorticity  $\tilde{\omega}(\vec{x}, t)$  is defined as  $\text{curl } \tilde{u}$  which gives the fine structure, and is twice the angular velocity of the material element. Taking curl of the N.S. equations, we have for a compressible flow

$$\rho \frac{D(\tilde{\omega})}{Dt} - (\text{grad } \tilde{u}) \cdot \tilde{\omega} = \frac{1}{\rho} \text{curl } \text{div} \bar{R} + \text{grad} \frac{1}{\rho} \times \text{div} \bar{S} \quad (2.1)$$

here for simplicity the body force is neglected, where  $\bar{R}$  is the viscous stress tensor, and

$$\bar{S} = \bar{R} - \bar{I}p \quad (2.2)$$

$\bar{I}$  is unit tensor and  $p$  is the pressure. The terms on the right side of (2.1) represent viscous torque and baroclinic torque respectively. The second term is usually considered as the contribution due to vortex line stretching. Since  $\bar{R}$  is a symmetric tensor, we may introduce another tensor quantity  $\bar{J}$  such that

$$\text{curl } \text{div} \bar{R} = \text{div} \bar{J} \quad (2.3)$$

It is easy to see then for the incompressible flow

$$(-\bar{J} \cdot \bar{n})_w \quad (2.4)$$

can represent the vorticity source at the wall, where  $w$  specifies the values evaluated at the wall. The idea of vorticity source at a wall was first introduced by Lighthill. Following Hornung we notice that at a non-accelerating no slip boundary, the last term i.e. the baroclinic term vanishes. So the compressibility effect at the wall comes only through the first term on the left. We may now write

$$\rho \frac{D(\tilde{\omega})}{Dt} = \frac{D\tilde{\omega}}{Dt} - \frac{1}{\rho} \frac{D\rho}{Dt} \tilde{\omega} \quad (2.5)$$

The second term on the right hand side is just a product of dilatation with vorticity which may represent the interaction of acoustic (longitudinal) and shear (transversal) flow fields. It is easy to verify, for a local plan wall with  $x, y$  as the cartesian coordinates in the plane and  $z$  the normal coordinate

$$\begin{aligned} (-\bar{J} \cdot \bar{n})_w &= -\bar{n} \times \left\{ \frac{d\tilde{u}}{dt} + \frac{1}{\rho} (\text{grad} p) \right\}_w \\ &+ \frac{1}{\rho_w} \bar{n} \left\{ (\text{curl } \tilde{\tau})_w \cdot \bar{n} \right\} \end{aligned} \quad (2.6)$$

where  $\bar{n}$  is unit vector normal to the surface.

A fourth term needs to be added for three dimensional flows in which the wall is curved in a plane normal to the wall shear stress  $\tau_w$ . This term is

$$\frac{1}{\rho_w} \text{grad} \bar{n} \cdot (\bar{n} \times \tau_w) \quad (2.7)$$

Despite of all these stated above the definition of vorticity flux in general is still ambiguous as raised by Lyman.

However so long as the boundary layer is attached to the body, the vorticity is convected downstream to form a wake and the viscous effect does not possess a global effect of the motion of the body as may be speculated from equations (2.6) that the first term on the right hand side essentially represent the non-viscous effect. So the real important problem is that the boundary layer under certain circumstances separates (breakaway) from the surface. Prandtl in his 1904 paper first described the separation flow and gave the separation criteria for 2D—steady boundary layer the vanishing of shear stress at the wall. He also proposed different methods for separation control which are still of great use today. For 2D unsteady boundary layer, Rott first noted that point of vanishing wall shear does not coincide with the point of separation and later we have MRS criteria which was verified by some experiments. Mohamed Gad-el-Hak and Bushnell gave a critical review on separation control. The question of three dimensional steady separation and the dispute about closed and open separation were clarified in a recent paper by Zhang and Deng. It is the breakaway type separation that is of great importance. This point of view was also adopted by Sears and Telionis. In the frame work of boundary layer theory, there must exist a singularity at the separation point. As was shown by Van Dommelen and Cowley in the unsteady 3D boundary layer problem, it is natural to use Lagrangian variables ( $\xi = (\xi, \eta, \zeta)$ ). Let the position coordinates  $x$  and  $z$  describe an orthogonal coordinate system on the surface of the body in question. The length of the line elements  $dx$  and  $dz$  are  $h_1 dx$  and  $h_3 dz$  respectively. The coordinate normal to the surface is denoted by  $y$ . The equation of continuity reads

$$\rho H(x, z) J(x, y, z) = \rho_0 H_0$$

where

$$J(x, y, z) = \begin{vmatrix} x_{,\xi} & x_{,\eta} & x_{,\zeta} \\ y_{,\xi} & y_{,\eta} & y_{,\zeta} \\ z_{,\xi} & z_{,\eta} & z_{,\zeta} \end{vmatrix}, \quad \rho_0 = \rho(\xi, \eta, \zeta, 0)$$

$$H(x, z) = h_1(x, z) h_3(x, z), \quad H_0 = H(\xi, \zeta)$$

We can also write down the momentum and energy equation. The details may be referred to Van Dommelen and Cowley's original paper. The essential points are in

the momentum equations there lacks the appearance of particle position  $y$  and the normal velocity  $v$ , therefore we may assume solutions to the momentum equations are regular and density also regular for compressible flow. The singularity comes from the continuity equation when the Lagrangian gradients of  $x$  and  $z$  become parallel. In two dimensions, it may be shown this corresponds to MRS criteria.

Open separation was convincingly demonstrated by K.C. Wang et al. Here both limiting stream lines on the opposite sides of the separation line come from upstream. The counter circumferential flow coming from the leeward side meets the flow coming from the windward side and then they lift up to form a free shear layer. The layer rolls up to form a vortex core, which entrains the surrounding rotating fluid and the fluid exits along the core axis. For the flow along the delta wing with sharp leading edge, the situation is similar, but here the sharp edge is the fixed separation line. The free shear layer or the free boundary layer is unstable and vortices is to be formed. The classical study of mixing layer by Brown and Roshko showed the rolled large vortex structure. The paper by Ho and Huerre gives that how perturbations affect the development of the shear layer such as vortex pairing by subharmonic resonance etc. which would provide us a great insight to the control of vortices, i.e. to reorganize the vortex structure. For a general review of vortex interactions, we refer to a paper by Saffman and Baker.

In short the separation control and vortex control are closely related subjects. Separation is not always harmful and certainly it is difficult (if not impossible) to avoid separation in all cases. What we can do, is to get an optimized result with mixed flow i.e. with flow regions containing both attached flow and separated flow. Finally it should be pointed out that as early as in 1950 Betz made a proposal relating to the production of concentrated vorticity region in a very low viscosity fluid.

### III. Steady methods of vortex control

"Moving in the air without causing any disturbed wind" is an ideal objective but not realistic. In the early development of aeronautics the guide line for the aerodynamic design is to keep the flow attached as long as possible. As the flight envelope enlarges this could not be achieved with a fixed configuration. So aircraft with variable geometry e.g. with variable sweepback was suggested, yet this is not always a feasible method. Discovery of vortex lift due to leading edge separation then changed the design philosophy. Now we can think of mixed type flow in general and consider the utilization of the separated vortex and some vortex control technique to meet our specific purpose. Excellent review papers were already in existence e.g. the one given by Lamar and the other by Rao and Campbell. Here we would give a brief summary of the development of vortex control technology.

### The idea of compartmentation

Fence, slot and pylon vortex generator etc. may all serve as leading edge compartmentation devices. These devices introduce local disturbance and the flow over a wing may be considered as consists of flow over several individual regions with an effect to prevent enlargement of separated region, at the same time to energize the flow in the boundary layer to prevent the early appearance of random eddies thus avoiding sudden change in the moment characteristics, another benefit is the increase in lift and drag ratio.

### Strake

The role of adding a strake to a wing body is well-known. The use of strakes is in certain sense a trade off of low speed performance and high speed performance of a supersonic aircraft. Besides we may have a favorable interference of the strake vortex and the vortex produced by main wing, thus steady stalling angle is increased and also the maximum lift coefficient.

Another effect of strakes might be of interest. It is known that at large angle of attack the forebody of an aircraft may produce an asymmetric vortex pattern at the leeside thus producing undesirable side force. Wind tunnel tests showed that with helical strakes or straight detachable strakes mounted on the two sides of forebody, the side force and the induced instability were greatly alleviated. Hinged strakes may also be used and deflecting the strakes to some anhedral angles leads to great alleviation of vortex break down effect at high angle of attack.

### Vortex flaps

With the increase of sweepback angle e.g. used for supersonic cruise fighter, the flow along the whole leading edge even at small angles of attack separates causing a complete loss of leading edge suction and a drag increase. Lamar et al. first found for a wing with warped leading edge the leading edge vortex could be reoriented resulting a high leading edge suction. Based on this principle and to comply requirements for different flight regimes the concept of vortex flap was introduced. By deflecting the leading edge vortex flap downward, the aerodynamic force acting on the flap will produce a thrustwise component effective in reducing the drag and in general increasing the lift and drag ratio. In order to get better results we naturally expect that the separated vortex would stay entirely above the vortex flap and the flow reattaches before the hinge line and that the flow will not separate again at the hinge line. In general, this could not be fully realized, even then say for example for a delta wing with sweepback angle  $74^\circ$  equipped with a rectangular vortex flap deflected at  $30^\circ$  downward, there is a significant increase in both lift and drag ratio and  $C_{L_{max}}$ , although the lift at small an-

gle of attack is slightly decreased. This has been found effective also at transonic speeds. Many geometrical modifications of vortex flap design could be made. The planform may be changed e.g. Rao demonstrated that reducing the length of vortex flap inboard improves efficiency, increasing flap size delays the inboard movement of vortex and the flap segmentation delays vortex bursting, achieving the same  $L/D$  as without segmentation but with a smaller flap area. It is seen that there are many geometrical parameters at our control to be used for optimizing the design of vortex flap. Also the vortex flap can be used in combination with trailing flaps.

However, the success of using vortex flaps on reducing the drag was at a penalty of lift reduction, because of the less local angle of attack and forward inclination of the lift acting on the downward deflected leading edge vortex flap. The lift reduction would impair the airplane performance in take off, landing and maneuvering, which ought to be improved. Many artifacts were proposed. One of the methods is to use the apex flap which is the foremost part of the wing that can be deflected about a transverse axis, and to tap beneficial interaction between the vortices separated from the downward deflections of apex flap and the leading edge vortex flap. The overall effect is that the induced pressure will yield higher suction on the leading edge vortex flap in the low speed as well as in the transonic and lower supersonic speed to produce a large thrust component conducive to the reduction of drag. It was demonstrated also the high effectiveness using apex flap on vortex flapped delta wing, double delta wing and with the wing-body combination even in side slip flow. It seems that adding apex flap on a vortex flapped wing will alleviate to some extent the disadvantage of reducing the lift while maintain the advantage of reducing the drag compared to the wing equipped merely with the vortex flap.

It is evident from experiments the separated vortex will burst beyond certain angle of attack. Therefore the use of vortex flap will no longer be effective as say  $\alpha$  is greater than  $40^\circ$  or so, i.e. we may have the conclusion that at still larger angles of attack the geometric modification alone is not sufficient to preserve a well organized vortex structure over the wing surface. To enlarge the envelope of flight regime especially at very high angle of attack, we must seek other methods, an immediate suggestion is to impart energy or momentum directly into the flow field through jets or to use some excitation system such as acoustic excitation and different kinds of unsteady excitations (which will mainly be discussed in the next section).

#### Vortex control with mass injection

In the early days suction were studied to achieve attached flows, however it was realized a suction penalty might outweigh the benefit derived from the control. In-

stead Cornish in the sixties introduced the spanwise blowing concept with the jet produced to lock a vortex above the stalled wing at a high angle of attack. This concept can be employed on a variety of geometrical configurations of wing. The lift augmentation is increased with the increase of the jet blowing momentum parameter  $C_\mu$

$$C_\mu = (\text{jet flux } G) \cdot (\text{jet velocity } V_n) / (g \cdot \frac{1}{2} \rho V_\infty^2 S)$$

$S$  is the reference area. Later we found that spanwise blowing is an effective measure to enhance the strength of the leading edge vortex over the wing surface and the delay of vortex bursting till higher angles of attack. However the application of spanwise blowing to airplane design is limited by the requirement of relatively high amount of momentum flux, if this is derived from a jet engine then we will have a large thrust loss. In order to keep low air consumption, a scheme using periodic blowing was suggested. Initial experiments showed that periodic blowing with frequencies 0, 1, 2 to 4 Hz does not produce apparent pulsative behavior for lift and pitching moment before stall occurs. It is remarkable to note that the leading edge vortex on a delta wing entrained by the downstream moving jet remains stable (almost steady) for a definite time after the spanwise blowing is shut down at moderate and high angle of attack and then becomes chaotic afterwards. Therefore periodic blowing will give the nearly same characteristics as with the steady blowing.

In combination with leading edge vortex flap, the mass injection is also able to raise its drag reduction effectiveness. Experimental results indicate that a single nozzle blowing near the hinge line is very effective in enhancing the vortex strength and at the same time to control the position of the separated vortex to stay longer over the flap thus improving the characteristics of a vortex flap.

Usually the ordinary flap efficiency can be greatly increased with a small mass flux spanwise blowing. The concept is different from the early developed jet flap concept which is related to circulation control. It is found that the flap shoulder vortex produced under the action of jet flow is generated over the flap and located always ahead of the jet with its vorticity gained from the flap shoulder separation and axial velocity provided by the entrainment effect of the jet. The lift augmentation caused by the flap spanwise blowing is attributed mainly to the induced vortex. Lift increment comes both from the pressure acting on the flap as well as on the wing, because the flow condition over there is also much improved under the action of jet flow.

Up to now one then can think of other types of blowing with different locations and orientations such that optimum choice can be made to our satisfaction. Studied leading edge blowing. Here the nozzle is located very near to the leading edge and the blowing is along the leading edge. The leading edge blowing now generates a jet leading edge vortex, which is stronger and more stable

than those produced by the nozzle located after the leading edge. This leading edge vortex and the spreading jet can enhance the outer panel vortex so the maximum lift coefficient is further increased.

Tangential leading edge blowing can also be used to profit. Actually this concept is derived from the concept of circulation control on two dimensional airfoils. The concept utilizes a thin high velocity tangential jet of fluid to control the location of the rear separation points on a rounded trailing edge airfoil. Gains in lift coefficient of the order of 80 times the injected momentum coefficient has been observed over a wide range of operational conditions. The possibility then exists to consider the use of the circulation control concept in a cross flow plane device to directly control the location of cross flow separation points and hence the trajectory of the ensuing vortices. An experiment conducted by Wood and Roberts shows the primary effect of the leading edge blowing upon the vortical flow. At  $40^\circ$  angle of attack without blowing, the upper surface flow exhibits the qualities associated with a separated stagnant bubble. With the addition of a small amount of blowing, the primary separation is delayed and the resulting vortical flow interact with the wing surface, producing the familiar vortical flow pressure signature. The corresponding increases in local normal force and rolling moment are obvious.

There exists a large amount of literature and many interesting experiments regarding the vortex control. Some such as using winglets to control wing tip vortices has already employed in the design of aircraft. It is not possible to cite all these experiments here. But before leaving it might be of some interest to mention another two experiments. Vakili and Wu experimentally demonstrated a locked vortex using periodic suction. In their preliminary and simplified experiment, periodic suction from the wing tip region of  $45^\circ$  swept wing in a uniform flow, in the water tunnel, resulted in the total elimination of flow separation on the wing which persisted when no suction was present. Finally Werlé's many experiments and beautiful pictures should be mentioned. In one of the experiments conducted by Werlé, the fully separated flow over a flat plate of small aspect ratio at  $\alpha = 20^\circ$  can be organized to realized the reattachment of the oncoming flow by the control of an aft-spanwise jet emitted near the upper surface of the plate.

Although the title of this section is listed as steady methods of vortex control, the fact is that we have already mentioned some unsteady control technique such as periodic blowing or suction. In fact unsteady vortex control may be more effective and this must be traced back to the formation of vortices. Almost all vortical flows are to some extent unsteady. We have already quoted the investigation of Brown and Roshko. They showed that in free shear layers originating at a splitter plate between two streams of different velocities, discrete vortices are observed. Winant and Browand showed that the growth of

such shear layers is the result of a pairing process between two neighboring vortices. They at first rotate around each other and then merge to form a single vortex of a larger diameter. The sharp leading edge of a delta wing may be considered as an edge of the splitter plate. The shear layer in this case wrapped up in a spiral fashion to form a large bound vortex. Gad-el-Hak in his experiments observed the formation of discrete vortices and at constant  $\alpha$ , the shedding frequency at the testing Reynolds numbers is proportional to the square root of the free stream velocity and the pairing phenomenon was also observed, even with the blunt leading edge. Since the vortex formation is sensitive to the initial perturbation, some resonance phenomena may occur. This will be discussed in the next section.

One final remark should be added. We have mentioned in this section some of the experiments were conducted at high speed, and naturally compressibility effect must come in. The baroclinic torque could play an important role. In a paper by Marble et al. gives us some insight into this problem. They studied shock enhancement of supersonic combustion processes. The mechanism rests upon the strong vorticity induced at the interface between a light and heavy gas by an induced pressure gradient. The specific phenomenon under investigation is the rapid mixing induced by interaction of a weak oblique shock wave with a cylindrical jet of hydrogen embedded in air. Because the combustion process will be carried out at supersonic velocity with respect to the engine, a rapid mixing is required and we expect the baroclinic vorticity would enhancing the mixing rate. The preliminary experiments and calculations are encouraging.

Thomann studied the method of reducing the base drag. In his experiments at high speed  $M = 0.566$ , the flow visualization pictures for a wedge without and with a splitter plate were taken. In the former case a larger wake exists while in the later case the wake is much smaller and hence less drag.

#### IV. Unsteady methods of vortex control

Strictly speaking nearly all vortex flows are unsteady or at least exhibit certain weak unsteady character. It is natural to employ unsteady excitation for its control. Relative to steady control, we might have some advantages namely less energy consumption, less penalty in structural weight and parasite drag and at the same time we can achieve significant global aerodynamic effects.

Up to present there appears three main type of unsteady excitation such as integral time dependent motion of bodies, motion of partial wall boundary and excitation with special flow perturbation devices. Of course certain combinations of excitations can be used if desired. In the following we will show by some typical examples to illustrate the potential benefits when using unsteady excitation.

A. Control for airfoil

a. Pitching oscillation of airfoil and dynamic Stall

For this problem it has already accumulated many theoretical and experimental research results. Some earlier results were summarized in a paper by McCroskey in 1977. Usually the pitching is sinusoidal and dynamic stall effects vary significantly with the amplitude of oscillation. L.W. Carr listed the events of dynamic stall on NACA 0012 airfoil and noticed there is a moment stall preceding the lift stall. To explain these phenomenon Ericsson offered to distinguish two different mechanism, one is due to the quasisteady effect i.e. a time lag and the pitching of the body modifying the boundary profile, the other is due to the instantaneous effect and during the upstroke, the breakaway point moves downstream and the leading edge bubble is more stable. The situation is just reverse during downstroke. As vortex forms near leading edge and sheds before stall, a large vortex stays long above the airfoil surface thus the lift slope increases a large amount. The dynamic stall vortex could be eliminated through the use of a leading slot and also by blowing see Carr.

b. Effect of oscillating pitch on vortical patterns in the wake of an airfoil

Koochesfahani found the vortical patterns are affected by the amplitude, frequency and the wave form of oscillations. An airfoil with NACA 0012 section was tested in a low speed water channel pitching around zero angle of attack. For a given frequency and amplitude, different complicated forms of vortex interaction were obtained by simply varying the wave form. A slight change in the wave form would result modifications of the vortex pairing process. What is more interesting is that Koochesfahani found as the frequency is high enough, an arrangement of vortices opposite to Kármán vortex street resulted. This pattern corresponds to a "jet". In fact the mean velocity profile, measured in a plane one chord length behind the airfoil, does not display with a usual wake profile i.e. with a velocity deficit, instead we now have a velocity excess that means we are able to convert the wake drag to a jet thrust. Although this fact of conversion was noticed by Kármán and Burgers (in the book Aerodynamic Theory Vol II edited by Durand) for the case of a flat plate in transverse oscillation, it was for a long time neglected and the present experiments were decisive.

c. Effects of Longitudinal Oscillations

At high angle of attack, benefits may be derived from longitudinal oscillations of an airfoil. Experiments were performed by Maresca et al. The testing model is a rectan-

gular wing with a NACA 0012 symmetric profile of which the steady stall angle of attack is about  $12^\circ$ . The measurements indicate an overshoot of the instantaneous lift and drag caused by strong vortex shedding during the dynamic stall experienced by the airfoil in decelerated motion. When the airfoil is moving forward in accelerated motion dynamic reattachment may be observed. The mean lift can be more than three times greater than the corresponding steady lift. An empirical formula is obtained which shows the relevant parameters, that the ratio of mean to steady lift ( $r$ ) depends on

$$r = 1 + A(\alpha)\varepsilon^\beta k^2 [1 - B(\alpha)k(\varepsilon)^{\varepsilon(\alpha)}]$$

where  $k$  the reduced frequency  $k = c\omega / V_\infty$  and define reduced amplitude  $\lambda = A\omega / V_\infty$  and  $\varepsilon = \lambda / k = 2(A / c)$ .  $A$  and  $\omega$  are respectively the amplitude and circular frequency of the oscillation,  $c$  the chord length,  $V_\infty$  the velocity of undisturbed stream and  $\alpha$  the angle of attack,  $5.7 \times 10^4 < Re_c < 4 \times 10^6$ . It is to be noted at angle of attack below the angle of static stall, the unsteady effects are weak. Beyond the steady stall angle, the optimization of the overshoot of mean lift coefficient may correspond to like resonance between the vortex shedding and the movement of the wing.

d. Pitching up airfoil at uniform rate

This problem contains all essential features of dynamic stall such as separation delay, the formation of separated vortices and their interactions. J. Wang made a series of experiments in a water tunnel with NACA 0015 airfoil at different rotational speeds  $\dot{\alpha}$ , which is normalized as  $\Omega = \dot{\alpha}c / V_\infty$ , and  $\alpha$  is from  $0^\circ$  to  $90^\circ$ . He found that according to the magnitude of  $\Omega$  three flow regimes can be recognized during the transient using the hydrogen bubble visualization technique namely.

1. Single leading edge vortex structure for  $0.64 < \Omega < 1.2$

2. Triple vortex structure (leading edge vortex, primary and secondary shear vortex) for  $0.14 < \Omega < 0.52$

3. Double vortex structure (leading edge vortex and one shear vortex) for  $0.037 < \Omega < 0.06$

It was further observed with the increase in  $\Omega$ , the breakaway from the leading edge delays and the leading edge vortex forms at a large angle of attack.

Some details of triple structure might be of interest. For the experiment with  $\Omega = 0.24$ , with the increase in angle of attack, reverse flow first appears on the upper surface near trailing edge, a shear layer is formed between the reverse flow and external flow and is destabilized to yield a shear vortex at  $\alpha \approx 34^\circ$ . As angle of attack further increases, the near surface reverse flow further extends upstream and the shear layer is across the whole upper surface and at a later time ( $\alpha \approx 45^\circ$ ) leading edge vortex is formed at  $x_0 / c \approx 0.20$ . The shear layer at the nose region continuously supply vorticity to the leading edge vortex enhancing its strength. This vortex will then move toward the trailing edge and during this time around the middle

part of the upper surface another shear vortex is formed thus we have a triple structure.

From the examples given above it is apparent in order to delay the dynamic stall or to alleviate the unfavorable interference caused by the dynamic stall vortex, we may adopt a control for the reverse flow region, e.g. using wall motion, tangential blowing or leading edge slot already mentioned. Yet care must be taken such that these measures would not introduce additional instability. It is also apparent that the diffusion and convection of vorticity which are directly related to the production of lift and circulation, the increase in the pitching-up rate will certainly delay the dynamic stall.

#### B. Control of vortex wake behind bluff bodies

Actually an airfoil at large angle of attack may be considered as a bluff body and for the bluff body we have a large amount of literature which might be of use to us.

Active control usually consists of longitudinal and lateral vibration of bodies and the periodic perturbation for the oncoming stream. In the latter case, if the amplitude of perturbation is larger than say the diameter of the body, the situation is equivalent to the longitudinal motion of the body.

When the natural vortex shedding frequency coincides with frequency for lateral cross flow oscillations, vortex resonance or "lock in" occurs. For longitudinal oscillations lock in phenomenon also occurs, though the frequency is about twice the value for lateral oscillations. Lock in phenomenon leads to drastic changes in the wake structure and also the forces exerted on the body. The occurrence of lock in is related to both the frequency and amplitude of oscillations and there exists a threshold value for the amplitude. Just quote here some figures about the variation of the base pressure to the strong resonance effect, a small perturbation amplitude of longitudinal oscillation  $2\Delta U / \omega D = 0.04$ ,  $C_{pb}$  varies from  $-1.44$  to  $-1.85$ . The conditional mean measurement of wake velocities show that the vortex strength increases by 29% and the distance between successive vortices decreases by 25%. These figures are indeed very significant see Armstrong et al. 1987 and Stansby 1976.

It is now known in the flow field there are two types of instability mechanism so called absolute and convective instability. In the absolute instability region, a small amplitude pulse disturbance will amplify exponentially and propagate both upstream and downstream, while in the convective instability region, the disturbance will propagate downstream only and after some time, in the region where original pulse disturbance is introduced, the flow recovers to the original state. And indeed there exists in the near wake of a bluff body a region of absolute instability. A careful study shows when a closed recirculation region exists behind the body the downstream stagnation point is just on the boundary sep-

arating the two instability region. Chomaz et al.(1988) pointed out the existence of absolute instability region is a necessary condition for the global instability. With a large instability region and a sufficient large amplitude disturbance say applied at downstream stagnation point, the wake structure can be changed to different forms of vortex structure, since the disturbances propagate upstream, so we now have a closed loop feedback system. At high Reynolds number we have three characteristic frequencies 1. the classical large scale Kármán mode frequency, 2. Kelvin-Helmholtz instability frequency of thin shear-layer and 3. Characteristic frequency of body vibration (either longitudinal or lateral). So with all these frequencies and the phase differences the near wake can be modified to a considerable extent. Either the rotation of cylinder or longitudinal (lateral) oscillations of cylinder can lead to a modification of Kármán mode with certain manipulations with the wake vortices aligned almost in a straight line corresponding to a transition from base drag to base thrust. Much more complicated modal structure may appear far different from a modified Kármán form and with the variation of amplitude and frequency of excitation switching phenomenon and the so called mode competition may occur, see Williamson and Roshko.

#### Motion of partial wall boundary

Although we do not have a complete theory for 3-D unsteady separation, we can at least draw some conclusions from the early unsteady boundary layer separation study on moving walls. For the downstream moving wall the growth of boundary layer thickness is suppressed because the relative velocity of wall with respect to the potential outflow decreases or in other words momentum is injected into the near wall boundary layer, thus separation is delayed. Here we give a simple example for illustration.

Modi et al. in their paper, "Effect of moving surfaces on the airfoil boundary layer control", describes experiments on a symmetric Joukowski airfoil equipped with a rotating cylinder at its leading edge. The flow field depends on the ratio of rotating cylinder surface speed to the free stream speed  $W / U_\infty$ . As this ratio increases from zero to four, the steady stall angle of attack is increased more than twice the original value and the value of maximum lift coefficient nearly doubles. Li et al. made numerical simulation of the problem and the results compared favorably with experiments.

#### Excitation with special flow perturbation devices

##### A. The use of spoiler-like flap.

Nagib et al. studied the dynamic scaling of forced unsteady separated flows in turbulent boundary layers. They found the existence of at least two distinct mechanism. At low reduced frequencies, the primary mechanism

corresponds to the momentum exchange induced by the modulation of separated shear layer, and leads to the periodic shedding of separation bubble. This mechanism scales with the characteristic height of the separation region. For larger values of the reduced frequency, the dominating mechanism is the formation and shedding of energetic vortices caused by the oscillation of the flap. This mechanism scales with the reduced frequency based on the flap height. Further experiments may be needed to verify the above arguments and the results obtained may serve directly for engineering applications.

#### B. Acoustic excitation

The role of acoustic excitation in tripping the boundary layer from laminar to turbulent thus delaying separation is well understood. However the mechanism of interaction between acoustic excitation and the separated vortex flow is not very clear. Zaman et al. studied effect of external acoustic excitation at high frequencies eliminates a prestall periodic shedding of large-scale vortices. Significant improvement in lift is also achieved during post stall, but with large amplitude excitation.

L.S. Huang et al. studied the effect of sound emanating from a narrow gap in the vicinity of the leading edge of a symmetric airfoil at a low  $Re \approx 3.5 \times 10^4$  based on the chord length. The shear layer is found to be very sensitive to acoustic excitation in the vicinity of separation point. The effect is to increase the entrainment in the early part of the shear layer, thus reducing the size of separation region. The results were already shown in the smoke visualization pictures. This may be understood as an acoustic streaming phenomenon, and a strong Helmholtz resonance streaming might be achieved.

#### C. Unsteady base bleed

Unlike the steady base bleed where we have net mass injected into the wake, with unsteady pulsating jet there is no mass added averaged over a cycle. D.R. Williams in the study of unsteady pulsing of cylinder wakes found that the jet are most effective in modifying the wake when pulsating at twice the Kármán shedding frequency. Also the amplitude plays an important role. The streaming flow acts to suppress the Kármán vortex street and reduce the momentum defect. At sufficient large amplitudes of pulsation, a negative momentum defect may be obtained.

#### D. Active surface heating

As may be expected that the shear layer is also very sensitive to localized surface heating in the vicinity of the separation point. In the numerical simulation given by Maestrello et al. the resulting effect is much related to the wave form, the amplitude and the bandwidth of the temperature function for a given pressure gradient. With sur-

face heating the separation region is reduced and the strong coupling causes the shear layer to resonant through its harmonics inducing additional vortical motion that remains close to the airfoil with increase in lift and decrease in drag.

#### Periodic forcing on mixing in turbulent shear layer and wakes

For internal flows with chemical reactions, it is important to know how the molecular mixing actually takes place and its relation to large scale coherent structure and what are the effects of a periodic disturbance applied to the free stream on large scale structure and mixing processes. Brown and Roshko first identified that organized vortical structures play a prominent role in those shear flows even when they are turbulent. Oster and Wygnanski found that these large structures could be controlled by introducing relatively small periodic perturbations into the flow and that the development of the shear layer could be drastically changed. A non-dimensional shear layer thickness growth law was given by Browand for the forced shear layer, expressed in terms of non-dimensional downstream distance  $\bar{X} = X / L$  where  $L = U_c / \epsilon F$ ,  $U_c$  is the convection speed,  $F$  is the forcing frequency and  $\epsilon$  is a parameter related to the velocities of the two streams. Three regions can be distinguished, 1. for  $\bar{X} < 1$ , the growth rate is enhanced by up to a factor 2; 2.  $1 < \bar{X} < 2$ , we have a frequency locked region, growth rate is inhibited, even reduced to zero. Reynolds stress are reversed; 3.  $\bar{X} > 2$ , we have a relaxation to unforced growth rate. The question is now "does the scalar product thickness behave similarly as the momentum thickness?". Here we cite some experimental results by Roberts and Roshko. In their high Reynolds number experiment, for the unforced flow the growth rate product thickness is small for small  $X$  and increasing monotonically with increasing  $X$  (downstream distance), and there is a region corresponding to approximately constant slope of the thickness, the beginning of this region apparently represent the beginning of the post-mixing transition region. An interesting data set was obtained at the forcing frequency 8 Hz. In this case frequency locked region existed in certain interval of  $X$ . The flow goes through the mixing transition in much the same way as the unforced case. However the product thickness stops growing at the beginning of the frequency locked region. Approximately half way into the frequency locked region the product thickness begins to grow again at a rate slightly larger than the asymptotic unforced rate. The frequency-locking phenomenon thus causes a temporary zero product thickness growth rate. It seems that entrainment stops there and without doing any mixing!

#### Unsteady aerodynamics of a stationary airfoil in periodic varying stream.



In Ho's invited paper at the First U.S. National Fluid Dynamics Congress, he described the effect of a controlled variation of free stream speed on the aerodynamics of a stationary airfoil. With time nondimensionalized with the period of free stream variation the lift coefficient converges together for different periods during accelerating phase, but during decelerating phase a large variation occurs and the lift coefficient may be very large compared to the steady case.

#### V. Conclusions

Although we have gone through many interesting experiments illustrating the utilization of vortex control technology, we cannot claim the review is exhaustive, nor we can claim the proposed method would be effective in every case, because the flow situations vary greatly, it will depend upon the concrete problem that we would like to solve. We are therefore satisfied if this would promote the production of new concepts of control. The rapid advances in computational fluid dynamics are really very impressive as manifested in the first two issues of *Aerospace America* this year, but "A statistical description of the effects of turbulence remains the pacing item for CFD." In January issue Moin mentioned direct numerical simulation plays an important role in the development of active and passive turbulence control strategies. It has been shown in one case a substantial drag reduction of 20% can be achieved by active control in fully developed turbulent flow when the wall boundary condition for the normal velocity was adapted to the instantaneous state of the flow within the boundary layer. This seems physically evident from the unsteady separation criteria because in this case bursting from the wall or breakaway could be suppressed. From engineering point of view this scheme could hardly be realized. Also now the commonly used Reynolds-averaged models are not successful in their applications to complex flows such as the highly vortical flows around fighter aircraft at high angles of attack. From practical point of view we would emphasize the role of well designed experiments, this is not only necessary for the understanding of physical mechanism but also is important in the development testing, and CFD code must also be validated. There is one difficult problem common to both experiments and numerical calculation i.e. the scale effect or Reynolds number effect, as most experiments were run at not too high Reynolds number, and numerical calculation at high Reynolds number is also not very reliable due to the requirement of small mesh size besides other problems. Moreover flows at high Reynolds numbers are usually turbulent. For free shear layer the effect of viscosity may not be as important as in the boundary layer. In any case caution must be taken in using the results obtained at low Reynolds number to extrapolate to high Reynolds numbers. Unsteady method of control is very promising and is also less studied. The important top-

ics to be studied might be switching effect, resonance effect and the phase and hysteresis effect; the relation between local control and global control, open loop control and feedback control and finally since this flow system is highly non-linear, we must carefully study the gradual time evolution and the sudden burst resulting from control response. No doubt we believe that the future exploitation of the vortex control technology will greatly enhance the performance of aircraft and its engine of next generation.

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