ANALYSIS OF METHODS FOR MODELLING REAL FLIGHT SITUATIONS

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INTRODUCTION

The real constructional-technical characteristics of aircraft are scattered to a great extent in the neighbourhood of the rated values prescribed in technical documentation, even at the end of their maufacturing. With the increase of service life,(1) the deviations mentioned above continue to increase stochastically mostly in a cumulative way as a function of:

- the physical-technical properties of the structural material applied;
- the peculiarities of their design and manufacture;
- the circumstances and the technical-economic conditions of operation (air traffic, maintenance, repair and overhaul), and
- the intensity of operation.

The changes in the constructional-technical characteristics naturally involve also the deviations in aerodynamics and flight engineering characteristics (performance data). Those deviations can be described by stochastic processes not controllable by means of simple methods on a reasonable cost-level, and influenced by the means of maintenance and repair, i.e. by stochastic processes controlled only in a limited way.

The real flight is considered to be the controlled three-dimensional motion of the aircraft as a flexible (non-rigid) body (or system of bodies), wich is determined by the given realization of the stochastic deviation process of aerodynamic and flight-engineering characteristics, and disturbed stochastically by the real environmental conditions (e.g. by atmospheric turbulence). The simulation of this motion on an acceptable level is considered today as a very important theoretical and practical problem.

In this paper, the possibilities, limitations and objective-orientation of modelling the real flight situations are dealt with.

1. THEORETICAL CONSIDERATION

1.1 Objectives of modelling

With respect model-formation,(2,3) it is important for what purposes the models are utilised, and what connections between the model and original, examined system can be established. Therefore in this case, a clear distinction should be made between the cases depending on whether the multitude of the real flights of a given aircraft type, or only a separate flight of a concrete piece of air- craft is to be simulated.

As for the purpose of utilisation, the following objectives requiring different attitudes can be set(3):

- research, investigation;
- state estimation, diagnosing;
- checking and determination of airworthiness;
- synthesis of control;
- investigation of flight events;
- operation of simulators;
- solution to the problems of air traffic control;
- examination of environmental problems.

1.2. General model

The state of aircraft as a dynamic system is usually given by state-vector x. State-vector x can include also characteristics describing the operation and technical state of the main unit-blocks in addition to the phase-vector elements describing the spatial position of the aircraft. In a general case, the model can be given in the following form(1,3,4):

$$dx(t) = f_x\left[\, x(t),\ x(t-\tau_x),\ p(\mu,t), z(\omega,t),\ u^*(t),\mu,\omega,t \right] dt + \sigma_x \left[\, x,\mu,t \,\right] dW_x$$

$$y(t) = f_y\left[\, x(t),\ x(t-\tau_y),\ p(\mu,t),\ u^*(t),\ \mu,\omega,t \right] + \sigma_y \left[\, y,\nu,t \,\right] \xi(t) \qquad (1)$$

$$u^*(t) = f_u\left[\, x(t),\ x(t-\tau_u),\ p(\mu,t), z(\omega,t),\ \mu,\ \omega,\ t \right]$$

$$x\ (t=t_o) = x_o\left[\, t=t_o,\ \mu_o,\ \omega_o \,\right]$$

$$y\ (t=t_o) = y_o\left[\, t=t_o,\ \mu_o,\ \omega_o \,\right]$$

where $x \in R^n$ is the state-vector, $p \in R^k$ is the parameter-vector characterizing

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the state of the aircraft, $z \in R^q$ is the environmental vector (vector of service conditions), $u \in R^m$ is the control (input) vector, $y \in R^r$ is the output /measurable/ signal vector, $W \in R^s$ and $\xi \in R^v$ are the Wienerian and Gaussian noise-vectors, respectively, σ_x, σ_y are the noise transfer matrices, t is the operational time. Vector **p** can be resolved into vector elements $p = [\ a,b,c,h\ ]^T$ "belonging" to vectors **x, u, z, y**. Vectors of constructional characteristics $[\ a,b,c\ ]^T$ and **h** as well as vector **z** of external disturbance in the field $\Omega_{[a,b,c]^T} = \Omega_\mu$, $\Omega_z = \Omega_\omega$, $\Omega_h = \Omega_\nu$ are characterized by density functions $f_\mu(.)$, $f_\omega(.)$, $f_\nu(.)$ and the actual values belonging to their given realiza- tion are assigned by random variables μ, ω and ν.

1.3 Deviations in characteristics

In a generalized sense, **p** is the vector of constructional characteristics, **u, z** are the vectors of service characteristics and **x, y** are those of operational characteristics. In the course of operation, the constructional characteristics (e.g. the technical ones, or in this case, the aerodynamic characteristics) are changing stochastically because of the reasons mentioned in the introduction, and they deviate badly (according to our examinations(1,3,5,6): by 5-10 %, sometimes even by 25-40 % (from the values recorded in the service documentation. Those deviations can be observed as early as at the end of manufacturing process and at the start of operation, and then, in the course of operation, they continue to increase first quickly, then a little more slowly, and, in most cases, by an accumulative way. The initial quick change represents the so called "wearing-in" process of the construction. (After some hours of flight, the constructional elements enter into smooth engagement, the geometric shape change by way of micro displacements and deformation).

In the course of our examinations, the changes experienced in the construction and the functional builld-up (operational logic) of the examined aircraft and its systems (such changes are: changes in the geometric shape, in dimensioning-failure, in failures of system elements), and the deviations in the characteristics connected with those (e.g. deviations in the coefficients of aerodynamic model) were considered as deviations in constructional characteristics.

The following changes were considered as deviations in service conditions (in environmental data): deviations from the rated (directive) values (e.g. extreme atmospheric conditions, loads exceeding the allowed limits, improper maintenance activity) prescribed in service documentation for the sake of controlling directly the aircraft operation, as well as from those prescribed in engineering activity for the sake of controlling service conditions and the operational process.

The deviations in the constructional, operational characteristics involve, as a matter of course, changes in service characteristics **x, y**.

With those above taken into consideration, the control and optimization of aircraft flight can be carried out according to Fig.1 provided that instead of model (1) its linearized form can be used:

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + C(t)z(t) + G_x(t)\eta(t) \qquad (2)$$

$$y(t) = H(t)x(t) + G_y(t)\xi(t)$$

where: **A** is the state-matrix of dimension (n n), **B** is the control-matrix of dimension (n,m), **C** is the influence-matrix of external disturbances of dimension (n,q), G_x and G_y are noise-transfer matrices belonging to noise-vectors $\eta \in R^s$ and $\xi \in R^v$ (Gaussian white noise), respectively, of dimension (n,s) or (r,v), respectively, while **H** is the measuring-matrix of dimension (r,n).

1.4 Structure of the entire model

It can also be seen from Fig.1, that the entire *mathematical model* of the aircraft can be formed only in case all the partial effect, too, are taken into consideration, and in case, the change brougth about by the partial effects can also be simulated.

Accordingly, the entire mathematical model of the aircraft should have the form(3) of the structure shown in Fig.2. It can be seen that the relationship between the models is partly vertical and partly horizontal. Some of the models fit into the other ones organically, or are embedded into them, respectively. Other models are connected only in a looser way with the description of other sub-processes (e.g. macro-economic models). At the same time, each partial model has its complicated influence exerted upon the other partial models individually in a demonstrable way.

2. PRACTICAL CONSIDERATION

2.1. Partial models to be applied

Some of the part-models shown in Fig.2 were investigated(5) one by one each to a depth of statisfactory extent. Especially, a lot of literature, monographies can be found concerning the determination of aerodynamic characteristics, the flight of aircraft, the mathematical models describing the aero engines, as well as the controlling of aircraft and its automatic and manual control systems. In connection with the latter, a lot of specialists deal today with the simulation of pilots' activity. In separate monographies, the simulation of the automatic systems on board and their single elements, respectively, as well as the simulation of their effects exerted on the aircraft's characteristics are dealt with independently. The environmental conditions can be given, on the one hand, by part-models elaborated thoroughly, e.g. with respect to the description of atmospheric turbulence, and, on the other hand, by stochastic models, which can be considered as primitive ones with respect to failures and their initial phase of occurance. Models of damage have been dealt with so far only to a small extent in relation to the aspects outlined above. Economic models can be considered as elaborated relatively well, however their interconnections with other models are not made clear sufficiently and uniequivocally.

It is a very difficult and complicated problem to combine the individual part-models and to decompose the general model, respectively, and it can not be generally solved but only approximately. The simulation is aggravated by the fact that the deviation in the single

characteristics exerts generally its influence on a number of other characteristics, or else, the deviations can not be given by simple statistical models.

2.2 Description of the characteristic deviations

The structural, operational and service characteristics can reach and even exceed the tolerance limits essentially in three different ways (Fig.3). In the first case, the change takes place under the influence of abrupt loads differing from the designed ones and exceeding them. In the second case, the characteristics are changing gradually, and are reaching the limit of the tolerance range in a predictable way. In the third case, the limit of the tolerance range in a predictable way. In this third case, the tolerance range becomes restricted for some other reason, e.g. by the effect of other failures, and the excess of the tolerance range can occur even under the influence of otherwise normal designs loads.

The deviations in the characteristics illustrated in Fig.3 can be considered as sudden (or abrupt) degradational (or parametric) and relaxation troubles

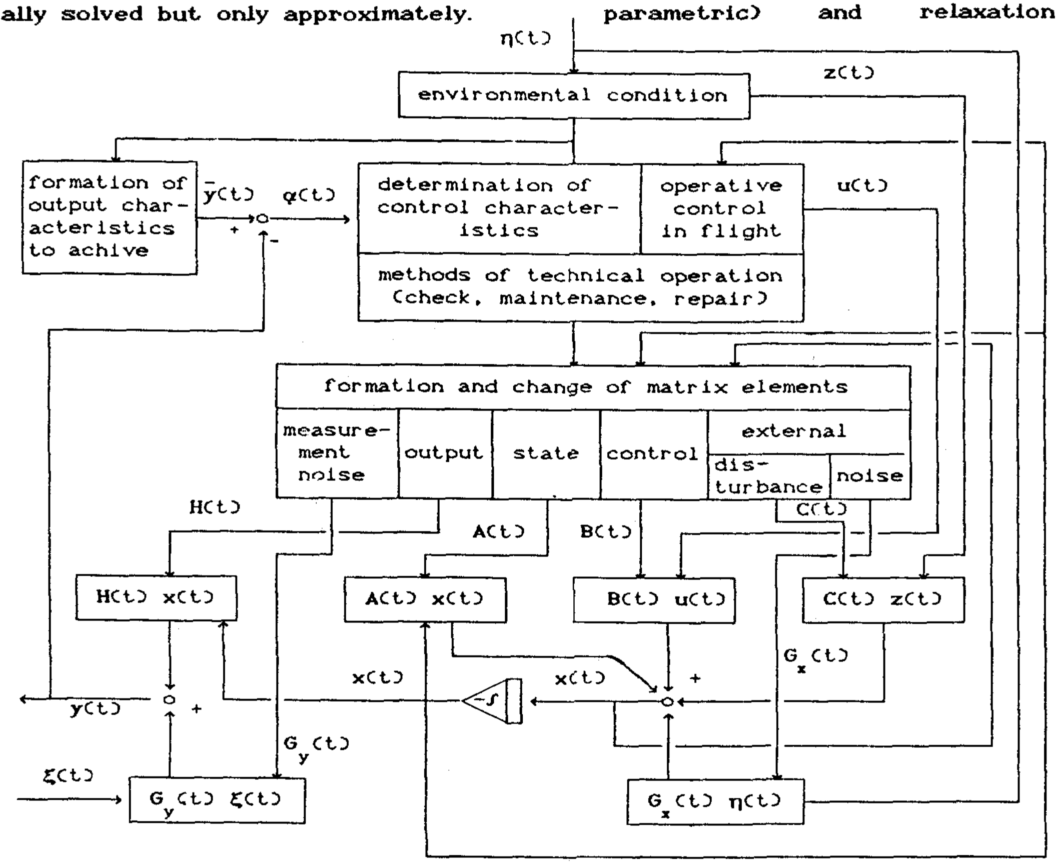


Fig.1: Block-diagram of the linearized model reflecting the operational process

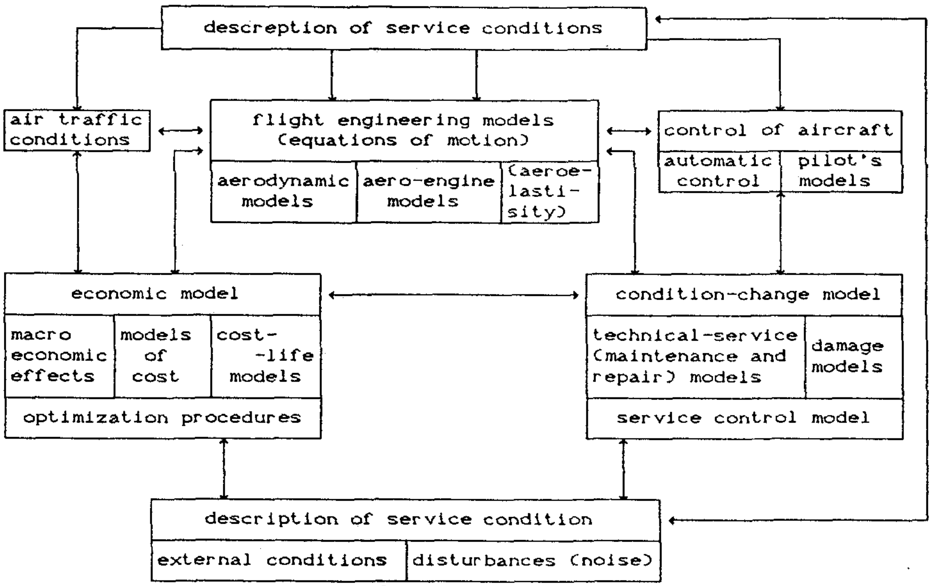


Fig.2: Structure of the entire model

(failures) based upon the analogy taken from the reliability theory. Consequently, the probability of the dwelling of the characteristics within the tolerance range can be given$^{(1,3)}$ with the help of exponential, normal and two-parameter exponential laws.

$$P_s(t) = e^{-\lambda_s t} \qquad (3)$$

$$P_p(t) = 1 - \frac{1}{\sigma_p\sqrt{2\pi}}\int_0^t e^{-\frac{(t-a_p)^2}{2\sigma_p^2}}\,dt \qquad (4)$$

$$P_r(t) = e^{-\delta\lambda_r(t-t_r)} \qquad (5)$$

where P_s, P_p, P_r are the probability of the dwelling of the characteristics within their tolerance range when exceeding the tolerance values in an abrupt, sudden degradational and relaxation way; λ_s, λ_r are the intensity of leaving the tolerance range with abrupt and relaxation changes; a_p, σ_p are the average time and variance, resp., when exceeding the tolerance range with gradual changes; t_r is the time of restriction of the tolerance range; δ is a constant ($\delta = 1$ if $t \geq t_r$, $\delta = 0$ if $t < t_r$ /.

In a way known from the reliability theory, probability P of the dwelling within the tolerance range - with the joint occurrance of all the three types of change - can be calculated by the following relationship:

$$P_o(t) = P_s(t) * P_p(t) * P_r(t)\ , \qquad (6)$$

or

$$P_o(t) = c_s P_s(t) + c_p P_p(t) + c_r P_r(t), \qquad (7)$$

where weighting coefficients c_s, c_p, c_r express the proportion of the sudden, degradational and relaxation changes.
When examining the change in the operational characteristics of a system, then - using the analogy taken from the reliability theory - the calculation principle of a system's reliability should be applied, or, respectively, the stochastic process describing the change in the characteristics should be approximated by some known relationship.

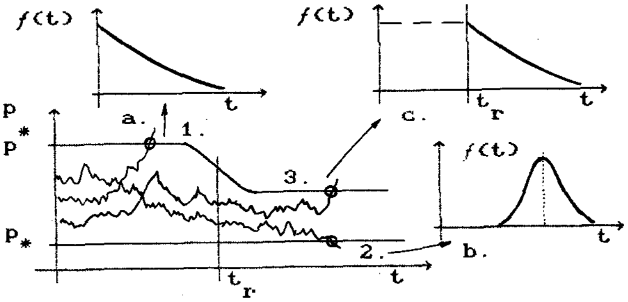


Fig.3: Typical cases of the characteristic parameters exceending the tolerance range (p-parameter, p^*, p_* - upper and lower values associated with the tolerance range, t- time of operation, t_r -time-parameter belonging to the restriction of tolerance range, 1.- sudden, 2. - degradation, 3.-relaxation failure: density functions of a. - exponential, b. - normal, c. - two-parameter exponential distributions).

2.3. Models recommended for evaluation of the deviation influences

By using the principle of control

engineering, the problem of the describing the characteristics deviations can be solved in another way, too. Let Ω_x, Ω_u, Ω_z, Ω_p, Ω_y denote the admissible vectorial field of each characteristics. Accordingly the use of the following expressions is recommended to indicate the deviations from the optional characteristics(1,5):

$$P_1 \left\{ y(t) \in \Omega_y \,\middle|\, t_o \le t \le t_o+\tau,\ x \in \Omega_x,\ u \in \Omega_u,\ z \in \Omega_z,\ p \in \Omega_p \right\} \tag{8}$$

$$P_2 \left\{ u(t) \in \Omega_u \mid t_o \le t \le t_o+\tau,\ x \in \Omega_x,\ z \in \Omega_z,\ p \in \Omega_p,\ y \in \Omega_y \right\} \tag{9}$$

If joint density function:

$$f_\Sigma = f \left[x(t),\ u(t),\ z(t),\ p(t),\ y(t) \right]$$

is known, then (8) and (9) can be calculated as follows:

$$P_1\left\{ y(t) \in \Omega_y | \ldots \right\} = \frac{\int_{\Omega_x}\int_{\Omega_u}\int_{\Omega_z}\int_{\Omega_p}\int_{\Omega_y} f_\Sigma \, dx\, du\, dz\, dp\, dy}{\int_{-\infty}^{\infty} dy \int_{\Omega_x}\int_{\Omega_u}\int_{\Omega_z}\int_{\Omega_p} f_\Sigma \, dx\, du\, dz\, dp} \tag{10}$$

$$P_2\left\{ u(t) \in \Omega_u | \ldots \right\} = \frac{\int_{\Omega_x}\int_{\Omega_u}\int_{\Omega_z}\int_{\Omega_p}\int_{\Omega_y} f_\Sigma \, dx\, du\, dz\, dp\, dy}{\int_{-\infty}^{\infty} du \int_{\Omega_x}\int_{\Omega_z}\int_{\Omega_p}\int_{\Omega_y} f_\Sigma \, dx\, dz\, dp\, dy} \tag{11}$$

It can be seen easly that the expressions similar to those in (8) - (11) can be written for vectors **x** and **p** too.

According to relationships (8) - (11), the models describing the deviations in the operational characteristics can be given only through approximate procedures and statistical data processing. However, these procedures can fairly well give the deviations in the operational characteristics. Therefore, their application can be recommended because of their information content.

2.4. Additional problems

The realization of modelling real flight situations is aggravated - in addition to the defectiveness of the part-models and their complicated interconnections little known so far, as well as the stochastic deviations(7) of the constructional and service characteristics - also by the following(3,5):

- the special distributions realized at the tail of the empirical density functions of the characteristics(8), and
- the inaccurate calculation and modelling of the joint occurrrence of several errors or failures.

In the course of statistical model-examinations, the random variable characteristics are generated within the confidence level associated with the given characteristics and according to a kind of type-distributions. However, the problems, i.e. the special flight situations are encountered mostly with the characteristic deviations occurring in the neighbourhood of the confidence-values assumed convertionally. In addition to the above said, there can occur a more considerable deviation between the theoretical and real functions at the tail of the density functions.

On the other hand, the occurence of the characteristics beyond their permitted values is considered as a failure, and is modelled by the emergence of independent events taking place with weak probability. However, according to the evidence of our examination records, the emergence of the flight events, catastrophic crashes are often brought about by nearly simultaneous, joint occurrence of 2-3, or even 4-6 faults and failures. Consequently, the faults, characteristic deviations and failures occurring with weak probability can not be considered as entirely independent events. In the course of our practical examinations, this fact can be taken into consideration in three ways(5):

- by weighting coefficient (by increasing proportionally the values of functions at the tail of density functions after the occurrence of the first fault, or failure),
- by the distortion of density function (by changing consecutively the density functions after the occurrence of the first, second, third faults or failures, respectively),
- by the decomposition of density functions into two parts (by decomposing the empiric density func-

tions of the occurrences of errors or failures into an independent density function representing the basic distribution, and a conditional density function depending on the occurrence of the given failures or characteristic deviations).

3. EXAMINATION RESULTS

3.1. Approximation of the parameter-deviations

There was a stady the approximation methods of structural characteristic deviations by stochastic processes with a view to the description of the characteristic deviations as a function of the operational time (flight hours)(1,9), applicated to the evalution and prediction of permanent deformation in delta wings of fighters in operation during levelling (Fig. 4, 5).

If the process describing the permanent deformation in aircraft is free from after-effects, and , consequently it is independent of the circumstances preceding the initial moment of the investigation, then the survey data are discrete sampling values of a monotonous, non-decreasing, continuous stochastic process, which can be approximated by the Markov process(9,10). From a mathamatical aspect, the non-steady state, continuous, random process of the permanent deformations is approximated by homogeneously continuous process having a steady-state increment and a discreate field of state.

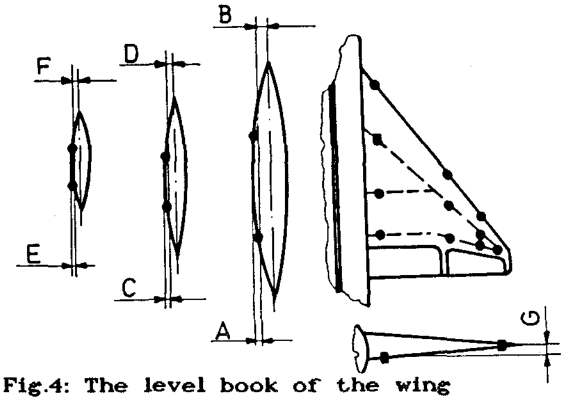


Fig.4: The level book of the wing

In Fig. 6,7 and 8 the some results of calculated level exceeding probability, of the influences of the permanent deformation on the wing section lift coefficient, and, the lift generated on the wing during the landing examined by the recomenden way in the part 2.3.

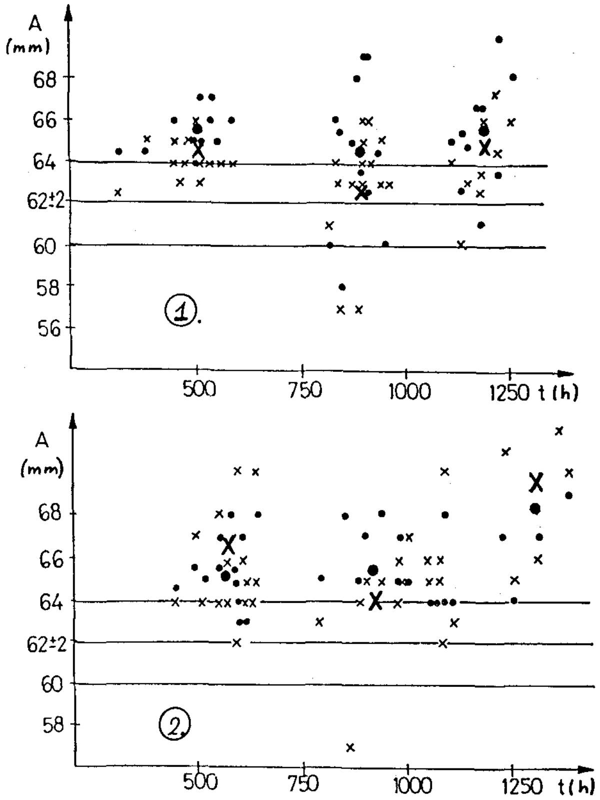


Fig.5:Dispersion of levelling value "A" on the basis of the values measured in the course of the aircraft repairs. 1.- single-seat fighter, 2. - double-seat fighter, x - left-hand side, • - right-hand side half-wing area, X, ● mean values of data konverted into average service - life up to repair, t - time of operation.

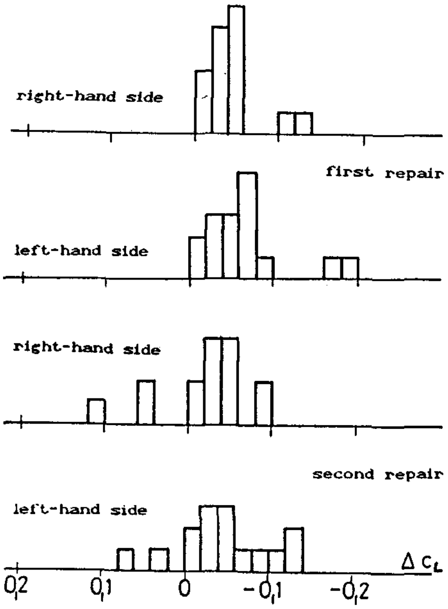
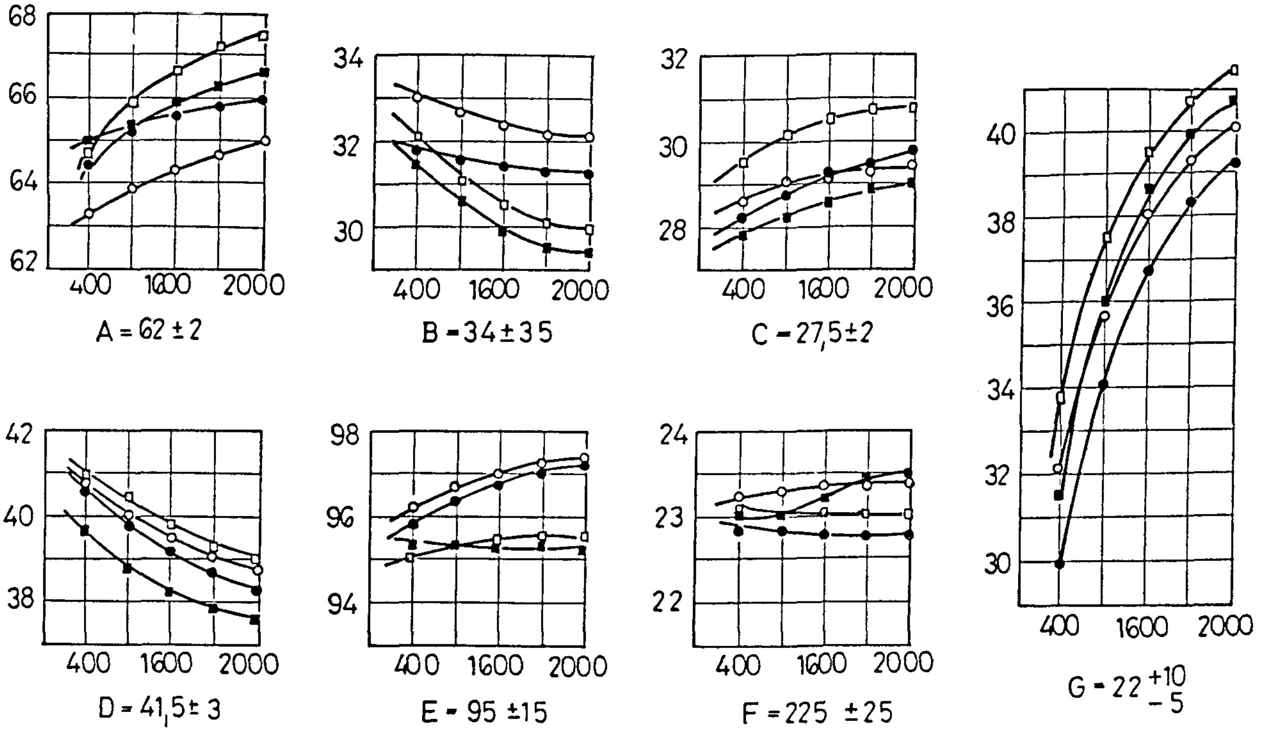


Fig.6: Calculated density functions of deviation in lift coefficient (c_L) due to permanent deformation in the airfoil section of levelling, close to the fuselage of the single-seat fighters.



A = 62 ± 2 B - 34 ± 3 5 C - 27,5 ± 2 G - 22 $^{+10}_{-5}$

D = 41,5 ± 3 E - 95 ± 15 F = 225 ± 25

Fig. 7: The change of the levelling mean values with tolerance in the wing during operation calculated from levelling measurement data approximated by Markov process
o - right-hand half-wing of single-seat fighters
- right-hand half-wing of double-seat fighters
o - left-hand half-wing of single-seat fighters
- left-hand half-wing of double-seat fighters
vetical axes - levelling values in mm,
horisontal axis - time of operation in flying hours.

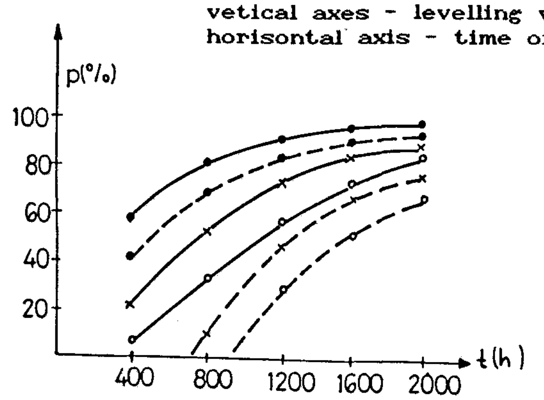


Fig.8: The probability of occurrance of the lack of lift (• - 2500 N, x - 4200 N, o - 4700 N) due to the deformation of to single-seat (———) and duble-seat fighters (··········)).

3.2. Statistical modelling

In our practice so far statistical model-examinations(2,11) approaching the real flight situations have been dealt with, with a view to the evaluation of the state-change processes, the investigation of the deviation efects for identification purposes, the prediction of the sensitivity parrameters of the functions the determination of the limitations in service and the development of the on board advisory system. In the course of our examinations(1,5), the density functions giving the distributions of the characteristics were provided, on the one hand, in the form of empirical density functions determined by the flight information recorded during normal flight operations, and, on the other hand, in the form of density functions of known distribution (normal, Weibulli, exponential) assumed on the basis of theoretical considerations.

As an example, some of our examination results associated with the landing of a medium-size aircraft are introduced here. In Figs. 9, 10, the results measurements in operation(1,5) are represented.

The statistical modelling was realised for the motion of the aircraft(12,13,14) in the vertical plane, only. The enviromental factors were modelling as air turbulence and wind-shear(5). In Fig. 11, the characteristic results of statistical modelling are shown.

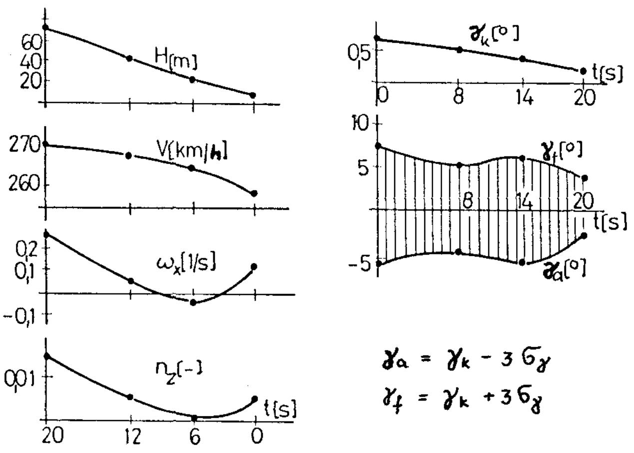


Fig. 9: Changes in the average values of the height of flight (H), velocity (V), roll rate, (P), roll angle, (γ) and lateral load factor (n) before touch-down. t- time befor touch-down.

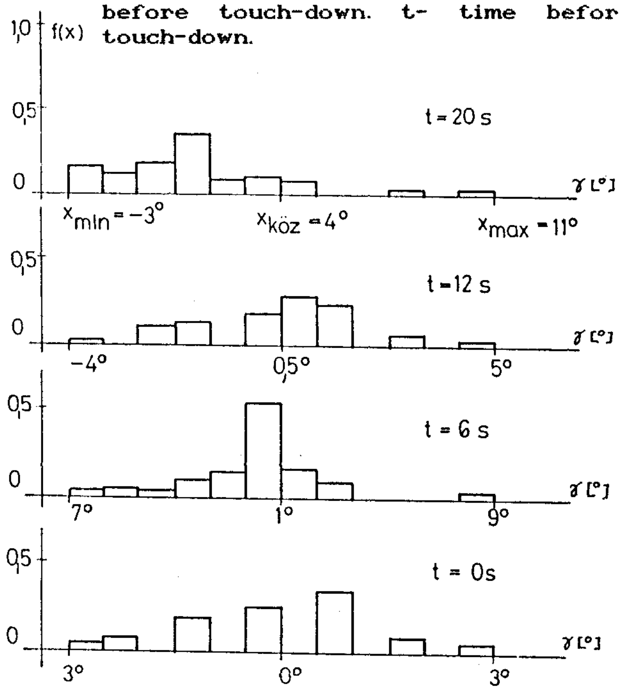


Fig.10: Empirical density functions of the roll angle (γ) measured in the course of landings for medium-size liners, t = 20, 12, 6, sec. before touching down, and at the moment of touch-down.

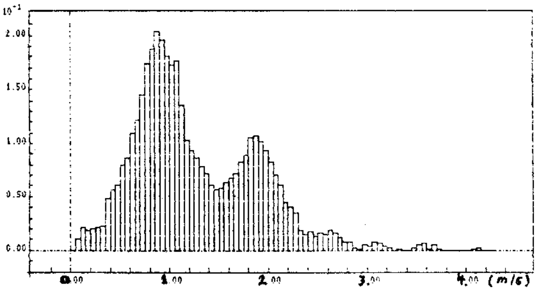


Fig.11: Density function of the vertical touchdown velocity at landing on the basis of statistical model-examinations of landing.

3.3. Modelling of individual flight situations

When investigating flight events, every time individual flight situations should be modelled, or else the simulated realization of a given flight should be formed on the basis of the information data stored by the data-recording system on board, an data gained from other sources of information very often defective (e.g. observation). On such occasions, the activity of the staff can be essentially considered as known, while the changes in the enviroimmental conditions, in external disturbances (e.g. atmospheric turbulence), the actual values of the constructional, service characteristics (e.g. aerodynamic coefficients) are unknown. Consequently the problem can be solved by the iterative interactive procedure with honologation(15) of the measurement data and with the identitification of the structural parameter deviations outlined in the block-diagram of Fig. 12.

SUMMARY

When modelling individual flight situations, in fact, a non-linear, stochastic model should be formed, which can be given by a system of differential equations having unstable parameters. In the model, the characteristic parameters (e.g. aerodynamic coefficients) are changing stochastically as a function of time, and partly in a cumulative way. An additive, stochastic noise brought about by the external disturbing effects (e.g. atmospheric turbulence). The formation of the model can be carried out only with the knowledge of the empiric density functions, or realizations of the distribution of characteristic parameters. The generalization of modelling can be promoted by the use of the models introduced in this paper, which can be applied to describe the parameter deviations, or by the use of the relationships recommended with a view to the evaluation of the effects of deviations, respectively.

Modelling of real flight situations should be carried out with a view to either statistical examinations, or to the simulation of a given flight situation. In the first case, statistical modelling should be performed, while in the second one, the problem can be solved on the basis of a complex iterative interactive program.

Reference

(1) Rohács J., Deviations of aerodynamic characteristics and performance data of aircraft in the operational process (Ph.D thesis) Institute of Civil Aviation Engineers, Kiev 1986 (in Russian)

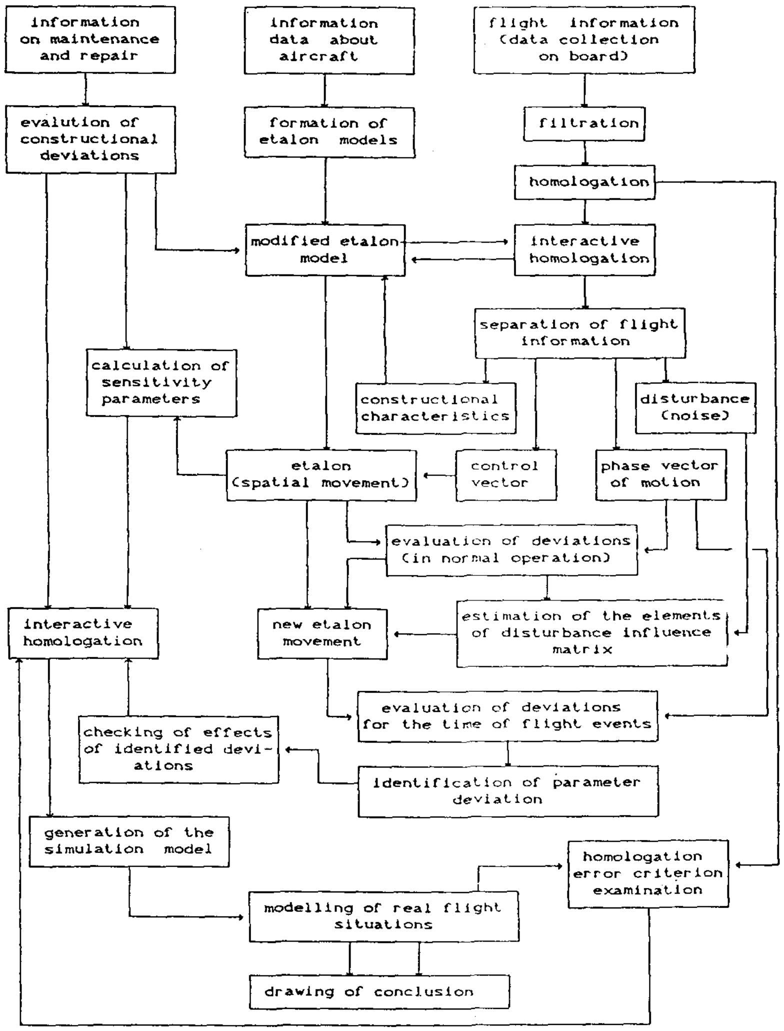


Fig.12. Simulation modelling of induvidual real flight dituation

(2) Лебедев А.А., Бобронников В.Т., Красилщиков М.Н., Малюшев В.В., Статистическая динамика и управление летательных аппаратов "Машиностроение", Москва, 1985

(3) Mathematical models for estimation of real characteristics of aircraft type Tu-154, LRI RTK Report,Budapest, 1989 (in Hungarian)

(4) Application of flight data for diagnostic purposes "Int. Conf. AIRDIAG'89 Rydzyna, Poland" 1989. 20.1-20.12. pp.

(5) Aerodynamical and flight engineering reliability of aircraft, LRI RTK Report, Budapest, 1984, 1985 (in Hungarian)

(6) Rohács J., The examination of theoperational characteric deviations of aircraft (Tasks and methods) IX. Magyar Repüléstudományi Napok, GTE-MALÉV, Budapest, 1988, 130-145 (in Hungarian)

(7) Ходько С.Т., Проектирование систем управления с нестабильными параметрами "Машиностроение" Ленинград, 1987.

(8) Боярский Г.Н., Касьянов В.А., Методы и опыт моделирования полетных ситуаций IX. Magyar Repüléstudományi Napok, GTE-MALÉV, Budapest, 1988, 519-530.

(9) Rohács J., The influence of the geometrical characteristic deviations of the aircraft on their performance data (Doctor tech. thesis) Technical University of Budapest, 1980 (in Hungarian)

(10) Evalution and prediction of permanent deformation in delta wings "Periodica Polytechnica Transportation Engineering" BME, 1986 No.1. 17 - 32. pp.

(11) Kushner H.J., Probability Methods for applications in stochastic control and for elliptic equations, Academic Press, New York - Son Francisco - London, 1977

(12) Горбатенко С.А., Макашов Э.М, и др., Механика полета "Машиностроение" Москва, 1969.

(13) Etkin B., Dynamics of atmospheric flight, Wiley, New York, 1972.

(14). Babister A.W., Aircraft stability and control Pergamen Press Oxford, London, New York, Paris, 1961.

(15) Касьянов В.А., Папченко О.М., суббота В.Н., Ударцев Е.П., Опыт идентификации аеродинамичецких характеристик продольного движения по данным летных испытаний "Некоторые вопросы прикладной аеродинамики" КИИГА Киев 1982.