AN ALGORITHM FOR DECOUPLING VERTICAL/HORIZONTAL MOTIONS OF NON-SYMMETRIC ROLLING AIRCRAFT

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Abstract:

This paper deals with the Vertical/Horizontal accelerations decoupling process of a rolling missile which is only non-symmetric relatively to inertias. The consequence of rolling an aircraft is to prohibit the separation of its motion equations into independent lateral and longitudinal groups, and also to affect its stability. Many vehicles bank or roll to accomplish a manoeuvre which can result in the presence of high roll rates, producing cross-couplings and its effects. If the designer adopts the objective of decoupling the pitch/yaw channels of the rolling aircraft, he can gain from the fact that a demanded manoeuvre could start to be applied at the same time as the vehicle starts the roll orientation task, saving time and also avoiding stability complications. Thus, in this paper a decoupling tecnique for non-symmetric aircraft is presented, which makes the global system (aircraft + autopilot) behave and have the characteristics of an axisymmetric aircraft. The result is that one can then apply all the theory developed for decoupling axisymmetric aircraft, using the Complex Summation Method as an useful tool.

Nomenclature:

A11, A12, ..., A43, A44 = State space components of acceleration and rotation rates;
a11, a12, ..., a43, a44 = State space components of acceleration and rotation rates, with feedback;
b11, b12, ..., b42 = Control components of acceleration and rotation rates;
D = d/dt operator;
(D+Pj) = (d/dt + Pj) - rotational operator;
$f = (f_z - j f_y)$ - Complex acceleration;
$f_x = (f_{zx} - j f_{yx})$ = Complex acceleration at the point x in the missile x-longitudinal axis;
f_y = y-axis component of acceleration;
f_z = z-axis component of acceleration;
Ix, Iy, Iz = Roll, Yaw and Pitch moments of inertia;
$j = \sqrt{-1}$;
KFZ, KQ, KFY and KR = Feedback gains;
L_p, L_r, L_v, L_ξ = Roll aerodynamic derivatives;
m = Missile mass;
M_q, M_w, M_v, M_η, M_ζ = Pitch aerodynamic derivatives;
N_r, N_w, N_v, N_η, N_ζ = Yaw aerodynamic derivatives;
p = Roll rate small perturbation;
P = Steady state roll rate;
q = Pitch rate;
r = Yaw rate;
T = Elevator and rudder servo time constant;
u = x-component of transverse velocity;
U = Steady state x-component of transverse velocity;
v = y-component of transverse velocity;
w = z-component of transverse velocity;
x = A point in the missile x-longitudinal axis;
X_q, X_u, X_w, $X_{propulsive}$ = x-direction force and trust aerodynamic derivatives;
Y_w, Y_v, Y_η, Y_ζ = y-direction force aerodynamic derivatives;
Z_w, Z_v, Z_η, Z_ζ = z-direction force aerodynamic derivatives;
$\Omega = (q + j\, r)$ - Complex rotation rate;
η = Elevator angle;
$\mu = (w - j\, v)$ - Complex transverse velocity;
ζ = Rudder angle;
ξ = Aileron angle;
$\delta = (\eta + j\zeta)$ - Complex control angle;

Subscripts:

$_D$ = Demanded;
$_I$ = In inertial axes of reference;
$_x$ = At the missile x-point in the x-longitudinal axis;
$_y$ = In the y-direction;
$_z$ = In the z-direction.

1. Introduction:

The fact that an aircraft has a high roll rate brings with it the introduction of coupling terms which prohibit the separation of the motion equations into independent lateral and longitudinal groups, and the six equations must be simultaneously treated[3] .

This coupling is what is in general terms referred to as inertial cross-coupling and, as is stated in most of the literature concerned with it, has only become of importance since the early fifties (its effects can be seen through a simmulation with results presented in Fig.3).

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This is not the only consequence of coupling due to high roll rates, but it affects the aircraft stability, which in general is significantly reduced as roll rate increases.

The most common aircraft to present this manoeuvre are remotely piloted vehicles (RPV) or unmanned vehicles, highly manoeuvrable airplanes or any aircraft having relatively large rotating masses.

The basic theory of inertia cross-coupling had been established as early as 1948 by Phillips[23], but it was only considered later by designers when the problem showed up strongly in flight testing.

There is nowadays a certain amount of published work on this subject, covering the stability hazards of the gyroscopic effects, and other effects such as magnus and asymmetry[3,9,10,11,13,14,16-19,21,23,24,28].

In parallel with rolling aircraft studies, studies of mathematical terms related to interconnected systems were developed[1,2,5,12,20,22,25,26].

Soon these two fields were brought together and with the necessity of identifying different types of couplings there was a great advance.

One of the first proposals for breaking down the total complexity was introduced by Lange and Fleming[21], who introduced the concepts of "complex symmetry" and "frequency symmetry", these being mathematical in concept rather than physical.

Others, such as Shinar and Merhav identified "low" roll rates as being those in which inertial and Magnus couplings may be ignored.

Other authors followed another line of research, which was the study of the influences of pitch and yaw on the roll motion giving origin to studies of other subjects such as roll-resonance.

Finally Fortescue[7,8] identified the different types of antisymmetric cross-couplings in axisymmetric missiles such as, Phasing Error Coupling, Magnus Effect, Control Coupling, Rotational Coupling and Gyroscopic Coupling.

The first three of these are aerodynamic in nature, the fourth one is of a geometric nature and is observed to depend on the system of reference axes used, and the fifth is of a gyroscopic nature.

With this classification and when considering linear systems, each type of cross-coupling can be separately studied.

The conventional aircraft types which use a Bank-to-turn steering policy and many cartesian missiles, which are designed not to roll will experience high roll torques and consequent roll rates due to aerodynamic effects at high angles of attack. These can cause premature loss of stability due to the pitch/yaw cross-couplings produced by the roll rate.

Most types of autopilot designs have dealt with this problem by controlling the roll/yaw coupling, since pitch in general has good stability conditions, and basically because most of the vehicles used have a preferential plane for application of demanding manoeuvre gs.

If we adopt the objective of decoupling the pitch/yaw channels, we could gain from the fact that we start to apply the demanded manoeuvre at the same time as the vehicle starts the roll orientation task, and save time and also avoid stability complications.

This idea was introduced by Fortescue[6,7] applied to the axisymmetric missile, and is followed here (the results of the application of this idea can be seen in Fig.7).

Objectives:

The objective is to decouple the vertical/horizontal aircraft response motions in non-rolling axes (in this case the vertical and the horizontal accelerations).

We will be only dealing with gyroscopic and rotational effects, which are the major ones and responsible for large angles of attack and side-slip[6-9,24].

It is considered that the vehicle has to respond to commands for manoeuvres in directions which are specified in Inertial (non-Rolling) axes, rather than in the vehicle's rolling (Body) axes. These commands and manoeuvres can be expressed as components in orthogonal axes, such as vertical and horizontal directions.

The result is a system which can be free to roll without loosing control.

As McGehee[15] says, the roll rates required for BTT policy in nonsymmetric missiles can be very high, so the application of the Complex Summation Method[6,7] is extended to the inertially non-symmetric aircraft case.

This technique is directed towards the control of RPVs. or other unmanned guided vehicles where the presence of a man (pilot) with all his feelings, sensitivities and response times are not part of the control loop (autopilot).

The general statement of the problem:

In this study the aircraft is considered rigid, or at least, their body flexure modes are very high, and thus there is no interference between them and the signals manipulated in the autopilot. It is then possible to separate the two subjects (body flexure and autopilot).

At this stage we could say that the baseline autopilot should be one which makes the lateral and normal channels similar, since one of our objectives is to obtain axial symmetry. This is possible by choosing the right baseline autopilot gains.

Then the system resulting from considering the autopilot will be vertically/horizontally and normally/laterally cross-coupled due to roll-rates and will have a behaviour somewhat similar to the axisymmetric case.

A system that is neither symmetric nor antisymmetric will be transformed into a similar and antisymmetrically coupled (SAC) one in order to be able to apply the methods developed for the axisymmetric aircraft case[6,7].

This will be done starting from the point where the system equations are defined in terms of body axes.

Mathematically, to obtain a SAC system means that we want to obtain a system expressed by complex equations equivalent to the equations which represent the axisymmetric aircraft.

We then consider the aircraft as two separate systems (the lateral and normal) and express the two outputs, the normal and lateral accelerations, in terms of demanded and cross-coupling perturbation inputs.

The acceleration differential equations for the aircraft, with the original baseline autopilot, which is the one shown in Fig.2, can be expressed in the form:

$$(D^2 + 2\xi_v\omega_v D + \omega_v^2) f_{zx} = (a_{v1}D+a_{v0})\eta_1+(b_{v1}D+b_{v0})f_{yx}+(c_{v1}D+c_{v0})r+(d_{v1}D+d_{v0})\zeta_1 \quad (1)$$

-normal acceleration differential equation;

$$(D^2 + 2\xi_\ell \omega_\ell D + \omega_\ell^2) f_{yx} = (a_{\ell 1}D+a_{\ell 0})\zeta_1+(b_{\ell 1}D+b_{\ell 0})f_{zx}+(c_{\ell 1}D+c_{\ell 0})q+(d_{\ell 1}D+d_{\ell 0})\eta_1 \quad (2)$$

-lateral acceleration differential equation.

The equations associated with the forward x-axis are neglected since the roll rate (p) and the forward velocity (u) have negligible effect on w, q, v and r [6,7].

In order to transform these equations into similar and antisymmetrically coupled ones means obtaining a system defined by one complex equation in airframe axes (refer to Fig.3), of the form:

$$[(D+Pj)^2 +2\xi_0\omega_0(D+Pj) + \omega_0^2) f_x = [a_1 (D+Pj)+a_0]\delta_D+ +j\{[b_1(D+Pj)+b_0]f_x-[c_1(D+Pj)+c_0]\Omega-[d_1(D+Pj)+d_0]\delta_D\} \quad (3)$$

where the variables are complex, defined as:

$$f_x = f_{zx} - j f_{yx} ; \quad \delta = \eta + j \zeta$$

Separating equation (3) into its real normal and lateral components leads to the following equations in airframe axes:

$$(D^2+2\xi_0\omega_0D+\omega_0^2)f_{zx}=(P^2-b_1P)f_{zx} + - [(2P-b_1)D+2\xi_0\omega_0P-b_0]f_{yx}+ +c_1Pq+(c_1D+c_0)r+ +(a_1D+a_0+d_1P)\eta_D+(d_1D+d_0-a_1P)\zeta_D \quad (4)$$

$$(D^2+2\xi_0\omega_0D+\omega_0^2)f_{yx}=(P^2-b_1P)f_{yx} + +[(2P-b_1)D+2\xi_0\omega_0P-b_0]f_{zx}+ -c_1Pr+(c_1D+c_0)q+ -(a_1D+a_0+d_1P)\zeta_D+(d_1D+d_0-a_1P)\eta_D \quad (5)$$

when the roll rate P is constant.

These last two equations (4 and 6) are the ones we need to obtain in order to have a SAC system, expressed in airframe axes.

Comparing these equations with the original ones when the baseline autopilot is included (1 and 2 plus autopilot), we obtain the control law needed in order to transform the original baseline system into a SAC system.

Once the system is transformed into a SAC system the decoupling techniques for such type of systems as for the axisymmetric aircraft can be applied by using the Complex Summation Method.

2. The Standard Missile

The missile considered here as non-symmetric for studying the cross-coupling effects between the horizontal and vertical motions is based on different inertias in pitch and yaw only.

The missile is of the tail controlled cartesian type, fitted with an accelerometer/rate gyro autopilot as in Fig.1.

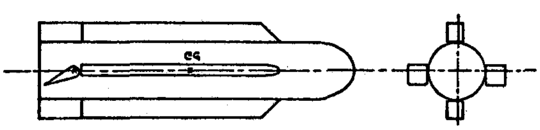

Fig.1 - Sketch of the non-symmetric missile used for cross-coupling studies.

3. The Equations of the Missile with High Roll Rates

The equations in their algebraic form are:

a) Equations associated with the forward x-axis:

$$(mD -X_u) u -X_w w -X_q q = X_{propulsive} \quad (6)$$
$$-L_v v +(IxD -L_p) p -L_r r = L_\xi \xi \quad (7)$$

b) Pitch equations:

$$(mD -Z_w) w -mU q +(mP -Z_v) v = Z_\eta\eta +Z_\zeta\zeta \quad (8)$$
$$-M_w w +(IyD -M_q) q -M_v v -(Iz-Ix)P r = M_\eta\eta+ M_\zeta\zeta \quad (9)$$

c) Yaw equations:

$$-(mP +Y_w) w +(mD -Y_v) v +mU r = Y_\eta\eta +Y_\zeta\zeta \quad (10)$$
$$-N_w w +(Iy-Ix)P q -N_v v +(IzD -N_r) r = N_\eta\eta +N_\zeta\zeta \quad (11)$$

In these equations it is considered that the Magnus effects as well as the control couplings are present and the missile is rolling with roll rate P and flying at a forward speed U and all the other state conditions are zero.

Some other effects are neglected such as the gravitational forces, the aerodynamic derivatives Y_r, $Y_{\dot v}$, $Z_{\dot w}$, Z_q, etc., and terms of second and higher order, the effects of which are small.

To a first approximation and for the same reasons as for the axisymmetric aircraft, the roll rate (p) and forward velocity (u) perturbations will have negligible effect on w, q, v and r, as was mentioned earlier. This can be said due to the fact that the non-symmetric aircraft or missile still maintains geometric axial symmetry.

Since the controlled outputs considered are the accelerations, it is better to change the state variables to accelerations and angular rates, instead of velocities and angular rates and the result is:

$$\dot f_{zx} =A11\ f_{zx}+A12\ q+A13\ f_{yx}+A14\ r+b11\ \eta +b12\ \zeta$$
$$\dot q =A21\ f_{zx}+A22\ q+A23\ f_{yx}+A24\ r+b21\ \eta +b22\ \zeta \quad (12)$$
$$\dot f_{yx} =A31\ f_{zx}+A32\ q+A33\ f_{yx}+A34\ r+b31\ \eta +b32\ \zeta$$
$$\dot r =A41\ f_{zx}+A42\ q+A43\ f_{yx}+A44\ r+b41\ \eta +b42\ \zeta$$

where A11, A12, ..., A43, A44 are the terms of the <u>state matrix</u> and b11, b12, ..., b42 the terms of the <u>control matrix</u>, after having transformed the equations (8, 9, 10 and 11) in terms of $\dot w$, $\dot q$, $\dot v$ and $\dot r$ to $\dot f_{zx}$, $\dot q$, $\dot f_{yx}$ and $\dot r$ using also the acceleration equations:

$$f_{zx} = D w + P v - (x D + U) q + x P r \quad \text{and}$$
$$f_{yx} = - P w + D v + x P q + (x D + U) r .$$

By considering that the lateral states are inputs to the normal ones and the same for the normal ones in relation to the lateral ones, we represent and deal with the equations as if they were representing two different systems with coupled inputs (f_{yx}, r and ζ into the normal motions f_{zx} and q; and f_{zx}, q and η into the lateral ones f_{yx} and r) plus independent inputs (η and ζ).

$$\dot f_{zx} =A11\ f_{zx}+A12\ q+b11\ \eta + A13\ f_{yx} +A14\ r+b12\ \zeta$$
$$\dot q =A21\ f_{zx}+A22\ q+b21\ \eta + A23\ f_{yx} +A24\ r+b22\ \zeta \quad (13)$$

and

$$\dot f_{yx} =A33\ f_{yx}+A34\ r+b32\ \zeta + A31\ f_{zx} +A32\ q+b31\ \eta$$
$$\dot r =A43\ f_{yx}+A44\ r+b42\ \zeta + A41\ f_{zx} +A42\ q+b41\ \eta \quad (14)$$

and are the independent control inputs of the normal and lateral equations respectively.

4. The Equations with Feedback when not Rolling

When the aircraft is not rolling, the system uses feedback from the rate gyros and accelerometers as represented by the diagram of Fig.2.

With this feedback control and when non-rolling, the equations can be separated into lateral and normal parts which are algebraically represented as in the axisymmetric case which follows:

Normal equations:

$$D \begin{bmatrix} w \\ q \end{bmatrix} = \begin{bmatrix} Z_w/m & U \\ M_w/Iy & M_q/Iy \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} + \begin{bmatrix} Z_\eta/m \\ M_\eta/Iy \end{bmatrix} \eta \qquad (15)$$

with the servo equation: $\dot{\eta} = (\eta_D - \eta) \ / \ T$ (16)

and the autopilot: $\eta_D = KFZ \ (f_{zD} - f_{zx}) + KQ \ q$ (17)

Lateral equations:

$$D \begin{bmatrix} v \\ r \end{bmatrix} = \begin{bmatrix} Y_v/m & -U \\ N_v/Iz & N_r/Iz \end{bmatrix} \begin{bmatrix} v \\ r \end{bmatrix} + \begin{bmatrix} Y_\zeta/m \\ N_\zeta/Iz \end{bmatrix} \zeta \qquad (18)$$

$$\dot{\zeta} = (\zeta_D - \zeta) \ / \ T \qquad (19)$$

and $\zeta_D = -KFY \ (f_{yD} - f_{yx}) + KR \ r$ (20)

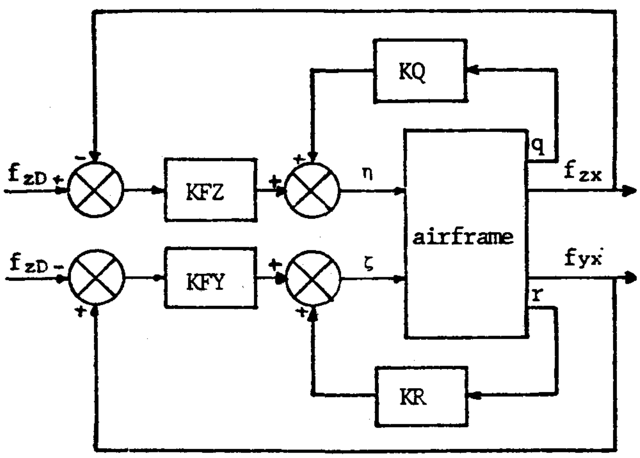

Fig.2 - Block diagram of the system formed by the non-rolling aircraft with autopilot to increase its stability and performance.

At this stage we can set KQ and KFZ in order to obtain a desired steady state gain and damping for the normal equations and the same for the lateral ones using KR and KFY.

5. The Equations with Feedback when Rolling

If now we leave the aircraft to roll, the result of the cross-coupling can be seen in Fig.3.

For the purpose of simplifying the study and understanding the process it will be assumed that the servos are ideal.

In this conditions if we apply the feedback of Fig.2 the equations (13 and 14) after being manipulated become:

$$[\ D^2 - (a11 + a22) D + a11 \ a22 - a12 \ a21 \ ] \ f_{zx} = $$
$$= (\ D \ b11 + a12 \ b21 - a22 \ b11 \ ) \ \eta_1 \ +$$
$$+ (\ D \ b12 + a12 \ b22 - a22 \ b12 \ ) \ \zeta_1 \ +$$
$$+ (\ D \ a13 + a12 \ a23 - a22 \ a13 \ ) \ f_{yx} \ +$$
$$+ (\ D \ a14 + a12 \ a24 - a22 \ a14 \ ) \ r \qquad (21)$$

$$[\ D^2 - (a33 + a44) D + a33 \ a44 - a34 \ a43 \ ] \ f_{yx} = $$
$$= (\ D \ b32 + a34 \ b42 - a44 \ b32 \ ) \ \zeta_1 \ +$$
$$+ (\ D \ b31 + a34 \ b41 - a44 \ b31 \ ) \ \eta_1 \ +$$
$$+ (\ D \ a31 + a34 \ a41 - a44 \ a31 \ ) \ f_{zx} \ +$$
$$+ (\ D \ a32 + a34 \ a42 - a44 \ a32 \ ) \ q \qquad (22)$$

Where the A11, A12, ..., A43 and A44 of equations (13 and 14) are substituted by a11, a12, autopilot.

At this stage the Complex Summation Method cannot still be applied because these normal and lateral equations do not represent a SAC system.

If, somehow, the aircraft control system (autopilot) could have been modified, resulting in a SAC system, then the Complex Summation Method could have been applied. This is what is done in the following sections, using a decoupling autopilot.

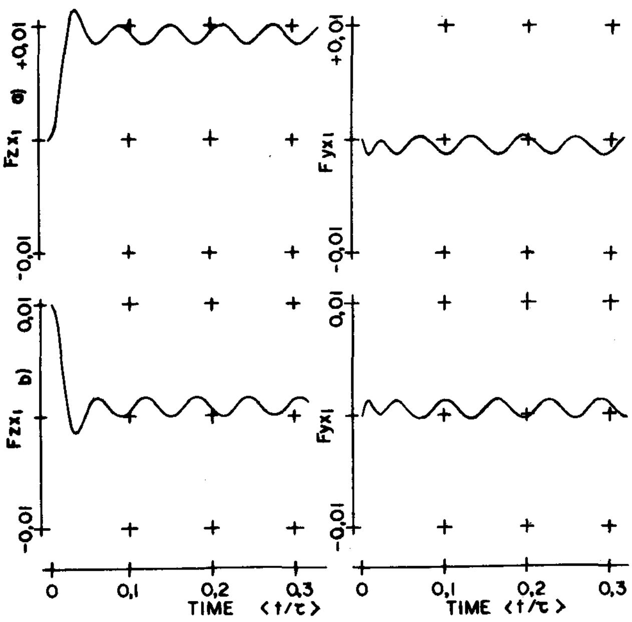

Fig.3 - Time responses of the rolling standard missile with only gyroscopic coupling and with the standard autopilot included, to a demanded step input acceleration f_{zD}=0.01

6. The Decoupling Algorithm for the Non-Symmetric Aircraft Responses in the Inertial Axes

To be able to apply the Complex Summation Method the equations representing the aircraft plus the autopilot must have equal characteristic equations, equal control dynamic coefficients and antisymmetric cross coupling dynamic coefficients.

A decoupling technique would be obtained by considering at an early stage the feedback of only q through KQ to the elevator (Fig.4) and using the value of KQ already determined in non-rolling conditions.

It is worth remembering that we want to obtain the lateral and normal equations with the same characteristic equation that means the normal and lateral eigenvalues will become equal.

The other parts such as control dynamics and cross-coupling dynamic coefficients can be made antisymmetrically coupled by extra feedback.

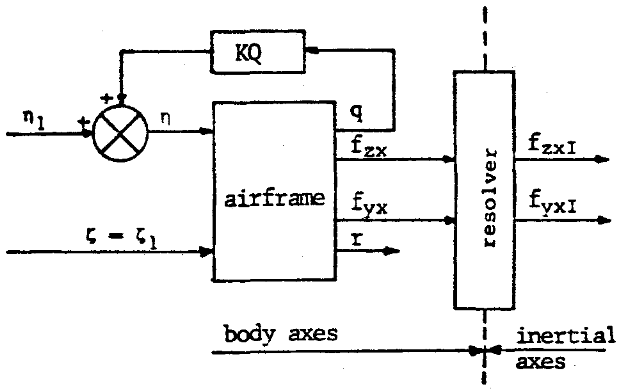

Fig.4 - Block diagram showing the aircraft with pitch rate q feedback only.

With this algorithm, the gains KQ and KFZ, which were already set for non-rolling conditions, are maintained in order to keep the non-rolling normal motion equations with the desired steady state gain and damping. With this done, the non-rolling normal motion equations are the reference ones when developing the next steps.

With the system gain KQ set and when not rolling, the system should be represented as in Fig.5.

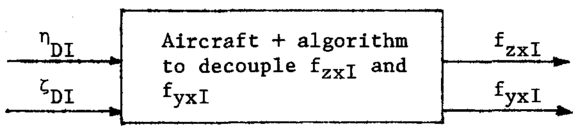

Fig.5 - Block of the decoupled aircraft system in inertial axes, considering elevator and rudder as the inputs.

This system, when rolling, should make the lateral and normal equations similar and antisymmetrically coupled in body axes and decoupled when in inertial axes.

The procedure will be to transform the acceleration equations, by control, in order to make the horizontal one similar to the vertical one in inertial axes.

What we originally have, with only feedback of q and when rolling, (that is equivalent to using KFZ=KFY=KR=0) is:

$$\begin{aligned}[D^2 -(a11 +a22)D +a11\ a22 -an12\ a21]\ f_{zx} &= (b11\ D +a12\ b21 -a22\ b11)\ \eta_1 + \\ &+ ap12\ a21\ f_{zx} + \\ &+ a13\ (D -a22)\ f_{yx} + \\ &+ (a14\ D +a12\ a24 -a14\ a22)\ r + \\ &+ b12\ (D -a22)\ \zeta_1 \end{aligned} \quad (23)$$

and

$$\begin{aligned}[D^2 -(a33 +a44)D +a33\ a44 -an34\ a43]\ f_{yx} &= (b32\ D +a34\ b42 -b32\ a44)\ \zeta_1 + \\ &+ a31\ (D -a44)\ f_{zx} + \\ &+ (a32\ D +a34\ a42 -a32\ a44)\ q + \\ &+ ap34\ a43\ f_{yx} + \\ &+ b31\ (D -a44)\ \eta_1 \end{aligned} \quad (24)$$

and what we want to obtain is:

$$[(D+Pj)(D+Pj)-(a11 +a22)(D+Pj)+a11\ a22-an12\ a21]f_x = \delta_D \quad (25)$$

a complex acceleration equation which is rotationally coupled in airframe or body axes (when rolling).

Equation (25) when expressed in inertial axes will be:

$$[D^2 -(a11 +a22)\ D +a11\ a22 -an12\ a21]\ f_{xI} = \delta_{DI} \quad (26)$$

which is totally uncoupled and is what is wanted.

The (D+Pj) is the differential operator that substitutes the D operator for inertial axes to rolling axes and the jP term of (D+Pj) is the Rotational Coupling term [7].

Thus, for this condition of decoupling in inertial axes, and with P as a constant, the complex equation (25) becomes:

$$[D^2-P^2+j2P\ D-(a11 +a22)(D+Pj)+a11\ a22-an12\ a21]\ f_x = \delta_D \quad (27)$$

where $f_x = (f_{zx} -jf_{yx})$ and $\delta_D = (\eta_D+j\zeta_D)$.

Substituting the complex variables by their real and imaginary parts, equation (27) becomes:

$$\begin{aligned}&[D^2 -(a11 +a22)D +a11\ a22 -an12\ a21]\ (f_{zx} -jf_{yx})= \\ &= (\eta_D +j\ \zeta_D) +[P^2 -j2P\ D +j(a11 +a22)P](f_{zx} -jf_{yx}) \end{aligned} \quad (28)$$

Now, separating the imaginary parts from the real parts, we obtain the two real acceleration equations that we want to achieve:

$$\begin{aligned}&[D^2 -(a11 +a22)D +a11\ a22 -an12\ a21]\ f_{zx} = \\ &= \eta_D +P^2\ f_{zx} -2P\ D\ f_{yx} + (a11 +a22)P\ f_{yx} \end{aligned} \quad (29)$$

and

$$\begin{aligned}&-j[D^2 -(a11 +a22)D +a11\ a22 -an12\ a21]\ f_{yx} = \\ &= j\zeta_D -jP\ \ f_{yx} -j2P\ D\ f_{zx} +j(a11 +a22)P\ f_{zx} \end{aligned} \quad (30)$$

To find the control law for obtaining these last equations we subtract member by member the equations we originally had (23 and 24) from the equations we want to obtain (29 and 30) and in that case we have:

<u>For the normal equations:</u>

$$\begin{aligned}&[D^2 -(a11 +a22)D +a11\ a22 -an12\ a21]\ f_{zx} + \\ &-[D^2 -(a11 +a22)D +a11\ a22 -an12\ a21]\ f_{zx} = 0 = \\ &\quad = \eta_D +P^2\ f_{zx} - 2P\ D\ f_{yx} +(a11 +a22)P\ f_{yx} + \\ &\quad -(b11\ D +a12\ b21 -a22\ b11)\ \eta_1 + \\ &\quad -ap12\ a21\ f_{zx} + \\ &\quad -a13\ (D -a22)\ f_{yx} + \\ &\quad -(a14\ D +a12\ a24 -a14\ a22)\ r + \\ &\quad -b12\ (D -a22)\ \zeta_1 \end{aligned} \quad (31)$$

and obtaining:

$$\begin{aligned}b12\ (D -a22)\ \zeta_1 &= \eta_D -(b11\ D +a12\ b21 -a22\ b11)\eta_1 + \\ &+(P^2 -ap12\ a21)\ f\ \ + \\ &+[-(2P +a13)\ D +(a11 +a22)\ P +a13\ a22]\ f\ \ + \\ &-(a14\ D +a12\ a24 -a14\ a22)\ r \end{aligned} \quad (32)$$

For the lateral equations:

$$
\begin{aligned}
&[\, D^2 -(a11 +a22)D +a11\, a22 -an12\, a21 \,]\, f_{yx} + \\
&-[\, D^2 -(a33 +a44)D +a33\, a44 -an34\, a43 \,]\, f_{yx} = \\
&= \zeta_D +P^2 f_{yx} +2P\, D\, f_{zx} -(a11 +a22)P\, f_{zx} + \\
&-(b32\, D +a34\, b42 -b32\, a44)\, \zeta_1 + \\
&-a31\, (D -a44)\, f_{zx} + \\
&-(a32\, D +a34\, a42 -a32\, a44)\, q + \\
&-ap34\, a43\, f_{yx} + \\
&-b31\, (D -a44)\, \eta_1 \qquad (33)
\end{aligned}
$$

and obtaining:

$$
\begin{aligned}
&b31\, (D -a44)\, \eta_1 = -\zeta_D -(b32\, D +a34\, b42 -b32\, a44)\zeta_1 + \\
&\quad +[\, (2P -a31)D -(a11 -a22)P +a31\, a44 \,]\, f_{zx} + \\
&\quad -(a32\, D +a34\, a42 -a32\, a44)\, q + \\
&\quad +[\, (a11 +a22 -a33 -a44)D -a11\, a22 +an12\, a21 + \\
&\quad +a33\, a44 -a34\, a43 +P^2 \,]\, f_{yx} \qquad (34)
\end{aligned}
$$

These terms are combinations of the aircraft characteristics, i.e. derivatives, mass, inertia, etc., and the gain KQ.

Then we have for the elevator:

$$
\begin{aligned}
&[\, (by1\, bz1 -cz1\, cy1)D^2 + \\
&\quad +(by1\, bz0 +bz1\, by0 -cz1\, cy0 -cy1\, cz0)D + \\
&\quad +by0\, bz0 -cz0\, cy0 \,]\, \eta_1 + \\
&+(by1\, D +by0)\, \eta_D +(cz1\, D +cz0)\, \zeta_D + \\
&-[\, cz1\, fz1\, D^2 +by1\, afz +cz1\, fz0 +fz1\, cz0)D + \\
&\quad -by0\, afz +cz0\, fz0 \,]\, f_{zx} + \\
&-[\, cz1\, q1\, D^2 +(cz1\, q0 +q1\, cz0)D +cz0\, q0 \,]\, q + \\
&-[\, (-by1\, fy1 +cz1\, afy1)D^2 + \\
&\quad +(-by1\, fy0 -fy1\, by0 +cz1\, afy0 +afy1\, cz0)D + \\
&\quad -by0\, fy0 +cz0\, afy0 \,]\, f_{yx} + \\
&+[\, by1\, r1\, D^2 +(by1\, r0 +r1\, by0)D +by0\, r0 \,]\, r = 0 \qquad (35)
\end{aligned}
$$

and for the rudder:

$$
\begin{aligned}
&[\, (cy1\, cz1 -bz1\, by1)D^2 + \\
&\quad +(cy1\, cz0 +cz1\, cy0 -bz1\, by0 -by1\, bz0)D + \\
&\quad +cy0\, cz0 -bz0\, by0 \,]\, \zeta_1 + \\
&+(cy1\, D +cy0)\, \eta_D -(bz1\, D +bz0)\, \zeta_D + \\
&-[\, bz1\, fz1\, D^2 +(-cy1\, afz +bz1\, fz0 +fz1\, bz0)D + \\
&\quad -cy0\, afz +bz0\, fz0 \,]\, f_{zx} + \\
&-[\, bz1\, q1\, D^2 +(bz1\, q0 +q1\, bz0)D +bz0\, q0 \,]\, q + \\
&-[\, (-cy1\, fy1 +bz1\, afy1)D^2 + \\
&\quad +(-cy1\, fy0 -fy1\, cy0 +bz1\, afy0 +afy1\, bz0)D + \\
&\quad -cy0\, fy0 +bz0\, afy0 \,]\, f_{yx} + \\
&+[\, cy1\, r1\, D^2 +(cy1\, r0 +r1\, cy0)D +cy0\, r0 \,]\, r = 0 \qquad (36)
\end{aligned}
$$

Where the new coefficients are expressed by:

$bz1 = -b11$	$by1 = -b32$
$bz0 = -a12\, b21 +a22\, b11$	$by0 = -a34\, b42 +b32\, a44$
$cz1 = -b12$	$cy1 = -b31$
$cz0 = b12\, a22$	$cy0 = b31\, a44$
$afz = P2 -ap12\, a21$	$afy1 = a11 +a22 -a33 -a44$
$fy1 = -(2P +a13)$	$afy0 = -a11\, a22 +an12\, a21 + +a33\, a44 -a34\, a43 +P2$
$fy0 = (a11 +a22)P +a13\, a22$	$fz1 = 2P -a31$
	$fz0 = -(a11 +a22)P + + a31\, a44$
$r1 = -a14$	$q1 = -a32$
$r0 = -a12\, a24 +a14\, a22$	$q0 = -a34\, a42 +a32\, a44$

After having added this control block (equations 35 and 36) in order to obtain .similar lateral and normal accelerations when starting by only considering feedback of q, we can compare the output accelerations with the demanded ones in inertial axes and feed the decoupled system with the input errors through the gain KFZ (already set for non-rolling conditions) and, ' in this way, restoring the baseline acceleration equations as if the aircraft was not rolling, and so with acceleration responses independent of P in inertial axes.

Equations (42 and 43) make up the basic algorithm for decoupling the vertical/horizontal accelerations.

A block diagram representing this control system which decouples the system just after the feedback of the pitch rate, makes the normal equation similar to the lateral one, and finally compares the output accelerations with the demanded ones in inertial axes as shown in Fig.6.

The result of a numerical example simmulation, after decoupling with the algorithm above can be seen in Fig.7.

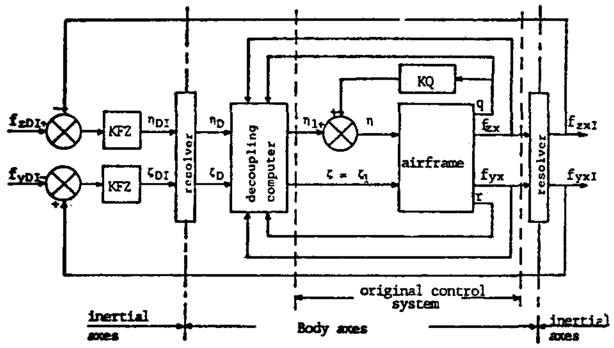

Fig.6 - Block diagram of the final decoupling control system.

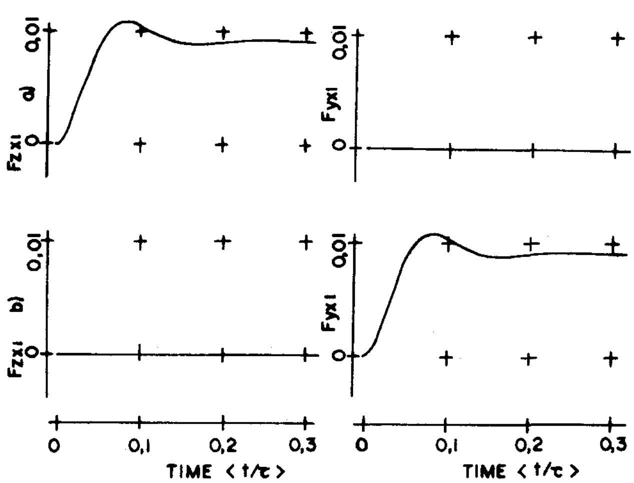

Fig.7 - Time responses of the decoupled and controlled non-symmetric rolling missile, to demanded accelerations of: a) f_{zDI}=0.01 and b) f_{yDI}=0.01.

7. Conclusions

As conclusion with respect to decoupling non-symmetric aircraft, we can say that the Complex Summation Method is not a straight forward technique to be applied for developing

Vertical/Horizontal decoupling algorithms.

The Pitch/Yaw channels are not antisymmetric, nor symmetric, and so their mathematical expressions cannot be added in complex fashion.

However we know that the Pitch/Yaw channels have something in common with SAC systems, since the couplings are processed in a similar way i.e. if Pitch is coupled by Yaw, Yaw is also coupled by Pitch but negatively or vice-versa.

These facts give rise to the idea that one could try to transform the system into a SAC one by means of feedback.

It has been shown that this can be done, but it is worth mentioning here that the solution is not unique. They will all have in common the necessity of accounting for the presence of the Rotational Coupling in rolling axes.

The process will always have the function of decoupling the zeros of the system, i.e. to annul the effect of a control input into the other channel, the one that is perpendicular to its corresponding channel; η_I into f_{yxI} or ζ_I into f_{zxI}.

By using Complex Summation, in order to express the complex equation we want to obtain for the accelerations in Inertial Axes, we can very easily identify the necessity of the presence of Rotational Coupling when in rolling axes. Then, we can very easily transform the complex equation we want to obtain into its real lateral and normal components.

After this, the process of obtaining the control law to transform the actual system to the one we want to obtain is a straightforward process. It can be done by comparing the equations we want with the ones we have (actual ones) as has been shown.

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