GAMING IN FUSELAGE AIMING

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Abstract

A head-on gunnery duel between two aircraft is considered as a game. The combatants have different qualities. One can decouple the aiming and shooting from trajectory shaping, while the other one can not. The opponent's position, velocity and acceleration is needed for aiming calculations as well as for calculations of the break away point, collision avoidance. An extra difficulty introduced emanates from the lag in the information from the radar used for this purpose. The aircraft which has the decoupling facility uses this delayed information to delude the opponent at the first hand in calculation of the break point and secondly to destroy the aiming to some extent.

1. Introduction

Modern fighter aircraft could have a possibility to use redundant control surfaces. This results in for example a capability of disconnecting the trajectory control from the direction of the fuselage pointing and then use this effect in head-on encounter combat. Consider the following scenario: Assume a head-on gunnery engagement between two aircraft, starting at two to three kilometers distance. One has the possibility to some extent to decouple the control of the trajectory from the aiming process and the other one has not this facility. The purpose for each aircraft is to aim on the hostile aircraft as long and good as possible. At the same time he must be sure that he can have a safe pass.

Decoupling of aiming and trajectory shaping can be performed in a couple of ways. Considering the pitch mode this can be very well utilized having an aircraft with a canard configuration, like the Swedish project JAS 39. There is a redundancy in pitch control using the canard and the elevons. The canard is very useful in maintaining aiming, it gives a fast and direct response. The digital flight control system, which is almost a must for this advanced flight mode, makes this, very fast and precision demanding, task possible to accomplish.

In an earlier study, [1] a passive target (flying on a straight line) is assumed. Perfect information and no delaying components besides the differential equations are also assumed. A three dimensional pointmass model is used and the process is optimized. An extension is found in [2] where the target is also maneuvering. A short period dynamic model in the vertical plane is assumed. The restriction due to the radar tracking is indicated in [3], where a simple filter model of the opponent's states is simulated.

The mathematical setup should require a short period model of the aircraft [2] as the process is of a short duration (three to five seconds) including physical constraints of for example the rate of deflection of the control surfaces. The opponent's velocity vector is not perfect it has a lag due to the Kalman filter which needs a certain time to update the target's velocity and acceleration. This fact makes the game a bit tricky, as the decoupled aircraft might trick the other one to break earlier than he needs. The filter used in this paper is simple straight forward and close to the filters used in practical aerial target tracking problems. The model used is rather simple as this study will concentrate on the game mechanism when including the filtering problem.

The aircraft in the decoupled mode is able to modulate the trajectory for maximum misleading of the opponent and still maintaining the aiming. This means he can aim to a closer point while satisfying the prescribed pass distance constraint. The less able aircraft has to give up aiming earlier in order to produce his contribution to the pass distance. This constitutes a new sort of a game. The game philosophy can be discussed, however. When taking practical constraints into account these will steer the discussion to a few choices.

The purpose of this paper is to describe the concept with a fuselage pointing aircraft decoupled from the trajectory. Particularly the effect of the lag in the measured data of the radar will be explored including the new concept of gaming under this incompleteness.

2. Kalman Filter

The models of the aircraft could be like in [2]. With the purpose to have a simple optimization performed we assume a simpler model, also the game is taking place in the plane (vertical plane without gravity). For each aircraft we have,

$$\dot{z} = v_z \quad (1)$$

$$\dot{v}_z = Acc \quad (2)$$

A simple Kalman filter is then,

$$\dot{z}_e = v_{ze} + K_1 \cdot (z_r - z_e) \quad (3)$$

$$\dot{v}_{ze} = Acc_e + K_2 \cdot (z_r - z_e) \quad (4)$$

$$\dot{Acc}_e = -B \cdot Acc_e + K_3 \cdot (z_r - z_e) \quad (5)$$

$$z_r = z + w, \quad w = N(0, \sigma) \quad (6)$$

where z is the position, v_z is the velocity in z-direction (both initially set to zero), Acc is the crossacceleration subindex e refer to estimated variables and w is, white, measurement noise, emanating from e.g. glinting and signal noise ratio. The Kalman gains are K_i; i=1, 2 or 3. The measurement noise is assumed to be stationary. In practice this is not the case, since, at closer distances the glint is dominating and further out the signal noise ratio will dominate the measurement noise, which constitutes a non-stationary process.

As the maneuver, Acc, is not known by the measuring aircraft, a model of the maneuver must be used in the filter. If we assume the target is switching from maximum turn in one direction to maximum turn in the other direction, $\pm Acc_{max}$, it will possess a Poisson process. An approximation of this is a Marcov model, Eqs.(1,2,7), where the input is a white noise, v. This is easier to handle than the Poisson process.

$$\dot{Acc} = B \cdot (-Acc + v), \; v = N(0, Acc_{max}) \quad (7)$$

In the literature like [4 - 7] filters of this type are studied and there is a theoretic background in determining the fictitious parameter B from the standard deviations σ and Acc_{max}. The filter used in this paper as well as in most aircraft implemented filters are theoretically simpler and the value of B is chosen empirically. The Kalman gains, K_i, are calculated with assumed noises and from a Riccati equation (see Appendix B). In practice this is backed up by simulations. A realistic filter response for a moderate noise environment is found in figure 1, where a step in target acceleration at a certain time is applied. We can notice the lag in the estimates of both the acceleration and the velocity, while the position is less affected. Initially, position and acceleration are zero and velocity is set to a positive value in this demonstration.

3. Optimal Control Problem

With the purpose to illustrate the possibility to take advantage of the lag in the filter a simple optimal control problem is,

$$\text{Maximize } \{ z(t_f) - z_p(t_f; \delta t) \} \quad (8)$$

A smart maneuver by the aircraft with decoupling facility can make him aiming and producing a predicted position, $z_p(t_f; \delta t)$ like a decoy in the other ones system, which deviates considerably from the real position, $z(t_f)$, at t_f. This forces the other aircraft to make his avoidance maneuver much earlier then he otherwise should have to. Each player will predict the opponents position, $z_p(t_f; \delta t)$, at an estimated pass time equal to the final time, t_f, determined by the closing velocity and present distance. The break time, t_b, is then

$$t_b = t_f - \delta t \quad (9)$$

The prediction is based on the measured variables from Eqs.(3-5) and the time it takes for the aircraft respectively to accomplish an avoidance maneuver, δt. The predicted position is then given by,

$$z_p(t_f; \delta t) = z_e(t_b) + v_{ze}(t_b) \cdot \delta t + \tfrac{1}{2} Acc_e(t_b) \cdot \delta t \cdot \delta t \quad (10)$$

In the cases below a common default value of δt is one second. Also, δt is called Delta Time in the figures, and the optimal difference between $z(t_f)$ and $z_p(t_f; \delta t)$ is denoted by Dz.

3.1 Maximize the object function

The problem is solved for one aircraft maximazing the object function (8). This means the other aircraft, if it was there, will be maximally misled about the first aircraft's final position. A typical optimal switching of the acceleration (±50 meters/second²) is given in figure 2, where we have three switching times. The very last one might coincide with the break point in the aiming problem below. In other cases it might be two or sometimes only one switch. We can notice the delay in the estimated acceleration, although the case in figure 2 has δt = .25 seconds, a relatively small number.

As expected the maximal miss prediction grows with the prediction time, see figure 3. The curves are produced for a certain noise environment. However, there is no measurement noise applied to Eq.(6) as this gives non-smooth curves, but the K_i in the Kalman filter corresponds to a given σ. The knees in figure 3 are due to the fact that the two first switching times disappear (goes to zero) at a certain prediction time value.

3.2 Maximize the object function with aiming

A passiv target is now introduced initially flying on a straight line. The aircraft which has decoupling facilities uses a small pitch deflection, $\delta\alpha$, added to the angle of attack in order to achieve aiming besides to some extent controlling the trajectory with the crossacceleration, Acc. Introduce an additional equation to Eqs.(1,2) for this aircraft,

$$\dot{\delta\alpha} = k_{\delta\alpha}\cdot(\delta\alpha_{in} - \delta\alpha),\ |\delta\alpha| \le \delta\alpha_{max} \quad (11)$$

The gain $k_{\delta\alpha}$ is in the range of .1 - .3, we use .2. A typical value of $\delta\alpha_{max}$ is two degrees. The two controls are Acc in Eq.(2) and $\delta\alpha_{in}$, where the latter one controls the aiming as it can not affect the trajectory and Acc is mainly used for trajectory shaping subject to the working range of $\delta\alpha$. We have a state constraint on $\delta\alpha$, as is shown in Eq.(11), and a control constraint on the acceleration, $\pm Acc_{max}$, set to 50 meter/seconds².

Obviously aiming can not be satisfied arbitrarily close to the target if we want to avoid collision. A break has to be made, the time for this corresponds to t_b above. This time is in the first examples considered as fixed (3.5 seconds). At least beyond this point maximum acceleration should be applied. The optimal control derived in Appendix B2 execute an acceleration large as possible including switching subject to the aimingcondition (C3). Typical control histories are depicted in figures 4,5, with $\delta\alpha_{max} = 2^\circ$ or 4° respectively. Notice that the noise comes through particularly apparent on the acceleration. Obviously, the acceleration stays longer on it´s limit when using a larger $\delta\alpha_{max}$. Also, the $\delta\alpha$ is plotted and demonstrates the effectiveness of the hyperplane tech-nique applied to this case. The switching function in Eq.(B12) has a shape as in figure 6 corresponding to the case in figure 5. The adjoints go through large changes when the different switchingtimes come through.

Maximal prediction errors versus δt are shown in figures 7,8. When no extra facility to aim is present, i.e. an ordinary aircraft, the prediction error was found to be 6.22 meters independent of δt. As a comparison we can look back to figure 3 corresponding to a very large $\delta\alpha_{max}$ (≈infinity). These observations together indicate the value of the control $\delta\alpha_{in}$. At a reasonable prediction time of 1.5 seconds the prediction error goes from 6.22, 55, 100 and 155 meters when $\delta\alpha_{max}$ goes from 0°, 2°, 4° and infinity.

In order to isolate the effect of break away the break time is swept in steps of .2 seconds, δt held fixed to one second. The result of this is seen in figures 9, 10. With no break away ($t_b = t_f = 4$ seconds) the prediction error is zero if $\delta\alpha_{max} = 0$, while a $\delta\alpha_{max}$ of 2° gives about 27 meters. Although, the z_f is zero in both cases.

An acceptable aiming error must be within $\pm 0.3^\circ$ (the limit at one kilometers distance) in order to obtain a good hit. The error is plotted in figure 11, which shows $|\epsilon| \le 0.18^\circ$ up to t_b. As the process is simulated in a computer it is possible to calculate the real aiming error, which is also plotted in figure 11. On the estimated error it is apparent how the disturbed z_e and v_{ze} comes through. The controls are using this error. The same run was checked without the measurement noise applied in Eq.(6). The error is then very close to zero. The aiming error profile looks the very same independent of $\delta\alpha_{max}$ as the control of the error is done equally by $\delta\alpha_{in}$ or Acc.

4. Game Problem

We consider now two aggresive aircraft, one has one control the Acc_1 and the other one has two controls the Acc_2 and the $\delta\alpha_{in}$. Refer to the aircraft as AC1 and AC2 respectively. The most difficult part is what to base the break on and particularly, which direction should it take. This is more detailed discussed in reference 8. In this paper one plausible case is studied. The game formulation is;

> Both produce at least a half pass distance relative to the predicted position of the opponent at pass and assume that the opponent does the same. Break direction is assumed to be à priori determined. Then both aim as long they can.

4.1 Maximize the object function

The prediction times used by each aircraft is a function of the opponent´s aiming time i.e. the break time.

$$\delta t_1 = t_f - t_{b2} \quad (12)$$

$$\delta t_2 = t_f - t_{b1} \quad (13)$$

where the subindex 1 and 2 indicate AC1 and AC2 respectively. The game will then be to match the break times such that each aircraft can produce at least a half of the given pass distance, which must be a fair assumption as both care to the same degree for non-collision. The optimization is first of all to maximize the prediction error for AC1 then adjust the break times till the pass condition is satisfied for both. From figures 9,10 it turns out that a smaller t_b is needed to achieve a larger side step distance, z_f ($z(t_f)$ in the objectfunction (8)). On the other hand the opponent then get a larger δt and this gives an even harder demand on the first aircraft, as he needs to turn away more as the prediction error grows (assuming that the turn is in a proper direction), figures 7,8, etc.

Both aircraft are supposed to make an up and a down turn for AC2 and AC1 respectively and produce at least 50 meters, z_{fj}, away from the predicted final position of the hostile aircraft, z_{pi} ($z_{pi}(t_f;\ \delta t_i)$ in the objectfunction (8)), where j=1 when i=2 and vice versa. In this case AC2 after a while reaches his limitations in both Acc_2 and $\delta\alpha_{in}$, which means he is not able to aim the full time out. When this happens his break is initiated. Hence, AC2 will obtain more than 50 meters in his contribution to the pass distance. The flight paths are shown in figures 12,13 note that there are different scales in z- and x-direction, which exaggerates the picture of the maneuvers. Clearly, AC2 deceives AC1's prediction and forces him to make a large avoidance as z_{p2} is much negative. The individual contributions to the pass distance, $z_{fi} - z_{pi}$, are for AC1 -50 meters while AC2 produces around 92 meters or 165 meters due to limitations in aiming, $|\delta\alpha| \leq 2^\circ$ or 4° respectively. The breaktimes are $t_{b1} = 2.035$ seconds and $t_{b2} = 2.9775$ seconds using $|\delta\alpha| \leq 2^\circ$ and $t_{b1} = 1.585$ seconds and $t_{b2} = 2.655$ seconds using $|\delta\alpha| \leq 4^\circ$.

Using this strategy in determining the break condition there is a considerable advantage of having an aircraft with decoupling facility. Assume the kill-probability [2], Eq.(14), is a good measure of how well one aircraft succed in this duel.

$$p_k = \text{Const}\cdot(R_o - R_{break})^3/R_o^2 \qquad (14)$$

The place where the gun is activated is R_o. The exchangeratio of killprobability will then be 3.14 and 4.70 using $|\delta\alpha| \leq 2^\circ$ and 4° respectively in advantage to AC2.

4.2 Aiming errors

As an illustration of the aiming errors we pick the examples corresponding to figure 12. Also, the measurement noise in Eq.(6) is not applied, since we like to isolate the effect on the errors emanating from the maneuvers. The estimated errors as well as the real errors are shown for both aircraft in figure 14. The controls of the aiming are based on the estimated errors as before. The results show the following: The real errors are larger than the estimated ones but still acceptable for AC2 till just after AC1 breaks, while AC1 has difficulties to keep the error within the $\pm 0.3^\circ$. The maneuvers by AC2, which original purpose were to deceive AC1 in prediction of AC2's pass position, also affect AC1's aiming significantly.

5. Summary

The results obtained are based on the assumptions made in this paper. Simplified models of the aircraft (pointmasses) and motion in a plane. Including more short period dynamic might somewhat smooth out the effect of switches in the accelera-tion. The game illustration will still be left. The game ingredients are more clearly visualized when using the simple models.

If there is a situation with a predic-tion time involved, a large error in prediction of a position can be performed. Introducing a $\delta\alpha$ increases the possibility of aiming and enlarging the prediction error for the opponent. A reasonable $\delta\alpha$, around 2°, will contribute with an amount in prediction error which is in a practi-cally useful range. The aiming error depends only on the measurement noise in Eq.(6) as far as the target is non-maneuvering.

The filter used is designed for other purposes than optimizing the predictor used, Eq.(10). A better predictor than Eq.(10) adapted to the filter used might be possible to find. However, the effects of the delayed information would still be a significant contributor to the game problem discussed.

The break problem is not trivial. It is reasonable to refer the size of the side step to the predicted pass position of the hostile aircraft. Which side of it or the turn direction is more questionable. If we base the turn direction on the à priori determined direction there is a large advantage with an aircraft, which can use a decoupled mode. There might be a motivation for using a sort of à priori determined direction. In the onboard autonomy system there must then be a calculation of what the opponent might do. This will include a simulation of the opponent's Kalman filter i.e. both knows each others prediction of position at pass.

One definite advantage with the decoupling facility is the irregular flight path such aircraft can perform causing a much larger aiming error in the opponent's measuring equipment than his more regular flight path returns to the decoupled aircraft. It takes a while till the system can detect that it does not aim perfectly. Then it is to late to switch strategy. The optimization used here is not designed for this particular deception aiming problem. Thus, if optimizing with respect to deceiving the aiming it might be possible to gain more. A nice target (passive) would not raise any aiming problem, while an optimally deceived radar can easily drive the aiming error out off what is acceptable for a hit.

The best to do for an aircraft without decoupling facility is to give up aiming and concentrate on jamming the opponent's aiming. This means the aircraft with decoupling facility can force other aircraft not to shoot in the head-on-encounter. Thus he has removed a threat.

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Appendix: A. Kalman Filter

The theory of the ordinary Kalman filter applied to the case studied gives the gains,

$$K_1 = R_{11o}/VarW \quad (A1)$$

$$K_2 = R_{12o}/VarW \quad (A2)$$

$$K_3 = R_{13o}/VarW \quad (A3)$$

where R_{1io} are stationary values (time goes to infinity) of the corresponding elements in the Riccati equation below. The variances are defined as,

$$VarW = \sigma^2 \quad (A4)$$

$$VarV = Acc_{max}^2 \quad (A5)$$

In this case the matrix Riccati equation turns out to be:

$$\dot{R}_{11} = 2\cdot R_{12} - R_{11}\cdot R_{11}/VarW \quad (A6)$$

$$\dot{R}_{22} = 2\cdot R_{23} - R_{12}\cdot R_{12}/VarW \quad (A7)$$

$$\dot{R}_{33} = -2\cdot B\cdot R_{33} - R_{13}\cdot R_{13}/VarW + VarV \quad (A8)$$

$$\dot{R}_{12} = R_{22} + R_{13} - R_{11}\cdot R_{12}/VarW \quad (A9)$$

$$\dot{R}_{13} = R_{23} - B\cdot R_{13} - R_{11}\cdot R_{13}/VarW \quad (A10)$$

$$\dot{R}_{23} = R_{33} - B\cdot R_{23} - R_{12}\cdot R_{13}/VarW \quad (A11)$$

The equations are integrated forward in time, initially starting from zero for all elements. This is done numerically by an Euler integration method. When integrating continuous equations in this way and they include noise component we have to divide the standard deviations with the square root of the integrationsteplength.

Appendix: B. Optimal Control Conditions

Substitute Eq.(10) into the object function (8), the object function to be maximized turns out to be

$$V = z(t_f) - z_e(t_b) - v_{ze}(t_b)\cdot\delta t - \tfrac{1}{2}Acc_e(t_b)\cdot\delta t\cdot\delta t \quad (B1)$$

That is an interior time control problem.

Forming the Hamiltonian

$$H = V_z\cdot v_z + Vv_z\cdot Acc + V_{ze}\cdot[v_{ze} + K_1\cdot(z + w - z_e)] + Vv_{ze}\cdot[Acc_e + K_2\cdot(z + w - z_e)] + VAcc_e\cdot[-B\cdot Acc_e + K_3\cdot(z + w - z_e)] \quad (B2)$$

The adjoint variables start by a capital V. The control is Acc and the noise, w, is considered as zero.

B1 No aiming

Maximizing the Hamiltonian gives the optimal control to be

$$Acc = Acc_{max}\cdot Sign\{V_{vz}\} \quad (B3)$$

The differential equations for the adjoints including the transversality conditions will then be

$$\dot{V}_z = -V_{ze}\cdot K_1 - V_{vze}\cdot K_2 - V_{Acce}\cdot K_3, \quad V_z(t_f) = 1 \quad (B4)$$

$$\dot{V}_{vz} = -V_z, \quad V_{vz}(t_f) = 0 \quad (B5)$$

$$\dot{V}_{ze} = V_{ze}\cdot K_1 + V_{vze}\cdot K_2 + V_{Acce}\cdot K_3, \quad V_{ze}(t_b) = -1 \quad (B6)$$

$$\dot{V}_{vze} = -V_{ze}, \quad V_{vze}(t_b) = -\delta t \quad (B7)$$

$$\dot{V}_{Acce} = -V_{vze} + V_{Acce}\cdot B, \quad V_{Acce}(t_b) = -\tfrac{1}{2}\delta t\cdot\delta t \quad (B8)$$

We also have

$$V_{ze}(t) = V_{vze}(t) = V_{Acce}(t) = 0 \text{ for all } t > t_b \quad (B9)$$

B1.1 The computational procedure. Assume δt is given. As the velocities in the x-direction of the aircraft can be considered constant and perfectly measurable as well as the present distance, the pass time t_f is known. Then we also know t_b, Eq.(9). The adjoint equations, Eqs.(B4-B8), can then be integrated backwards in time. Then we pick up the timevalues when the control in Eq.(B3) changes sign and store them. These are called switchingtimes. The state equations Eq.(1-5) can now be integrated forward in time using the stored switchingtimes to create the control associated with Eq.(B3). The predicted position from Eq.(10) and the final position can then be determined. All this is done in one backward and one forward sweep. As a matter of fact this case is possible to solve analytically so far. The numerical approach is more convenient also to solve this, as the extended problem below can not be analytically solvable.

B2 With aiming

First the state constraint in Eq.(11) has to be included in the optimal control problem. This is done by the Hyperplane technique [9] transferring it to a state-control constraint,

$$\delta\alpha - A_o\cdot(\delta\alpha + \delta\alpha_{max}) \le \delta\alpha_{in} \le \delta\alpha - A_o\cdot(\delta\alpha - \delta\alpha_{max}) \quad (B10)$$

The parameter A_o is chosen as large as practically useful. Too large might raise numerical problem, too low will satisfy the constraint pessimistic and not use the full working area of it. A value of 4 gave a distinct approach, with a smooth arc, of $\delta\alpha$ to it's maximal value, see figures 4,5. A larger A_o will give a sharper arc.

When $t \geq t_b$ we have the same conditions as in section B1. Then we have to consider if the acceleration is on it's limit or not. The $\delta\alpha_{in}$ is on it's limit when the acceleration is off it's limit and vice versa. Consider acceleration as the control variable maximizing the Hamiltonian and let the other control, $\delta\alpha_{in}$, satisfy Eq.(C3). Form the new Hamiltonian with the interesting terms written down,

$$H = V_{vz}\cdot Acc + V_{\delta\alpha}\cdot(Eps - Acc/v_x - k_{\delta\alpha}\cdot\delta\alpha) + \text{remaining terms from Eq.(B2)} \quad (B11)$$

The optimal acceleration will then be

$$Acc = Acc_{max}\cdot Sign\{V_{vz} - V_{\delta\alpha}/v_x\} \quad (B12)$$

The adjoint equations to be modified are those for V_z and V_{vz}, also an extra adjoint variable for $\delta\alpha$ has to be introduced, $V_{\delta\alpha}$. Two cases have to be considered with respect to saturation or not of the acceleration.

The case when $|Acc| = Acc_{max}$;

$$\dot{V}_z = -V_{ze}\cdot K_1 - V_{vze}\cdot K_2 - V_{Acce}\cdot K_3 + V_{\delta\alpha}\cdot(1 + K_{bul})\cdot(1/Dt + (v_x + v_{xT})/x)/x, \quad V_z(t_f) = k_z \quad (B13)$$

$$\dot{V}_{vz} = -V_z + V_{\delta\alpha}\cdot(1/Dt/v_x + (1 + K_{bul})/x), \quad V_{vz}(t_f) = 0 \quad (B14)$$

$$\dot{V}_{\delta\alpha} = V_{\delta\alpha}/Dt, \quad V_{\delta\alpha}(t) = 0 \text{ if } t \geq t_b \quad (B15)$$

where v_x is the velocity of the aircraft (in x-direction), x is the distance to the target (initially 2000 meters), subindex T stands for target and K_{bul} is the ratio of target velocity and the mean velocity of a fired bullet, v_{bul}. Numerically, K_{bul} is in the order of 0.2. See Appendix C for the time increment Dt.

The case when $|Acc| < Acc_{max}$;

$$\dot{V}_z = -V_{ze}\cdot K_1 - V_{vze}\cdot K_2 - V_{Acce}\cdot K_3 + V_{vz}\cdot v_x\cdot(1 + K_{bul})\cdot[1/Dt + (v_x + v_{xT})/x]/x, \quad V_z(t_f) = k_z \quad (B16)$$

$$\dot{V}_{vz} = -V_z + V_{vz}\cdot[1/Dt + v_x\cdot(1 + K_{bul})/x], \quad V_{vz}(t_f) = 0 \quad (B17)$$

$$\dot{V}_{\delta\alpha} = k_{\delta\alpha}\cdot(V_{\delta\alpha} - V_{vz}\cdot v_x)\cdot A_o + V_{vz}\cdot v_x/Dt, \quad V_{\delta\alpha}(t) = 0 \text{ if } t \geq t_b \quad (B18)$$

<u>B2.1 The computational procedure</u>. The procedure in section B1.1 is almost applicable on these adjoint equations, Eqs.(B13-B18). Besides storing the switching times during backward integration the timeintervals while the acceleration is saturated must be stored during forward integration and then be used to determine, which of the sets of adjoints above is applicable. Unfortunately, this makes the procedure iterative. When initiating the backward integration a guess of time intervals must be done. Still no need for a more complicated optimization method.

Appendix: C. Aiming Condition

The two aircraft are closing with constant velocity (250 meters/seconds each) in a near collision course. Then angles and deviations can be considered small. The geometry in figure C1 is exaggerated. The gun may point within $\pm\delta\alpha_{max}$ out from the velocity vector. An aiming error, ϵ, is introduced, which has to be close to zero on a given tolerance level for a good hit. This error can be derived from figure C1 using the sinus theorem.

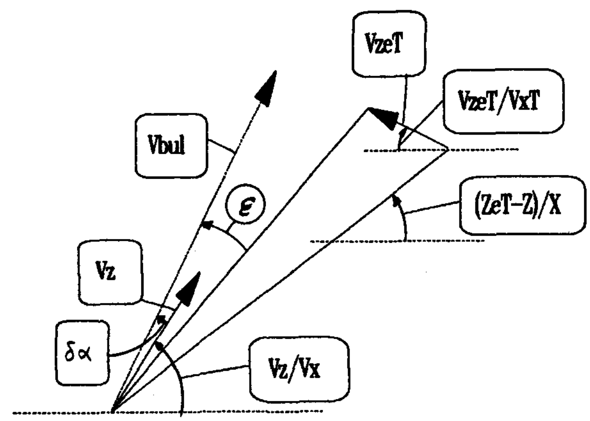

Figure C1. Aiming Geometry

$$\epsilon = v_z/v_x + \delta\alpha - (1 + K_{bul})\cdot(z_{eT} - z)/x - K_{bul}\cdot v_{zeT}/v_{xT} \quad (C1)$$

In order to close the aiming triangle we have to drive ϵ to zero. As there is no direct control in Eq.(C1) we have to make use of time derivative of ϵ and use the technique in reference 2, Eq.(7), yielding the aiming condition

$$0 = \epsilon + \{Acc/v_x + k_{\delta\alpha}\cdot(\delta\alpha_{in} - \delta\alpha) - (1 + $$
$$+ K_{bul})\cdot[(v_{zeT} - v_z)/x + (v_x + v_{xT})\cdot $$
$$\cdot(z_{eT} - z)/x^2] - K_{bul}\cdot Acc_{eT}/v_{xT}\}\cdot Dt \quad (C2)$$

where Dt is the time increment it is supposed to take to drive an error to zero, [2]. It is convenient to rewrite Eq.(C2) with the controls explicit and summarize the remaining terms in an auxiliary function Eps(•).

$$Acc/v_x + k_{\delta\alpha}\cdot\delta\alpha_{in} + Eps(\epsilon/Dt,\ \delta\alpha,\ z_{eT},$$
$$,\ z,\ v_{zeT},\ v_z,\ v_{xT},\ v_x,\ Acc_{eT},\ x,\ k_{\delta\alpha},$$
$$,\ K_{bul}) = 0 \quad (C3)$$

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Figures

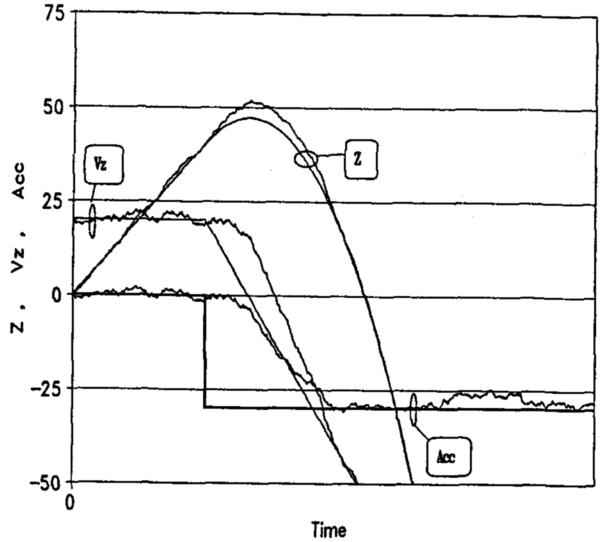

Figure 1. Stepresponse of the filter.

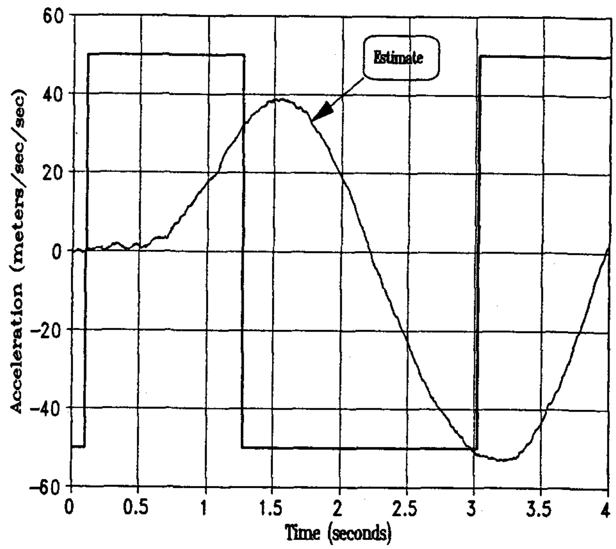

Figure 2. Optimal switching of the acceleration, δt=0.25sec.

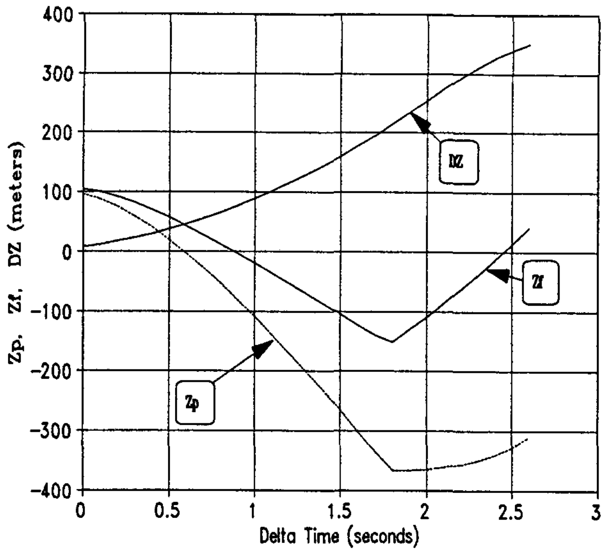

Figure 3. $z_p(t_f;1)$, $z(t_f)$ and Dz vs δt.

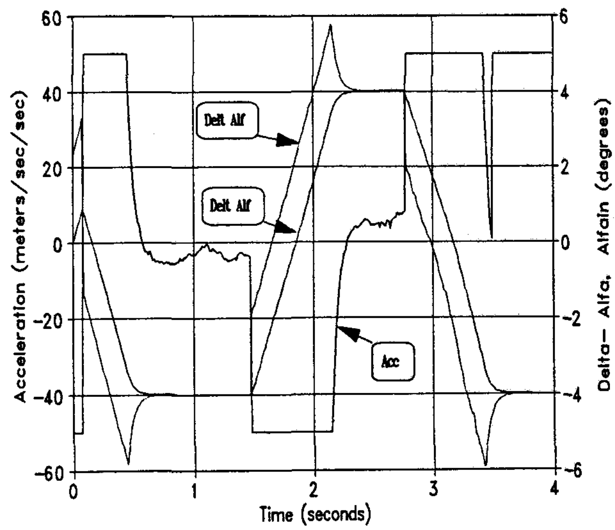

Figure 5. Control histories and $\delta\alpha$ vs time, $\delta\alpha_{max}=4^{\circ}$.

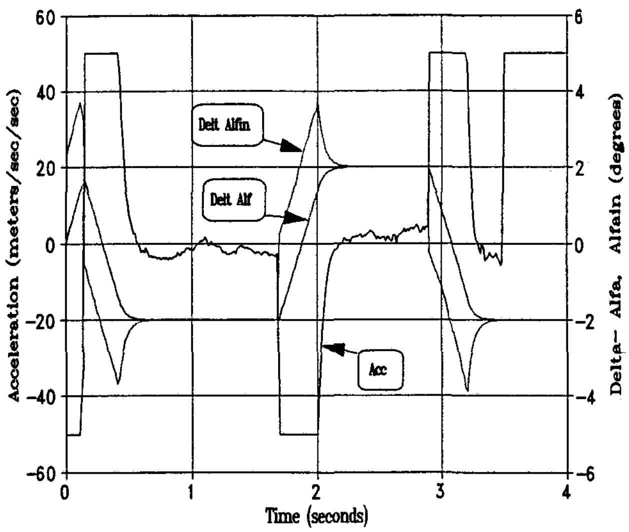

Figure 4. Control histories and $\delta\alpha$ vs time, $\delta\alpha_{max}=2^{\circ}$.

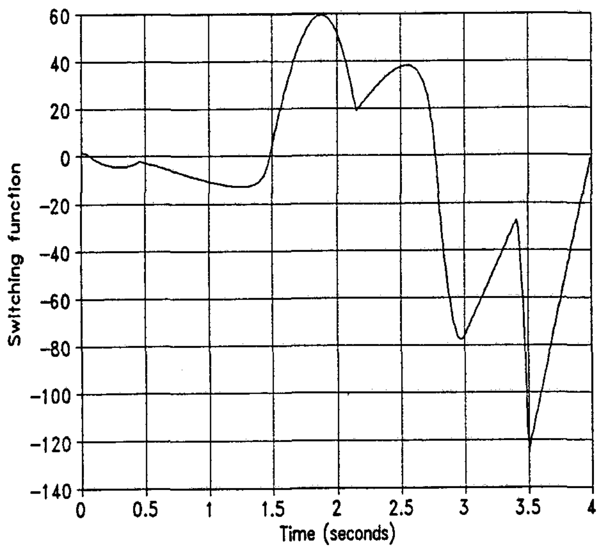

Figure 6. Switching function vs time, $\delta\alpha_{max}=4^{\circ}$.

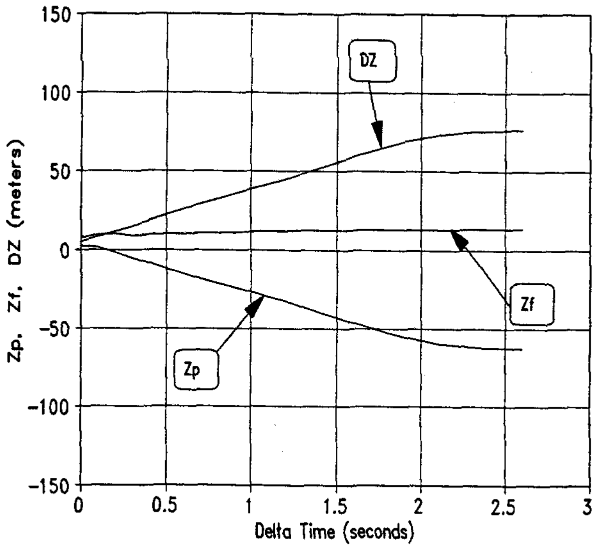

Figure 7. Aiming; $z_p(t_f;{}^{\bullet}_{\circ})$, $z(t_f)$ and Dz vs δt, $\delta\alpha_{max}=2^\circ$.

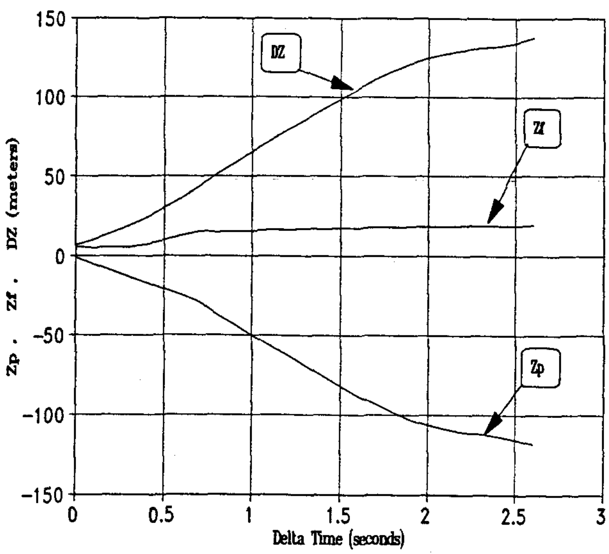

Figure 8. Aiming; $z_p(t_f;{}^{\bullet}_{\circ})$, $z(t_f)$ and Dz vs δt, $\delta\alpha_{max}=4^\circ$.

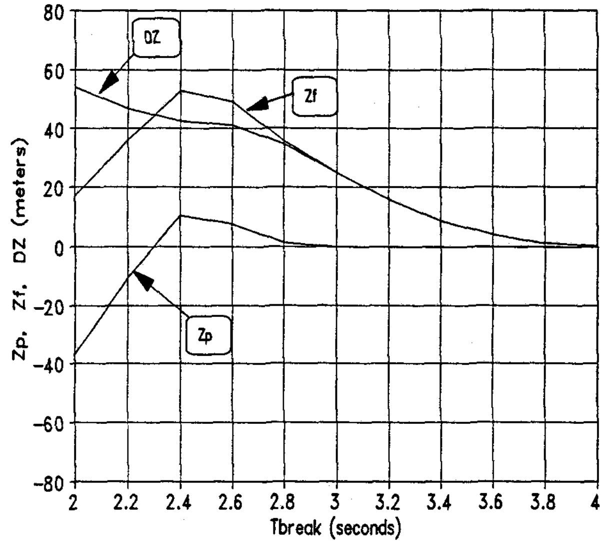

Figure 9. Aiming; $z_p(t_f;{}^{1}_{\circ})$, $z(t_f)$ and Dz vs t_b, $\delta\alpha_{max}=0^\circ$.

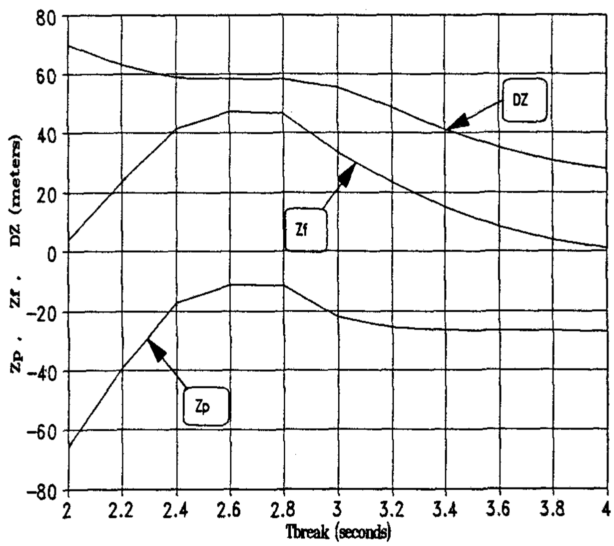

Figure 10. Aiming; $z_p(t_f;{}^{1}_{\circ})$, $z(t_f)$ and Dz vs t_b, $\delta\alpha_{max}=2^\circ$.

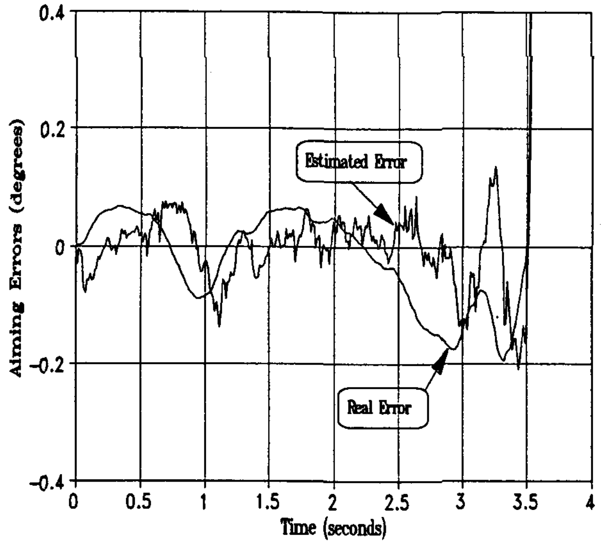

Figure 11. Estimated and real aiming error vs time, $\delta\alpha_{max}=2^\circ$.

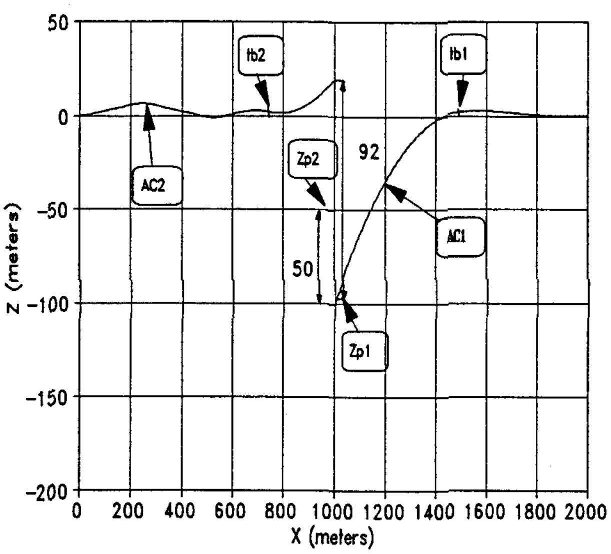

Figure 12. Optimal flight paths, $\delta\alpha_{max}=2^\circ$.

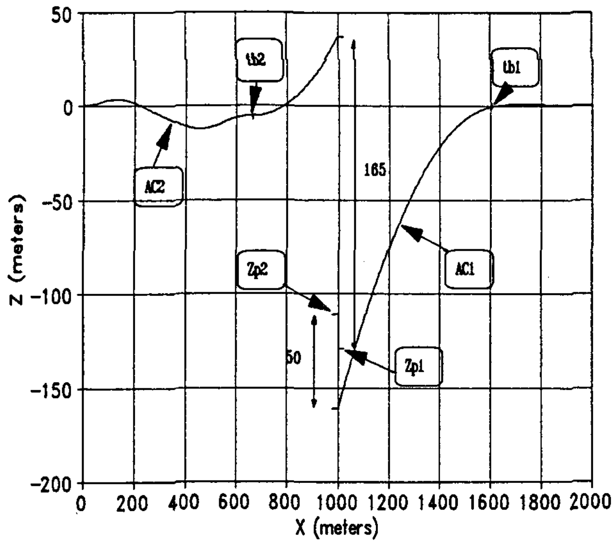

Figure 13. Optimal flight paths, $\delta\alpha_{max}=4^\circ$.

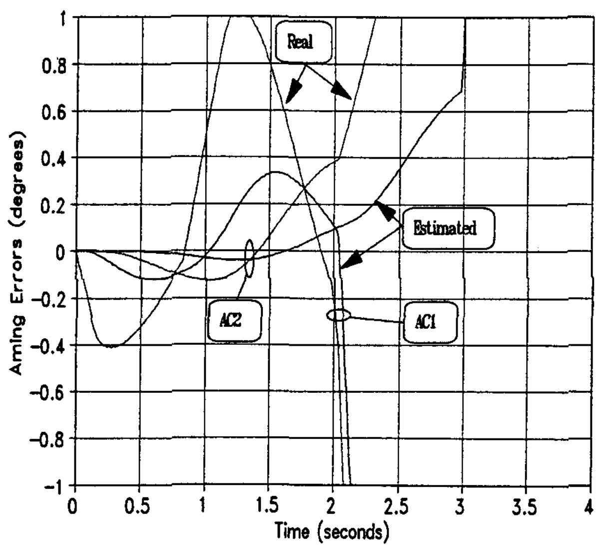

Figure 14. Estimated and real aiming errors for AC1 and AC2 vs time.