ROBUST CONTROL SYSTEM DESIGN WITH MULTIPLE MODEL APPROACH AND ITS APPLICATION TO FLIGHT CONTROL SYSTEM

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Abstract

In this paper, an approach to robust flight control system design is proposed and examined with a view to applying it to the flight control system for an in-flight simulator. Robustness of the control system is obtained by considering multiple models that represent an uncertain dynamical system. A delay element of uncertain delay time is used to introduce arbitrarily assigned bandwidths for a multiple input control system. The quadratic performance index that is directly given from the design objective in a simple manner makes the system design straightforward. Constraint of output feedback is posed to introduce a practical control law. The design approach is applied to a robust model following flight control of an in-flight simulator, where precise flight control is necessary. Numerical results that were obtained for research airplanes of the National Aerospace Laboratory, Japan, are shown to demonstrate the feasibility of the approach.

Introduction

Control system design for uncertain dynamical systems and systems with changing parameters has been one of the major research subjects in linear control theory. In particular, new theoretical results have been recently introduced using frequency response, such as the H^{∞} approach. The multiple model approach discussed here has been developed with the same objective[1]. The plant dynamics is not uniquely given but it is described with multiple candidates, or multiple models.

The term "multiple model approach" is used in control engineering, but the same concept is common in other fields of engineering, and it is generally referred to as the multiple design point method. The concept of the multiple model or multiple design point is often used in flight control system design, too. Changes of parameters, such as dynamic pressure, Mach number, weight and balance, and configuration, have a significant influence on the dynamic property of aircraft, and consideration of every necessary point in the flight envelope is important for the design of a flight control system, such as stability augmentation system, auto-pilot, and other sophisticated flight control systems. Although these attempts at design have often been carried out in a trial and error manner, extension of the control theory with the multiple model approach has made it possible to design more efficiently. For instance, the multiple model approach has been combined with the LQR(Linear Quadratic Regulator)[2], LQG(Linear Quadratic Gaussian)[3,4] and the pole assignment[5].

For flight control system design, not only prescribed changes of parameters but also uncertain dynamics should be considered. Uncertainty in the high frequency range, which comes from various sources such as flexible modes, non-linearity of the actuator system, uncertain time lags due to digital signal processing, and effects of filters, should be carefully considered. This is one of the major motivations of the recent robust control research. In practice, designers often find that high gain feedback control introduces a high crossover frequency, where phase information is quite uncertain, and this makes the closed-loop system unstable. In order to obtain the maximum performance while suppressing the frequency band, a multi-model approach using uncertain delay time has been proposed[6]. By considering different delays, bandwidths can be adjusted for each input channel. A prefixed control structure, such as proportional output feedback of a fixed gain, can give a practical control law.

This approach was applied to a model-following problem. In particular, its application to in-flight simulation with a research airplane is investigated in the present paper. For in-flight simulation with a research airplane, precise flight control is necessary, and this is apt to introduce high gain feedback, where compromise between control performance and robustness is important. Flight experiments with an in-flight simulator are generally conducted under designed flight conditions, where the dynamics of the airplane is given, and robustness against uncertainty of the control response is more serious than robustness against changing parameters. Furthermore, an in-flight simulator generally has a precise measurement system for its flight dynamics and it can be assumed that all states are directly measured and can be used for feedback control.

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The present paper is based on the author's previous research with E.H. Dowell. Research on the optimality of the multiple model approach for the modeling and estimation problem indicates that some kinds of constraint, such as proportional output feedback, are necessary to avoid acute dependency on the selected design grid points[7]. The multiple model approach was successfully applied to an active flutter control problem[8], and its application to flight control system design has recently been presented[6]. In the present paper, the multiple model approach is first reviewed briefly, and afterwards the application of this approach to a model-following control problem will be discussed.

Multiple model approach

The multiple model approach adopted here is the same as that is discussed in the previous papers[6,8]. The LQR method is extended to the multiple model problem while posing constraint of proportional output feedback control.

The plant dynamics is given by the following multiple models.

$$dx_i/dt=A_ix_i(t)+B_iu_i(t)$$
$$y_i(t)=C_ix_i(t), \quad i=1,M \tag{1}$$

where $x_i \ni R^{n_i}$, $u_i \ni R^m$, $y_i \ni R^r$ are the state variables, control variables, and output variables, respectively. The subscript i denotes the i-th model, and the system matrices (A_i, B_i, C_i) are constant matrices of adequate size. The control law is defined as follows.

$$u_i(t)=K_ix_i(t), \quad K_i=K_o+KC_i \tag{2}$$

The feedback gains K_o and K are common to all models, where K_o is a given fixed matrix, and K is an arbitrary matrix to be designed. The feedback control has two constraints, i.e. an unchangeable feedback gain and proportional output feedback. The first constraint appears in the model matching problem. The proportional output feedback control constraint is necessary in parasitic uncertain dynamics, which is the main subject of the present paper. It can also introduce a practical control law. The output may be a variable that is directly measured, or a variable that is accurately reconstructed.

The performance index is defined as a weighted summation of performance indices for each model.

$$J=\sum_{i=1}^{M} p_i J_i,$$

$$J_i=\int_0^\infty x_i(t)^TQ_ix_i(t)+u_i(t)^TRu_i(t)dt \tag{3}$$

where p_i (>0, $\Sigma p_i=1$) is the probability of the i-th model; or it can be used as an adjustable design parameter. Weighting matrices $Q_i(\geq 0)$ and $R(>0)$ are appropriately given as in an ordinary LQR problem. Here it is assumed that the closed-loop system characterized with $A_i+B_iK_i$ is asymptotically stable. The performance index can be rewritten by using the solution of the matrix Lyapunov equation as follows.

$$J=\sum_{i=1}^{M} p_i \, tr\{(Q_i+K_i^TRK_i)X_i\} \tag{4}$$

$$(A_i+B_iK_i)X_i+X_i(A_i+B_iK_i)^T+W_i=0 \; ; \quad i=1,M \tag{5}$$

where W_i is an appropriately defined distribution of the initial state, i.e. $W_i=E[x_i(0)x_i(o)^T]$.

The optimal gain K that minimizes the performance index satisfies the following necessary conditions, or the optimality conditions.

$$\sum_{i=1}^{M} RK_iX_iC_i^T+B_i^TY_iX_iC_i^T=0 \tag{6}$$

$$(A_i+B_iK_i)^TY_i+Y_i(A_i+B_iK_i)+Q_i+K_i^TRK_i=0; \; i=1,M \tag{7}$$

Although there is no direct computational algorithm for the optimal solution, gradients with respect to the parameters, or the feedback gain K, are easily calculated. Therefore, any kind of gradient method can be applied to this problem. An exception is the case where there is no initial feedback gain that stabilizes all models. A penalty function method that was proposed by the author and Dowell[8], is suitable for such a case. Furthermore, this method is quite reliable, and with it obtaining a converged solution for general cases is quite easy. The only demerit is that it requires some computational time.

The formulation of the LQR is constructed with a deterministic model here, i.e. the initial value response is evaluated. The same formulation, however, is possible using a stochastic model such as in Reference[8], where W_i can be considered as the intensity of the white noise input, and averages of the quadratic costs are evaluated.

Robust control system with time delay element

As stated in the Introduction, there have been many research reports on the application of the multi-model approach to the flight control problem, where multiple models represent dynamics of the flight vehicle under different flight conditions. Robustness against uncertain dynamics, however, has not been so extensively discussed using multiple models. In this paper, it is proposed that an uncertain delay time element to be used to design a robust flight control system. This approach has been successfully applied to an aeroelastic control problem[8].

The first order delay element that is given by the following transfer function is proposed to present the uncertainty of the plant dynamics in the high frequency range.

$$\frac{u^*}{u} = \frac{1-Ts/2}{1+Ts/2} \quad (8)$$

This is the [1,1] Pade approximation of the pure time delay element of time T(sec). The frequency response has a unity gain for any frequency, and the phase lag increases along with increased frequency, as shown in Figure 1. At the frequency $\omega=2/T$ (rad/sec), it gives a 90 degree phase lag, and at the higher frequency it approaches to an asymptote of 180 degree phase lag. Two models that are $T_1=0$ and $T_2=2/\omega_0$, are adequate to present system uncertainty for the higher frequencies, where ω_0 could be called the boundary frequency of the controlled system. The phase difference between the two models is less than 90 degrees below the boundary frequency, and more than 90 degrees above it. In the high frequency range, the difference is almost 180 degrees, and this means that even the sign of the response is not certain. Therefore, two models are adequate to present uncertainty in the high frequency range. Roughly speaking, the boundary frequency might be close to the crossover frequency of the controlled system, when the control system is introduced with a small penalty on the control cost.

Although the first order time delay element considered here does not claim any optimality for presenting uncertain dynamics in the high frequency range, it is one of the most simple dynamics, and it can be applied to most general cases. For other kinds of uncertainty, another model may be suitable. The multi-model approach is flexible, and it can be adapted to wider problems by considering appropriate models.

Application to in-flight simulation problems

The National Aerospace Laboratory, Japan (NAL) procured a Dornier Do-228-200 aircraft for general purposes of research on navigation and flight control problems in 1988. One of the major functions that will be installed in the airplane is in-flight simulation. The NAL had already developed the VSRA (Variable Stability and Response Airplane), an in-flight simulator based on Beechcraft Model 65, which has been used for basic research on pilot related flight dynamics problems[9]. The effectiveness of an in-flight simulator was proven through flight tests with the VSRA, and it is considered that the Do228 in-flight simulator, which will have more payload, will be valuable for the future activity of the NAL.

In-flight simulation is a somewhat mature technology. The United States of America and the Federal Republic of Germany have long and bright histories of in-flight simulation. CALSPAN in the USA, especially, has a pioneering history in the technology. In Japan, in addition to the NAL VSRA, the Self Defence Force developed the VSA(Variable Stability Airplane) based on the P2VJ, and it has been effectively used for pilot training.

Since precise control of the airplane is necessary for in-flight simulation, design of the flight control system is an interesting problem for flight control engineers, and it has motivated flight control researchers to investigate many practical problems of flight control. Much effort has been expended by flight control researchers on each in-flight simulator, and each flight control system is considered be optimally designed. For each given condition and objective, methods of flight control system design were proposed[9-12], based on theoretical research on the model-following and model-matching problems, e.g.[13-17]. One of the objectives of the present paper is to provide a control system design tool with which the optimal control system can be efficiently obtained for the design of the NAL Do228 in-flight simulator.

Model matching

When there is a sufficient number of controllers corresponding to each degree of freedom of rigid body motion, perfect in-flight simulation is possible. When there are not enough controllers, design of a model-following control system depends on the objective of each simulation. Kawahata proposed a model-matching method, where some important outputs of the model and aircraft are selected to be matched[12]. The control system has free design parameters for feedback gains of the plant that come from the uncontrollable space of the total system of the plant and model with respect to the model input. In principle, model matching is possible by making uncontrollable spaces for the total system, such as $y=y_m$. Furthermore, it was shown that the derivatives of the output with respect to time can be equal when the condition $v<n-m$ is satisfied, where n is the order of the plant dynamics, m is the number of the output to be matched, and v is the number of transmission zeros of the plant. Since the number of the uncontrollable space is n-v,

$m\times(n-\nu)$ feedback gains can be adjusted for design objectives other than the model matching.

The free feedback gains can be used for suppressing the effect of disturbances such as gust and uncertainty of the plant dynamics. Therefore, its design is a regulator problem. Kawahata gave the feedback gain with a pole assignment method based on the engineering insight that the closed-loop system should have enough damping with a moderate feedback gain and that the moderate gain can be realized by setting the bandwidth of the closed-loop system close to that of the open-loop system. An example of longitudinal motion was shown and the result was installed in the NAL VSRA. This was very successful, but in general a rather exact mathematical model for the plant, or the research airplane is necessary. Actually, this was accomplished by extensive flight tests for identification of the flight dynamics[18]. Determining the free feedback gain is an ordinary regulator problem and the multiple model approach can be naturally applied. The multiple model approach introduces more active feedback control while maintaining the function of the feedforward control. Before the discussion, the result of the model matching is briefly reviewed by following the result of Kawahata[12].

The dynamics of the airplane used for the in-flight simulation is given by the following state equation.

$$dx/dt=Ax(t)+Bu(t)$$
$$y(t)=Cx(t) \tag{9}$$

where $x\in R^n$, $u\in R^m$, $y\in R^r$ are the state, input, and output of the plant, respectively. The model to be simulated is also described with the following state equation.

$$dx_m/dt=A_m x_m(t)+B_m u_m(t)$$
$$y_m(t)=C_m x_m(t) \tag{10}$$

The subscript m denotes the model, and $x_m\in R^n$, $u_m\in R^m$, $y_m\in R^r$ are the state, input and output of the model, respectively. The k-th element of the output y_k is to be matched with the k-th element of the model output y_m. The relative order of the k-th output, defined by the following has an important role.

$$\sigma_k=\min\{\, i,\ C_k A^{i-1}B\neq 0 \,\},\ \ k=1,r \tag{11}$$

The error vector, which consists of the output errors and their derivatives, are defined as follows.

$$e_j=d^{i-1}y_k/dt^{i-1} - d^{i-1}y_{mk}/dt^{i-1},$$
$$j=i+\sigma_1+\ldots+\sigma_{k-1}\ ,\ \ k=1,r,\ \ i=1,\sigma_k \tag{12}$$

Due to the condition of the relative order, the error vector e(t) can be written,

$$e(t)=Mx(t)-M_m x(t) \tag{13}$$

M and M_m are appropriately defined constant matrices. The error dynamics is introduced as,

$$de/dt=M(Ax(t)+Bu(t))-M_m(A_m x_m(t)+B_m u_m(t)) \tag{14}$$

The differential equation of vector form (14) has r non-trivial equations and they introduce the model following control law as,

$$u(t)= -B^{*-1}(A^{*}x-A_m{}^{*}x_m-B_m{}^{*}u_m)$$
$$-B^{*-1}K^{*}(Mx-M_m x_m) \tag{15}$$

where B^*, A^*, $A_m{}^*$, and $B_m{}^*$ are appropriately defined matrices[12]. K^* is used for K in Reference [12]. The control law can be rewritten as,

$$u(t)=K_x x(t)+K_m x_m(t)+K_m u_m(t) \tag{16}$$

The structure of the control system is shown in Figure 2. The error vector is uncontrollable with respect to the model input, and its feedback to the plant does not influence the property of uncontrollability. Therefore, the feedback gain K^* is independent from the model matching condition, and it can be arbitrarily assigned.

As shown in Figure 2, the block diagram of the model matching control system, the model dynamics and feedforward control are realized in the in-flight simulation computer and there is no space of uncertainty. On the other hand, the state equation of the plant dynamics is only a mathematical model and the difference between it and the actual flight system is unavoidable. As pointed out in Reference [12], the error dynamics due to the model control input is decoupled in the closed-loop plant dynamics and it is described by a system of order $n-\nu$, where ν is the number of the transmission zeros of the plant transfer function $C(sI_n-A)^{-1}B$, as previously discussed. However, the output error due to the modelling error of the plant and that due to disturbances to the plant, such as gusts, are governed by the closed-loop plant dynamics, i.e.

$$dx/dt=(A+BK_x)x(t). \tag{17}$$

The error due to external disturbance and that due to the modelling error driven by the model control input should be maximally suppressed, and this design objective introduces high gain feedback. This system, however, is vulnerable to the uncertainty in the high frequency range. Stability of the closed-loop control system against uncertainty is most important; in particular, bandwidths of each channel of the control input should be taken care of. This can be accomplished by the multiple model approach.

Robustness of the closed-loop system should be obtained while maintaining the model matching condition at the nominal plant dynamics. The feedback component of the control law that satisfies the model matching condition is given by the following form.

$$u_{feedback}(t)=K_x x(t)=(K_0+KM)x(t) \qquad (18)$$

where $K_0=B^{*-1}A^*$, $K=B^{*-1}K^*$, M is an appropriately defined matrix [12]. The feedback gain K is a free design parameter. From the dynamics of the controlled system (17) and the constraint of the feedback gain (18), the feedback control can be determined with the multiple model approach and uncertain delay model. This is demonstrated in the following examples.

Numerical examples

Example #1

In order to demonstrate the properties of the multiple model approach, the same example as Reference [12] was calculated, i.e. the longitudinal motion of a large transport aircraft, the Boeing 747, is simulated with the NAL VSRA based on the Beechcraft Model 65. The feedback gains were determined with the multiple model approach. Figure 3 shows the block diagram of the feedback control with uncertain delay models. Four models of uncertain delay time are considered as follows.

model #1	$T_e=T_T=0$
model #2	$T_e=0.5$(sec), $T_T=0$
model #3	$T_e=0$, $T_T=2$(sec)
model #4	$T_e=0.5$(sec), $T_T=2$(sec)

where the subscripts e and T denote elevator and throttle, respectively. The corresponding boundary frequencies for non-zero delay times are $\omega_e=4$(rad/sec) and $\omega_T=1$(rad/sec). These figures are defined with engineering intuition only for a numerical example. In general, however, uncertainty of the response due to throttle input occurs at a lower frequency than elevator input, and the bandwidth of throttle should be made lower than that of elevator.

In the example of in-flight simulation[12], the outputs to be matched with the elevator and throttle are velocity u and flight path angle γ. Since the error due to external disturbances in these variables should be small, the performance index is simply defined as,

$$J=\frac{1}{4}\sum_{i=1}^{4} J_i$$

$$J_i=\frac{1}{2}\int_0^\infty (u_i/U_0)^2+(\gamma_i/\gamma_0)^2 +r_e(\delta_{e_i}/\delta_{e_0})^2+r_T(\delta_{T_i}/\delta_{T_0})^2\,dt \qquad (19)$$

In the performance index, outputs and control variables are non-dimensionalized with balanced nominal values[19], for instance, $\gamma_0=1$(rad), $\delta_{e_0}=1$(rad), $\delta_T=100$(mm) are given, and U_0 is the flight velocity. r_e and r_T in the performance index are the weighting parameters on the control cost, and here appropriately small values are given in order to maximize the regulator performance, such as, $r_e=r_T=10.^{-4}$ When the control cost in the performance index is less evaluated, the bandwidths for each input could be close to the given boundary frequencies. The following initial condition is simply used for the calculation.

$$W_i=\mathrm{diag}\{\,U_0^2,\ 0,\ 0,\ \theta_0^2\,\} \qquad (20)$$

where the state variables are defined as $x^T=[\,u,w,q,\theta\,]$, and θ_0 is simply given as $\theta_0=1$(rad).

The feedback gain K_x was obtained with minimizing the performance index (19), and then the feedforward gains K_u and K_m were introduced with the model-matching condition. The poles of the closed-loop system are obtained as, -5., -1.34, -0.59, -1.26±1.60j. The crossover frequencies and phase margins at each break point are $\omega_c=4.25$(rad/sec), PM=105^o and $\omega_c=0.75$ (rad/sec), PM=100^o. The break points are the elevator input and throttle input, respectively. In this numerical example, the crossover frequencies of each input are close to the boundary frequencies, which can be arbitrarily given as design parameters.

Figure 4 shows the stability region plotted against changes of delay time. As shown in Figure 3, the first order delay models are used. The abscissa is for delay time in the elevator input and the ordinate is for that in the throttle input. The symbol denotes four design points with the multiple models. At the design points, the closed-loop system is stable. For reference, the stability region of the feedback control given in Reference [12] is also plotted. The gain is small compared with the present gain and the stability region is wider than the present result. In particular, the gains for the throttle input are small.

The objective of the present multiple model approach is disturbance attenuation while maintaining an adequate stability margin. Figure 5 shows time histories of error dynamics due to initial errors; i.e., $u_i/U_o=0.05$ and $\theta_i/\theta_o=0.05$ are chosen for the plot. With the present multi-model control, the attenuation of the initial error is a few times faster than that of Reference[12]. Comparing the set of feedback gains, it is understood that the present result uses more throttle input than the result of Reference [12] Although time histories of the control variables are not shown, they are in a moderate level. This regulator performance can be considered almost maximum under the constraint of the bandwidths for each control input.

Example #2

The next example is for the planned NAL Do228 in-flight simulator. The parameters for the dynamics of the airplane are as follows.

$m=5700$ (kg)	$U_o=67.36$(m/s)	
$X_u=-0.0285$	$X_\alpha=6.53$	$X_{\delta_e}=0$
$X_{\delta_P}=0.936$	$X_{\delta_{DLC}}=0$	$Z_u=-0.291$
$Z_\alpha=-84.4$	$Z_q=-1.0$	$Z_{\delta_e}=-5.3$
$Z_{\delta_P}=-0.187$	$Z_{\delta_{DLC}}=-11.8$	$M_u=0$
$M_\alpha=-6.70$	$M_{\dot\alpha}=-0.57$	$M_q=-1.69$
$M_{\delta_e}=-4.44$	$M_{\delta_P}=0$	$M_{\delta_{DLC}}=0.230$

Notations follow Reference [20]. The aerodynamic parameters are calculated from References [21,22] that are based on wind tunnel tests and flight tests. The flap-in-flap type direct lift control (DLC) is planned to be installed. There is a complete number of control inputs, i.e. elevator, power, and DLC, for three degrees of freedom of the longitudinal motion, and they are enough for perfect model following.

In this example as well as the previous example, there is no transmission zero. Since the matrix M in (13) is non-singular, the full state feedback gain can be arbitrarily designed under the condition of model-matching. The feedback gain was introduced with multiple models of uncertain delay time as demonstrated in Example #1. Eight models, which consist of two uncertain delay models for three control inputs, are considered. The following two delay times for each input are given for a numerical example.

elevator;	$T_e=0$,	$T_e=2/3.14$ (sec)
power;	$T_p=0$,	$T_p=2/0.628$ (sec)
DLC;	$T_{DLC}=0$,	$T_{DLC}=2/1.26$ (sec)

Since all state variables of the longitudinal motion should be simulated with three control inputs, the following performance index is defined.

$$J=\frac{1}{8}\sum_{i=1}^{8}J_i$$

$$J_i=\frac{1}{2}\int_0^\infty (u_i/U_o)^2+(\alpha_i/\alpha_o)^2+(\theta_i/\theta_o)^2$$
$$+r_e(\delta_{e_i}/\delta_{e_o})^2+r_P(\delta_{P_i}/\delta_{P_o})^2+r_{DLC}(\delta_{DLC_i}/\delta_{DLC_o})^2dt \quad (21)$$

where the character with subscript o denotes the appropriately balanced nominal values as given in Example #1. r_e, r_P, and r_{DLC} are weighting parameters for the control cost. In the present example, they are given as $r_e=r_P=r_{DLC}=10^{-2}$.

For a numerical example, the same aircraft as Example #1, i.e. the Boeing 747, is chosen for in-flight simulation. Figures 6a-6e are typical numerical results of flight simulation with the Do228. Figure 6a shows the input to the model aircraft. The elevator is driven by the so called 3211 input, and other inputs to the model are equal to zero. Figures 6b-6d show the responses of the model and plant. As previously mentioned, the model-matching is possible for any output, if the plant dynamics is exactly the same as the given mathematical model. In this numerical simulation, pure time-delay elements are inserted to each control input in order to evaluate performance robustness of the simulation. The delay times are given as, $T_e=0.3$(sec), $T_P=1.5$(sec), $T_{DLC}=0.75$(sec). These figures are approximately half of the values that are given for design points in the multiple model approach. Although solid lines and dotted lines show the responses of the model and plant, respectively, the difference is quite small, and it is difficult to distinguish it in most of the part. Figure 6e shows the plant control inputs generated by the model-following control law. Most component of the control comes from the model-matching condition, and influence of the delay elements is small.

Concluding Remarks

An approach that is characterized by three concepts, multiple models, the LQR, and the constraint of proportional output feedback control, is proposed to apply to flight control system design for an in-flight simulator. A delay model that presents uncertainty in the high frequency range is especially used for the robust control. This approach introduces the control law of maximum feedback gain, or maximum disturbance attenuation, while maintaining stability under the given uncertainty in the high frequency range. The approach was applied to flight control design for an in-flight simulator that is planned in the NAL. The result demonstrates that control system design can be accomplished by giving a small number of parameters, including the bandwidths of each channel for a closed-loop system.

The multiple model method was examined with a view to applying it to the synthesis of an in-flight simulation flight control system. No one can claim that the flight control system can be introduced with minimizing a single performance index. Flight control system design requires much experience and deep knowledge of flight dynamics and hardware. The multiple model approach, however, can introduce a reasonable feedback control law with a small number of design parameters, and it can be a strong tool for flight control system design, with which designers can effectively approach the optimal solution.

As a delay model was successfully used in the multi-model approach, the approach is flexible, and by giving appropriate models it can be applied to various problems other than robust control, such as a non-linear control problem, system failure problem, and system integrity problem. A sophisticated computer aided design environment would make the approach more powerful. In that case, a reliable and efficient computational algorithm is essential. A penalty function method is suitable for this purpose.

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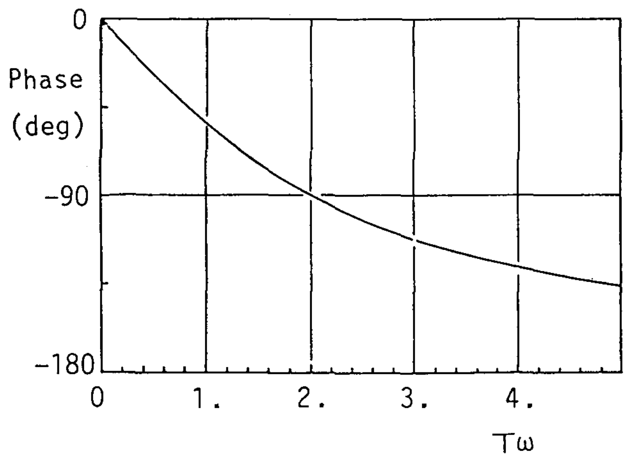

Figure 1 Frequency response of the 1st order approximant of pure time-delay.

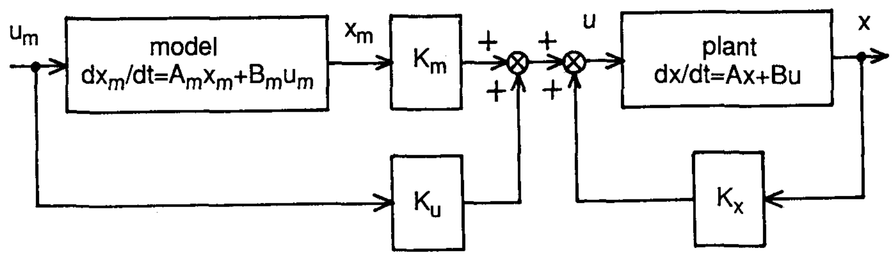

Figure 2 Block diagram of model-matching control system.

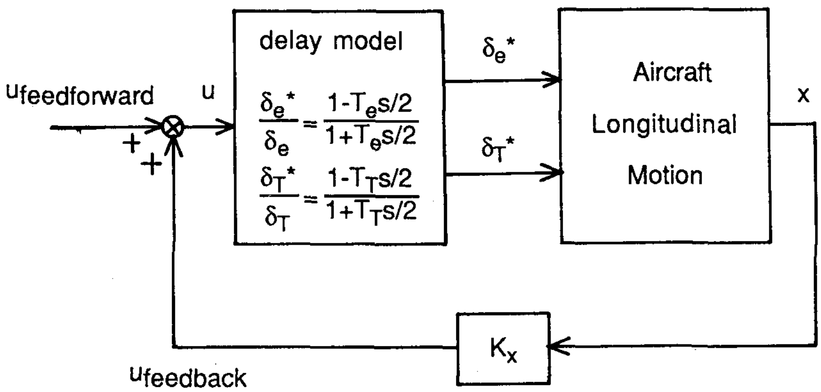

Figure 3 Uncertain delay time model for in-flight simulation. (Example #1)

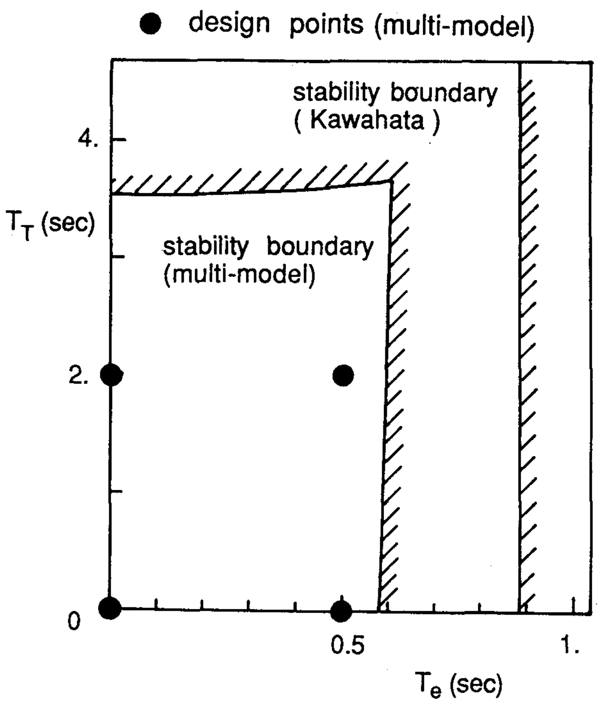

Figure 4 Stability map for Example #1.

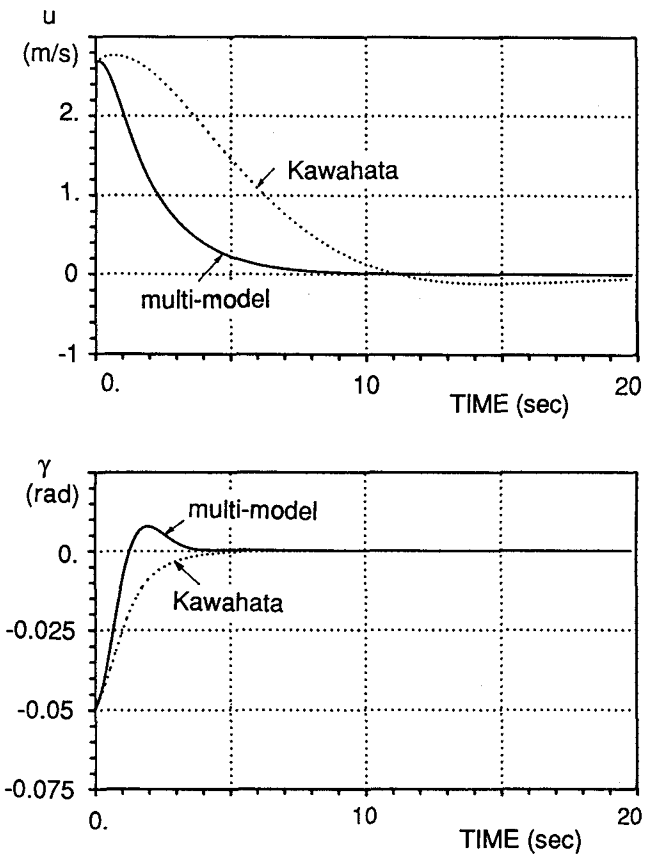

Figure 5 Time histories of response due to an initial deviation. (Example #1)

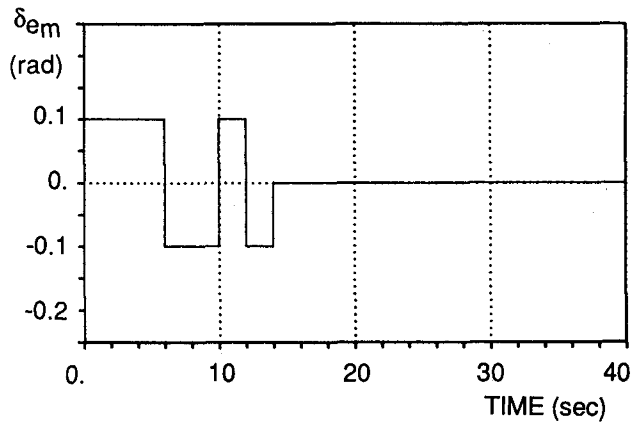

Figure 6a Input to the model. (Example #2)

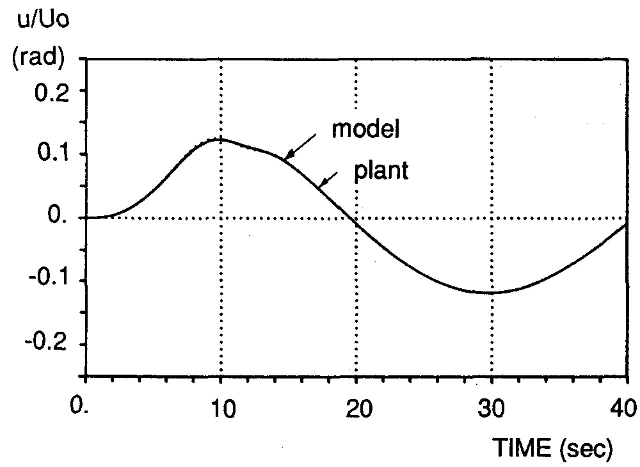

Figure 6b Responses of the model and plant. (Example #2)

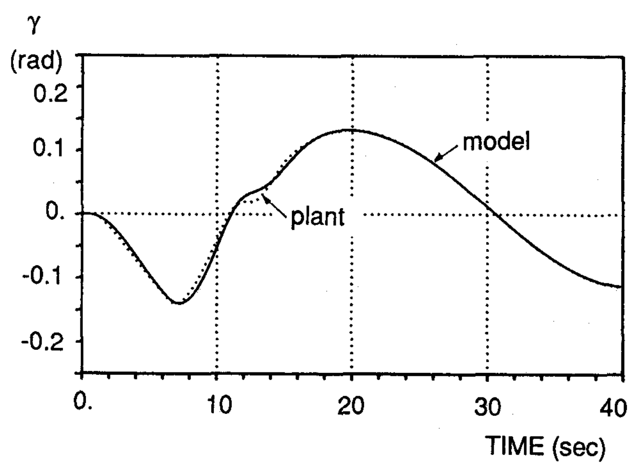

Figure 6c Responses of the model and plant. (Example #2)

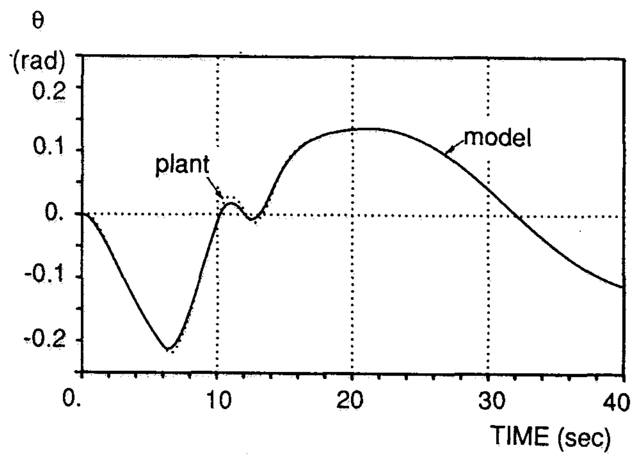

Figure 6d Responses of the model and plant. (Example #2)

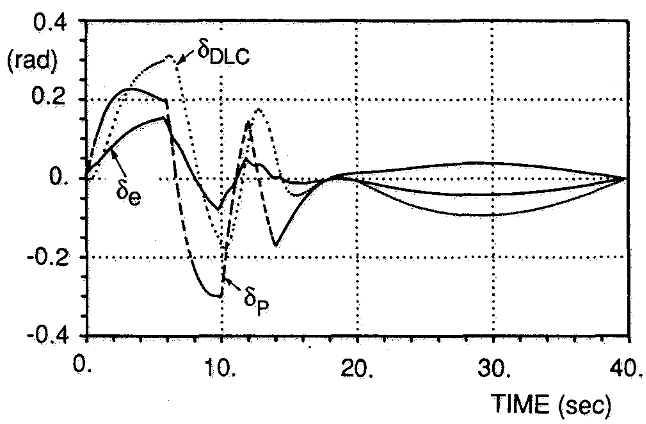

Figure 6e Control inputs to the plant. (Example #2)