BIFURCATION THEORY IN FLIGHT DYNAMICS
AN APPLICATION TO A REAL COMBAT AIRCRAFT

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Abstract

Several years ago, ONERA undertook the development of a methodology, based on Bifurcation Theory, devoted to the analysis of the asymptotic behaviour of non linear differential equations depending on parameters.

The aim of this paper is to present some significant results obtained, in Aircraft Flight Dynamics, when this methodology is applied to a real combat aircraft, ie the German-French Alpha-Jet. The paper is essentially related to oscillatory motions and to sensitivity analysis of spin behaviour to several parameters. Comparisons between predictions and flight tests are shown.

Nomenclature

C_X, C_Y, C_Z	: dimensionless body axes force coefficients
C_l, C_m, C_n	: dimensionless rolling, pitching and yawing moment coefficients
F_X, F_Y, F_Z	: body axes thrust components
L^A, M^A, N^A	: body axes aerodynamic momentum
L^F, M^F, N^F	: body axes thrust momentum
p, q, r	: roll, pitch and yaw rates
u, v, w	: body axes components of aircraft velocity
X^A, Y^A, Z^A	: body axes aerodynamic forces
α	: angle of attack
β	: sideslip angle
δ_l, δ_m, δ_n	: aileron, elevator and rudder deflections
δ_Y	: lateral offset of the center of gravity
θ, Φ	: pitch and roll angles
g	: gravitational acceleration
m	: aircraft mass
A, B, C	: moments of inertia for X, Y and Z axes
E	: product of inertia for X and Z axes

Introduction

The request for a highly manoeuvrable aircraft requires a preliminary study of its high angle of attack dynamics. This subjet is connected with the definition of the flight enveloppe boundary, itself connected with stability problems and with the assessment of post stall behaviour.

In these areas, thanks to computer capabilities and to some mathematical results in the field of non linear differential equations, known as Bifurcation Theory, significant progress were achieved recently and in the past few years [1, 2, 3, 4, 5].

The purpose of this new approach of non linear Flight Dynamics is to evaluate, as extensively as possible, the risks of temporary aircraft control losses, while avoiding usual systematic or Monte Carlo numerical simulations which may lead to prohibitive number of cases to be examined.

More precisely, all possible steady states of the aircraft, including various steady spins, are first computed. Then, all the possible jumps from an equilibrium state to another one are localized and the analysis of the related local transient motion is performed.

In the opposite with previous papers which mainly aimed at presentation of computation methods used for the determination of equilibrium surfaces, bifurcation surfaces and envelope of periodic orbits exhibited by a non linear aircraft model, the aim of this paper is to present the results obtained when this methodology is applied to a real combat aircraft, ie the German-French Alpha-Jet.

The study is specifically devoted to oscillatory motion and to sensitivity analysis of departure and spin predictions to a given set of parameters.

After a brief description of the methodology and associated numerical procedures, the oscillatory flight cases such as "agitated" spins are studied by means of checking the stability characteristics of periodic orbits related to oscillatory unstable equilibria. Complex oscillatory modes are pointed out.

The synthesis of all these results shows the existence of different spin modes for some given control deflections. It illustrates too that the lack of a realistic non linear model may lead to great difficulties for flight analysis when the motion is quasi-periodic or chaotic. Comparisons between prediction and flight tests are shown.

Analysis of the sensitivity of predictions to some model parameters modifications is also presented. The first sensitivity analysis deals with the influence of lateral offset of the center of gravity on normal spin recovery. The second one deals with the influence of gyroscopic torques induced by engine rotors.

Finally, the interest of this methodology is discussed in the conclusion.

Methodology

Based on Bifurcation Theory, the methodology developped at ONERA is essentially devoted to the analysis of asymptotic behaviour of non linear differential systems depending on parameters

$$\frac{dX}{dt} = F(X,U) \qquad (1)$$

in which X : n dimensional state vector
U : m dimensional parameter vector
F : n non linear functions of X and U

Bifurcation Theory is a set of mathematical results which aims at the analysis and explanation of unexpected modifications in the asymptotic behaviour of non linear differential systems when parameters are slowly varying.

For a fixed U, two types of asymptotic state are commonly encountered. The first is given by the solution of the following equation :

$$0 = F(X,U)$$

and is named steady state. The second is given by the relation :

$$X(T) = X(0)+\int_0^T F(X,U)dt$$

in which the period of the orbit (T) is unknown when the system is autonomous.

The stability of these asymptotic states is provided by an eigenvalue analysis of linearized system. Steady states are stable if all the eigenvalues have a negative real part. According to Floquet Theory, periodic orbits are stable if all the eigenvalues of the transition matrix have a modulus less than one with the exception of one eigenvalue which is real and equal to one.

Bifurcation Theory

It is not the purpose of this paper to substitute itself for specialized works dealing with mathematical basis of Bifurcation Theory. Nevertheless, it appears to be essential to describe a few aspects encountered in Aircraft Flight Dynamics.

For steady states, the most common phenomena appears when at least one negative real eigenvalue (μ) crosses the imaginary axis when U varies. Two cases are to be considered. (See Fig 1).

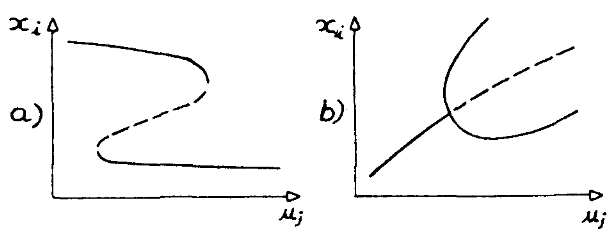

Figure 1. "Bifurcations" of steady states for at least a null real eigenvalue: (a) limit point; (b) pitchfork.
—— stable branch, - - unstable branch.

If the state is regular, ie the implicit function theorem works, the equilibrium curve goes through a limit point. It should be noted that a limit point is structurally stable under uncertainties of the differential system studied.

If the state is singular, several equilibrium curves cross a pitchfork bifurcation point. This bifurcation point is structurally unstable It breaks in limit points under uncertainties. In Aircraft Flight Dynamics this type of particular point appears only for discrete values of parameters and only if the differential system studied is a symmetrical one.

Another very interesting bifurcation point, named Hopf bifurcation, occurs when a pair of complex eigenvalues ($\mu_{1,2}$) crosses the imaginary axis, for $U = U_0$, when U varies. After crossing this point, a periodic orbit appears. (See Fig 2).

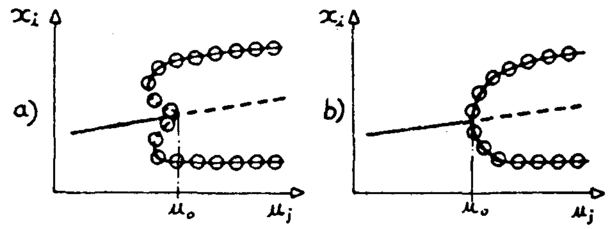

Figure 2. Bifurcation of steady states for, a pair of pure imaginary eigenvalues: (a)subcritical and (b) supercritical Hopf bifurcation.
—— stable branch, - - unstable branch,
o-o- periodic orbit envelope.

Depending of the nature of system nonlinearities, the bifurcation may be subcritical or supercritical. In the first case, the stable periodic orbit amplitude is large even in the vicinity of U_0. In the second case the amplitude of the orbit grows in proportion to the distance in U to U_0.

Finally, the last domain of interest concerns the behaviour of the system when periodic orbits loose their stability. (See Fig 3).

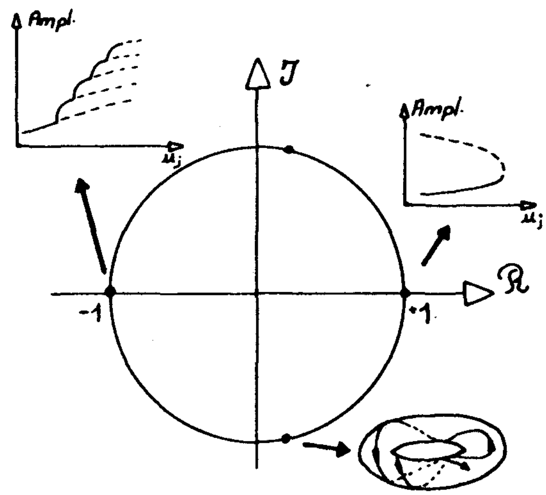

Figure 3. Bifurcations of periodic orbits.
—— stable branch, - - unstable branch.

Three possibilities are commonly found:

- A real eigenvalue crosses the point +1. In this case a periodic limit point appears.

- A real eigenvalue crosses the point -1. Then, a period doubling bifurcation occurs. In the vicinity of this point, the stable periodic orbit of period T becomes unstable and a new stable periodic orbit of period 2T appears. Generally this type of stability loss is followed by others of the same type (Feigenbaum cascades) which conduct to chaotic motion.

- Two conjugate imaginary eigenvalues leave the unit circle. After this torus bifurcation, motion lies on stable or unstable torus surrounding the unstable orbit.

For more details on Bifurcation Theory see Ioos [6], Guckenheimer [7] or Kubicek [8].

Numerical procedures

The computation of all asymptotic states mentionned above is performed by a numerical code developped at ONERA. It uses a continuation method initally derived from Kubicek [8].

Starting with an approximation of an asymptotic state, for a given value of parameters, the code determines, by a continuation process, the curve X(U) solution of a set of non linear algebric equations (G) which is computation case dependant.

$$
(G)\begin{cases}
\text{Equilibrium points} : F(X,U) = 0 \\
\text{Limit points} : F(X,U) = 0,\quad \mu = 0 \\
\text{Hopf points} : F(X,U) = 0,\quad \mu_{1,2} = \pm 2i\pi/T \\
\text{Periodic orbits} : X(T) = X(0)+\int_0^T F(X,U)dt
\end{cases}
$$

It should be observed that continuation process assumes, a priori, continuity and derivability conditions for G.

It can be noticed that the numerical code developped at ONERA is in a general form. Users have only to express their own problem in the form (1).

Application to a real aircraft

Alpha-Jet is a tandem two seat German-French aircraft for close support and battlefield reconnaissance. With narrow strake on each side of nose, it is also an advanced jet trainer.

Considering its great ability to safely demonstrate numerous and various high angle of attack behaviours and for flight tests correlation, the training version was chosen to investigate the interest of the methodology.

Aircraft Model

Each of six global coefficients (C_l, C_m, C_n, C_X, C_Y, C_Z) is expressed independantly as a function of influencing parameters. They have been measured on a rotary balance in the vertical wind tunnel at ONERA/IMFL and tabulated over a wide state and control domain,

$$0 < \alpha < 180°$$
$$-90° < \beta < 90°$$
$$-600°/s < \Omega < 600°/s.$$

In order to prevent continuation problem and fictive localized deformations of equilibrium surfaces, aerodynamic coefficients are usually smoothed to ensure continuity and derivability conditions for G.

In our application, no preliminary smoothing was done. Coefficients are evaluated by linear interpolation of the tabulated data. The following results will illustrate the robustness of our continuation algorithm.

Equations of Motion

The adopted system of equations represents a six degrees of freedom motion. Since there will be found :

- three momentum equations, assuming D = F = 0

$$A\frac{dp}{dt} - E\frac{dr}{dt} + (C-B)qr - Epq = L^A + L^F$$
$$B\frac{dq}{dt} + (A-C)rp + E(r^2-p^2) = M^A + M^F$$
$$C\frac{dr}{dt} - E\frac{dp}{dt} + (B-A)pq + Erq = N^A + N^F$$

- three force equations

$$m(\frac{du}{dt}+qw-rv) = X^A + F_x + mg\sin\theta$$
$$m(\frac{dv}{dt}+ru-pw) = Y^A + F_x + mg\cos\theta\sin\Phi$$
$$m(\frac{dw}{dt}+pv-qu) = Z^A + F_x + mg\cos\theta\cos\Phi$$

- two kinematic equations. Eulers's kinematic equation for heading angle is not taken into account

$$\frac{d\Phi}{dt} = p + tg\theta(q\sin\Phi+r\cos\Phi)$$
$$\frac{d\theta}{dt} = q\cos\Phi - r\sin\Phi$$

- four unsteady aerodynamic equations.

The nonlinearities of this system are many and can be classified into two groups.

The first one includes those which are intrinsic to the system and which are due to the motion equations of a solid in space (trigonometrical lines and gyroscopic momentum).

The second one includes those from the aerodynamic model of Alpha-Jet. In this category, distinction should be made between curvature nonlinearities of coefficients ($C_Z(\alpha)$, $C_l(\beta)$, etc..) and coupling nonlinearities such as certain coefficients which connect longitudinal and lateral variables ($C_m(\beta)$, $C_l(\alpha)$, etc..).

Results and discussion.

The results presented here are related to spin, spin entry and spin recovery. In order to simplify the interpretation of computations, only typical cases will be shown. More precisely, inverted spin and spin recovery using low negative angles of attack are not considered here.

Starting from a straight level flight at low angle of attack, when the pilot moves the elevator for a full nose up attitude (δ_m = -20°), multiple steady states appear at high angle of attack when both aileron and rudder deflection varies. The projection of the equilibrium surface in three characteristic sub-spaces (α, δ_l, δ_n), (r, δ_l, δ_n) and (p, δ_l, δ_n) allows easily to recognize spin and roll regimes domains. (See Fig 4). Before having a

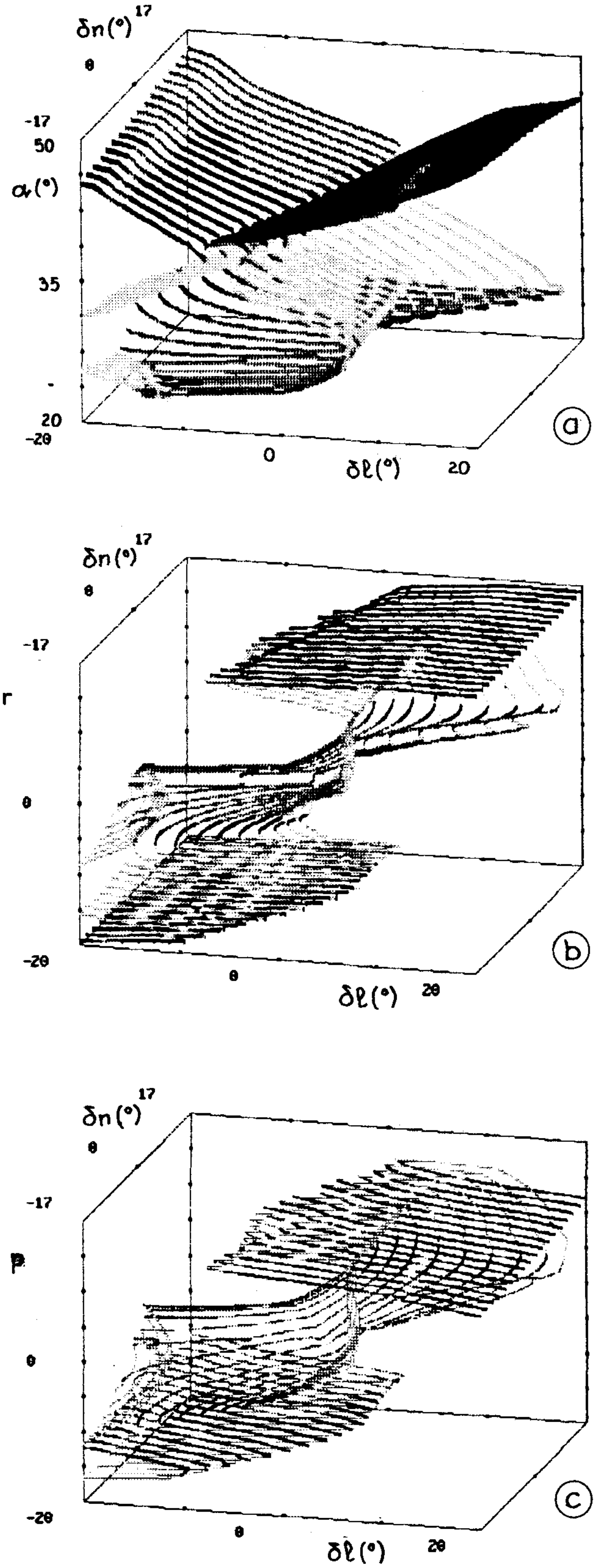

Figure 4. Equilibrium surface projection in characteristic sub-spaces: (a) (α, δ_l, δ_n); (b) (r, δ_l, δ_n); (c) (p, δ_l, δ_n). black stable, grey oscillatory unstable, little grey unstable divergent.

look at these different flight regimes, it should be observed that the lack of symmetry of this surface is due to non symmetrical aerodynamic data for symmetrical aileron and rudder deflections at high angle of attack.

As it can be seen on the previous figure, the stability is very different from a point to another. More precisely, it seems that left spins, related to negative δ_l,are much more unstable than right spins. Perhaps this low degree of stability can explain pilot's difficulties to demonstrate left steady spins on Alpha-Jet. The third type of steady states encountered on this surface corresponds to an important roll motion at moderate angle of attack. In flight, this kind of motion occurs mainly when pilots fail spin entry or fail the transition from one spin on one side to another spin on the other side.

In order to better define these motions, let us consider the equilibrium curve corresponding to full rudder deflection (δ_n = 17°). (See Fig 5).

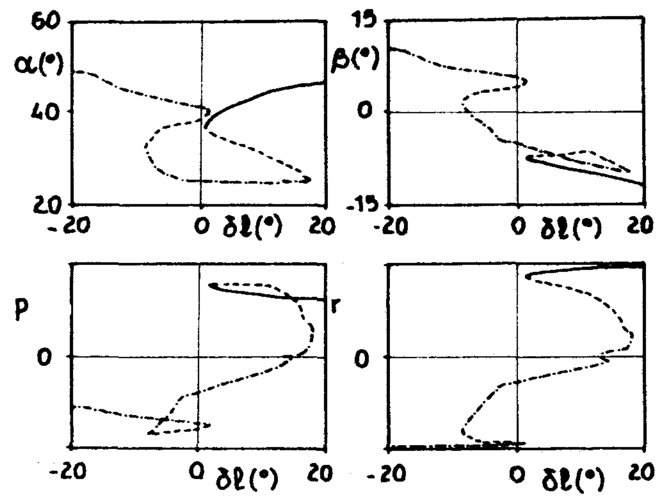

Figure 5. Equilibrium curve for δ_n = 17°.
--- stable, - - unstable divergent
-.- oscillatory unstable.

It can be observed that right spin is stable while left spin is always oscillatory unstable excepted for a few positive aileron deflections. In the vicinity of this last equilibrium branch (δ_l negative), there exist several periodic orbits when aileron deflection decreases from δ_l = +1.26° to δ_l = -20°. (See Fig 6).

Two distinct branches can be observed. On the first one, the limit points are numerous. Between δ_l = -0.308° to δ_l = -7.°, two convergent series of -1 periodic bifurcations determine a region in which an Alpha-Jet can exhibit a chaotic behaviour. On the contrary with typical chaotic behaviours exhibited by well known particular differential equations, in our case there are only little differences in amplitude between the different orbits of period T, 2T, etc... . Then, it seems that this behaviour will be very difficult to observe and to characterize in flight. Another important phenomena on this branch of the envelope is the rapid variation of orbits amplitude when δ_l is less than -10°. This could explain the sensibility of spin agitations versus aileron which is well known by pilots. Finally, one has to notice that, for an aileron deflection less than -10°, there exist several stable orbits for a given control deflection.

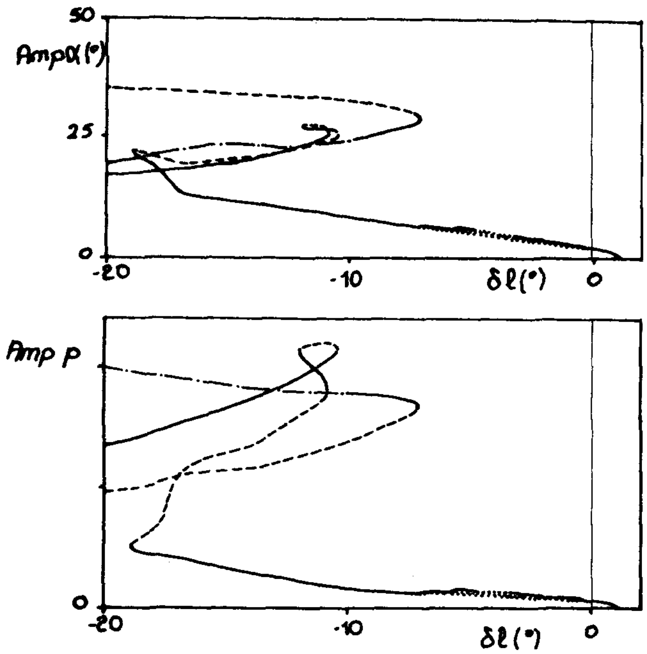

Figure 6. Envelope of periodic orbits when δ_l varies for δ_m = -20° and δ_n = 17°.
—— stable, - - unstable divergent
-.- oscil. unstable, ... unstable (-1°).

On the other branch of the periodic envelope, there exist oscillatory unstable orbits with great amplitude when δ_l is less than -10.4°. (See Fig 6). Around them, the motion takes place on a toroïdal surface if it is stable. Nevertheless, the existence, for same values of δ_l, of an invariant torus and stable orbits can lead to non similar flight behaviours. These different behaviours depend of the initial state and of the history of control deflections during the manoeuvre.

All these phenomena have been demonstrated in flight. In order to make correlations between predictions and flight, flight tests have been done at the French Flight Test Center in Istres. The results, which will be commented further, have been obtained at the end of 1988 and analyzed with standard flight test techniques at ONERA/IMFL in Lille during 1989.

For full elevator and rudder deflections (δm = -20°, δ_n = 17°) quiet left spin is demonstrated for δ_l = -4°. (See Fig 7).

For an aileron deflection in the vicinity of δ_l = -5°, chaotic motion was not demonstrated. This result is due to the short duration of spin tests and to the absence of great differences between the orbits in presence as it was previously mentionned.

When aileron deflection is close to -10°, Alpha-Jet can exhibit different motions due to the simultaneous existence of stable orbits and a stable invariant torus. (See Fig 8, 9 and 10).

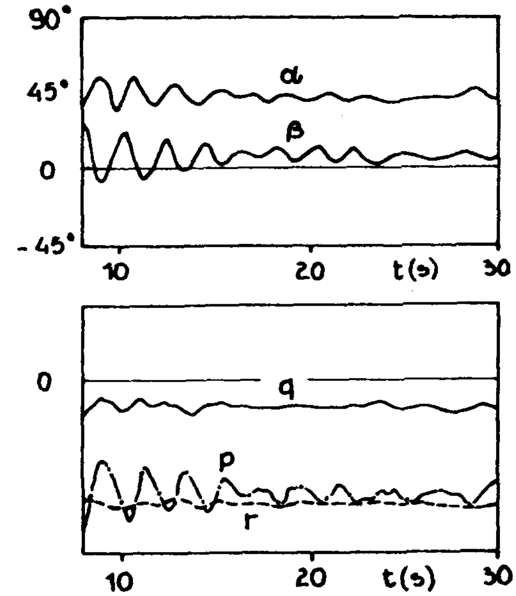

Figure 7. Quiet left spin for δ_l = -4°

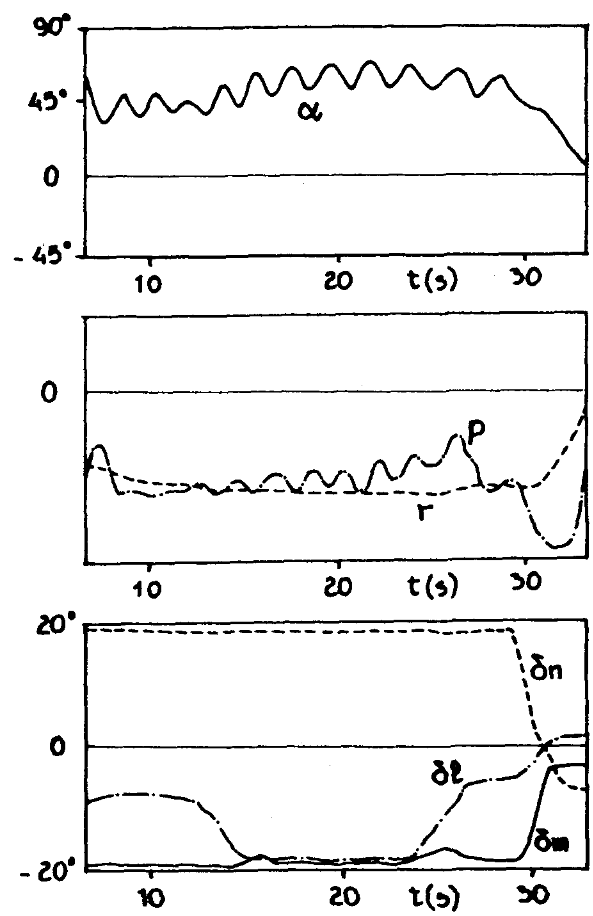

Figure 8. Quiet spin for δ_l moving from -20° to 0°

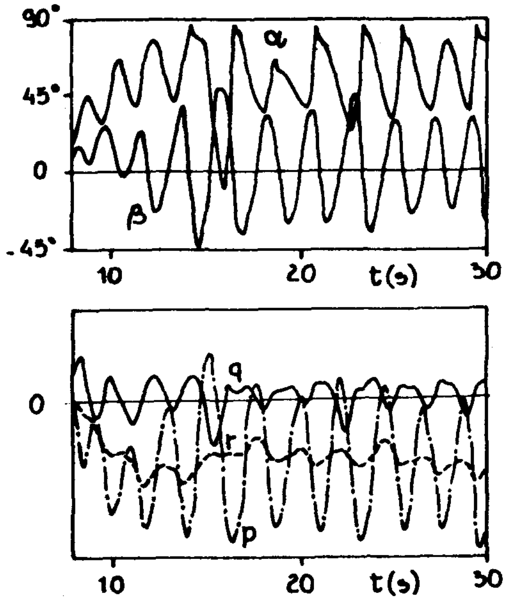

Figure 9. "Regular agitated" spin for δ_l =-10°.

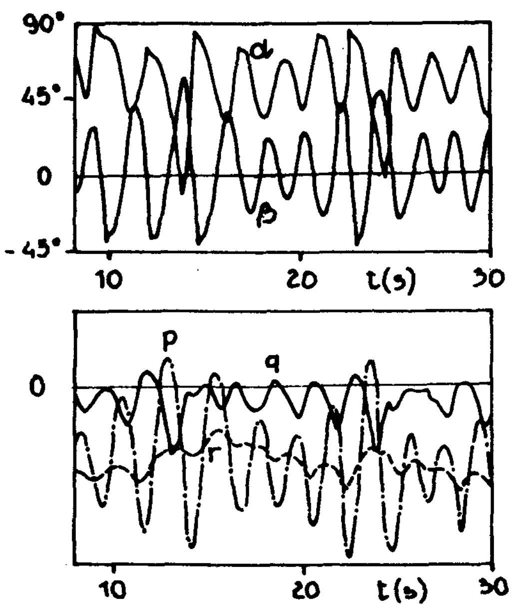

Figure 10. Motion on a toroïdal surface for δ_l =-10°. See p versus time.

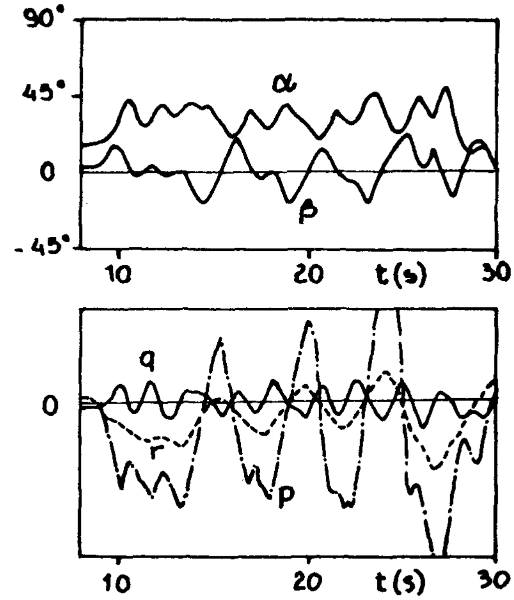

Figure 11. Unsuccessful spin entry.

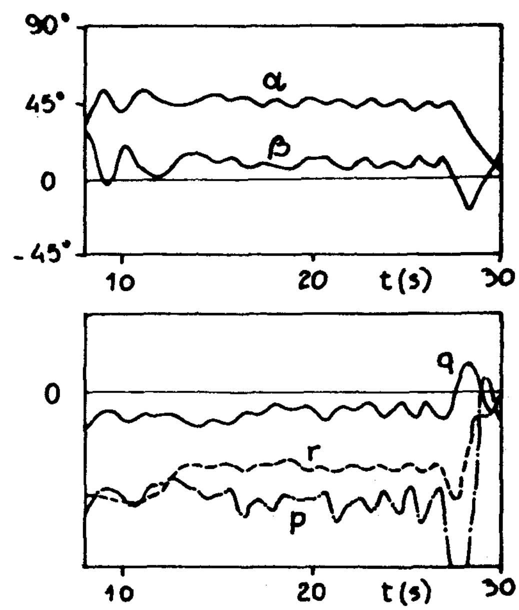

Figure 12. Quiet left spin for δ_n = -17° and negative aileron deflection.

Another very interesting behaviour is what happens when pilots fail left spin entry. In this case, an oscillatory motion at a moderate angle of attack appears. (See Fig 11). It corresponds to orbits which surround the lower oscillatory unstable equilibrium branch of Fig 4.

Starting with the previous quiet left spin for δ_l = -4° and reversing rudder deflection, another quiet spin can be observed for a rather unusual combination of lateral control deflections. (See Fig 12). However, there is a very good aggreement with the stable equilibrium branch obtained for δ_n = -17° and negative δ_l. (See Fig 4).

Going back to full positive positive rudder deflection, quiet spin turns into flat spin when pilots push on the stick according to equilibrium states computations for different elevator deflections. (See Fig 13 and Fig 14).

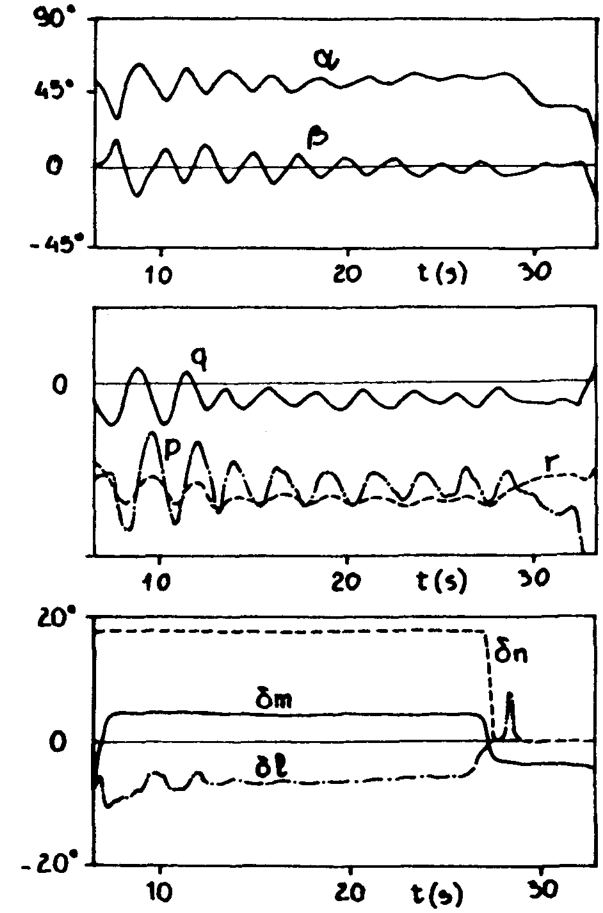

Figure 13. Flat spin for δm = 4°.

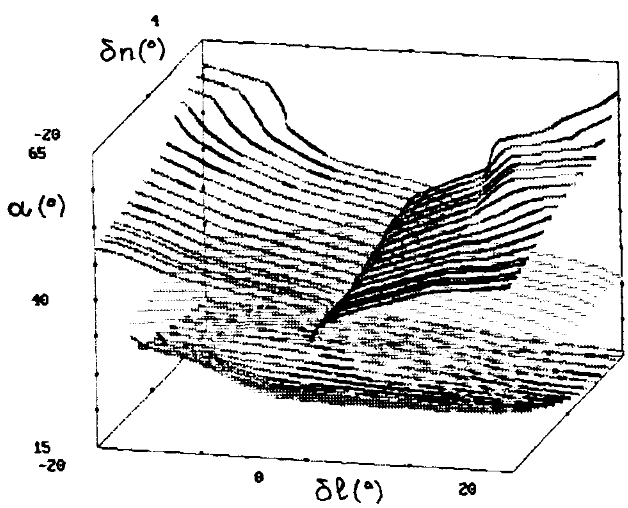

Figure 14. Equilibrium surface for $\delta_m = 4°$ in the (α, δ_l, δ_n) space.
black stable, grey oscillatory unstable, little grey unstable divergent.

As it can be seen on the figures 8 and 14, for example, spin recovery is always satisfactory achieved by putting lateral control deflections at neutral, after a transient small positive aileron deflection, and pushing on the stick in order to decrease elevator deflection.

Influence of a lateral offset of the cdg

During previous flight tests on Alpha-Jet and spin tests in the vertical wind tunnel at ONERA/-IMFL, it has been found that spin is very sensitive to a lateral offset of the center of gravity.

In the following, one are going to show that Bifurcation Theory is able to study this influence of the position of the cdg on spin recovery.

It is assumed that the effect of a lateral offset of the cdg is similar to a shift (δ_Y) of the cdg outside the symmetry plane, along the Y body axis. Then, only moments due to exterior forces are modified.

Rudder at neutral and for a positive value of elevator deflection, spin recovery from left spin is achieved through a limit point for $\delta_l < 4°$. When lateral offset of cdg increases, the limit point moves in the (δ_l, δ_Y) plane in a such way that spin recovery may be less easy to obtain because of the stabilizing effect of positive δy on left spin. (See Fig 15).

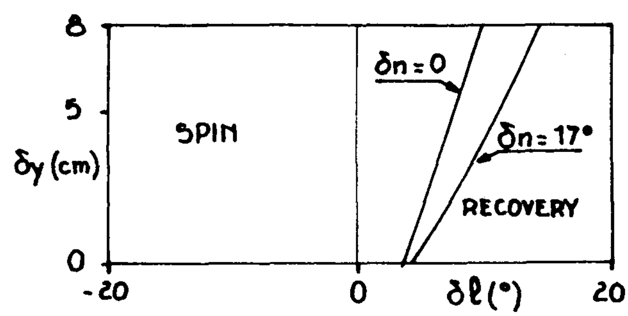

Figure 15. Influence of lateral offset of the cdg on left spin recovery in the (δ_l, δ_Y) plane.

Influence of gyroscopic torques due to engines

Rotor speed effects appear in the momemtum motion equation :

$$\frac{d\sigma}{dt} = M$$

with $\sigma = \sigma_{\text{fixed parts}} + \sigma_{\text{rotor(s)}}$
and $\sigma_{\text{rotor(s)}} = \Sigma(I_{\text{rotor}} \cdot \Omega_{\text{rotor}})$.

In the following, we have considered constant rotor speed. Then, these effects are easier to take into account. It is just like introducing additional moments in the previous motion equations.

Considering inertia characteristics and rotor speed of LARZAC engines which were used on Alpha-Jet, it seems that these additional terms are non negligible damping terms similar to cross coupling aerodynamic coefficients due to angular rates.

Equilibrium computations show no significant influence of this effect excepted only few localised deformations of equilibrium surfaces.

In simulation, when only one stable equilibrium exists, no difference can be seen with or without rotor effects. However, when several stable equilibria are in competition, the influence of rotor speed is more important. In certain cases, it can lead to very different final states for the same control deflections. (See Fig 16).

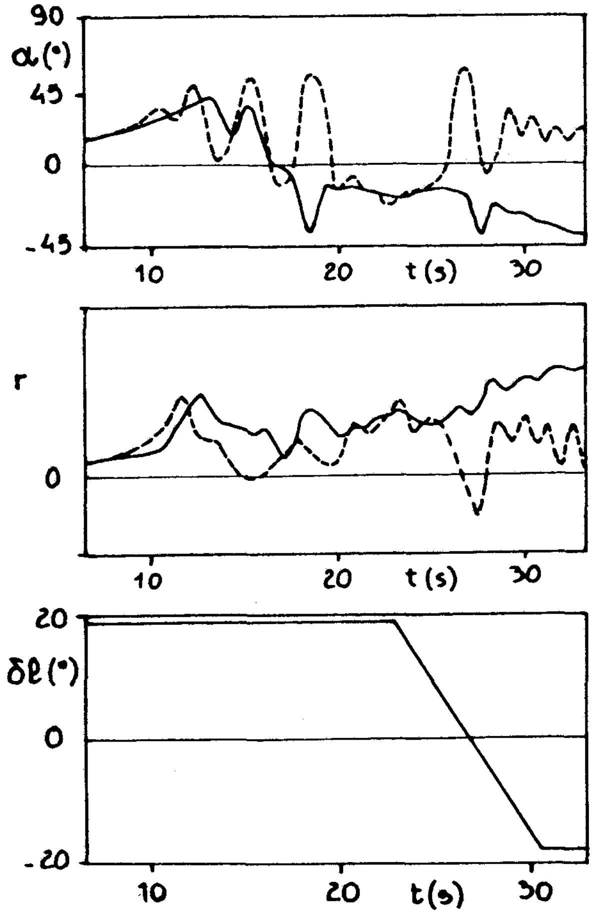

Figure 16. Influence of engines rotor gyroscopic torque on final state.
— without, - - with gyros. torque.

This difference cannot be explained by a possible instability of one equilibrium state. This phenomena is more surely related to the stabilizing effect of rotor speed, mentionned above, during the transient motion which depends also greatly of the time history of control deflections. Then, it can be said that, if no significant effect is found on equilibrium state, gyroscopic torques due to rotor speed have to be taken into account, in simulation, in order to increase the reality of spin entry.

CONCLUSION

In the high angle of attack flight regime, fighter aircraft behaviour is so complex that it is very difficult to predict it exhaustively. Usually, this flight domain is investigated by mean of systematic or Monte Carlo numerical simulations before the first flight and by mean of extensive and expensive flight tests.

However, in spite of these tedious efforts, it remains that an analysis of such phenomena is still very delicate due to their complexity and apparently random character.

Thanks to Bifurcation Theory and to computers capabilities, ONERA has developped a methodology and a tool to investigate asymptotic behaviour of non linear differential equations depending on parameters. This methodology has been used to study high angle of attack behaviour of an Alpha-Jet.

After computing equilibrium surfaces, periodic orbits envelopes and predicting aircraft behaviour, flight tests have been done to verify predictions. Thanks to flight test pilots, to which it has been asked to perform rather unusual flight tests, very good correlations with results predicted by the Theory have been obtained.

The results presented in this paper show the richness of the informations provided by this methodology. However, one cannot forget that the quality of predictions is directly connected to the accuracy of the aerodynamic data base put in the aircraft model.

Then, considering all these results, it can be said that this technique has a great potentiality and is ready to investigate aircraft behaviour, using only wind tunnel data, in order to help aircraft designers.

Finally, for a complete understanding of non linear systems behaviour, transient motion have also to be studied to understand the immediate behaviour and the influence of speed variation of controls on the motion. But, independantly of transient motion, one has always to keep in mind that, if asymptotic states are not always achieved by a limited number of simulations and flight tests, they exist and probably will happen, at least one time, during aircraft life.

In the future, due to its ability to treat non linear differential equations, this methodology would be successfully applied to other highly non linear systems like, just for example, high perfomance missiles, helicopters or submarines.

ACKNOWLEDGEMENTS

This study has been carried out under STPA contract. It was done in connection with Dassault Aviation for the aircraft, and with the Flight Test Center in Istres and ONERA/IMFL in Lille for the realization and the analysis of flight tests.

I would like to thank them all for having got confidence in this study and for having given the opportunity to show, from predictions to flight tests, the interest of the methodology developped at ONERA.

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