A STUDY OF WING ROCK

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Abstract

So far, the study on wing rock has been carried out along two approaches. One is to analyze the fluid mechanism of wing rock, and the other is to study the characteristics of wing rock motions.

According to the later approach, this paper begins with the general equations of unsteady aircraft motion. And then, the aerodynamic model which describes aircraft manoeuvers at high angles of attack is set up. Furthermore, by means of the qualitative theory of ordinary differential equations and the propagational matrix analysis method, the reasons of causing wing rock and its characteristics are discussed. The results show that wing rock is a Hopf bifurcation phenomenon of nonlinear dynamic systems. For aircraft with moderate sweptback wings, the main reason of causing wing rock is the variation in roll damping moment with the angle of attack and the sideslip angle.

I. Introduction

To improve maneuverability and enlarge flight envelope, modern high performance aircraft are usually designed for flight to high angles of attack (AOA). Therefore, some special phenomena such as wing rock, stall, nose slice, departure, and spin etc. which dangerously occur in high AOA region have received widespread attention. Wing rock is an uncommanded oscillation mainly around the roll axis. Occuring near stall AOA, wing rock will degrade weapon's aiming accuracy and aircraft's turn effectiveness. Moreover, it will cause safety problems in air combat. So, it is necessary to study the mechanisms of wing rock.

So far, the study on wing rock has been carried out along two main approaches. One is to analyze its fluid mechanism, i.e. to set up the relation between wing rock and the formation, burst, separation and reattachment of the vortices on aircraft. Theoretically, wing rock can be classified as three types: slender wing rock, conventional wing rock and wing-body rock. Slender wing rock can happen to airborne vehicles with flying wing type configuration. It results from asymmetrical leading edge vortices, while the burst of the vortices confines its amplitude. Conventional wing rock usually happens to aircraft with moderate sweptback wings. Asymmetrical leading edge stall of the wings (dynamic stall) results in roll damper lowering and even becoming negative. That in turn results in wing rock and the variation in roll damping moment with various roll rates keeps wing rocking on. Wing-body rock results from the asymmetrical vortices shedding from the slender forebody, and the interaction between the vortices and the airflow over wings and tails as well as the unsteady nature of flow (vortex delay)sustains rocking. (Refs.(1) & (2))

According to the characteristics of wing rock, the other approach is to use approximate mathematical models of aircraft (usually three or less degrees of freedom) and calculate time histories for analysis (Refs.(1) & (3)). However, the airflow's nature under high AOA condition is so sensitive to small changes in aircraft's geometry that the characteristics of the three types of wing rock motions mentioned aboved are rather different and the foregoing approximate mathematical models can only give limit results. To overcome the limitness, this paper utilizes the mathematical model of nonlinear dynamic equations of full six degrees of freedom, in which aerodynamic data are superimposed by static data from wind tunnel test and dynamic data from estimation. The time histories of a spin motion computed from the mathematical model have been compared with those recorded from a flight test under the same conditions and the model verified.

Furtherly, the qualitative theory of ordinary differential equations (ODE) has been applied to search for the mathematical mechanism and the propagational matrix analysis method to analyze the main physical factors of giving rise to and sustaining wing rock.

II. Mathematical Model of Aircraft Motion

While wing rocking, the variation of aircraft's motion parameters interconnects with that of airflow's characters over the aircraft. Especially, wing rock motions have much to do with the formation, burst, separation and reattachment of vortices. So, the direct approach to research wing rock is to solve the unsteady flow equations and the equations of aircraft motion simultaneously. However, to do that is rather difficult at the present time. In the former work (Refs.(1) to (3)), the equations of aircraft motion used to be simplified according to the characteristics of wing rock motions and nonlinear aerodynamic models were set up. And then, the equations of motion were solved to replay the wing rock motions. For instance, Hsu (Ref.(1)) used lateral equations of three degrees of freedom from the viewpoint that wing rock is mainly a roll oscillation:

$$\left.\begin{aligned} &\frac{m}{\bar{q}S}(\dot{V}-wp+ur-g\sin\phi\sin\theta)=C_y(t)\\ &\frac{I_x}{\bar{q}Sb}\dot{p}-\frac{I_{xz}}{\bar{q}Sb}\dot{r}=C_l(t)\\ &\frac{I_z}{\bar{q}Sb}\dot{r}-\frac{I_{xz}}{\bar{q}Sb}\dot{p}=C_n(t)\end{aligned}\right\} \quad (1)$$

where aerodynamic terms are:

$$\left.\begin{aligned} &C_y(t)=C_{y0}+C_{y_\beta}\beta+C_{y_p}p+C_{y_r}r+C_{y_{\delta a}}\delta_a+C_{y_{\delta_r}}\delta_r\\ &C_l(t)=C_{l0}+C_{l_\beta}\beta+(C_{lp0}+C_{lp\beta}|\beta|+C_{lpp}|p|)p\\ &\qquad +C_{l_r}r+C_{l_{\delta a}}\delta_a+C_{l_{\delta_r}}\delta_r\\ &C_n(t)=C_{n0}+C_{n_\beta}\beta+(C_{n_{p0}}+C_{n_{p\beta}}|\beta|+C_{n_{pp}}|p|)p\\ &\qquad +C_{n_r}r+C_{n_{\delta a}}\delta_a+C_{l_{\delta_r}}\delta_r\end{aligned}\right\} \quad (2)$$

whereas

$C_{i0}=C_i(\alpha_s,\ \beta=0)$, $i=y,\ l,\ n$;

α_s AOA of steady state;

p, r nondimentional angular rates.

Eq.(2) shows that he thought the main nonlinear factor of giving rise to and sustaining wing rock is the variation in roll damper with sideslip angle and roll rate. Similarly, there are also models of one (Ref.(1)) and two (Ref.(3)) degrees of freedom. Though these models can test wing rock motions of some aircraft, their common defect is that they can only deal with motions of some special configurations. In addition, especially in the earlier stages of aircraft design, the nonlinear aerodynamic derivatives in these models are difficult to obtain. To keep clear of such difficulties, the models that describe large amplitude maneuvers at high AOA can be approximately utilized. Seeing that wing rock occurs near stall AOA, this paper takes the following dynamic equations of rigid body in the body axis system:

$$\left.\begin{aligned} &\dot{\alpha}=q+\Big[-\Big(\frac{\bar{q}S}{mV}C_x-\frac{g}{V}\sin\theta+r\sin\beta\Big)\sin\alpha+\Big(\frac{\bar{q}S}{mV}C_z\\ &\qquad+\frac{g}{V}\cos\theta\cos\phi-p\sin\beta\Big)\cos\alpha\Big]\sec\beta\\ &\dot{\beta}=-\Big[\Big(\frac{\bar{q}S}{mV}C_x-\frac{g}{V}\sin\theta\Big)\sin\beta+r\Big]\cos\alpha+\Big(\frac{\bar{q}S}{mV}C_y\\ &\qquad+\frac{g}{V}\cos\theta\sin\phi\Big)\cos\beta-\Big[\Big(\frac{\bar{q}S}{mV}C_z+\frac{g}{V}\cos\theta\cos\phi\Big)\sin\beta-p\Big]\sin\alpha\\ &\dot{p}=-\Big[\Big(\frac{I_z-I_y}{I_x}-\frac{I_{xz}^2}{I_xI_z}\Big)qr+\Big(1-\frac{I_y-I_x}{I_z}\Big)\frac{I_{xz}}{I_x}pq\\ &\qquad+\frac{\bar{q}Sb}{I_x}\Big(C_l+\frac{I_{xz}}{I_z}\Big)C_n\Big]\Big/\Big[1-\frac{I_{xz}^2}{I_xI_z}\Big]\\ &\dot{q}=\frac{\bar{q}S\bar{c}}{I_y}C_m+\frac{I_z-I_x}{I_y}pr+\frac{I_{xz}}{I_y}(r^2-p^2)\\ &\dot{r}=\Big[\Big(\frac{I_{xz}^2}{I_xI_z}-\frac{I_y-I_x}{I_z}\Big)pq-\Big(1+\frac{I_z-I_y}{I_x}\Big)\frac{I_{xz}}{I_z}qr\\ &\qquad+\frac{\bar{q}Sb}{I_z}\Big(\frac{I_{xz}}{I_x}C_l+C_n\Big)\Big]\Big/\Big[1-\frac{I_{xz}^2}{I_xI_z}\Big]\end{aligned}\right\} \quad (3)$$

where the tangential force equation is removed on account of the velocity variation negligible in the motion type considered.

The kinematic equations concerned are:

$$\left.\begin{aligned} &\dot{\theta}=q\cos\phi-r\sin\phi\\ &\dot{\phi}=p+q\,\mathrm{tg}\,\theta\sin\phi+r\,\mathrm{tg}\,\theta\cos\phi\\ &\dot{\psi}=q\sin\phi\sec\theta+r\cos\phi\sec\theta\end{aligned}\right\} \quad (4)$$

The aerodynamic terms in Eq.(3) are composed of static data from wind tunnel test and dynamic data from estimation with the strip theory or penal method (Refs.(4) to (6)), which can be expressed as:

$$\left.\begin{aligned} &C_x=C_x(\alpha,\beta,\vec{\delta}=0)+C_{x_{\delta_e}}(\alpha,\beta)\delta_e\\ &C_y=C_y(\alpha,\beta,\vec{\delta}=0)+C_{y_{\delta_e}}(\alpha,\beta)\delta_e+C_{y_{\delta a}}(\alpha,\beta)\delta_a\\ &\qquad+C_{y_{\delta r}}(\alpha,\beta)\delta_r+\frac{b}{2V}\Big[C_{y_p}(\alpha)p+C_{y_r}(\alpha)r\Big]\\ &C_z=C_z(\alpha,\beta,\vec{\delta}=0)+C_{z_{\delta_e}}(\alpha,\beta)\delta_e\\ &C_l=C_l(\alpha,\beta,\vec{\delta}=0)+C_{l_{\delta_e}}(\alpha,\beta)\delta_e+C_{l_{\delta a}}(\alpha,\beta)\delta_a\\ &\qquad+C_{l_{\delta r}}(\alpha,\beta)\delta_r+\frac{b}{2V}\Big[C_{lp}(\alpha)p+C_{l_r}(\alpha)r\Big]\\ &C_m=C_m(\alpha,\beta,\vec{\delta}=0)+C_{m_{\delta e}}(\alpha,\beta)\delta_e+\Big(\frac{\bar{c}}{2V}\Big)C_{mq}(\alpha)q\\ &C_n=C_n(\alpha,\beta,\vec{\delta}=0)+C_{n_{\delta e}}(\alpha,\beta)\delta_e+C_{n_{\delta a}}(\alpha,\beta)\delta_a\\ &\qquad+C_{n_{\delta r}}(\alpha,\beta)\delta_r+\frac{b}{2V}\Big[C_{n_p}(\alpha)p+C_{n_r}(\alpha)r\Big]\end{aligned}\right\} \quad (5)$$

Eq.(5) shows that the nonlinear aerodynamic forces consist of three parts: 1) static aerodynamic forces arising from variation in angle of attack and sideslip angle; 2) the forces from deflections of controls; 3)rotating forces. Here the first two parts are usually given by wind tunnel results in tabular form. In the concrete computation , the bi-cubic spline method is employed for value inserting. The third part can be numerically estimated. To check the practicality of the chosen mathematical model, a combat aircraft is taken as example. The time histories of a spin computed from the model are compared with those recorded from flight test under the same conditions. The results are shown in Fig.1.

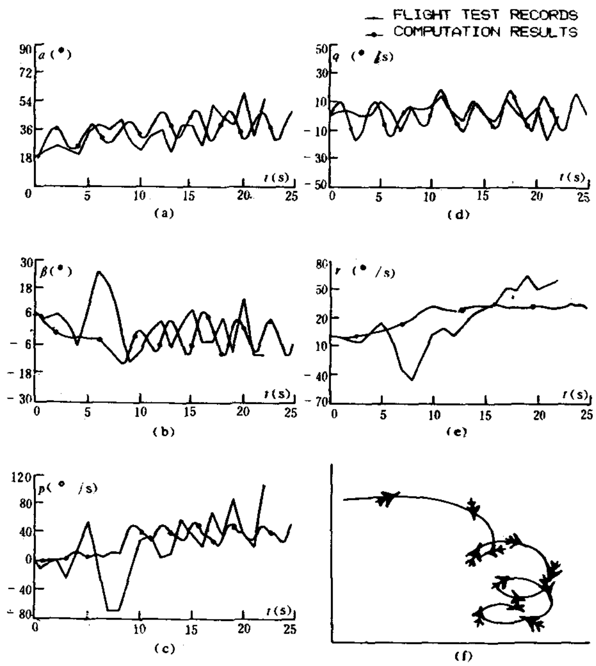

Fig.1 Comparison between Flight Test Records and Computation Results of Spin Entering

They show that the parameters from computation have similar regularities vs. time with their counterparts from flight test. Of course, small discrepancies exist on account of the differences between computation and flight test conditions, such as the errors between the model and the real fighter. Anyway, the illustrations have verified the practicality of the model.In particular, Fig.1(5) which is based on the computed time histories vividly brings before sight the procedure of spin entering. It supports the above verification of practicality from another side. Furtherly, the propagational matrix analysis method is employed to analyze the first seconds of the time histories and the seconds after spin entering. And then new time histories are recomputed after some dynamic derivatives and the motor gyro moment varied according to the analysis results. The general conclusion is that static aerodynamic forces play major roles in the earlier stages of high maneuver flight, while dynamic derivatives become more important in later seconds. For spin motions, the gyro moment and the roll damping moment also give much effect.

Here should be pointed out that if unsteady and dynamic aerodynamic information is available from rotatory balance, bending airflow wind tunnel and other ways, concerned terms can be added in the aerodynamic model of Eq.(5).Thus, the above-mentioned mathematical model of aircraft motion is still practicable.

III. Equalibrium Surface of Aircraft

The foregoing mathematical model for wing rock analysis is a high dimensional set of nonlinear differential equations. If gravity negligible (Refs.(7) and(8)), Eq.(3) decouples from Eq.(4). However, the equations are still of five dimensions and can't be solved analytically up to the present time. So, the ODE qualitative theory is applied by means of the equalibrium surface analysis (Refs.(6) to (8)). Eq(3) can be simply rewritten as:

$$\dot{X} = F\ (X, C) \qquad (6)$$

where X is the state vector $(\alpha\ \beta\ p\ q\ r)^T$ and C is the control vector $(\delta_a\ \delta_e\ \delta_r)^T$.

The surface stretched by variables α, β, p, q and r which satisfy the equation $\dot{X} = 0$ under various values of C is called the equalibrium surface (ES). Any point on the surface is an equalibrium point (EP), which denotes a reference (equalibrium) flight state. According to the ODE qualitative theory, the local stability behavior around a EP depends on the eigenvalues of the Jaccobian matrix of the point, or on the roots of the equation:

$$\det \left| \frac{\partial F}{\partial X} \right| = 0 \qquad (7)$$

If the real parts of all the roots are negative, the point is locally stable, otherwise unstable.

In view of data at hand, a moderate sweptback combat aircraft is taken as example. The geometric and weight parameters are as following:

mass = 22,700 Kg , $S = 48.77\ m^2$,
I_x = 67,790 Kgm^2 , b = 19.20 m ,
I_y = 427,348 Kgm^2 , $\bar{c}$ = 2.76 m ,
I_z = 476,564 Kgm^2 ,
$I_{xy} = I_{yz} = I_{zx} = 0$.

The computed state is height = 10,000m and velocity = 184 m/sec.

The ES curves of the sample aircraft are plotted in Fig.2. The designator S indicates that the Jaccobian matrix of any point on the branch has no eigenvalues with positive or null real parts, or all the points on the branch are locally stable. U indicates that the Jaccobian matrix has a positive real eigenvalue. And L indicates that the matrix has a complex pair of roots with positive real parts. Fig.2 shows that the designator varies from S to L when the control variable δ_e goes from positive to between -16° and -17° . The corresponding AOA is about 32°, which is close to the stall AOA of the aircraft.

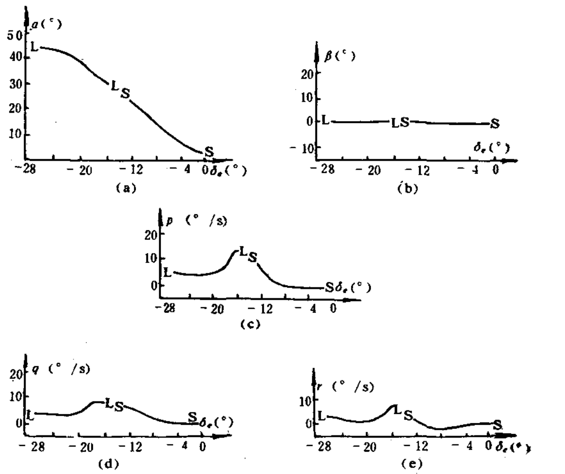

Fig.2 The Equalibrium Surface of the Sample Aircraft

Calculation experience (Ref.(6)) reveals that an nonlinear attractor, which corresponds to a stable limit cycle in two dimensional systems, may occur near the EP when it is a critical point of the stability behavior on a EP branch, along which the equalibrium points vary from locally stable behavior to unstable oscillatory behavior (designator from S to L). In the ODE qualitative theory this phenomenon is named Hopf bifurcation. To test its existance, the EP corresponding to $\delta_e = -20.5°$, $\delta_a = \delta_r = 0$ is chosen. Here,

$$(\alpha , \beta , p, q, r) = (41°, 0.4°, 4.5°/s, -1.4°/s, 3.83°/s)$$

Taken control law as in Table 1, the time histories are computed with four order Runge-Kutta method as in Fig.3. If an input of step elevator deflection is exerted as disturbance, the aircraft responds violently after t>5 sec. After 10 seconds, it enters an oscillatory motion with cycle about 9 seconds, and each parameter oscillates round its equalibrium value, though each has its own style. Here comes out a so-called attractor. The roll oscillatory motion from longitudinal disturbance accompanied by pitch and yaw oscillatory motions is called wing rock. Fig.3 illustrates the projection on the horizontal plane of the trajectory of the aircraft's center of gravity. This projection curve is not close. Fig.4 shows the varying procedure of the aircraft's attitude during t = 12 ~ 18 sec from the time histories in Fig.3. The characteristics of wing rock clearly appear. As mentioned above, according to the ODE qualitative theory this is a Hopf bifurcation phenomenon.

Table 1

t (sec)	< 5	5 ~ 6.1	> 6.1
δ_e (deg)	-20.5	-10.5	-20.5
$\delta_a = \delta_r = 0$			

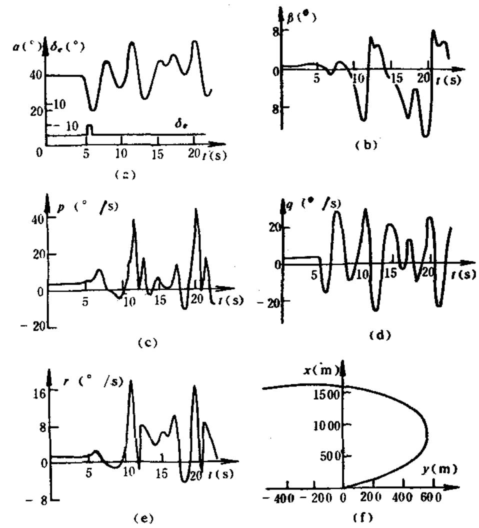

Fig.3 Time Histories of Wing Rock

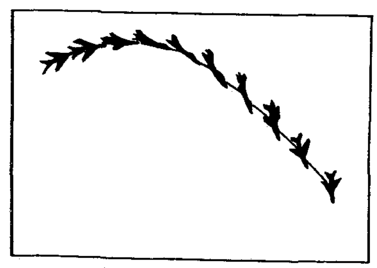

Fig.4 Wing Rock of Sample Aircraft

IV. Physical Reasons of Wing Rock

In the following discussion, the propagational matrix analysis method (Ref.(9)) is employed to explore the mechanical mechanism of wing rock. The basic concepts on propagational matrix are briefly stated here.

For an nonlinear system

$$\dot{X} = F(X,t) ,$$

the time histories can be obtained by Runge-Kutta methed. Its iterative formula is:

$$x_{k+1}=x_k+h[F(x_k,\ t_k)+2F(a_k,\ t_k+\frac{h}{2})+2F(b_k,\ t_k+\frac{h}{2})+F(c_k,\ t_k+h)]/6 \quad (8)$$

After a series of derivation, we get:

$$\frac{\partial X_{K+1}}{\partial X_K} = U_K + M_K \quad (9)$$

where $U_K + M_K$ is called propagational matrix. The matrix U_K is the transfer matrix of the corresponding linear system. Its elements can be expressed as:

$$\left.\begin{aligned} &U_{ij}=\exp\left[h\left(\frac{\partial f_i}{\partial x_j}\right)_k\right]-(1-\delta_{ij}) \\ &\qquad\qquad i,\ j=1,\ 2,\ \cdots\cdots,\ n \\ &\delta_{ij}=\begin{cases}1, & i=j\\ 0, & i\neq j\end{cases}\end{aligned}\right\} \quad (10)$$

The matrix M_K is the correctional matrix taking the nonlinear effect of two order partial derivative terms into account. Its formula can be:

$$M_K = (h^2/6)E_K(N_K^{(1)}+N_K^{(2)}+N_K^{(3)}) \quad (11)$$

where each term is explained in Ref.(9).

From Eq.(9) we know that the propagational matrix reflects the varying tendency with time of each motion parameter. So, some typical points that represent the motion varying nature can be selected and their propagational matrices are arranged in line with magnitude of their elements to find the main factors giving rise to, developing and sustaining wing rock.

From Fig.3, the typical points at t = 6.1, 7.8, 10, 12.4 and 16 sec which represent the emerging, developing and lasting of wing rock are selected. The elements of the propagational matrix at each corresponding point are computed and rearranged. As an example, for point at t = 6.1 sec the elements of the matrices U_K and M_K which affect the variation of the roll rate p are shown in Table 2.

Table 2

U_{31}	U_{32}	U_{33}	U_{34}	U_{35}
−0.303E−01	0.379E−02	0.988E+00	0.267E−03	0.515E−01
M_{31}	M_{32}	M_{33}	M_{34}	M_{35}
0.171E−03	−0.317E−03	0.150E−04	−0.340E−05	0.463E−04

From this table, the elements in matrix M_K are all less than those in matrix U_K. In U_K the element U_{33} is the largest, and followed by U_{31} and U_{35}; the other elements are less in the lower orders of quantity. Finding the maximum term from the expressions of these three U_K elements, we can see that the major factor giving rise to the oscillation of roll rate p is the variation in roll damping moment with AOA. To Support this argument, Fig.5 illustrates C_{lp} vs. AOA curve of the sample

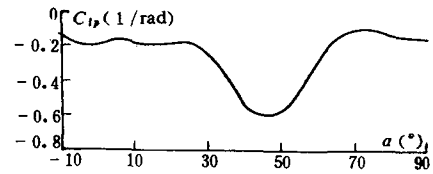

Fig.5 C_{lp} vs. α Curve of the Sample Aircraft

aircraft. Note that the negative value of C_{lp} gradually lessens along both two sides of AOA = 40°, and the variation is sharp until AOA gets to about 25° or 70°. From Fig.3(1), the elevators deflect to -10.5° at t = 6.1 sec, so the aircraft's AOA decreases to 25° and just drops into the varying region of C_{lp}. Out of question, that will lead to roll damping moment descreasing and hence roll oscillation amplitude increasing. With oscillation the AOA increasing, reaching and even surpassing the former equalibrium value, the roll damping derivative will gradually increase negatively. After the AOA surpasses the former equalibrium value, the derivative will then decreases negatively. Such reciprocation causes, develops and sustains wing rock. Analyzing the propagational matrices of several other points also prove this conclusion. The above conclusion is consistent with that stated in the introduction of this paper, which is that the variation in roll damping moment from asymmetrical leading edge stall (dynamic stall) of wings gives rise to wing rock.

V. Conclusions

1) The mathematical model which describes the aircraft flying at high AOA is the base for researching wing rock. The key point is how to set up appropriate aerodynamic models.

2) The aerodynamic model which consists of static aerodynamic forces from wind tunnel test and dynamic forces from estimation can be employed for wing rock analysis.

3) From the viewpoint of the ODE qualitative theory, wing rock is a Hopf bifurcation phenomenon of nonlinear dynamic systems.

4) For the sample aircraft , the variation in roll moment from dynamic wing stall at high AOA may cause wing rock.

5) The equalibrium surface and propagational matrix analysis method is a powerful tool for studying high AOA motions of aircraft.

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