

**FINITE ELEMENT APPLICATION TO INTERIOR NOISE PREDICTION  
IN AIRCRAFT FUSELAGE**

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**Abstract**

The requirement for low noise levels inside a propeller-powered commuter airliner results in the demand for easily applicable analysis tools to analyze complex fuselage structures and the acoustic cavity inside the passenger compartment, respectively.

Therefore, the finite element method, combined with a mechanical-acoustic analogy was used to predict the noise distribution in the Dornier 328 cabin volume resulting from the propeller induced harmonic vibration of the whole fuselage.

To study the influence of structure modifications as well as the cavity behavior (mainly the resonant frequencies due to different cabin cross-sections) detailed finite element models were generated.

In the present paper, the modal synthesis technique to solve coupled structural-acoustic problems is summarized. Two- and three-dimensional finite element models of the acoustic cavity and the surrounding aircraft structure are given. The coupling procedure and final results are described on the basis of the full-scale fuselage of the Dornier 328 commuter aircraft. Dynamic results of the structure and the cavity are compared with experimental test data of a fuselage test section. Effects, that have a significant influence are discussed, to improve the basic understanding of structural-acoustic problems, as well as to show the practicability of this finite element approach.

**1. INTRODUCTION**

Up to now, the finite element method FEM has been successfully applied to structural analysis in statics and dynamics. Moreover, it is also well suited to solve coupled structure-acoustic problems.

In general, the interaction between a vibrating structure and the fluid, e.g. air, that is enclosed by or surrounds the structure, may be neglected. In the particular cases, where a more detailed investigation of structural vibrations or just the dynamic behavior of the fluid medium itself is of interest, then the coupling effects cannot be disregarded any more.

For the present problem, the air enclosed by the fuselage walls is a second dynamic system, which can strongly couple with the structure. That is why the interaction between the two systems has to be taken into account.

First finite element formulations were derived some 20 years ago, but powerful computers are essential to calculate such large numerical problems. Furthermore, in recent years, some new algorithms have been developed, to carry out these coupled tasks efficiently. Based upon a mechanical analogy, it is currently possible to use only slightly modified, but nevertheless general purpose finite element codes. For a detailed analytical description of the coupling approach and a discussion of coupling effects see Reference [1].

The presented finite element calculations for the Dornier 328 fuselage resulted in the prediction of frequency-dependent noise levels inside the passenger compartment and the coupled vibration of the fuselage walls, respectively.

As a consequence, these results not only yield a basic understanding of interaction effects, but also provide detailed information on this very individual, vibrating fuselage structure. With this information, an acoustical fine tuning and damping of the primary structure can be performed. Additionally, in the design phase, the selection of an appropriate propeller, which should avoid the effect of noise amplification, when coupled fuselage resonances and critical propeller harmonics are of about the same frequency, was assisted. The comprehensive interior noise control program for the Dornier 328 is outlined in [2,3].

**2. FINITE ELEMENT FORMULATION OF  
STRUCTURAL-ACOUSTIC ANALYSIS**

Based on a structural-acoustic analogy it has been shown in Reference [1] that general purpose finite element codes, which include elements for anisotropic elastic material, can be used to obtain the matrix formulation of the acoustic equations of motion for an enclosed cavity :

$$[G] \{\dot{p}\} + [H] \{p\} = -[A] \{\ddot{u}\} \quad (1)$$

where  $[G]$  is the acoustic "mass" matrix (proportional to the reciprocal of the fluid bulk modulus),  $[H]$  is the acoustic "stiffness" matrix (proportional to the reciprocal of the fluid density),  $\{p\}$  gives the pressure at the grid points of the finite element mesh,  $\{u\}$  is the structural displacement vector, and  $[A]$  is the matrix of surface contact areas.

The equations of motion for a structure, coupled with the acoustic field through the pressure, damping neglected, becomes

$$[M] \{\ddot{u}\} + [K] \{u\} = \{F\} + [A]^T \{p\} \quad (2)$$

where  $[M]$  and  $[K]$  represent structural mass and stiffness matrices, and  $\{F\}$  is the vector of external forces applied to the structure.

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The vector  $\{u\}$  includes the normal displacement on the interface area which is coupled with its equivalent cavity node, as well as all other structural displacements.

The formulation of the coupled equations in a modal basis,

$$\begin{bmatrix} [G] & [A] \\ [0] & [M] \end{bmatrix} \begin{Bmatrix} \{\ddot{p}\} \\ \{\ddot{u}\} \end{Bmatrix} + \begin{bmatrix} [H] & [0] \\ -[A]^T & [K] \end{bmatrix} \begin{Bmatrix} \{p\} \\ \{u\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{F\} \end{Bmatrix} \quad (3)$$

needs a considerable amount of computer time for systems with a large number of degrees of freedom.

A modal synthesis method is used to reduce the large system of asymmetrical equations. By expanding the structural equations of motion in terms of their modes of vibration in vacuo  $[X_s]$  and the acoustic equations in terms of the cavity hard-walled modes  $[X_a]$

$$\{p\} = [X_a]\{q_a\}; \{u\} = [X_s]\{q_s\} \quad (4)$$

the equations (3) can be transformed, as pointed out in [1], into the known form for the equations of motion

$$[\hat{M}]\{\ddot{q}\} + [\hat{K}]\{q\} = \{\hat{F}\} \quad (5)$$

with the symmetric system matrices

$$[\hat{M}] = \begin{bmatrix} [I] & [0] \\ [0] & [\lambda_s] \end{bmatrix} \quad (6)$$

$$[\hat{K}] = \begin{bmatrix} [\lambda_a] + [B][B]^T & -[B][\lambda_s] \\ -[\lambda_s][B]^T & [\lambda_s^2] \end{bmatrix} \quad (7)$$

the assembled modal coordinates

$$\{q\} = \begin{Bmatrix} \{q_a\} \\ \{q_s\} \end{Bmatrix} \quad (8)$$

and the generalized forces

$$\{\hat{F}\} = \begin{Bmatrix} -[B][X_s]^T\{F\} \\ [\lambda_s][X_s]^T\{F\} \end{Bmatrix} \quad (9)$$

The coupling matrix  $[B]$  is obtained by transforming the matrix of surface contact areas  $[A]$  with the eigenvectors of the uncoupled systems

$$[B] = [X_a]^T[A][X_s] \quad (10)$$

$[\lambda_s]$  and  $[\lambda_a]$  are diagonal matrices with squared values of the uncoupled eigenfrequencies of the structure and the cavity.

The coupled structural-acoustic equations (5) can be solved with standard features of commercial FE codes, provided that these codes allow modifications of the matrices to the forms of eqs. (6) to (9).

In summary, the flow charts (fig. 1, 2) show, how structural-acoustic analyses can be performed, using common structural analysis programs e.g. the Finite Element Code COSA, developed by Dornier.

The structure model is idealized in the usual way, with degrees of freedom that are directed nor-

mal to the cavity surface. The finite element model for the cavity with hard-walled boundaries has associated nodes on the interface area. It is calculated using the mechanical-acoustic analogy relations, as described in [1].

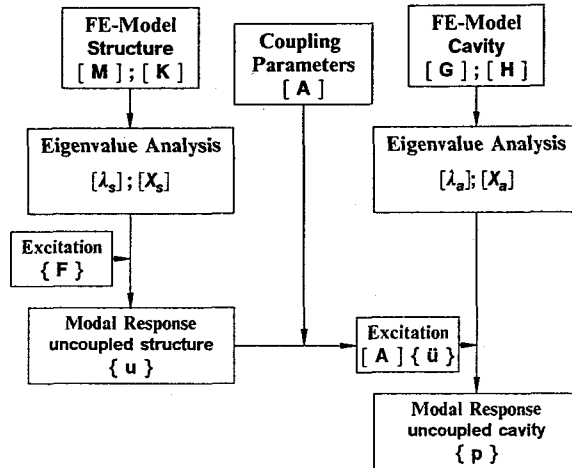


Fig. 1: Structural-Acoustic Analysis, Uncoupled Solution

The coupling matrix  $[A]$  may be obtained in a separate FE calculation, where a pressure load of the intensity 1 Pa acts on all structure nodes, that will be used for coupling. This gives as a result the coupling area for each nodal point.

An eigenvalue analysis supplies the uncoupled eigenfrequencies and modes for the cavity and the structure, which are used for the modal synthesis.

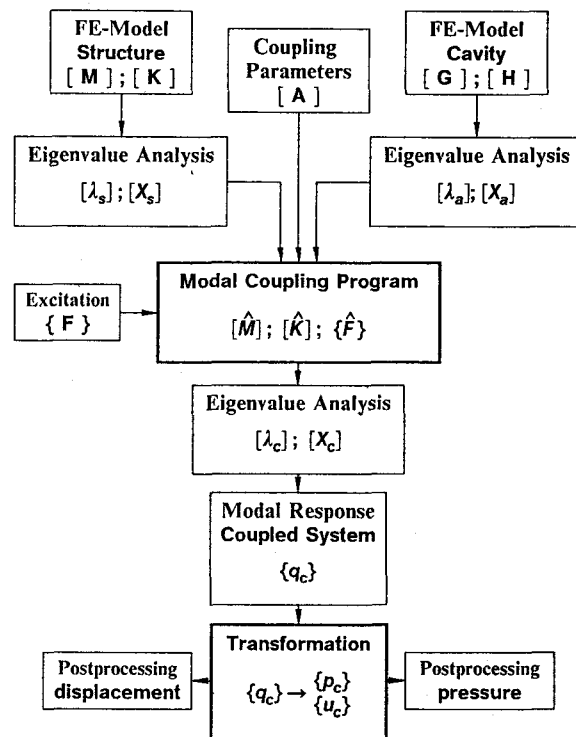


Fig. 2: Structural-Acoustic Analysis, Coupled Solution

A first approximation of fluid-structure analysis can be obtained by neglecting the coupling matrix [B] in eq. (5). In this case, uncoupled structural displacements are determined in a modal response analysis and the resulting outputs are applied as excitation forces to the cavity, to obtain the uncoupled pressure (see fig. 1).

To solve the coupled system (5), the symmetric matrices (6), (7) and the transformed excitation forces (9) are set up in the modal coupling program. An eigenvalue analysis, followed by a modal response analysis yield the modal vector  $\{q_i\}$ , which can be transformed back to physical coordinates, the coupled displacements and the pressure.

It should be noted, that with the exception of the modal coupling program and the transformation subroutine only standard features of the FE-Code COSA were used.

### 2.1 Semi-Analytical Eigenvalue Analysis of 3-dimensional Acoustic Cavity

Modal properties for the acoustic cavity inside an ideal, circular cylinder can be determined in a closed form by an analytical solution of the wave equation in cylindrical coordinates. If there is any deviation from the ideal circular geometry, e.g. due to cabin-floor or head-racks, then the three-dimensional mode-shapes and the related natural frequencies can be obtained by using a semi-analytical method.

If a three-dimensional cavity has to be idealized with a constant cylindrical geometry, the three-dimensional problem reduces to a two-dimensional one, because the modal characteristics in the axial direction are already known for the cylinder. The plane cross-section can be idealized by simple anisotropic membrane elements.

The eigenvalue analysis of this two-dimensional cavity yields eigenvalues  $\lambda_{a_i}$  and the eigenvectors  $\{x_a\}_i$ , which are used to calculate the modal parameters for the three-dimensional cavity

$$\lambda_{a_{k,j}} = \left(\frac{kc}{2L}\right)^2 + \lambda_{a_i} \quad (11)$$

$$\{x_a\}_{k,j} = \{x_a\}_i \cos\left(k\pi \frac{z}{L}\right) \quad (12)$$

with the speed of sound  $c$ , the length of the cylinder  $L$  and the axial coordinate  $z$ .

## 3. STUDIES ON THE UNCOUPLED CAVITY

To build a coupled system, following the modal synthesis technique, separate eigenvalue analyses for the uncoupled structure and the cavity have to be carried out.

For complex three-dimensional structures, like a fuselage with different frames, it is usually not possible to build a 2-dimensional finite element model with a dynamic behavior that is fully equivalent to the real structure. Therefore, the finite element calculation and the eigenvalue analysis must be based on a detailed 3-dimensional FE model.

Contrary to the real structure, cavity properties do not change from one point to another. Hence, plane-modes of an acoustic volume can be described exactly by a 2-dimensional cavity model. Taking this into account, particular effects in a plane, such as the influence of geometry modifications, may be studied on relatively simple 2-D models and not on complex 3-D models. 3-D frequencies can also be obtained by using equations (11,12).

### 3.1 Eigenvalue Analysis of Two-dimensional Acoustic Cavity

Plane models of cavities ( finite element mesh similar to fig. 3 ) were generated with a different geometry, to study, in detail, the influence on the natural frequencies and the corresponding mode-shapes of the acoustic cavity. Each approximately circular model had a diameter of  $d=2.4$  [m], and the idealization should represent different cross-sectional cuts through the passenger compartment of the airplane.

The first model was a cavity which did not reflect the presence of a cabin floor or other structural installations. The second idealization considered the floor as a hard-walled, and in no way acoustically transparent boundary. In the third cavity, passenger seats and the head-racks were build as rigid walls.

FE calculations as well as (uncoupled) eigenvalue analyses were carried out and the results, mode-shapes together with their corresponding resonant frequencies, are given in Table 1.

Cavity modes can be classified by their number of axial/longitudinal (l), cross-sectional (m) and radial (n) number of nodal lines. Structure modes are named according to the ESDU specifications for circular-cylindrical shells [4]. Axial modes, e.g. for a cylindrical shell (both ends free) are counted 'number of axial half-waves+1'. To distinguish between structure and cavity in the following, three-dimensional structure modes are described by 'l-m-n' and cavity modes by 'l,m,n'; two-dimensional modes by 'm-n' or 'm,n', respectively.

Modes that are symmetrical about the vertical median line are indicated by an 'S', anti-symmetrical modes by an 'A'. This characterization cannot be chosen for the cavity model III, because of the no longer symmetrical geometry. There, modes are described by their mode-number.

For all cavity idealizations, the first mode with the natural frequency  $f=0$  Hz is not listed. This mode is often called 'Helmholtz-mode', and equals a constant pressure in the whole cavity. It is important for the modal synthesis procedure, not to omit this special mode, in any case !

As can be seen in Table 1, from the comparison of cavity models I and II, the cabin floor will not always exert a significant influence on the mode-shape. For the shown modes it seems to be only a minor disturbance. Nevertheless, the corresponding natural frequencies will be shifted. Especially for symmetrical modes a considerable shift to higher values can be noted. Structural installations ( model III in comparison to model II ) completely change the mode-shape, the frequency and thereby the sequence of the modes.

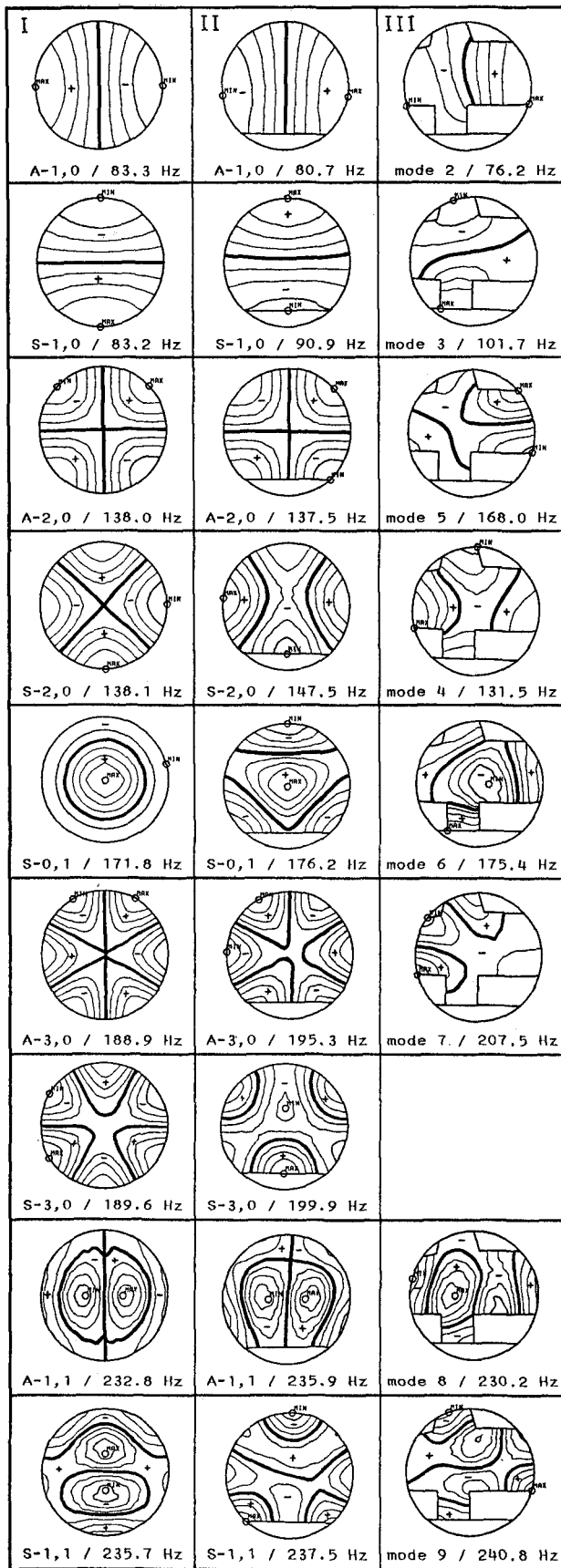


Table 1: Comparison of Mode-Shapes, Resonant Frequencies for different two-dimensional Cavities

### 3.2 Eigenvalue Analysis of Three-dimensional Acoustic Cavity

In the next phase, a 3-dimensional, cylindrical cavity with a cross-sectional idealization corresponding to fig. 3 was generated, using anisotropic tetrahedron elements. It was the model of the acoustic volume in the passenger compartment with a length of 10.25 [m]. Because of the assumption of symmetrical propeller-induced noise, only modes that are symmetrical about the vertical median line would be excited. Therefore it was sufficient to idealize a half-model. Results of the eigenvalue analysis (mode-shapes together with their resonant frequencies) are listed in Table 2 and may be compared directly with the 2-dimensional cavity results of model II in Table 1.

Mode-Shape	Mode-Number l, m, n	Calculated Frequency 3-D Model	Calculated Frequency 2-D Model
	S-0,0,0	0.0 Hz	0.0 Hz
	1,0,0	16.6 Hz	16.6 Hz
	2,0,0	33.2 Hz	33.2 Hz
	3,0,0	49.7 Hz	49.8 Hz
	4,0,0	66.4 Hz	66.3 Hz
	S-0,1,0	91.9 Hz	90.9 Hz
	1,1,0	93.4 Hz	92.4 Hz
	2,1,0	97.7 Hz	96.8 Hz
	3,1,0	104.5 Hz	103.6 Hz
	4,1,0	113.4 Hz	112.5 Hz
	S-0,2,0	149.1 Hz	147.5 Hz
	1,2,0	150.0 Hz	148.4 Hz
	2,2,0	152.7 Hz	151.2 Hz
	3,2,0	157.1 Hz	155.7 Hz
	4,2,0	163.1 Hz	161.7 Hz
	S-0,0,1	173.2 Hz	176.2 Hz
	1,0,1	173.9 Hz	177.0 Hz
	2,0,1	176.1 Hz	179.3 Hz
	3,0,1	179.8 Hz	183.1 Hz
	4,0,1	184.7 Hz	188.3 Hz
	S-0,3,0	203.4 Hz	199.9 Hz
	1,3,0	204.1 Hz	200.6 Hz
	2,3,0	205.7 Hz	202.6 Hz
	3,3,0	209.1 Hz	206.0 Hz
	4,3,0	213.2 Hz	210.6 Hz
	S-0,1,1	235.6 Hz	237.5 Hz
	1,1,1	236.2 Hz	238.1 Hz
	2,1,1	237.5 Hz	239.8 Hz
	3,1,1	240.2 Hz	242.7 Hz
	4,1,1	243.7 Hz	246.6 Hz

Table 2: Mode-Shape and Resonances for 3-dimensional Cavity Comparison with semi-analytical 2-dimensional Solution

To demonstrate the practicability of the semi-analytical solution described in section 2, calculated frequencies, derived from the 2-dimensional model, are also listed. The good agreement shows that it is rather simple to approximate the natural frequencies, but this is valid only for homogenous cylinders with invariant axial geometry.

#### 4. COUPLING PRECONDITIONS AND EFFECTS

##### 4.1 Analyzing Coupled System Modes

When analyzing the coupled system, the influence of the formerly uncoupled components - structure and cavity - is of principal concern in understanding its new dynamic response. Because the coupled eigenvector is composed of both elements, the structure and the cavity parts, a modal decomposition of the vector will be performed to isolate the individual influence of each involved mode-part.

As a first result, for every specific resonant frequency, this approach enables one to find out the most dominant uncoupled component, which is directly responsible for the existence of this resonance. Furthermore, as a second result, a detailed list of all involved mode-parts, together with their quantitative weight, is obtained.

In the case of a 'cavity-dominated' coupled resonance, one cavity mode has an outstanding relative share of more than 50% (often > 90%) in the cavity part of the coupled eigenvector. All other cavity modes have a much lower value. An analogous rule is used to define 'structure-dominated' coupled frequencies. Strongly coupled modes are characterized by the fact, that the relative weight of the main cavity mode is about of the same value as the relative weight of the main structure mode.

Knowing the contribution of each single mode-part, it is possible to analyze all coupled resonant frequencies of interest more accurately. Especially when three-dimensional structure and cavity models with large numbers of degrees of freedom and coupled resonant frequencies are to be studied, this mode-part information will become an important auxiliary interpretation means.

##### 4.2 Frequency- and Mode-Shape Interaction

In a certain frequency range, depending on structure properties such as stiffness, mass distribution or simply its dimensions and the cavity volume, the number of uncoupled modes and resonances can vary significantly. As the number of resonances increases, the probability that appropriate modes of the structure and the cavity fit together, and couple, will be more likely, too. A growth in apparently coupled modes is to be expected, because merging the uncoupled eigenvectors of a structure and the corresponding cavity, results in a higher modal density. Moreover, resonant frequencies of the modes that have the tendency to couple, may now be lying close together.

But the relative position of the frequencies is only a minor aspect for coupling, since coupled modes were found, even when the resonances of the mode parts lay far apart. Above all, the weight of each mode part, i.e. the quantitative contribution to the coupled mode, is influenced by the relative frequency distance.

In the following, it is shown that the most important precondition that two modes will actually couple is the conformity in their mode-shapes.

##### 2-D Example of Mode-Shape Interaction

A finite element calculation was carried out with a circular frame that has a stiffness and mass distribution, comparable to the realistic airplane frames. The structure completely encloses a two-dimensional cavity ( FE models see fig. 3 ). The shown FE idealization corresponds also to a side view on a cross-section of the three-dimensional cavity inside the fuselage walls.

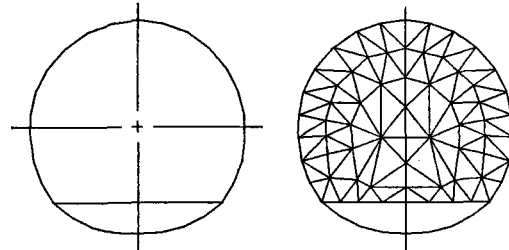


Fig. 3 : FE Idealization of two-dimensional Structure and Cavity

Subsequently, a coupled eigenvalue analysis was performed. Fig. 4 shows some typical structure and cavity modes that have the inclination to couple.

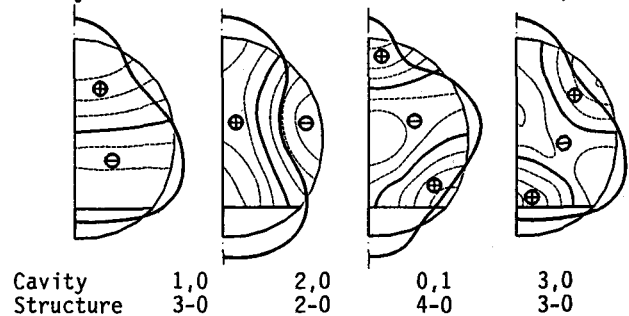


Fig. 4 : Mode-Shape Interaction between two-dimensional Structure and Cavity

##### 3-D Example of Mode-Shape Interaction

Analyzing three-dimensional systems is far more complicated, for the modal density in the interesting frequency range ( here : up to 250 Hz ) increases significantly. As an example, the diagram in fig. 5 gives the number of modes for the uncoupled fuselage structure, the cavity as well as the coupled system.

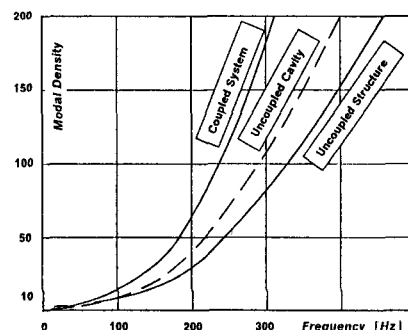
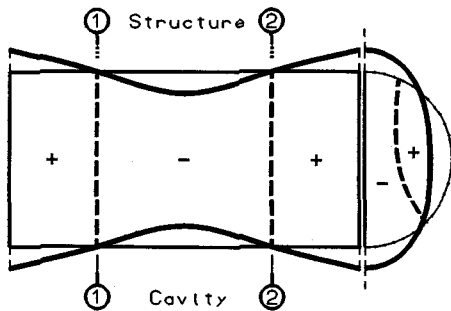


Fig. 5 : Number of Modes depending on the Frequency

The interpretation of a coupled 3-D mode may be further difficult, if many mode-parts with nearly the same percentage contribution build a coupled system mode.



Cavity (1,m,n) 2,2,0 <sup>(1)</sup>  
 Structure (1-m-n) 3-2-0

Fig. 6 : Mode-Shape Interaction between three-dimensional Structure and Cavity, Coupled Mode at  $f = 51.9$  Hz

As an example, Fig. 6 shows the structure-dominated coupled system mode of the actual airplane with the resonant frequency  $f = 51.9$  Hz. Because the number of nodal lines in the axial direction is the same for the structure and the cavity, an obvious axial coupling was found. Furthermore, the cross-sectional mode-shapes show an additional strong coupling tendency.

#### 4.3 Coupling Effects and Influence on the Dynamic Response of the Excited System

In general, the effect of coupling is a shift in the coupled frequencies, compared to the uncoupled solution, which has been briefly described in section 2. Cavity-dominated, coupled resonant frequencies, are normally shifted to higher values, whereas structure-dominated resonances are usually reduced. For a thorough description of this frequency shift on the basis of one- and two-dimensional examples see [1].

Subsequent to the harmonic excitation and the calculation of the coupled structural-acoustic dynamic response, an obvious noise amplification was found, especially when critical, coupled modes were excited. Commonly, not all of these critical modes yield automatically higher noise levels, because this effect depends most on the actual excitation forces as well as on their related phase displacement.

As a conclusion we can say that the location and the mode parts of critical resonances are known after the coupled eigenvalue analysis. But only an excitation with realistic dynamic loads shows us the really critical modes, which are responsible for the actual dynamic response, i.e. the final interior noise.

<sup>(1)</sup> Definition of  $\{ \}$  : see section 3 and [4]

## 5. EXAMPLES FOR COUPLED SYSTEMS

### 5.1 Two-dimensional Structure and Cavity

To demonstrate the coupling effects, a plane cavity (model I, Table 1) and a circular frame structure were used for structural-acoustic calculations. According to the flow-charts in fig. 1 and fig. 2, eigenvalue analyses were performed and the uncoupled as well as the coupled dynamic response due to harmonic excitation with external forces was calculated.

Table 3 gives uncoupled resonant frequencies for the cavity and the structure as well as the resulting resonances of the coupled system. A classification of cavity- or structure-dominated frequencies is also included.

Uncoupled Cavity		Coupled System	Uncoupled Structure	
Mode-Shape	Frequency [ Hz ]	Frequency [ Hz ]	Frequency [ Hz ]	Mode-Shape
0,0	0.0 ->	0.0	<- 0.0	rigid
(2,0)		20.0	<- 20.3	2-0
(3,0)		58.3	<- 59.0	3-0
1,0	83.2 ->	84.4		(3-0/4-0)
(4,0)		109.6	<- 110.8	4-0
2,0	138.1 ->	139.7		(2-0/4-0)
0,1	171.8 ->	171.0		(5-0/0-1)
(0,1)		176.7	<- 178.6	5-0
3,0	189.6 ->	191.6		(3-0/5-0)
1,1	235.7 ->	235.0		(0-1/6-0)
4,0	238.5 ->	239.9		(6-0/4-0)
(4,0)		259.8	<- 262.3	6-0

Table 3 : Natural Frequencies of the Uncoupled Cavity and the Structure with Reference to the Coupled System

The cavity pressure was averaged over the whole area yielding frequency dependent curves, which are given in fig. 7.

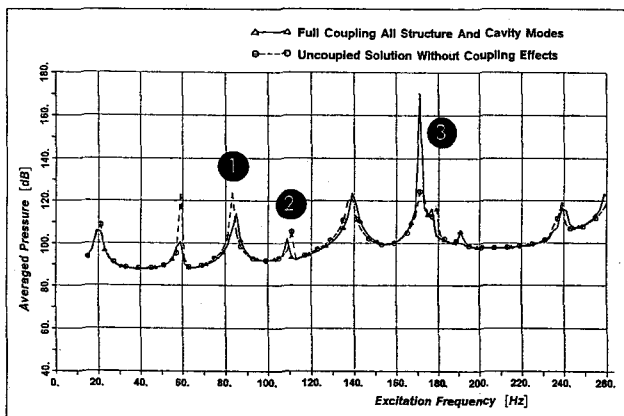


Fig. 7 : Comparison of Coupled and Uncoupled Solution Dynamic Response of the Cavity  $\{p_c\} / \{p\}$

The comparison of the uncoupled and the coupled solutions shows three typical effects of coupling. A shift in the resonant frequencies and an obvious noise amplification can be seen.

- ① a cavity-dominated frequency was shifted to a higher value : Cavity mode '1,0' : 83.2 -> 84.8 Hz
- ② a structure-dominated frequency was reduced : Structure mode '4-0' : 110.8 -> 109.6 Hz
- ③ shows an evident pressure increase of about 45 [dB] ( modal damping factor used in the calculation: 0.1% ) at the cavity-dominated resonance  $f=171.0$  Hz, which was the cavity breathing mode '0,1'. This cavity mode coupled strongly with the structural breathing mode '0-1' ( uncoupled resonant frequency : 269.4 Hz) and the applied external forces greatly increased the dynamic response.

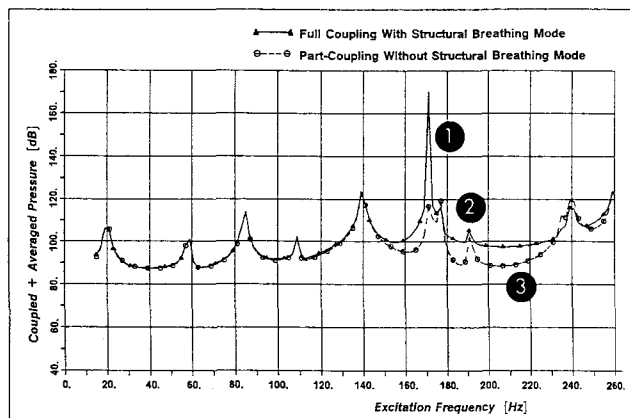


Fig. 8 : Comparison of Coupled Dynamic Response Cavity  $\{p_c\}$   
 1) Using all Modes up to 600 Hz for Modal Synthesis  
 2) Omitting the Structural Breathing Mode '0-1' (269.4 Hz)

Fig. 8 is a comparison of two coupled solutions. In one calculation, the first structural breathing mode '0-1' was used for the modal synthesis, while the other calculation was performed without this mode. The influence of this special mode is evident. Without the structural mode-part '0,1', the cavity-dominated resonance is excited to a lesser extent and the dynamic response is reduced ( ① ).

It was found that the same effect occurs even when the resonant frequency of the structure breathing mode is far more distant (here:  $\Delta f \approx 100$  Hz). This fact shows clearly, how essential it is for the dynamic response of a coupled system built by the modal synthesis technique, to consider not only the modes in the interesting frequency range.

Particular modes, such as the breathing modes, that show a strong tendency for 'mode-shape-coupling', must be used in every case. If the resonant frequency of such a mode is far outside the interesting frequency range, the exact calculation of this frequency is normally not possible, because of the finite element idealization. But this is not so essential, as long as the mode-shape itself is used in the modal coupling procedure.

Although not every coupled mode is influenced in the same way (see structure-dominated mode '5-0' at 176.7 Hz or cavity-dominated mode '3,0' at 191.6 Hz ( ② ), as a general tendency, lower pressure values over a large frequency range ( 145 Hz - 260 Hz ( ③ )) are the result of omitting the essential structure mode '0-1'.

The cavity central point is influenced more than any other by the structure breathing modes ('0-1', '0-2', etc.), for these are the only mode-parts that excite the cavity directly at this location and lead to a coupled response. Fig. 9 shows the averaged pressure as well as the coupled pressure at the central point for both modal syntheses.

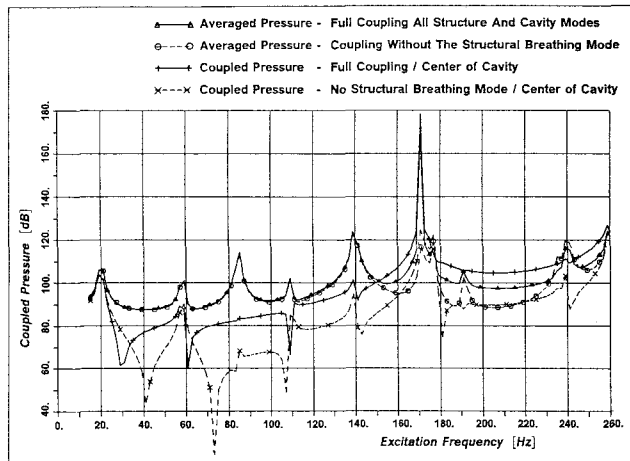


Fig. 9 : Comparison of Coupled Dynamic Response - Cavity  
 1/2) Averaged Coupled Pressure  
 3/4) Coupled Pressure at Cavity Central Point

## 5.2 Three-dimensional Fuselage Test-Section

A representative fuselage test-section of the Dornier 328 commuter aircraft was built with an overall length of 3.6 [m] and a diameter of 2.4 [m]. Wooden endplates were attached on both ends of the fuselage section to enclose completely the cavity volume. Finite element calculations were performed to evaluate the interior noise distribution and to compare these results with experimental test data ( see [2] ). The finite element meshes for the symmetrical half-models of the structure and the cavity are shown in fig. 10.

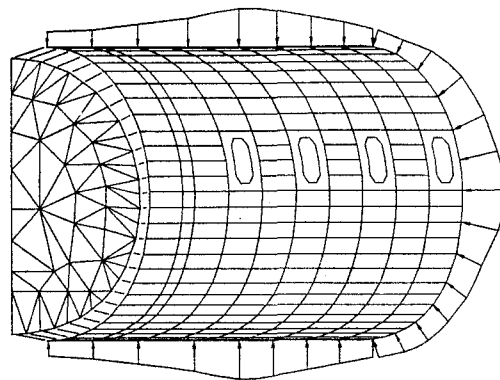


Fig. 10 : Finite Element Models of Fuselage Test-Section / Cavity  
 Propeller-Induced Excitation-Forces on Structure

Uncoupled eigenvalue analyses and the coupling procedure were performed using all modes up to 400 Hz for the modal synthesis. Forces equivalent to propeller noise were used to excite the fuselage structure and the coupled dynamic response was calculated. Fig. 11 shows a typical pressure distribution in the vertical mid-plane and in a cross-section of the cavity as well as the corresponding coupled displacement of the test-section.

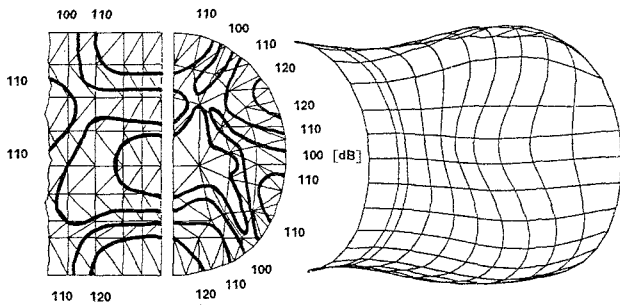


Fig. 11 : Coupled Dynamic Response; Cavity : Pressure  $\{p_c\}$   
Fuselage Test-Section : Displacement  $\{u_c\}$

To compare these calculation results and the related experimental measurements, the maximum exterior noise level ( $p_{max,ext}$ ) was used as a reference value for the calculated interior pressure. Subsequently, for every excitation frequency, the pressure values were spatially averaged over the whole volume. The resulting *noise reduction curve* is given in fig. 12 ( $NR = p_{max,ext} - p_{interior}$ ).

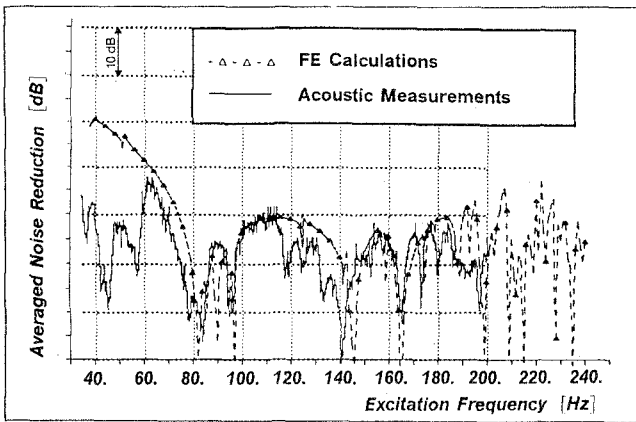


Fig. 12 : Comparison of FE Calculations and Measurements of the Dornier 328 Fuselage Test-Section

As can be seen, theoretical results and test data agree very well, especially in the frequency range that is not affected by the wooden endplates (above 60 Hz).

### 5.3 Full-Sized Fuselage

In consideration of the influence of coupling, a structural-acoustic calculation of the full-scale Dornier 328 passenger compartment was performed to predict the locations of critical structure and cabin cavity resonances.

#### Uncoupled Systems, FE Models

An elaborate finite element model of the fuselage structure (see fig. 13) was generated with the intention of predicting all uncoupled resonant frequencies up to 500 Hz. In the idealized half-model, with symmetric boundary conditions in the vertical mid-plane, the individual influences of the frames, the stringers, the floor, the wall panels, the passenger front door and the windows were taken into account. The overall cabin length was 11.0 [m] and the average diameter 2.4 [m].

On the other hand, for the cavity, a finite element model was built with a cross-sectional element mesh similar to the cavity idealization in fig. 3. The axial idealization was chosen in such a way that all cavity nodes on the surface could be coupled with the corresponding structure nodes. Major parameters of the two models are listed in Table 4.

After separate finite element calculations were performed, a static condensation was used to reduce the large number of degrees of freedom (DOF's) for both models. Uncoupled eigenvalue analyses with the remaining *dynamic DOF's* were carried out and two typical structural modes are presented in fig. 14 and fig. 15.

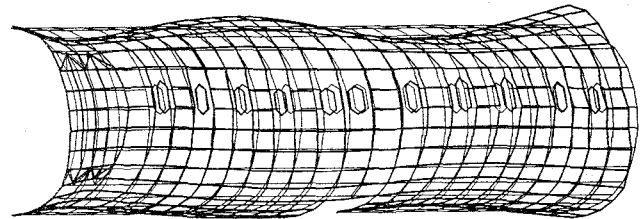


Fig. 14 : Uncoupled Structure Mode '5-2-0' (91.2 Hz)

Fig. 14 shows a structure mode with the mode-number  $l=5$  and  $m=2$  (resonance  $f=91.2$  Hz). This mode will couple mainly with the cavity cross-sectional modes  $m=2$  and  $m=1$ .

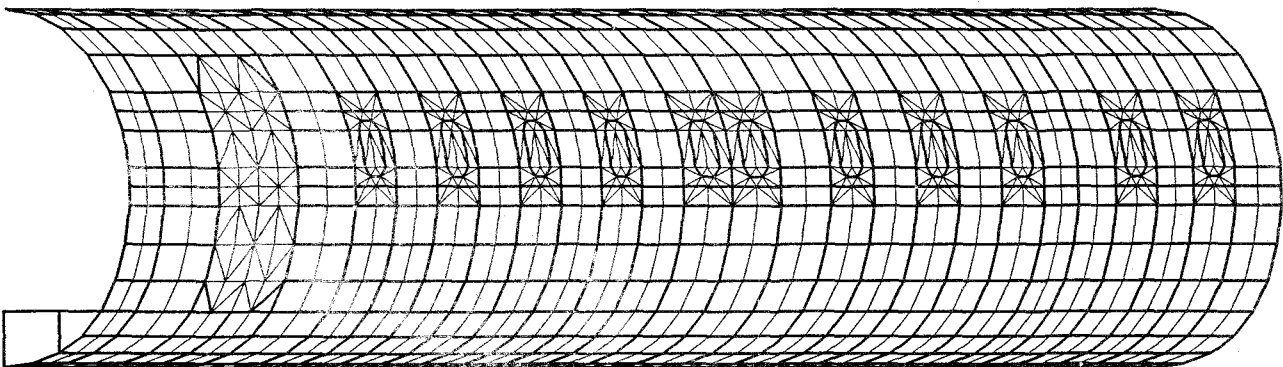


Fig. 13 : Finite Element Model of the Full-Sized Dornier 328 Fuselage



FE Parameters	Structure	Cavity
Number of nodes	4390	12524
Degrees of freedom (DOF)	20347	12524
Dynamic DOF's	2132	1887
Coupled DOF's	765	765
Symmetric supports	136	-

FE Elements	Structure	Cavity
Triangular Shell $\Delta$	876	-
Rectangular Membrane $\square$	706	-
Triangular Membrane $\Delta$	132	-
Tension/Compression Bar $\leftrightarrow$	44	-
Bending Beam $\leftrightarrow$	112	-
Bending/Torsion Beam $\leftrightarrow$	1665	-
Supporting Elements $\leftrightarrow$	375	-
Anisotropic Solid (TETA)		7650

Table 4 : Major Parameters of Finite Element Idealizations

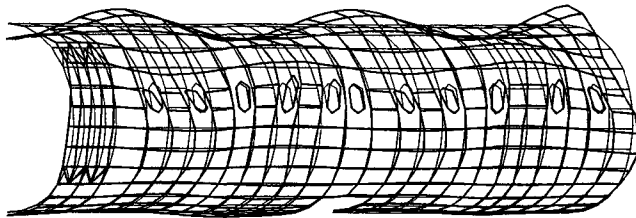


Fig. 15 : Uncoupled Structure Mode '6-3-0' ( 159.2 Hz )

Fig. 15 presents the uncoupled structure mode '6-3-0' and, contrary to fig. 14, it can be seen that some windows (from left to right : 1,2,7,8) show a separate motion. Nevertheless, this mode is mainly influenced by the frames. It will be seen that this structure mode couples strongly with the cavity cross-sectional mode-part  $m=2$  and also with the longitudinal mode-part  $l=5$ .

### Coupled System, Results

The coupling procedure was carried out using all 200 calculated structure modes up to 470 Hz and 100 cavity modes up to 300 Hz for the modal synthesis. Subsequent to the eigenvalue analysis, the system was excited by external forces equivalent to sound pressure levels on the surface. The principal exterior noise distribution, caused by the propellers, can be seen in fig. 16.

The system response, that was calculated in the frequency range 0-250 Hz, was transformed and thereby separated into the coupled pressure and structure displacement, respectively.

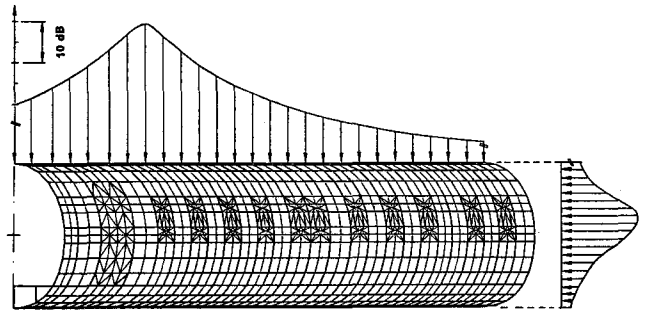


Fig. 16 : Propeller-Induced Forces on the Fuselage Structure, Excitation of the Coupled System

A typical result, the calculated pressure distribution in a cavity cross-section and the corresponding structure displacement at the coupled resonance  $f = 174.0$  Hz is given in fig. 17. This strongly coupled and also strongly excited cavity-dominated system mode, which is mainly composed of the first cavity breathing mode '0,0,1' ( $f_c=173.2$  Hz, see Table 2 ) and the structure mode '5-4-0' ( $f_s=173.5$  Hz) is also indicated in fig. 19 ( 4 ).

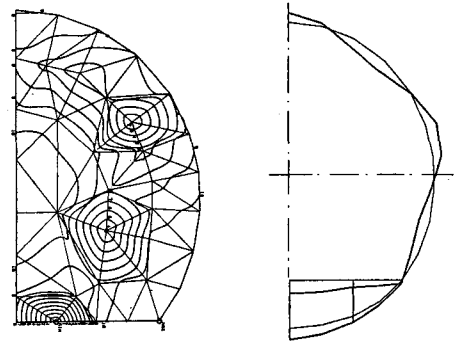


Fig. 17 : Coupled System Mode at the Resonance  $f=174.0$  Hz Cavity and Structure Mode-Parts

For every excitation frequency, the pressure was averaged over 51 different cross-sectional planes as well as the whole volume. Fig. 18 shows the averaged pressure for all 51 fuselage sections.

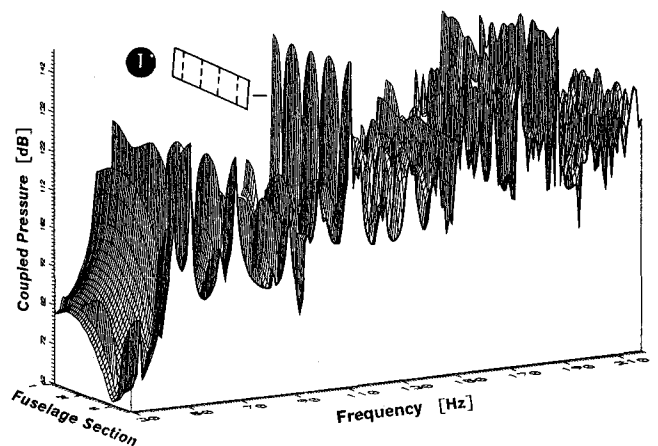


Fig. 18 : Cavity Dynamic Response : Averaged Pressure  $\{p_c\}$  Average at Different Fuselage Cross-Sections

This presentation is particularly suited for identifying axial cavity modes, such as '4,1,0' at the coupled resonant frequency  $f=111.9$  Hz ( 1 ).

The overall averaged pressure is to be seen in fig. 19, showing clearly those dominant resonances of the coupled system that were excited by the applied external forces.

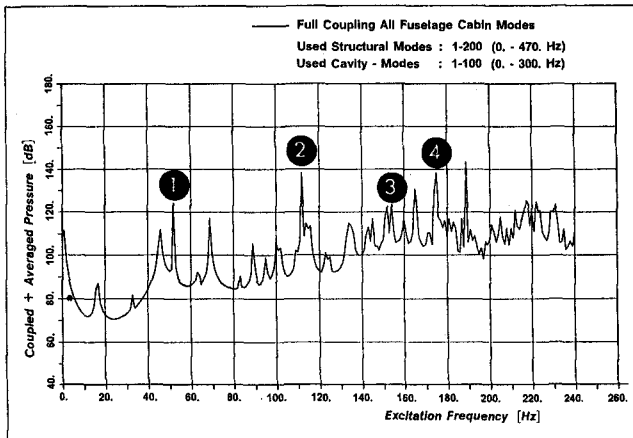


Fig. 19 : Cavity Dynamic Response : Averaged Pressure  $\{\bar{p}_c\}$  Spatial Average Whole Passenger Compartment

Some modes and the shift in the resonant frequencies due to coupling are :

- 1 structure-dominated : '3-2-0' (see fig. 6)  
f: 52.8 Hz -> 51.9 Hz
- 2 cavity-dominated : '4,1,0' (see fig. 18)  
f: 113.4 Hz -> 111.9 Hz
- 3 structure-dominated : '6-3-0' (see figs. 15,22)  
f: 159.2 Hz -> 154.5 Hz
- 4 cavity-dominated breathing mode : '0,0,1'  
f: 173.1 Hz -> 174.0 Hz (see fig. 18)

To compare the cavity response with measurements, noise reduction values as defined in section 5.2 were computed in the interesting frequency range up to 250 Hz and averaged over the whole passenger compartment. The diagram is shown in fig. 20. The reference pressure was  $p_{max,ext} = 128.5$  [dB].

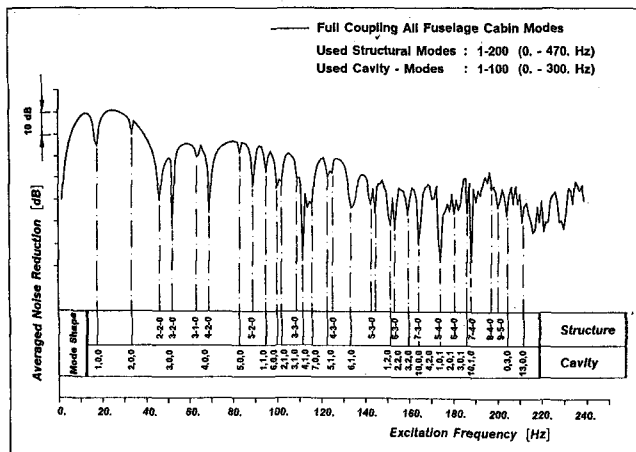


Fig. 20 : Spatially Averaged Noise Reduction together with Mode-Shape List for Major Cavity / Structure Mode-Parts

As can be seen in this plot and the mode-shape list of the dominant mode-parts, in comparison to fig. 12, the number of resonances considerably increases because of the three times longer cavity and fuselage. Nevertheless, with the appropriate analysis tools (the described mode-part information), critical structure- or cavity-dominated system resonances could be clearly identified.

### Coupling Effects

The modal decomposition of a coupled eigenvector separates the cavity and the structure parts, yielding for each mode-part the relative contribution to the coupled mode, which is additionally a direct measure for the kinetic energy.

It has been observed that cavity mode-parts with a cross-sectional mode number  $m=2$  have a great influence on the coupled pressure, because structural motions are directed normal to the nodal line of the acoustic mode. Therefore, a modal coupling calculation was performed, in which these cavity modes were not used (see Table 5, Version <2>). Although the first cavity mode with  $m=2$  ('0,2,0') has an uncoupled resonant frequency of 149.1 Hz, the coupling influence can be seen in a large frequency range from 25 up to 190 Hz. Some modes (e.g. the structure-dominated mode '3-2-0',  $f=51.9$  Hz, see fig. 21 1)  $p=28$  dB) were strongly influenced. But a more interesting result was a direct superposition of separate coupling effects.

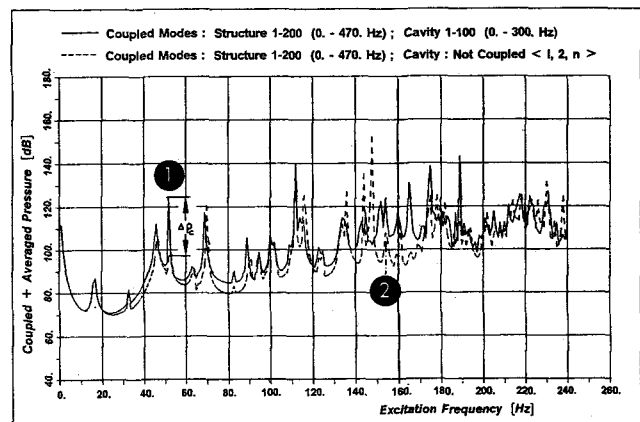


Fig. 21 : Comparison of Coupled Dynamic Response - Cavity  
1) Using all Cavity Modes up to 300 Hz for Modal Synthesis  
2) Omitting the Cavity Cross-Sectional Modes 'm=2'

Analyzing the coupled system mode at  $f=154.5$  Hz (fig. 21 2), two coupling effects were found. In comparison to the uncoupled solution, a relatively large frequency shift (159.2 -> 154.5 Hz) and an increase in noise can be seen. If only the dominant mode-parts of the uncoupled systems are taken into account, this structure-dominated resonance is a combination of structure mode '6-3-0' and cavity mode '2,2,0' (see figs. 15,22 / Table 2).

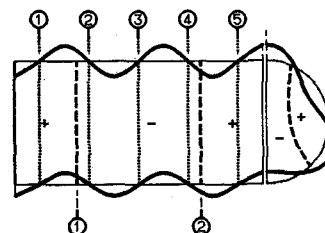


Fig. 22 : Mode-Shape Interaction between Cavity Mode '2,2,0' and Structure Mode '6-3-0' (Coupled Resonance  $f=154.5$  Hz)

Further mode-parts, their uncoupled resonant frequencies and their percentage contribution to the system mode are listed in Table 5. In version <1> all cavity modes were used for the modal synthesis, in version <2> all cavity modes with a mode number  $m=2$  were omitted. The result is a pressure decrease of 17 [dB] for the coupled mode '6-3-0' (fig 21 ②).

Mode-Shape	Frequency [Hz]	Modal Synthesis Version	
		< 1 >	< 2 >
		Mode - Part Contribution	Mode - Part Contribution
2,2,0	152.7	27.1 %	-
1,2,0	150.0	16.4 %	-
5,3,0	218.5	5.0 %	16.0 %
5,0,1	190.8	4.7 %	18.2 %
6,2,0	178.9	4.6 %	-
5,4,0	248.1	2.2 %	7.0 %
3,1,0	104.5	1.8 %	4.1 %

Table 5 : Structure-dominated System Mode '6-3-0',  $f = 154.5$  Hz  
Percentage Contribution of Major Cavity Mode-Parts

With the mode-part information in Table 5 and the comparison of the pressure curves in fig. 21, two separate coupling effects could be identified that did not influence each other. The structure mode-shape  $m=3$  coupled strongly with the cavity cross-sectional mode-shape  $m=2$ , that was responsible for the noise level. On the other hand, the axial mode-shape coupling was directly responsible for the shift in the resonant frequency, hence cavity modes with the axial number  $l=5$  were found in the eigenvectors of version <1> as well as version <2>, here with a dominant weight.

Fig. 23 shows the final results of two coupled dynamic response analyses. The difference in the number of modes used for the modal coupling step is the reason for different pressure levels. Although all modes were used in the frequency range below 150 Hz, coupling effects can be seen clearly ( $f > 20$  Hz).

The data presented in fig. 23 demonstrates that it is essential in a *modal* coupling calculation to use even modes with a much higher resonant frequency, as long as they show a strong coupling tendency with other modes in the frequency range to be analyzed.

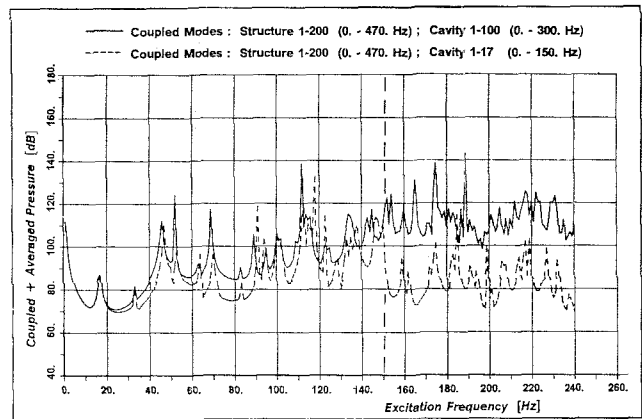


Fig. 23 : Comparison of Coupled Dynamic Response - Cavity  
1) Using all Cavity Modes up to 300 Hz for Modal Synthesis  
2) Using all Cavity Modes up to 150 Hz for Modal Synthesis

### Conclusions

On the basis of accurate idealizations of the fuselage structure as well as the cavity, with the help of the structural-acoustic analogy combined with a common finite element program, it was possible to obtain detailed theoretical results. Furthermore, the good agreement between calculations and experiments resulted in a basic understanding of coupled systems that are greatly influenced by modal interaction effects. The identification and analysis of critical structural and acoustic modes is the direct way to a successful active noise control program.

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