THE DESIGN OF INTERCONTINENTAL SUPERSONIC TRANSPORT AIRCRAFT OF SECOND GENERATION BY USING GLOBAL OPTIMIZATION TECHNIQUES

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Abstract: The optimum-optimorum configuration of the supersonic transport aircraft is the configuration for which the shapes of its surface and also of its planprojection are simultaneously determined in such a manner that its drag attains its minimum at a given cruising Mach number M_∞. The problem of the determination of the optimum-optimorum configuration of a supersonic aircraft with small movable leading edge flaps which presents a minimum drag at two cruising Mach numbers M_∞ and M_∞^* are here also considered.

1. INTRODUCTION

The optimum-optimorum theory introduced by the author in [1] - [5] was used for the determination of the optimum-optimorum shape of the wing alone at a given cruising Mach number M_∞ as in [1] - [5], [8], [16] and of the wing alone of variable geometry [1], [9], [10] and [16], which is of minimum drag at two cruising Mach numbers M_∞ and M_∞^*.

More recently, the variational problem concerning the determination of the optimum-optimorum shape of the integrated wing-fuselage configuration (at a cruising Mach number M_∞) was considered by the author in [11] - [15]. For the integrated wing-fuselage configuration all its geometrical parameters i.e. the distributions of cambers, twists, thicknesses and also the shapes of the plan-projections of the wing and of the fuselage are optimized in order to obtain a minimum drag (at a given cruising Mach number M_∞).

The further step in the optimization of the entire configuration of the aircraft is to consider that the integrated wing-fuselage configuration is of variable geometry. This variable geometry can be realized with the help of small movable leading edge flaps. The integrated wing-fuselage configuration with movable flaps can be optimized at two supersonic cruising Mach numbers M_∞^* and M_∞ ($M_\infty^* < M_\infty$). At the higher supersonic Mach number M_∞ the integrated configuration of the aircraft is flying with the flaps in retracted position. The aircraft, flying with the flaps in open position, is adapted for the second lower supersonic Mach number M_∞^*.

The determination of the optimum-optimorum configuration of the aircraft with variable geometry leads to the solving of
- two three-dimensional boundary-value problems for the axial disturbance velocity u on the aircraft with flaps (in retracted and open positions) and of
- two successive enlarged variational problems (with free boundary) for the aircraft with small leading edge flaps (in retracted and in open positions).

The wing-fuselage configuration with flaps in retracted position is here considered as a wing alone, for which the surface is discontinuous along the junction lines between the wing and the fuselage. The wing-fuselage configuration with flaps in open position is considered also as a wing alone for which the surface is now discontinuous along the junction lines between the wing and the fuselage and between the wing and the leading edge flaps.

2. THE OPTIMUM-OPTIMORUM THEORY

The author has proposed in [1] - [7] a method for the design of fully-optimized shape of the supersonic aircraft configuration which she called it optimum-optimorum theory. This theory allows the simultaneous determination of the optimal shapes of the surface and of the plan-projection of the supersonic aircraft in order to obtain a minimum drag. The determination of the shape of the optimum-optimorum supersonic aircraft leads to an extended variational problem for the drag functional C_d, i.e.

$$C_d \equiv \int_{S(x_1,x_2)} F\left[\, x_1,\, x_2,\, Z(x_1,x_2)\right] dx_1\, dx_2 = \min. \qquad (1)$$

Here the function $Z(x_1,x_2)$ and also the boundary $S(x_1,x_2)$ of the integral are a priori unknown and are determined by the solving of this extended variational problem. The optimum-optimorum shape of the supersonic aircraft is chosen among a set of aircraft, which are defined through some common properties. In the frame of the optimum-optimorum theory of the author, two aircraft belong to the same set if:

- their surfaces can be piecewise approximated through a superposition of homogeneous polynomes of the same degree;

- their planprojections are polygons which can be related through affine transformations, and

- the shapes of the aircraft of the set fulfill the same auxiliary conditions (of geometrical or aerodynamical nature).

The parameters of the optimization are the coefficients Z_{ij} of homogeneous polynomes of the equations of the surfaces and the similarity parameters ($\nu_1, \nu_2, \ldots, \nu_n$) of the planprojections of the aircraft of the set. In order to solve this enlarged variational problem for the determination of the extremum of the drag functional $C_d^{(t)}$ with free boundary the author uses her hybrid, numerical-analytical method.

This method starts with the remark, that the dependence of the drag functional $C_d^{(t)}$ versus the coefficients Z_{ij} of the polynomes, which piecewise approximate the surfaces of the aircraft, is a quadratic form, while the dependence versus the similarity parameters of the planform are nonlinear and very complicated. The method presents two steps.

- In the first step the set of similarity parameters of the planform ($\nu_1, \nu_2, \ldots, \nu_n$) are considered as given. The boundary of the drag functional $C_d^{(t)}$ is now a priori known. The optimal value of the coefficients of polynomial expansions of the surface of the aircraft are obtained by solving a linear, algebraic system. These optimal coefficients determine uniquely the value of the drag functional $(C_d^{(t)})_{opt}$, for the prescribed set of similarity parameters of the planform. This value of $(C_d^{(t)})_{opt}$ represents a "point" of what is called here lower limit hypersurface of the drag functional $C_d^{(t)}$ i.e.

$$(C_d^{(t)})_{opt} = f(\nu_1, \nu_2, \ldots, \nu_n) \quad (2)$$

Each of these points can be analytically determined.

- In the second step, through systematical variation of the set of similarity parameters the "position" of the minimum of this hypersurface is numerically (or graphically) determined and gives the best set of similarity parameters ($\nu_1, \nu_2, \ldots, \nu_n$) of the planform, as presented in (Fig. 1), for two similarity parameters. The optimal set of similarity parameters together with a chosen area S_0 of the plan-projection determine the shapes of the planform and of the surface of the optimum-optimorum aircraft of a given set of aircraft. The optimum-optimorum aircraft is exactly the optimal aircraft corresponding to this optimal set of similarity parameters. The minimum value of the "ordinate" of the hypersurface represents the drag coefficient of the optimum-optimorum aircraft of the set. The above theory was successfully used by the author for the effective design of the shape of optimum-optimorum delta wing Adela [1] - [7], and of the optimum-optimorum shape of the integrated wing-fuselage configuration (at cruising Mach number $M_\infty = 2$) as in [11] - [15] , [25] . The shape of delta wing Adela is given in (Fig. 2) and the modification in the shape of the wing Adela, due to the fuselage integration (in the section $\tilde{x}_1 = 0{,}6$) is given in (Fig. 3). A further new application of the optimum-optimorum theory taken here into consideration is the determination of the shape of the entire aircraft which is an integrated wing-fuselage-flaps configuration of minimum drag at two cruising Mach numbers M_∞ and M_∞^* . Two variational problems in cascade are here occurring

- one for the determination of the optimum-optimorum shape of the aircraft at the higher cruising Mach number M_∞ with the flaps in retracted position and

- the second for the determination of the optimum-optimorum shapes of the flap-surface and its planprojection in such a manner, that the entire aircraft (with the flaps in open position) is of minimum drag at the lower supersonic cruising Mach number M_∞^* .

3. DETERMINATION OF THE AXIAL DISTURBANCE VELOCITIES

Let us refer the integrated thick, lifting integrated delta wing to a three-orthogonal system of axes $Ox_1x_2x_3$ having the apex O of the wing as origin. The plane Ox_1x_2 is the plane of symmetry of the integrated wing and the axis Ox_1 is the bisectrix of the angle of the integrated wing, in the plane Ox_1x_3, at its apex (the shockfree entry direction). The integrated thick, lifting delta wing surface is supposed to be flattened in the plane Ox_1x_2 (Fig. 3) and is considered in a parallel stream with the undisturbed velocity $\vec{V}_\infty$ at a moderate angle of attack α (measured between the Ox_1 - axis and $\vec{V}_\infty$).

In the framework of linearised theory for flattened integrated thick, lifting delta wings at moderate angle of attack α , in the boundary value problem concerning the determination of the axial disturbance velocity u the effect of lift can be separated from the effect of thickness. Further the following two delta wing components will be separately considered. The <u>thin integrated delta wing</u> which is the skeleton surface of the thick, lifting integrated delta wing and is considered at the same angle of attack α and the thick-symmetrical integrated delta wing which has the same thickness distribution as the thick, lifting integrated delta wing but its skeleton surface is a plane. This component is considered at zero angle of attack.

The skeleton surface $Z(x_1,x_2)$ of the integrated delta wing is supposed to be continuous but, for the sake of generality, the thickness distributions $Z^*(x_1,x_2)$ on the lateral sides OA_1C_1 and OA_2C_2 (corresponding to the wing) and $Z'^*(x_1,x_2)$ on the central part OC_1C_2 (corresponding to the fuselage) are supposed to be different. Further this wing will be called initial integrated delta wing. The author introduced, as in [1] - [15] a well-suited affine transformation in order to obtain dimensionless coordinates

$$\tilde{x}_1 = \frac{x_1}{h_1}, \quad \tilde{x}_2 = \frac{x_2}{\ell_1}, \quad \tilde{x}_3 = \frac{x_3}{h_1}, \quad B = \sqrt{M_\infty^2 - 1} \quad (3)$$

$$\left(y = \frac{y}{\ell}, \quad \ell = \frac{\ell_1}{h_1}, \quad \nu = B\,\ell, \quad \bar{\nu} = B\,c', \quad \bar{k} = \frac{\bar{c}'}{\ell} \right)$$

A transformed integrated delta wing is obtained, which has the maximal depth 1 and the half-span 1 (Fig. 4). The traces $\tilde{C}_1$ and $\tilde{C}_2$ of the junction lines $\tilde{O}\tilde{C}_1$ and $\tilde{O}\tilde{C}_2$ (between the wing and the fuselage) have the following positions on the axis $\tilde{C}\tilde{y}$ (parallel to axis Ox_2): $y_c = \pm \bar{k}$. The transformed integrated delta wing is placed in a supersonic flow with the cruising Mach number $\tilde{M}_\infty = \sqrt{1 + \nu^2}$. Between the dimensionless axial disturbance velocities u, u^* and $\tilde{u}$, $\tilde{u}^*$ and the dimensionless downwashes w, w^*, w'^* and $\tilde{w}$, $\tilde{w}^*$, $\bar{w}^*$ of the initial and transformed integrated delta wing components there are the following relations:

$$u = \ell\,\tilde{u}, \quad w = \tilde{w}, \quad u^* = \ell\,\tilde{u}^*, \quad w^* = \tilde{w}^*, \quad w'^* = \bar{w}^* \quad (4)$$

Further the assumption is made, that the downwashes $\tilde{w}$, $\tilde{w}^*$ and $\bar{w}^*$ are expressed in form of superpositions of homogeneous polynomes in $\tilde{x}_1$ and $\tilde{x}_2$ i.e. - on the thin component of the transformed integrated wing

$$\tilde{w} = \sum_{m=1}^{N} \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \tilde{w}_{m-k-1,k}\, |\tilde{y}|^k \quad (5)$$

and on the thick-symmetrical component of the transformed integrated wing

$$\tilde{w}^* = \sum_{m=1}^{N} \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \tilde{w}^*_{m-k-1,k}\, |\tilde{y}|^k \quad (6)$$

if $\bar{k} < \tilde{y} < 1$ (here $\bar{k} = \bar{\nu}/\nu$ is supposed constant) and

$$\bar{w}^* = \sum_{m=1}^{N} \tilde{x}_1^{m-1} \sum_{k=0}^{m-1} \bar{w}^*_{m-k-1,k}\, |\tilde{y}|^k, \quad (7)$$

if $\tilde{y} < \bar{k}$. The coefficients $\tilde{w}_{ij}$, $\tilde{w}^*_{ij}$ and $\bar{w}^*_{ij}$ and the similarity parameter ν are unknown and will be determined through the fully-optimization process. The axial disturbance velocity $\tilde{u}$ on the thin component of the transformed integrated thick-lifting delta wing with subsonic leading edges and a central ridge is of the form

$$\tilde{u} = \sum_{n=1}^{N} \tilde{x}_1^{n-1} \left\{ \sum_{q=0}^{E(\frac{n}{2})} \frac{\tilde{A}_{n,2q}\, \tilde{y}^{2q}}{\sqrt{1 - \tilde{y}^2}} + \sum_{q=1}^{E(\frac{n-1}{2})} \tilde{C}_{n,2q}\, \tilde{y}^{2q} \cosh^{-1} \frac{1}{\sqrt{\tilde{y}^2}} \right\} \quad (8)$$

The axial disturbance velocity $\tilde{u}^*$ of the thick-symmetrical integrated delta wing component of the transformed integrated thick, lifting delta wing with subsonic leading edges is according to [7] - [9]

$$\tilde{u}^* = \sum_{n=1}^{N} \tilde{x}_1^{n-1} \left\{ \sum_{q=0}^{n-1} \tilde{H}^*_{nq}\, \tilde{y}^q \left[\cosh^{-1} M_1 + (-1)^q \cosh^{-1} M_2 \right] \right.$$

$$+ \sum_{q=1}^{E(\frac{n-1}{2})} \tilde{C}^*_{n,2q}\, \tilde{y}^{2q} \cosh^{-1} \frac{1}{\sqrt{\nu^2 \tilde{y}^2}} + \sum_{q=0}^{E(\frac{n}{2})} \frac{\tilde{P}^*_{n,2q}\, \tilde{y}^{2q}}{\sqrt{1 - \nu^2 \tilde{y}^2}} +$$

$$\left. + \sum_{q=0}^{n-1} \tilde{G}^*_{nq}\, \tilde{y}^q \left[\cosh^{-1} R_1 + (-1)^q \cosh^{-1} R_2 \right] \right\} \quad (9)$$

Here the following notations will be made

$$R_1 = \sqrt{\frac{(1+\bar{\nu})(1-\nu\tilde{y})}{2(\bar{\nu}-\nu\tilde{y})}}, \quad R_2 = \sqrt{\frac{(1+\bar{\nu})(1+\nu\tilde{y})}{2(\bar{\nu}+\nu\tilde{y})}} \quad (10a)$$

$$M_1 = \sqrt{\frac{(1+\nu)(1-\nu\tilde{y})}{2\nu(1-\tilde{y})}}, \quad M_2 = \sqrt{\frac{(1+\nu)(1+\nu\tilde{y})}{2\nu(1+\tilde{y})}}. \quad (10b)$$

The coefficients of $\tilde{u}$ for the thin transformed integrated delta wing are related to the coefficients of the downwash $\tilde{w}$ and the coefficients of $\tilde{u}^*$ for the thick-symmetrical transformed integrated delta wing are related to the coefficients of the downwashes $\tilde{w}^*$ and $\bar{w}^*$ through the following linear and homogeneous relations

$$\tilde{A}_{n,2q} = \sum_{j=0}^{n-1} \tilde{a}^{(n)}_{2q,j}\, \tilde{w}_{n-j-1,j} \quad (11)$$

$$\tilde{P}^*_{n,2q} = \sum_{j=0}^{n-1} \left(\tilde{p}^{*(n)}_{2q,j}\, \tilde{w}^*_{n-j-1,j} + \bar{p}^{*(n)}_{2q,j}\, \bar{w}^*_{n-j-1,j} \right) \quad (12)$$

The constants $\tilde{a}^{(n)}_{2q,j}$, $\tilde{p}^{*(n)}_{2q,j}$, $\bar{p}^{*(n)}_{2q,j}$ etc. are functions only of the similarity parameter ν.

Let us now consider the open integrated delta wing (with flaps in open positions) (Fig. 5), at cruising Mach number M^*_∞ ($B^* = \sqrt{M_\infty^{*2} - 1}$). The downwashes on the wing and fuselage are unchanged and are given for the transformed configuration in the formulas (3), (4) and (5) and for the initial configuration as in the formulas (4). The downwashes $\tilde{\tilde{w}}$ and $\tilde{\tilde{w}}^*$ on the thin- and thick-symmetrical components on the flaps of the transformed open integrated wing (Fig. 6) are supposed to be expressed in the form of superposition of homogeneous polynomes i.e.

$$\tilde{\tilde{w}} = \sum_{n=1}^{N} \tilde{x}_1^{n-1} \sum_{k=0}^{n-1} \tilde{\tilde{w}}_{n-k-1,k} \, |\tilde{y}|^k \qquad (13)$$

on the thin transformed flaps component and

$$\tilde{\tilde{w}}^* = \sum_{n=1}^{N} \tilde{x}_1^{n-1} \sum_{k=0}^{n-1} \tilde{\tilde{w}}^*_{n-k-1,k} \, |\tilde{y}|^k \qquad (14)$$

on the thick-symmetrical transformed flaps component. Between the downwashes and axial disturbance velocities on the initial and transformed flaps there are the following relations

$$\tilde{\tilde{w}}' = \tilde{\tilde{w}} \;, \quad \tilde{\tilde{w}}'^* = \tilde{\tilde{w}}^* \;, \quad \tilde{\tilde{u}}' = \ell \tilde{\tilde{u}} \;, \quad \tilde{\tilde{u}}'^* = \ell \tilde{\tilde{u}}^* \qquad (15)$$

The following notations are further made:

$$\bar{\nu}^* = B^* c' \;, \quad \tilde{\nu}^* = B^* \ell \,, \quad \nu^* = B^* L \,, \quad k^* = \frac{L}{\ell} \qquad (16)$$

The transformed open integrated delta wing is placed in a supersonic flow with the cruising Mach number $M^*_\infty = \sqrt{1 + \nu^{*2}}$ (Fig. 7). The axial disturbance velocities $\tilde{\tilde{u}}$ and $\tilde{\tilde{u}}^*$ on the thin and thick-symmetrical transformed open integrated delta wing with subsonic leading edges at the cruising Mach number M^*_∞ are obtained by the author here under the following form

$$\tilde{\tilde{u}} = \sum_{n=1}^{N} \tilde{x}_1^{n-1} \left\{ \sum_{q=0}^{n-1} \tilde{\tilde{K}}_{nq} \tilde{y}^q \left[\cosh^{-1} N_1 + (-1)^q \cosh^{-1} N_2 \right] + \sum_{q=0}^{E(\frac{n}{2})} \frac{\tilde{\tilde{A}}_{n,2q} \tilde{y}^{2q}}{\sqrt{k^{*2} - \tilde{y}^2}} + \sum_{q=1}^{E(\frac{n-1}{2})} \tilde{\tilde{C}}_{n,2q} \tilde{y}^{2q} \cosh^{-1} \sqrt{\frac{k^{*2}}{\tilde{y}^2}} \right\} \qquad (17)$$

and

$$\tilde{\tilde{u}}^* = \sum_{n=1}^{N} \tilde{x}_1^{n-1} \left\{ \sum_{q=0}^{n-1} \tilde{\tilde{K}}^*_{nq} \tilde{y}^q \left[\cosh^{-1} N^*_1 + (-1)^q \cosh^{-1} N^*_2 \right] + \sum_{q=0}^{E(\frac{n}{2})} \frac{\tilde{\tilde{P}}_{n,2q} \tilde{y}^{2q}}{\sqrt{1 - \nu^{*2} \tilde{y}^2}} + \sum_{q=1}^{E(\frac{n-1}{2})} \tilde{\tilde{C}}^*_{n,2q} \tilde{y}^{2q} \cosh^{-1} \sqrt{\frac{1}{\tilde{\nu}^{*2} \tilde{y}^2}} \right.$$
$$+ \sum_{q=0}^{n-1} \tilde{\tilde{G}}^*_{nq} \tilde{y}^q \left[\cosh^{-1} R^*_1 + (-1)^q \cosh^{-1} R^*_2 \right]$$
$$\left. + \sum_{q=0}^{n-1} \tilde{\tilde{H}}^*_{nq} \tilde{y}^q \left[\cosh^{-1} M^*_1 + (-1)^q \cosh^{-1} M^*_2 \right] \right\} \qquad (18)$$

In the formulas (17) and (18) the following notations have been made:

$$N_1 = \sqrt{\frac{(1+k^*)(k^* - \tilde{y})}{2k^*(1-\tilde{y})}} \;, \quad N_2 = \sqrt{\frac{(1+k^*)(k^* + \tilde{y})}{2k^*(1+\tilde{y})}} \qquad (19a)$$

$$R^*_1 = \sqrt{\frac{(1+\bar{\nu}^*)(1-\tilde{\nu}^*\tilde{y})}{2\tilde{\nu}^*(\bar{k} - \tilde{y})}} \;, \quad R^*_2 = \sqrt{\frac{(1+\bar{\nu}^*)(1+\tilde{\nu}^*\tilde{y})}{2\tilde{\nu}^*(\bar{k} + \tilde{y})}} \qquad (19b)$$

$$M^*_1 = \sqrt{\frac{(1+\tilde{\nu}^*)(1-\tilde{\nu}^*\tilde{y})}{2\tilde{\nu}^*(1-\tilde{y})}} \;, \quad M^*_2 = \sqrt{\frac{(1+\tilde{\nu}^*)(1+\tilde{\nu}^*\tilde{y})}{2\tilde{\nu}^*(1+\tilde{y})}} \qquad (19c)$$

$$N^*_1 = \sqrt{\frac{(1+\nu^*)(1-\tilde{\nu}^*\tilde{y})}{2\nu^*(k^* - \tilde{y})}} \;, \quad N^*_2 = \sqrt{\frac{(1+\nu^*)(1+\tilde{\nu}^*\tilde{y})}{2\nu^*(k^* + \tilde{y})}} \qquad (19d)$$

These new formulas are obtained by the author by using the results of high conical flow theory of Germain [17], the hydrodynamic analogy of Carafoli [18] , [19] and the principle of minimum singularities [10] , [21].

The coefficients of the axial velocities $\tilde{\tilde{u}}$ and $\tilde{\tilde{u}}^*$ are related to the coefficients of the downwashes $\tilde{w}$, $\tilde{\tilde{w}}$ and $\bar{w}^*$, $\tilde{w}^*$ and $\tilde{\tilde{w}}^*$ through linear and homogeneous relations of the form:

$$\tilde{\tilde{A}}_{n,2q} = \sum_{j=0}^{n-1} \left(\tilde{a}^{(n)}_{2q,j} \tilde{w}_{n-j-1,j} + \tilde{\tilde{a}}^{(n)}_{2q,j} \tilde{\tilde{w}}_{n-j-1,j} \right) \qquad (20)$$

$$\tilde{\tilde{P}}_{n,2q} = \sum_{j=0}^{n-1} \left(\tilde{p}^{*(n)}_{2q,j} \tilde{w}^*_{n-j-1,j} + \bar{p}^{*(n)}_{2q,j} \bar{w}^*_{n-j-1,j} + \tilde{\tilde{p}}^{*(n)}_{2q,j} \tilde{\tilde{w}}^*_{n-j-1,j} \right) \qquad (21)$$

The coefficients $\tilde{a}^{(n)}_{2q,j}$, $\tilde{\tilde{a}}^{(n)}_{2q,j}$, $\bar{p}^{(n)}_{2q,j}$, $\tilde{p}^{(n)}_{2q,j}$, $\tilde{\tilde{p}}^{*(n)}_{2q,j}$ etc. are functions only on the similarity parameters $\bar{\nu}^*$, $\tilde{\nu}^*$ and ν^*.

The theoretical determined pressure coefficient C_p according to the present theory (i.e. by using the formulas (8) and (9) with $G^*_{nq} = 0$) are in good agreement with experimental results for a large range of Mach numbers (M_∞ = 1,25 - 2,2) and angles of attack α ($|\alpha| < 10°$) as it can be seen in Fig. (8a,b) and Fig. (9a,b) for the longitudinal central section ($\tilde{x}_1 = 0$) and for the transversal section $\tilde{x}_1$ = 0,599 of the upper side of the optimum-optimorum delta wing Adela (Fig. 2) and for the angles of attack α = - 8°, +8°.

This agreement between theory and experiment is due to the accuracy of the solutions of the boundary value problems for the axial disturbance velocities $\tilde{u}$ and $\tilde{u}^*$ given in formulas (8) and (9). These solutions for $\tilde{u}$ and $\tilde{u}^*$ present the following advantages in comparison with the ones obtained in the frame of slender body theory [26] - [29] :

- they fulfil the full-linearised partial differential equation, which is hyperbolic, includes the influence of Mach number M_∞ and does not need any restrictions concerning the magnitude of span;

- the boundary conditions along the characteristic surface, i.e. the Mach cone of the apex of the integrated wing, and at the infinity (forward) are satisfied;
- according to the hydrodynamic analogy of Carafoli

[18] , [19] the singularities in these solutions of u and u* are <u>located only along the singular lines</u> (i.e. along the leading edges of the wing, along the junction lines of the wing-fuselage configuration etc.) and therefore are easier to be applied as the solutions for axial disturbance velocities given in [28] , [29] , which are obtained by using singularities located on the whole wing surface;

- these singularities are chosen according to the principle of minimum singularities [14] , [15] and therefore the potential solutions for u and u* given here are <u>matched with a boundary layer solution</u> and are zonal solutions similar as in [31] ;

- the solutions (8) and (9) for u and u* can be also used for the calculation of pressure distribution and of aerodynamic characteristics of an aircraft which shape is given in discrete form. The surface of the integrated wing can be piecewise approximated in form of polynomial expansions which are obtained by using the two-dimensional minimal quadratic error similar as in [7] .

4. OPTIMIZATION OF THE AIRCRAFT WITH RETRACTED FLAPS

The optimization of the shape of the thin and thick-symmetrical integrated wings, components of the thick, lifting integrated delta wing are further treated. The variational problem for the thin integrated delta wing component (for a given value of the similarity parameter ν) leads to the determination of the coefficients $\tilde{w}_{ij}$ of the downwashes $\tilde{w}$ in such a manner, that the drag coefficient C_d ,

$$C_d \equiv \ell \sum_{n=1}^{N} \sum_{m=1}^{N} \sum_{k=0}^{m-1} \sum_{j=0}^{n-1} \tilde{\Omega}_{nmkj} \tilde{w}_{n-j-1,j} \tilde{w}_{m-k-1,k} = \text{min.} \quad (22)$$

In addition the following auxiliary conditions must be fulfilled. The lift coefficient ($C_\ell = \ell \tilde{C}_\ell$) is given

$$\tilde{C}_\ell \equiv \sum_{n=1}^{N} \sum_{j=0}^{n-1} \tilde{\Lambda}_{nj} \tilde{w}_{n-j-1,j} = \frac{C_{\ell_0}}{\ell} \quad (23)$$

The pitching moment coefficient ($C_m = \ell \tilde{C}_m$) is also given

$$\tilde{C}_m \equiv \sum_{n=1}^{N} \sum_{j=0}^{n-1} \tilde{\Gamma}_{nj} \tilde{w}_{n-j-1,j} = \frac{C_{m0}}{\ell} \quad (24)$$

The axial disturbance velocity u vanishes along the leading edge

$$\tilde{F}_t \equiv \sum_{j=0}^{E(\frac{t}{2})} \tilde{\Psi}_{tj} \tilde{w}_{t-j-1,j} = 0 \ , \quad (t = 1,\dots,N) \quad (25)$$

The coefficients $\tilde{\Omega}_{nmkj}$, $\tilde{\Lambda}_{nj}$, $\tilde{\Gamma}_{nj}$ and $\tilde{\Psi}_{tj}$ are only functions of the similarity parameter ν. The corresponding Hamilton's operator H is

$$H \equiv \ell \tilde{H} = \ell \left[\tilde{C}_d + \lambda^{(1)} \tilde{C}_\ell + \lambda^{(2)} \tilde{C}_m + \sum_{t=1}^{N} \lambda_t \tilde{F}_t \right] \quad (26)$$

In this formula the Lagrange's multipliers $\lambda^{(1)}$, $\lambda^{(2)}$ and λ_t are functions of the similarity parameter ν. By cancellation of the coefficients of each independent variation $\delta\tilde{w}_{\theta\sigma}$ entering in the first variation of H the following equations are obtained

$$\sum_{n=1}^{N} \sum_{j=0}^{n-1} \left[\tilde{\Omega}_{n,\theta+\sigma+1,\sigma,j} + \tilde{\Omega}_{\theta+\sigma+1,n,j,\sigma} \right] \tilde{w}_{n-j-1,j} +$$

$$+ \lambda^{(1)} \tilde{\Lambda}_{\theta+\sigma+1,\sigma} + \lambda^{(2)} \tilde{\Gamma}_{\theta+\sigma+1,\sigma} + \lambda_{\theta+\sigma+1} \tilde{\Psi}_{\theta+\sigma+1,\sigma} = 0$$

$$(1 \le \theta+\sigma+1 \le N \ , \ \theta = 0,1,\dots,(N-1) \ ) \quad (27)$$

These equations together with the auxiliary conditions (23), (24) and (25) form a linear algebraic system of equations which determines uniquely the optimum values of the coefficients $\tilde{w}_{ij}$ as well as the Lagrange's multipliers $\lambda^{(1)}$, $\lambda^{(2)}$ and λ_t for a given value of the similarity parameter ν. Similarly, the optimization of the shape of thick-symmetrical integrated delta wing component leads to the determination of the values of the $\tilde{w}^*_{ij}$ and $\bar{w}^*_{ij}$ of the downwashes $\tilde{w}^*$ and $\bar{w}^*$ (on the wing and on the fuselage) in such a manner, that the drag coefficient

$$C^*_d \equiv \ell \sum_{n=1}^{N} \sum_{m=1}^{N} \sum_{k=0}^{m-1} \sum_{j=0}^{n-1} \left\{ \left[\tilde{\Omega}^*_{nmkj} \tilde{w}^*_{n-j-1,j} + \right. \right.$$

$$\left. + \tilde{\Omega}'^*_{nmkj} \bar{w}^*_{n-j-1,j} \right] \tilde{w}^*_{m-k-1,k} + \left[\bar{\Omega}^*_{nmkj} \tilde{w}^*_{n-j-1,j} + \right.$$

$$\left. \left. + \bar{\Omega}'^*_{nmkj} \bar{w}^*_{n-j-1,j} \right] \bar{w}^*_{m-k-1,k} \right\} = \text{min.} \quad (28)$$

Additionally, the following auxiliary conditions must be fulfilled:

- the cancellation of the thickness of the wing along its leading edges

$$\bar{F}^*_t \equiv \sum_{m=t+1}^{N} \sum_{k=0}^{m-1} \tilde{d}^{(t)}_{mk} \tilde{w}^*_{m-k-1,k} = 0 \quad (29)$$

- the continuity of class C_1 of the surface along the junction line between the wing and the fuselage

$$\tilde{E}^*_t \equiv \sum_{m=t+1}^{N} \sum_{k=0}^{m-1} \tilde{c}^{(t)}_{mk} (\tilde{w}^*_{m-k-1,k} - \bar{w}^*_{m-k-1,k}) = 0 \quad (30)$$

$$\tilde{G}^*_t \equiv \sum_{m=t+1}^{N} \sum_{k=0}^{m-1} \tilde{g}^{(t)}_{mk} (\tilde{w}^*_{m-k-1,k} - \bar{w}^*_{m-k-1,k}) = 0 \quad (31)$$

$$\tilde{L}^*_t \equiv \sum_{m=t+1}^{N} \sum_{k=0}^{m-1} \tilde{\ell}^{(t)}_{mk} (\tilde{w}^*_{m-k-1,k} - \bar{w}^*_{m-k-1,k}) = 0 \quad (32)$$

$$(t = 0,1,\dots,(N-1) \ )$$

- the given relative volume of the wing

$$\tilde{\tau} \equiv \sum_{m=1}^{N} \sum_{k=0}^{m-1} \tilde{\tau}_{mk} \, \tilde{w}^{*}_{m-k-1,k} = \tilde{\tau}_0 \sqrt{\ell} \qquad (33)$$

- the given relative volume of the fuselage

$$\bar{\tau} \equiv \sum_{m=1}^{N} \sum_{k=0}^{m-1} \bar{\tau}_{mk} \, \bar{w}^{*}_{m-k-1,k} = \bar{\tau}_0 \sqrt{\ell} \qquad (34)$$

The corresponding Hamilton's operator H^* of this variational problem is

$$H^* \equiv \ell \tilde{H}^* = \ell \left[\tilde{C}^*_d + \mu^{(1)}\tilde{\tau} + \mu^{(2)}\bar{\tau} + \sum_{t=1}^{N} (\mu_t \tilde{F}_t + \right.$$

$$\left. + \bar{\mu}_t \bar{E}_t + \eta_t \tilde{G}_t + \bar{\eta}_t \tilde{L}_t) \right] \qquad (35)$$

Here $\mu^{(1)}$, $\mu^{(2)}$, μ_t , $\bar{\mu}_t$, η_t , $\bar{\eta}_t$ are Lagrange's multipliers. If the first variation of H^* is cancelled, the following equations are obtained

$$\sum_{n=1}^{N} \sum_{j=0}^{n-1} \left\{ \left[\tilde{\Omega}^*_{n,\theta+\sigma+1,\sigma,j} + \tilde{\Omega}^*_{\theta+\sigma+1,n,j,\sigma} \right] \tilde{w}^*_{n-j-1,j} + \right.$$

$$\left. + \left[\tilde{\Omega}'^*_{n,\theta+\sigma+1,\sigma,j} + \bar{\Omega}^*_{\theta+\sigma+1,n,j,\sigma} \right] \bar{w}^*_{n-j-1,j} \right\} +$$

$$+ \mu^{(1)} \tilde{\tau}_{\theta+\sigma+1,\sigma} + \sum_{t=1}^{N} \left[\mu_t \, \tilde{d}^{(t)}_{\theta+\sigma+1,\sigma} + \bar{\mu}_t \, \tilde{c}^{(t)}_{\theta+\sigma+1,\sigma} + \right.$$

$$\left. + \eta_t \, \tilde{g}^{(t)}_{\theta+\sigma+1,\sigma} + \bar{\eta}_t \, \tilde{\ell}^{(t)}_{\theta+\sigma+1,\sigma} \right] = 0 \qquad (36)$$

and

$$\sum_{n=1}^{N} \sum_{j=0}^{n-1} \left\{ \left[\bar{\Omega}^*_{n,\theta+\sigma+1,\sigma,j} + \tilde{\Omega}'^*_{\theta+\sigma+1,n,j,\sigma} \right] \tilde{w}^*_{n-j-1,j} + \right.$$

$$\left. + \left[\bar{\Omega}'^*_{n,\theta+\sigma+1,\sigma,j} + \bar{\Omega}'^*_{\theta+\sigma+1,n,j,\sigma} \right] \bar{w}^*_{n-j-1,j} \right\} +$$

$$+ \mu^{(2)} \bar{\tau}_{\theta+\sigma+1,\sigma} - \sum_{t=1}^{N} \left[\bar{\mu}_t \, \tilde{c}^{(t)}_{\theta+\sigma+1,\sigma} + \eta_t \, \tilde{g}^{(t)}_{\theta+\sigma+1,\sigma} + \right.$$

$$\left. + \bar{\eta}_t \, \tilde{\ell}^{(t)}_{\theta+\sigma+1,\sigma} \right] = 0 \qquad (37)$$

$$(1 \le \theta+\sigma+1 \le N \, , \quad \theta = 0,1,..,(N-1) \,)$$

These equations together with the auxiliary conditions (29) - (34) form a linear algebraic system of equations which determines uniquely the optimum values of the coefficients $\tilde{w}^*_{\theta\sigma}$ and $\bar{w}^*_{\theta\sigma}$ as well as the values of Lagrange's multipliers for a given value of ν. By using the hybrid analytical-numerical method of the author [1] - [4] the optimum-optimorum shape of the thick, lifting integrated delta wing is determined. The limit line I of the drag functional $C_d^{(t)}$ of the thick, lifting integrated delta wing is introduced in (Fig. 7) i.e.

$$(C_d^{(t)})_{opt} = f(\nu) \quad , \qquad (C_d^{(t)} = C_d + C_d^*) \qquad (38)$$

Each point of the limit line is analytically determined by solving a classical variational problem for a given value of parameter ν. The value $\nu = \nu_{opt}$, for which the limit line I attains its minimum is obtained numerically by systematical variation of the similarity parameter ν as in [4] , [14] , [15] . The fitting of an integrated fuselage produces an important modification of the optimal thickness distribution as it can be seen in (Fig. 8), for the section $\tilde{x}_1 = 0{,}6$.

5. OPTIMIZATION OF THE FLAPS SHAPE

The optimization of the shape of the thin and thick-symmetrical open integrated wing (i.e. the wing-fuselage configuration with flaps in open position) at the second, lower Mach number M^*_∞ is here considered (Fig. 13). The shape of the integrated wing-fuselage configuration (with flaps in retracted position) is determined by the solution of the precedent variational problem and remains unchanged in this second variational problem, which consists in the determination of the shape of the flap in such a manner, that the entire aircraft (with flaps in open position) is of minimum drag at the second cruising Mach number M^*_∞ . The variational problem of the thin open integrated delta wing is firstly considered. The downwashes $\tilde{w}$ (on the transformed thin integrated wing-fuselage configuration) and $\tilde{\tilde{w}}$ on the flap are given as in formulas (5) and (13). The coefficients $\tilde{w}_{ij}$ of w are previously determined by the precedent variational problem and are here supposed known and constant. The optimization of the thin flap component (for a given value of ν) leads to the determination of the coefficient $\tilde{\tilde{w}}_{ij}$ of the downwash $\tilde{\tilde{w}}$ in such a manner, that the drag at the cruising Mach number M^*_∞ attains its minimum i.e.

$$\tilde{\tilde{C}}_d \equiv \sum_{n=1}^{N} \sum_{m=1}^{N} \sum_{k=0}^{m-1} \sum_{j=0}^{n-1} \left\{ \left[\tilde{\Omega}_{nmkj} \, \tilde{w}_{n-j-1,j} + \right. \right.$$

$$\left. + \tilde{\tilde{\Omega}}_{nmkj} \, \tilde{\tilde{w}}_{n-j-1,j} \right] \tilde{\tilde{w}}_{m-k-1,k} + \left[\tilde{\Omega}'_{nmkj} \, \tilde{w}_{n-j-1,j} + \right.$$

$$\left. \left. + \tilde{\tilde{\Omega}}'_{nmkj} \, \tilde{\tilde{w}}_{n-j-1,j} \right] \tilde{\tilde{w}}_{m-k-1,k} \right\} = \min. \qquad (39)$$

with the following auxiliary condition (at the cruising Mach number M^*_∞).

- The lift coefficient C'_ℓ is given:

$$\tilde{C}_\ell \equiv \sum_{n=1}^{N}\sum_{j=0}^{n-1}(\tilde{\Lambda}'_{nj}\,\tilde{w}_{n-j-1,j} + \tilde{\tilde{\Lambda}}_{nj}\,\tilde{\tilde{w}}_{n-j-1,j}) = \frac{C'_{\ell 0}}{\ell} \quad (40)$$

- The pitching moment coefficient C'_m is given

$$\tilde{C}_m \equiv \sum_{n=1}^{N}\sum_{j=0}^{n-1}(\tilde{\Gamma}'_{nj}\,\tilde{w}_{n-j-1,j} + \tilde{\tilde{\Gamma}}_{nj}\,\tilde{\tilde{w}}_{n-j-1,j}) = \frac{C'_{m0}}{\ell} \quad (41)$$

- The axial disturbance velocity $\tilde{\tilde{u}}$ vanishes along the leading edge

$$\tilde{\tilde{F}}_t \equiv \sum_{j=0}^{E(\frac{t}{2})} \tilde{\tilde{\Psi}}_{tj}\,\tilde{\tilde{w}}_{t-j-1,j} = 0 \quad , \quad (t = 1,\dots,N) \quad (42)$$

- The wing and the flap surface are continuous of class C_1 along the junction line between the wing and the flap

$$\tilde{\tilde{E}}_t \equiv \sum_{t=1}^{N}\sum_{k=0}^{m-1}\tilde{\tilde{c}}^{(t)}_{mk}\,(\tilde{\tilde{w}}_{m-k-1,k} - \bar{w}_{m-k-1,k}) = 0 \quad (43)$$

$$\tilde{\tilde{G}}_t \equiv \sum_{t=1}^{N}\sum_{k=0}^{m-1}\tilde{\tilde{g}}^{(t)}_{mk}\,(\tilde{\tilde{w}}_{m-k-1,k} - \bar{w}_{m-k-1,k}) = 0 \quad (44)$$

$$\tilde{\tilde{L}}_t \equiv \sum_{t=1}^{N}\sum_{k=0}^{m-1}\tilde{\tilde{\ell}}^{(t)}_{mk}\,(\tilde{\tilde{w}}_{m-k-1,k} - \tilde{w}_{m-k-1,k}) = 0 \quad (45)$$

All the coefficients $\tilde{\Lambda}'_{nj}$, $\tilde{\tilde{\Lambda}}_{nj}$ etc. are depending only on the similarity parameters ν^* and $\tilde{\nu}^*$ ($\tilde{\nu}^*$ is here constant). The corresponding Hamilton's operator of this variational problem is

$$\tilde{\tilde{H}} \equiv \ell\,\tilde{\tilde{H}}' = \ell\Big[\tilde{C}_d + \tilde{\tilde{\lambda}}^{(1)}\tilde{\tilde{C}}_\ell + \tilde{\tilde{\lambda}}^{(2)}\tilde{\tilde{C}}_m + \sum_{t=1}^{N}(\tilde{\tilde{\lambda}}_t\tilde{\tilde{F}}_t + $$

$$+ \tilde{\mu}_t\tilde{\tilde{E}}_t + \bar{\mu}_t\tilde{\tilde{G}}_t + \tilde{\tilde{\mu}}_t\tilde{\tilde{L}}_t)\Big] \quad (46)$$

<u>Remark:</u> The Lagrange's multipliers $\tilde{\tilde{\lambda}}^{(1)}$, $\tilde{\tilde{\lambda}}^{(2)}$, $\bar{\lambda}_t$, $\tilde{\mu}_t$, $\bar{\mu}_t$ and $\tilde{\tilde{\mu}}_t$ are only functions of the similarity parameter ν^*. If the first variation of the Hamilton's operator $\tilde{\tilde{H}}$ is cancelled ($\delta\tilde{\tilde{H}} = 0$) the following equations are obtained:

$$\sum_{n=1}^{N}\sum_{j=0}^{n-1}\Big\{\Big[\tilde{\tilde{\Omega}}'_{n,\theta+\sigma+1,\sigma,j} + \tilde{\tilde{\Omega}}'_{\theta+\sigma+1,n,j,\sigma}\Big]\tilde{\tilde{w}}_{n-j-1,j} + $$

$$+\Big[\tilde{\Omega}'_{n,\theta+\sigma+1,\sigma,j} + \tilde{\tilde{\Omega}}_{\theta+\sigma+1,n,j,\sigma}\Big]\tilde{w}_{n-j-1,j}\Big\} + $$

$$+\tilde{\tilde{\lambda}}^{(1)}\tilde{\tilde{\Lambda}}_{\theta+\sigma+1,\sigma} + \tilde{\tilde{\lambda}}^{(2)}\tilde{\tilde{\Gamma}}_{\theta+\sigma+1,\sigma} + \tilde{\tilde{\lambda}}_{\theta+\sigma+1,\sigma}\,\tilde{\tilde{\Psi}}_{\theta+\sigma+1,\sigma} + $$

$$+\sum_{t=1}^{N}\Big[\tilde{\mu}_t\,\tilde{\tilde{c}}^{(t)}_{\theta\sigma} + \bar{\mu}_t\,\tilde{\tilde{g}}^{(t)}_{\theta\sigma} + \tilde{\tilde{\mu}}_t\,\tilde{\tilde{\ell}}^{(t)}_{\theta\sigma}\Big] = 0 \quad (47)$$

$$(1 \le \theta+\sigma+1 \le N\,,\ \theta = 0,1,..,(N\text{-}1)\,)$$

This algebraic system, together with the auxiliary conditions (26)-(31) determine uniquely the coefficients $\tilde{\tilde{w}}_{n-j-1,j}$ of the downwashes and the Lagrange's multipliers $\tilde{\tilde{\lambda}}^{(1)}$, $\tilde{\tilde{\lambda}}^{(2)}$, $\bar{\lambda}_t$, $\tilde{\mu}_t$, $\bar{\mu}_t$ and $\tilde{\tilde{\mu}}_t$ (as functions of the similarity parameter ν^*).

Let us consider now the second variational problem concerning the optimization of the thick-symmetrical component of the flap by cruising Mach number M^*_∞. The downwashes coefficients $\tilde{\tilde{w}}^*_{ij}$ are determined in such a manner that the drag coefficient $\tilde{\tilde{C}}^*_d$ of the thick-symmetrical component of the open integrated wing attains its minimum (at cruising Mach number M^*_∞) i.e.

$$\tilde{C}^*_d \equiv \sum_{n=1}^{N}\sum_{m=1}^{N}\sum_{k=0}^{m-1}\sum_{j=0}^{n-1}\Big\{\Big[\tilde{\Omega}^{**}_{nmkj}\,\tilde{w}^*_{n-j-1,j} + $$

$$+\bar{\Omega}^{**}_{nmkj}\,\bar{w}^*_{n-j-1,j} + \tilde{\tilde{\Omega}}^{**}_{nmkj}\,\tilde{\tilde{w}}^*_{n-j-1,j}\Big]\,\tilde{w}^*_{m-k-1,k} + $$

$$+\Big[\tilde{\Omega}'^{**}_{nmkj}\,\tilde{w}^*_{n-j-1,j} + \bar{\Omega}'^{**}_{nmkj}\,\bar{w}^*_{n-j-1,j} + $$

$$+\tilde{\tilde{\Omega}}'^{**}_{nmkj}\,\tilde{\tilde{w}}^*_{n-j-1,j}\Big]\,\bar{w}^*_{m-k-1,k} + \Big[\tilde{\Omega}^{(**)}_{nmkj}\,\tilde{w}^*_{n-j-1,j} + $$

$$+\bar{\Omega}^{(**)}_{nmkj}\,\bar{w}^*_{n-j-1,j} + \tilde{\tilde{\Omega}}^{(**)}_{nmkj}\,\tilde{\tilde{w}}^*_{n-j-1,j}\Big]\tilde{\tilde{w}}^*_{m-k-1,k}\Big\} = \text{min.} \quad (48)$$

Additionally auxiliary conditions are considered:

- the flap is of null-thickness along its leading edges

$$\tilde{\tilde{F}}^*_t = \sum_{m=t+1}^{N}\tilde{\tilde{d}}^{*(t)}_{mk}\,\tilde{\tilde{w}}^*_{m-k-1,k} = 0 \quad (49)$$

- the thick-symmetrical flap is integrated i.e. along the junction line between the wing and the flap the surface must be continuous of class C_1 i.e.

$$\tilde{\tilde{E}}'^*_t \equiv \sum_{m=t+1}^{N}\sum_{k=0}^{m-1}\tilde{\tilde{c}}^{*(t)}_{mk}\,(\tilde{\tilde{w}}^*_{m-k-1,k} - \tilde{w}^*_{m-k-1,k}) = 0 \quad (50)$$

$$\tilde{\tilde{G}}^*_t \equiv \sum_{m=t+1}^{N}\sum_{k=0}^{m-1}\tilde{\tilde{g}}^{*(t)}_{mk}\,(\tilde{\tilde{w}}^*_{m-k-1,k} - \bar{w}^*_{m-k-1,k}) = 0 \quad (51)$$

$$\tilde{\tilde{L}}^*_t \equiv \sum_{m=t+1}^{N}\sum_{k=0}^{m-1}\tilde{\tilde{\ell}}^{*(t)}_{mk}\,(\tilde{\tilde{w}}^*_{m-k-1,k} - \tilde{\tilde{w}}^*_{m-k-1,k}) = 0 \quad (52)$$

$$(\,t = 0,1,..,(N\text{-}1)\,)$$

- the relative volume τ' of the flap is given:

$$\tilde{\tilde{\tau}}^* \equiv \sum_{m=1}^{N}\sum_{k=0}^{m-1}\tilde{\tilde{\tau}}^*_{mk}\,\tilde{\tilde{w}}^*_{m-k-1,k} = \tau'_0\,\sqrt{\ell} \quad (53)$$

The Hamilton's operator $\tilde{\tilde{H}}$ of this variational problem is

$$\tilde{\tilde{H}}^* = \ell\,\tilde{\tilde{H}}'^* = \ell\,\Big[\tilde{\tilde{C}}^*_d + \mu^*\tilde{\tilde{\tau}}^* + \sum_{t=1}^{N}(\tilde{\mu}^*_t\tilde{\tilde{F}}^*_t + \bar{\mu}^*_t\tilde{\tilde{E}}_t + $$

$$+\tilde{\eta}^*_t\tilde{\tilde{G}}^*_t + \bar{\eta}^*_t\tilde{\tilde{L}}^*_t)\Big] \quad (54)$$

Here μ^{*}, $\tilde{\mu}_{t}^{*}$, $\bar{\mu}_{t}^{*}$, $\tilde{\eta}_{t}^{*}$ and $\bar{\eta}_{t}^{*}$ are the Lagrange's multipliers. If the first variation of $\bar{\tilde{\tilde{H}}}^{*}$ is cancelled the following equations are obtained

$$\sum_{n=1}^{N} \sum_{j=0}^{n-1} \Bigg\{ \left[\bar{\tilde{\tilde{\Omega}}}^{(**)}_{n,\theta+\sigma+1,\sigma,j} + \bar{\tilde{\tilde{\Omega}}}^{(**)}_{\theta+\sigma+1,n,j,\sigma} \right] \bar{\tilde{\tilde{w}}}^{*}_{n-j-1,j} +$$

$$+ \left[\tilde{\Omega}^{(**)}_{n,\theta+\sigma+1,\sigma,j} + \bar{\tilde{\tilde{\Omega}}}^{**}_{\theta+\sigma+1,n,j,\sigma} \right] \tilde{\tilde{w}}^{*}_{n-j-1,j} +$$

$$+ \left[\bar{\Omega}^{(**)}_{n,\theta+\sigma+1,\sigma,j} + \bar{\tilde{\tilde{\Omega}}}'^{**}_{\theta+\sigma+1,n,j,\sigma} \right] \bar{w}^{*}_{n-j-1,j} \Bigg\} +$$

$$+ \mu^{*} \bar{\tilde{\tilde{\tau}}}^{*}_{\theta+\sigma+1,\sigma} + \sum_{t=1}^{N} \Big[\tilde{\mu}^{*}_{t} \bar{\tilde{\tilde{d}}}^{*(t)}_{\theta+\sigma+1,\sigma} + \bar{\mu}^{*}_{t} \tilde{\tilde{c}}^{*(t)}_{\theta+\sigma+1,\sigma} +$$

$$+ \tilde{\eta}^{*}_{t} \tilde{\tilde{g}}^{*(t)}_{\theta+\sigma+1,\sigma} + \bar{\eta}^{*}_{t} \bar{\tilde{\tilde{\ell}}}^{*(t)}_{\theta+\sigma+1,\sigma} \Big] = 0 \qquad (55)$$

$$(1 \leq \theta+\sigma+1 \leq N\ ,\ \ \theta = 0,1,..,(N-1)\ )$$

These equations, together with the auxiliary conditions (52)-(56) form a linear algebraic system which determines uniquely the values of the coefficients $\bar{\tilde{\tilde{w}}}^{*}_{\theta\sigma}$ of the downwash $\bar{\tilde{\tilde{w}}}^{*}$ and the values of the Lagrange's multipliers μ^{*}, $\tilde{\mu}_{t}^{*}$, $\bar{\mu}_{t}^{*}$, $\tilde{\eta}_{t}^{*}$ and $\bar{\eta}_{t}^{*}$ as function of the similarity parameter ν^{*}. The best value of the similarity parameter ν^{*} for the thick-lifting flap (at cruising Mach number M_{∞}^{*}) can be also determined by using the hybrid numerical-analytical method of the author as in [1] - [4]. The optimal value of ν ($\nu = \nu_{\mathrm{opt}}$) is the position of the minimum of the lower limit line (Fig. 7) of the drag functional $(\tilde{\tilde{C}}_{d}^{(t)})_{\mathrm{opt}}$ i.e.

$$(\tilde{\tilde{C}}_{d}^{(t)})_{\mathrm{opt}} = f(\nu) \qquad (\tilde{\tilde{C}}_{d}^{(t)} = \tilde{\tilde{C}}_{d} + \tilde{\tilde{C}}_{d}^{*}) \qquad (56)$$

6. AGREEMENT WITH EXPERIMENTAL RESULTS

The aerodynamic characteristics of the optimum-optimorum wing Model Adela (Fig. 2) were measured in the framework of DFG research contracts, by the author and collaborators, in trisonic wind tunnel (section 60 x 60 cm^2) of the DFVLR-Köln. The theoretically predicted values of the lift and pitching moment coefficients C_ℓ and C_m according to the above theory are in very good agreement with the experimental results for the all range of Mach numbers (M_∞ = 1,25 - 2,2) and angles of attack α ($|\alpha| < 14°$) taken here into consideration as in [7] , [23] , [34]. The dependence of lift- and pitching moment coefficients C_ℓ and C_m versus the angle of attack α is linear in supersonic flow also at higher angle of attack α as in (Fig. 11a,b), but nonlinear with respect to the Mach number M_∞ as in (Fig. 12 a,b). Recently the measurements of C_ℓ and C_m on wedged delta wing given in (Fig. 14), performed [37] at angles of attack $|\alpha| \leq 16°$ and higher supersonic Mach numbers (M_∞ = 2,4 ÷ 4,0) are in good agreement with the values of C_ℓ and C_m predicted by the present theory for supersonic leading edges (Fig. 15a,b) for all the ranges of angles of attack α (i.e. $|\alpha| < 16°$) for C_ℓ and C_m at M_∞ = 2,4 ÷ 3,2 . At higher Mach numbers i.e. M_∞ = 3,6 ÷ 4,0 the range of angles of attack α for a good agreement is more reduced i.e. $|\alpha| \leq 10°$ for C_ℓ and $|\alpha| \leq 12°$ for C_m . The time of calculation of the aerodynamic characteristics of the aircraft, by using own softwares according to this theory is less than 4 seconds on Cyber 175 !

7. CONCLUSIONS

The optimum-optimorum theory of the author can be successfully applied for the global-optimization of the entire configuration of the aircraft, which shape presents the following advantages: a) it is total integrated (i.e. wing-fuselage and wing-flap integration) and therefore has no drag due to corners; b) it is of minimum drag for two different supersonic/hypersonic cruising Mach numbers and therefore is useful for the supersonic aircraft flying at higher supersonic cruising Mach number (i.e. M_∞ = 3 ÷ 4); c) it is of high lift due to Kutta auxiliary conditions along the leading edges; d) it presents a reduced drag and increased lift for a large range of Mach numbers and angles of attack. The hybrid numerical-analytical method of the author which allows the effective determination of the optimum-optimorum shape of the supersonic aircraft presents the following advantages: a) it is accurate because it allows the simultaneous optimization of all geometrical parameters of its shape; b) it is flexible while it can be applied to the optimization of complex shape of supersonic aircraft and allows, to add or to suppress some auxiliary conditions and to change the cruising Mach number chosen for the optimization; c) it is able to determine the shape of the aircraft of variable geometry in order to obtain a minimum drag at two, very different, supersonic/hypersonic cruising Mach numbers; d) it is fast (6 sec. computer time at Cyber 175) for the full-optimization of the shape of supersonic aircraft !

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FIGURES

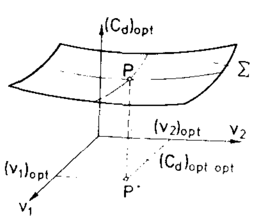
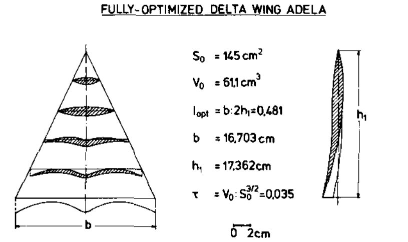


Fig. 1,2



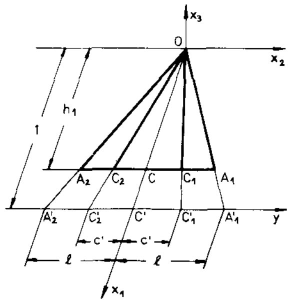
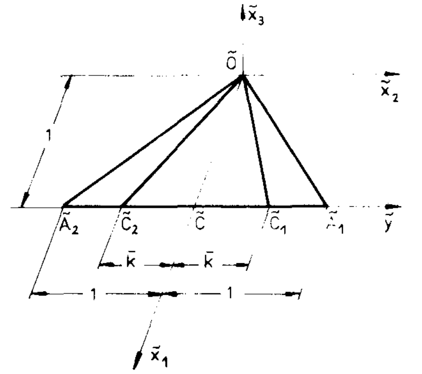


Fig. 3,4



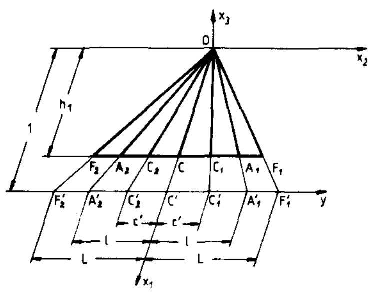
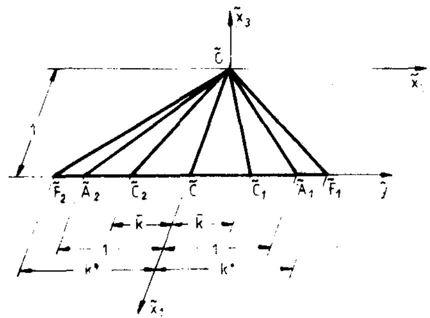


Fig. 5,6



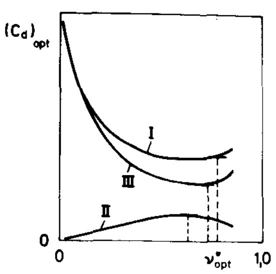
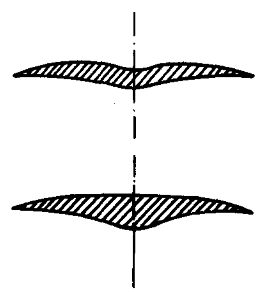


Fig. 7,8



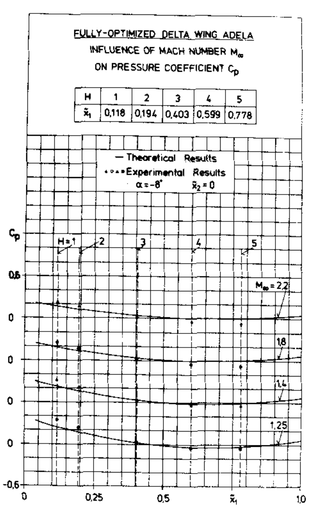
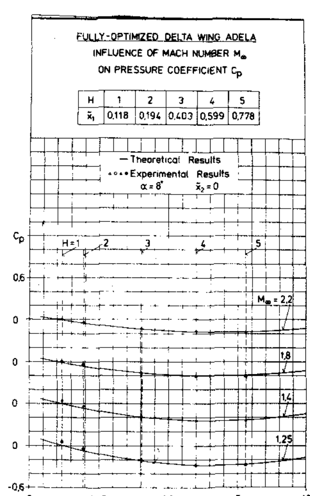


Fig. 9a,b



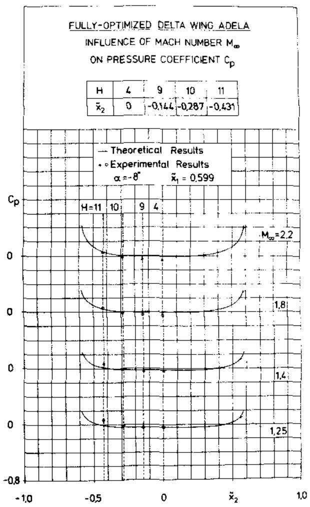
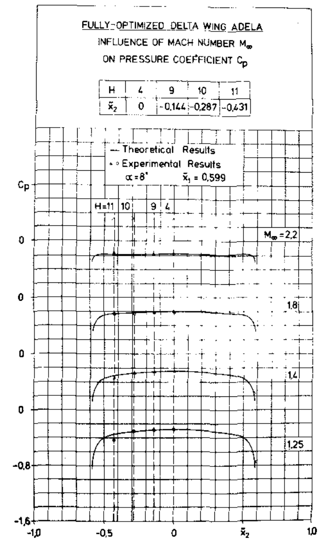


Fig. 10a,b



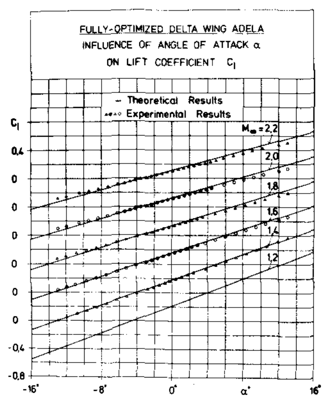
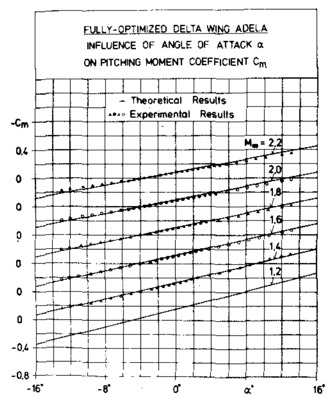


Fig. 11a,b



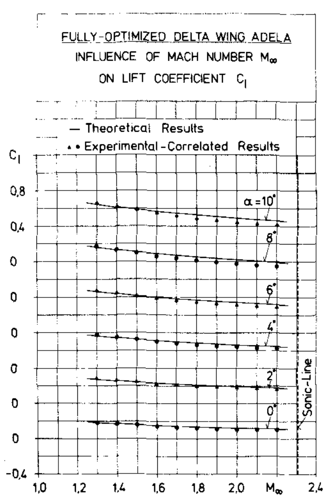
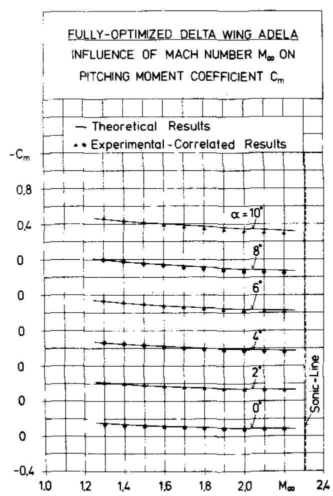


Fig. 12a,b



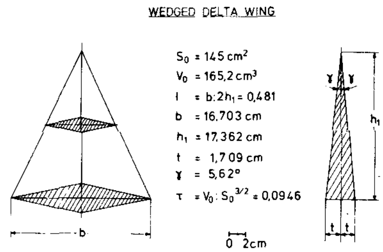


Fig. 13,14

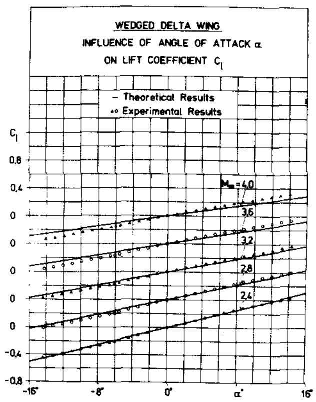


Fig. 15a,b