NONLINEAR FLUTTER ANALYSIS OF WINGS AT HIGH ANGLE OF ATTACK

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Abstract

In this paper, two methods are presented to analyze nonlinear flutter of wings with separated vortex at high angle of attack. One of the methods is a Time Integration Method(TIM). Combined with the calculated unsteady aerodynamic forces for wings at high angle of attack, the structural dynamic equations of the wing are integrated by Runge-Kutta method in time domain, and the wing motion can be simulated at any flying speed. Another method is a Describing Function Method (DFM). In the DFM, the nonlinear generalized aerodynamic forces are linearized by using the concept of describing function. Then, the structural dynamic equations of the wing are solved by conventional V-g method, and the critical flutter speed can be obtained.

To verify the numerical methods, flutter tests for wings at high angle of attack are carried out in a low speed wind tunnel. The wing models are a rectangular wing and a delta wing. The wings can move in rolling and pitching. The basic angles of attack in the experiment are 14° and 18°. It is shown that the higher the basic angle of attack, the lower the critical flutter speed. The results calculated by the above mentioned methods are in agreement with the experiment.

Nomenclature

$C(\xi,\tau)$ = generalized aerodynamic force coefficient
$D(\xi,ik)$ = describing function
[D] = generalized aerodynamic force coefficient matrix
{e} = state vector
{f} = generalized aerodynamic force vector
$H(\xi,s)$ = indicial response of generalized aerodynamic force coefficient
[I] = unit matrix
[K] = generalized stiffness matrix
k = reduced frequency
[M] = generalized mass matrix
[0] = zero matrix
$\Delta P(x,y)$ = pressure difference on the wing surface
q(t) = generalized coordinate
S = wing area
s = laplace variable
t = time
U = freestream velocity
V = critical flutter speed
x,y = corresponding coordinates
z = deformation of the wing
ξ = a kind of deformation of the wing
τ = nondimensional time
ρ = density
ω = eigen frequency
α_0 = static angle of attack
ψ = nondimensional rolling angle
θ = nondimensional pitching angle

subscript
s = static
i = ith
j = jth

Introduction

For nonlinear flutter analysis of wings at high angle of attack, Strganac and Mook [1] developed an integration method in time domain. With the unsteady airloads on wings obtained by an unsteady Vortex-Lattice Method, the equation of motion is integrated. In Ref.[1], only the results of a rectangular wing with big aspect ratio AR=10 were given. For another low aspect ratio rectangular wing, no detailed results were supplied. The integration method in Ref.[1] can give the vibration history of wings at any flying speeds, but the computational work is time consuming.

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By introducing a describing function for the nonlinear generalized aerodynamic force, Ueda and Dowell [2] analyzed the flutter of airfoils at transonic flow in frequency domain with better computational efficiency.

Up to now, the phenomenon of nonlinear flutter for wings with separation at high angle of attack has not been thoroughly investigated. So, wind tunnel tests for this problem would be much helpful for further research.

In this paper, both Time Integration and Describing Function Methods are developed for nonlinear flutter analysis of wings at high angle of attack. The nonlinear aerodynamic forces are provided by a subsonic unsteady numerical method— Potential Difference Method [3] developed recently by the authors. Besides, the experiments of flutter for the wings at high angle of attack were carried out, and the test results confirm the feasibility of the above mentioned methods.

Time Integration Method

The small deformation of wing structure can be expressed by

$$z(x,y,t) = \sum_{i=1}^{N} \Phi_i(x,y) \cdot q_i(t) \qquad (1)$$

where $\Phi_i(x,y)$ is the ith eigen mode, and $q_i(t)$ is the corresponding generalized coordinate.

Then the equation of motion can be written as

$$[M]\{\ddot{q}\}+[K]\{q\} = \{f\} \qquad (2)$$

where the dot means d/dt, [M] is the generalized mass matrix, [K] is the generalized stiffness matrix, {f} is the generalized aerodynamic force vector.

By introducing the state vector $\{e\}=(q_1, q_2, q_3, \cdots q_N, \dot{q}_1, \dot{q}_2, \dot{q}_3, \quad \dot{q}_N)^T$. Eq.(1) is transformed to

$$\{\dot{e}\} = \begin{bmatrix} [0] & [I] \\ [M]^{-1}[K] & [0] \end{bmatrix} \{e\} + \begin{bmatrix} [0] \\ [M]^{-1} \end{bmatrix} \{f\} \qquad (3)$$

For a given initial vector $\{e_0\}$, the aerodynamic pressure difference $\Delta P(x,y)$ on wing surface is obtained by the Potential Difference Method, and the generalized aerodynamic forces are calculated by

$$f_i = \iint_S \Delta P(x,y) \cdot \Phi_i(x,y) dxdy$$

then, Eq.(3) can be solved by Runge-Kutta method. The state vector {e} obtained at the end of each time step provides a new boundary condition for the next time step to calculate the aerodynamic pressure. By this procedure, the wing motion can be simulated with a step-by-step discrete time history.

For a given basic angle of attack α_0, the static deformation $\{q_s\}$ must be calculted at first by an iteration process from the governing equation

$$[K]\{q_s\}=\{f_s\}$$

where $\{f_s\}$ is the static aeroelastic airload. Then, the initial state vector $\{e_0\}$ for the above Runge-Kutta procedure is formed with $\{q_s\}$ superposed by a certain disturbance of generalized displacements and velocities.

Describing Function Method

For a harmonic vibration of the wing

$\xi = \bar{\xi} e^{ik\tau}$, where τ is the nondimensional time and k is the reduced frequency, the corresponding generalized aerodynamic force f is expressed by

$$f = \frac{1}{2} \rho U_\infty^2 \cdot S \cdot C(\xi, \tau) \qquad (4)$$

where $\frac{1}{2}\rho U_\infty^2$ is the dynamic pressure, S is the wing area, and $C(\xi, \tau)$ is the coefficient of generalized aerodynamic force.

For present problem, $C(\xi, \tau)$ is a nonlinear function with respect to ξ. For harmonic motion, it can be linearized by introducing a discribing function $D(\bar{\xi}, ik)$ and the following approximate relation holds:

$$C(\bar{\xi}, \tau) = D(\bar{\xi}, ik) \cdot \xi = D(\bar{\xi}, ik) \cdot \bar{\xi} e^{ik\tau} \qquad (5)$$

The corresponding relation in the Laplace domain is

$$\hat{C}(\bar{\xi}, s) = D(\bar{\xi}, s) \cdot \bar{\xi}/(s-ik) \qquad (6)$$

where the superscript ^ represents the Laplace transformation of a function, s is the Laplace variable and $i=\sqrt{-1}$.

If k=0, the Eq.(6) represents an indicial response relationship

$$H(\bar{\xi}, s) = D(\bar{\xi}, s) \cdot \bar{\xi}/s \qquad (7)$$

In practical application, the indicial response time history of aerodynamic force is first calculated by certain nonlinear aerodynamic code with a step input $\bar{\xi}$. Then, by aid of curve fitting, this indicial response is approximated by a polynomial of exponential functions $\Sigma a_n \cdot \exp(b_n \tau)$, from which it is straightforward to get an closed form expression of $H(\bar{\xi}, s)$ in Laplace domain. According to Eq.(7), the describing function is obtained

$$D(\bar{\xi}, ik) = ik \cdot H(\bar{\xi}, ik)/\bar{\xi} \qquad (8)$$

By introducing the describing functions, the generalized aerodynamic force for wings at high angle of attack formally has a linear relation to the generalized displacements

$$\{f\}=\frac{1}{2}\rho U_\infty^2 S\cdot[D]\{q\} \quad (9)$$

where the generalized aerodynamic force coefficient matrix [D], different from the linear problem, depends on not only the reduced frequency but also the vibration amplitudes.

So, a linear flutter equation is gotten

$$[M]\{\ddot{q}\}+[K]\{q\}=\frac{1}{2}\rho U_\infty^2[D]\{q\} \quad (10)$$

This equation must be solved by an iteration process. At first, a set of vibration amplitudes $\{\bar{q}_0\}$ are chosen. After the steady flow for the wing at a basic angle of attack is computed, a step deformation $\Phi_j(x,y)\cdot\bar{q}_{0j}$ is superposed upon the wing impulsively, and a set of indicial reponses of generalized aerodynamic force coefficients are obtained in the time domain. Then, the D_{ij} (i=1,N) are calculated by the procedure mentioned above.

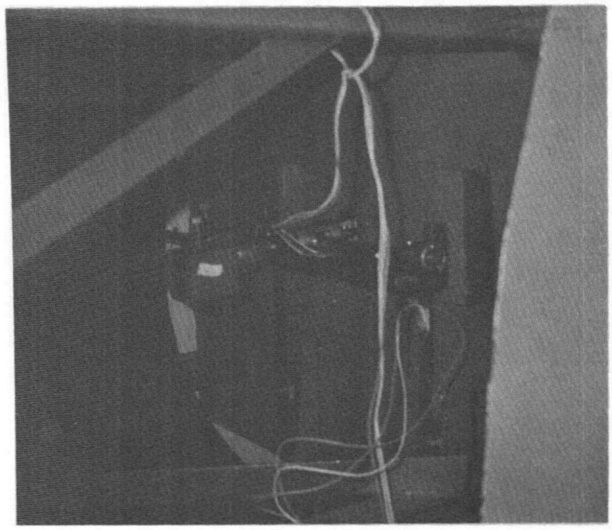
Having got the aerodynamic matrix [D], the conventional V-g method is adopted to solve Eq.(10), and the result provides a critical flutter speed V_F and flutter mode $\{\bar{q}_F\}$, which usually is not consistent with the preassigned $\{\bar{q}_0\}$. Then, the resultant mode $\{\bar{q}_F\}$ is used as the initial $\{\bar{q}_1\}$ for the second run of flutter calculation. Such an iteration procedure will lead to the final results, i.e. the limit cycle flutter mode and the flutter speed.

Experiments

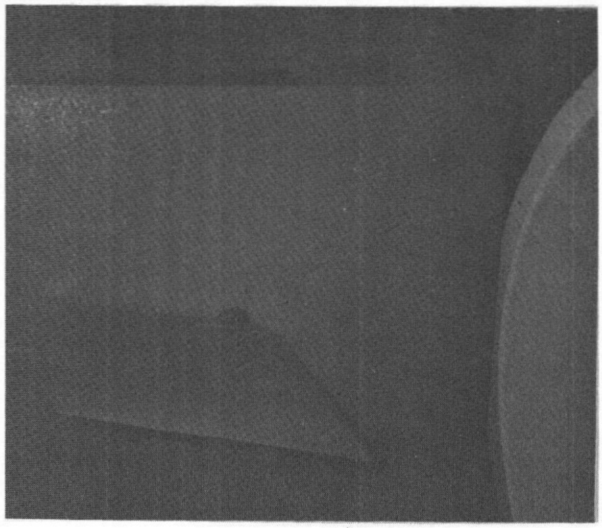
The models for wind tunnel test are a solid wooden rectangular wing and a delta wing. A supporting system provides rolling (ψ) and pitching (θ) degrees of freedom for the rigid models. The rectangular wing has a sharp side edge with aspect ratio AR=2, and the chord length is 300 mm. The pitching axis is 68 mm behind the leading edge. The delta wing has a sharp leading edge with aspect ratio AR=2.61, and the root chord length is 460 mm. The pitching axis is 285 mm before the trailing edge. The basic angles of attack in the experiments are 14° and 18° for both wings. At these attitudes, vortex separation occurs from the side edge for the rectangular wing and from the leading edge for the delta wing.

A low speed wind tunnel with the diameter 1.m of the test section is used, its maximum airspeed is 60 m/s. Fig.1 is the photograph of the test model.

The supporting springs are adjustable, and hence can provide different eigen frequencies for the model. The ω_ψ and ω_θ denote the rolling and pitching frequency respectively. For the rectangular wing, two sets of ω_ψ, ω_θ are selected. One set is ω_ψ=1.875 Hz and ω_θ=2.5 Hz. Another set is ω_ψ=1.875 Hz and ω_θ=2.75 Hz. For the delta wing, only one set (ω_ψ=2.0 Hz, ω_θ=2.375 Hz) is used.



(a)



(b)

Fig.1 The test system
(a) the supporting system
(b) the rectangular wing model in the wind tunnel

Results and Conclusions

The test results of the critical flutter speed and the results calculated by both TIM and DFM are listed in Table-1, where α_0 is the basic angle of attack.

TABLE-1 COMPARISION OF CALCULATED RESULTS WITH EXPERIMENT

CASE No.	TEST WING	α_o^o	ω_ψ (Hz)	ω_θ (Hz)	V_F (m/s)		
					TEST	TIM	DFM
1	RECT.	14	1.875	2.5	11	10.25	10.47
2	RECT.	18	1.875	2.5	10	9.25	9.129
3	RECT.	14	1.875	2.75	13	11.30	11.9635
4	RECT.	18	1.875	2.75	11	10.20	10.1936
5	DELTA	14	2.0	2.375	21	19.50	18.6414
6	DELTA	18	2.0	2.375	20	18.50	17.56

where the RECT. means rectangular.

The vibration time history obtained by TIM for case-3 at the airspeeds 11 m/s and 11.5 m/s are shown in Fig.2 and Fig.3 respectively. The former is a subcritical state, and the latter is a supercritical state.

Fig.4 and Fig.5 depict the results of the TIM for case-5 and case-6 at the airspeeds 19.5 m/s and 18.5 m/s respectively. Both states are at the critical flutter point.

For the calculation of the generalized aerodynamic force coefficients in the DFM, an eleven terms polynomial of exponential functions is used in the curve fitting process for the indicial response.

Since there is a gap between the wing root and the shield plate, and the influence of this gap cannot be taken into account accurately in the numerical analysis. This is one of the reasons for the discrepancy between the test and calculation results. Nevertheless, the numerical results may be considered to be satisfactory.

Compared with the TIM, the advantage of the DFM lies in the much less computational time needed to get a flutter point. But the DFM can only give the critical flutter point, while the TIM can give not only the critical flutter point, but also the subcritical and supercritical responses. Besides, the linearized approximation of the generalized aerodynamic forces in DFM is not involved in the TIM. The superposition of modal generalized aerodynamic forces used in the DFM is tenable only upon the engineering consideration.

From the results of the present work, it can be seen that the higher the basic angle of attack, the lower the critical flutter speed. This means that the vortex separation for wings at high angle of attack deteriorates the flutter characteristics of the wing, which is important for combat aircraft in maneuver flight.

References

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2. Ueda,T. and Dowell, E. H., Flutter Analysis Using Nonlinear Aerodynamic Forces. Journal of Aircraft, Vol.21 No.2, Feb.1984, PP.101.

3. Ye, Z.,Yang, Y.and Zhao,L., Subsonic Steady, Unsteady Aerodynamic Calculation For Wings at High Angle of Attack. 17th ICAS Congress.

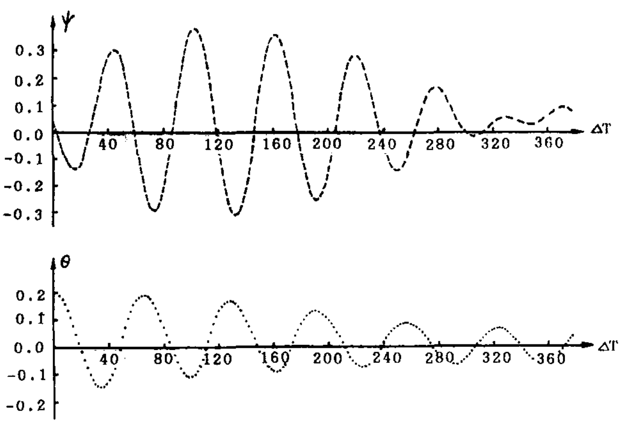

Fig.2 The ψ and θ versus the nondimensional time (U_∞ = 11.0 m/s)

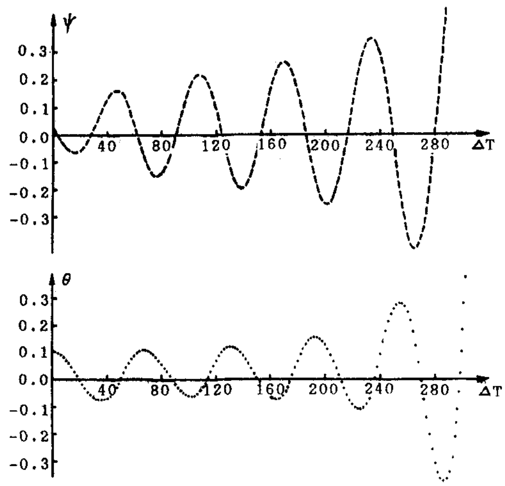

Fig.3 The ψ and θ versus the nondimensional time (U_∞ = 11.5 m/s)

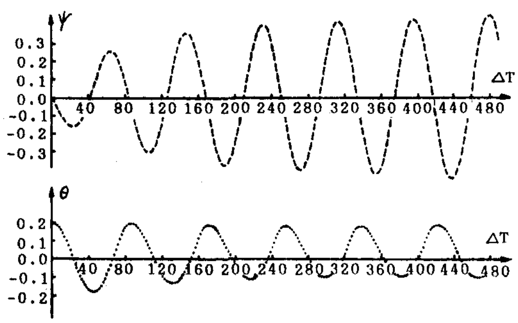

Fig.4 The ψ and θ versus the nondimensional time (U_∞ = 19.5 m/s)

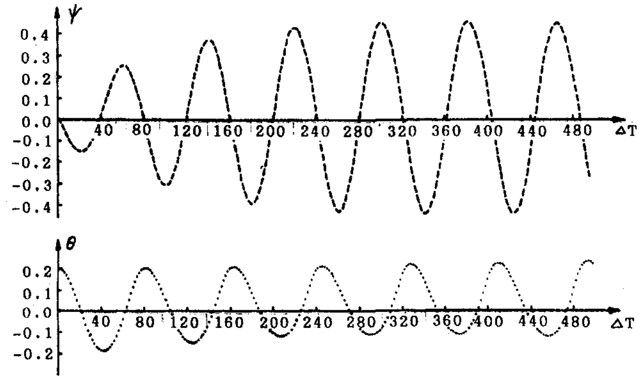

Fig.5 The ψ and θ versus the nondimensional time (U_∞ = 18.5 m/s)