ICAS-88-6.9.1

STRESS INTENSITY FACTOR OF THREE DIMENSIONAL CRACK AT THE EDGE OF A HOLE

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ABSTRACT

Crack growth data of a corner crack at the edge of a hole in a plate loaded with cyclic tension were used by Schijve [1] to derive empirical stress intensity factors (K) which he then compared with K values interpolated from Finite Element results. In the present study, the differences between Schijve's empirical and calculated K values are investigated, and a simple curve fitting procedure is presented to improve the calculated K for application to crack growth analysis. The present procedure gives results that are in excellent correlation with Schijve's empirically derived K factors.

I. INTRODUCTION

The studies of corner cracks at the edge of a hole in a plate are of interest in engineering applications due to their common appearance in practice. The MIL A-83444 recognizes this type of cracking and requires the assumption of an initial quarter circular crack at the edge of a hole for damage tolerance analysis of certain aircraft geometries.

A recent study of cracks at the edge of a hole in 4340 steel and 7075-T651 aluminum lugs, subjected to pin load and uniform load is presented in [4]. The analysis therein uses the approximate Green Functions in crack growth calculations for cyclic constant amplitude loading, and also includes the complicated transition from quarter elliptical to a straight through crack. The results are presented alongside the data that was derived from crack growth tests for the same geometries and loading. The life predictions are reported to be satisfactory to good, whereas crack geometry aspect ratio predictions were poor to satisfactory.

The good life prediction presented in [4] seems as an outstanding accomplishment in view of the complexity of the test and the analysis. For the same reason, the poor to satisfactory crack aspect ratio prediction is not surprising and might suggest that the actual cracks were growing through shapes different from those that resulted from the analysis. Hence, it would be desireable to have the means to make more accurate crack growth shape predictions, leading to improved accuracy in crack growth life calculations. This is significant in the determination of inspection intervals of aircraft, and has maintenance and economical implications.

A recent publication, by Schijve [1], presents a study of a similar crack configuration in a plate. Schijve utilized test data that were generated in [2] and presented in [3] for an almost quarter elliptical corner crack. The crack was initiated (as a quarter circular) at the edge of a hole in a polymethylmethacrylate plate and then fatigue grown with remote uniform cyclic tensile loading. By considering the measured crack increments at nine points along the front of the growing crack and then applying the baseline fatigue crack growth data of the same material, from [3], Schijve derived empirical stress intensity factors along the front of ten crack sizes. He compares the empirical ΔK results with calculated K values which he obtains using interpolations between finite element (F.E.) K results by Rajue and Newman [5] along with minor geometrical correction factors. Schijve concludes that a satisfactory agreement was found between the empirical and the calculated stress intensity factors but he states that the "fairly sharp drop of the calculated K values at $\theta = 90°$ (hole surface, Figure 1), is more difficult to understand". He also concludes that this drop in K is not evident from the empirical data.

The quarter elliptical corner crack has received a considerable attention from various investigators. A comparison of the various results is illustrated in Figure 9 of [5] showing differences up to 100%. In view of the lack of a closed form solution for this crack, the selection by Schijve of the F.E. results by Rajue and Newman [6] as the anlytical baseline for comparison with the experimental values appears a logical approach. Hence, the present study will focus on the same two sets of results.

The intent of the present study is to examine the differences between the empirical and calculated K values in [1], to evaluate their significance in making crack growth predictions and to propose a simple analytical procedure to improve the results of the K factor calculations in [1]. This is in order to obtain K values that give improved correlation with the empirical values and thereby allow to make more accurate crack growth predictions. The focus of attention in the present study is at the crack front intersection with the hole surface, $\theta = 90°$, see Fig 1a, because the K values calculated in [1] at this point are about 18% to 28% lower than the empirical values.

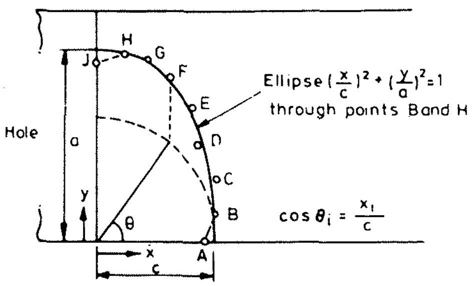

Figure 1a - Geometry Definition of Corner Crack. Extracted from [1]

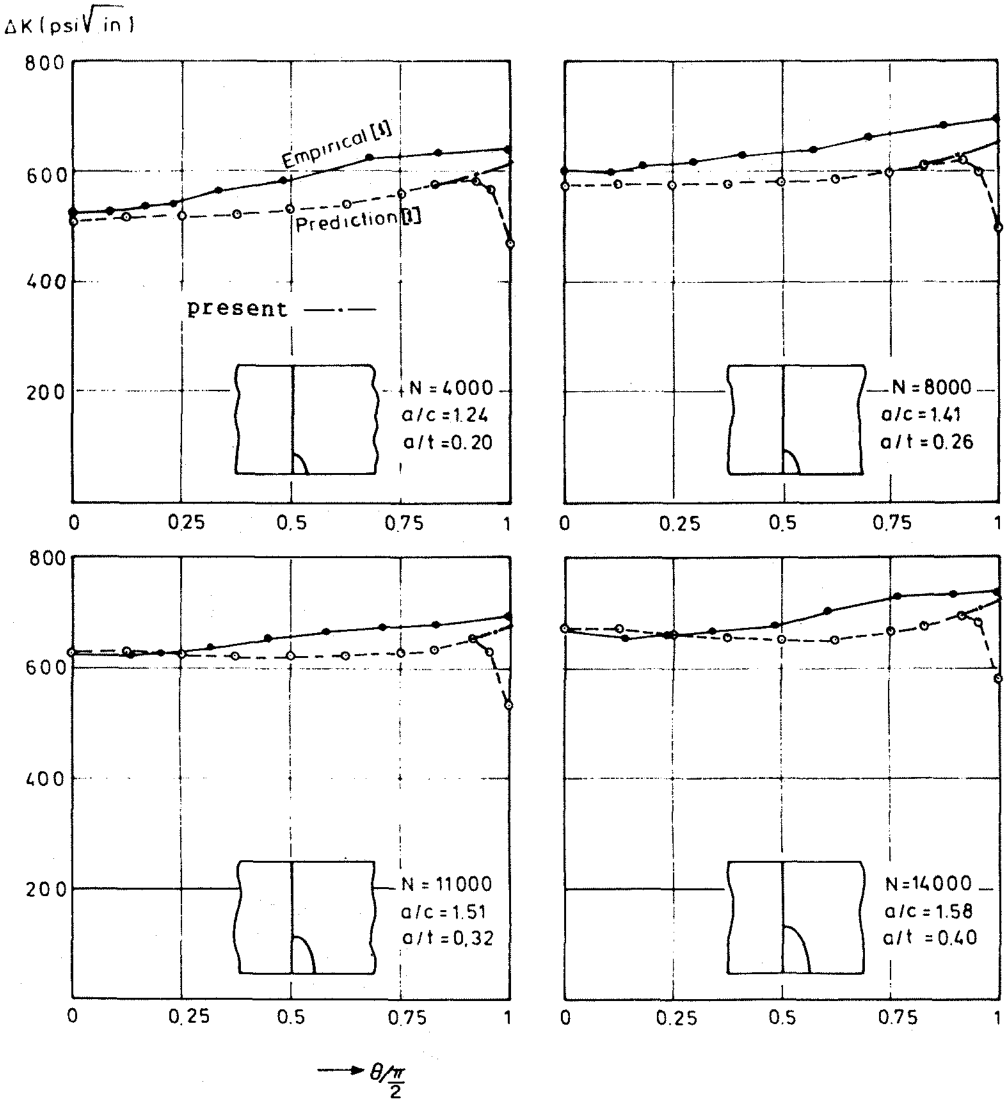

Figure 1b - Comparison Between Empirical and Predicted K-Values for Increasing Crack Size. Extracted from [1]

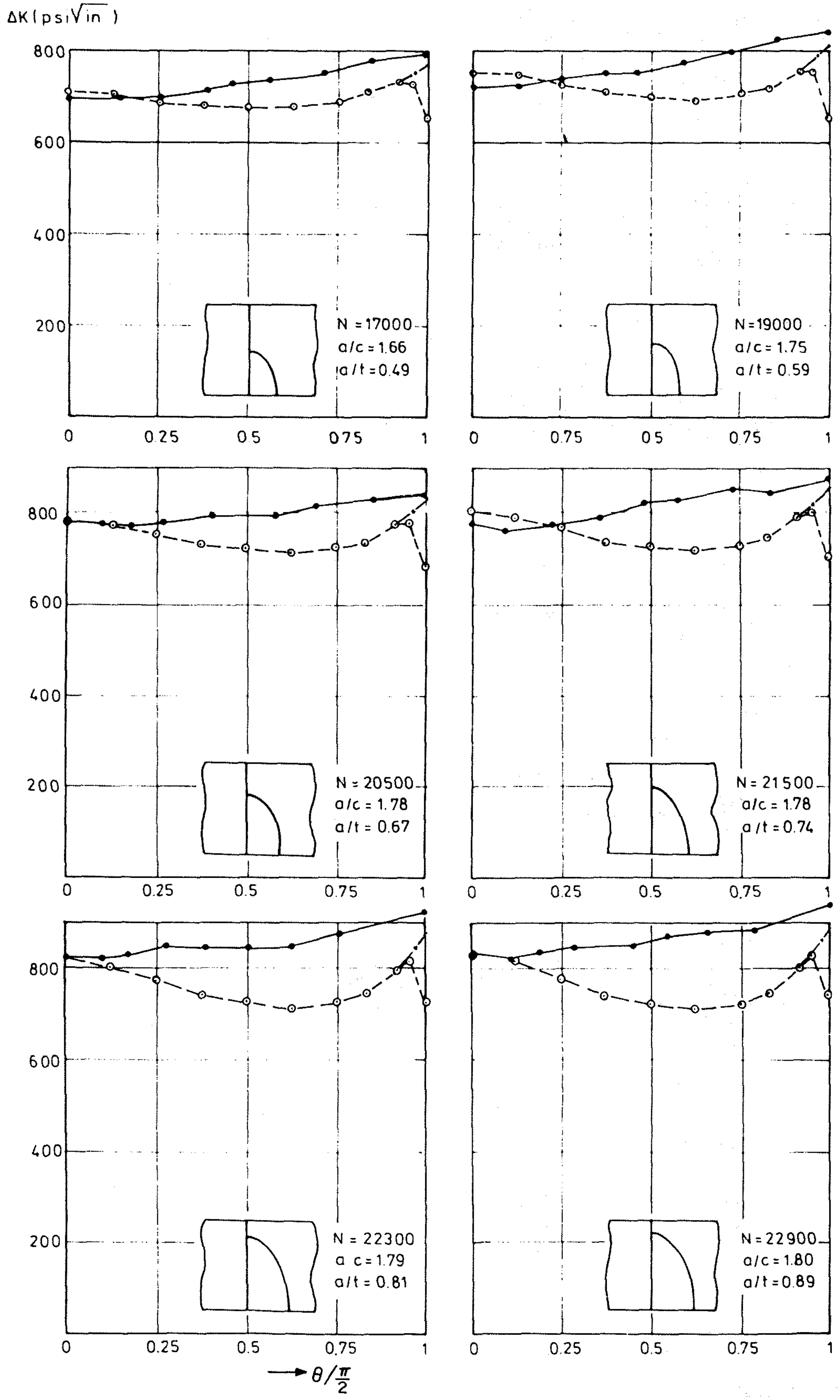

Figure 1b (Cont'd)

II. REVIEW AND DISCUSSION OF DATA AND RESULTS FROM REFERENCE [1]

In the following text both the stress intensity factor K, as well as the range of the stress intensity, ΔK, will be designated by K.

The procedures and details of the systematic empirical derivations of the K factors are presented in [1] and will not be repeated here. The plots of the empirical and calculated K factors are taken from [1] and are presented here in Figure 1b. It has to be noted that each set of empirical K results along the crack front corresponds to an average shape of two actual consecutive crack fronts. However, the calculated corresponding K values are for an exact quarter elliptical crack, curve fitted through two points, corresponding to the average empirical crack shape. (See Figure 1a).
The quarter ellipse is fitted such that it intersects both the plate surface and the hole surface at right angles and its origin is at the intersection of these surfaces. Hence, the calculated and the empirical K values refer to very similar crack geometries, but not identical.

The distribution of the empirical K results along the crack front is relatively smooth and reasonable (Figure 1b). On the other hand, the distribution of the calculated K values exhibits a sudden drop in the vicinity of the crack front intersection with the hole surface. This sharp drop seems unexpected for the following reasons:

1. By analogy to the corresponding embedded elliptical crack subjected to uniform tension, the reduction in K from $\theta = .917\,\pi/2$ to $\theta = \pi/2$ is 0.5%. Here, along the same crack front shape in the quarter elliptical configuration the calculated K drops about 18% between these angles.

2. The stress field, approaching a hole in an uncracked material, increases due to stress concentration; consequently this should increase the K as θ approaches $\pi/2$.

Although this type of sudden drop in K near the surface has been reported by some investigators presenting finite element derived K factors, [7], it has also been indicated that this could be mesh dependent. As the mesh becomes more refined, a higher K value is sustained along the crack front closer to the plate surface [8]. Nonetheless, at the closest point to the surface the numerically (F.E.) calculated K is lower than for points remote from the surface. This could be explained by the analytical study in [9] that shows a change of the 1/SQRT(r) singularity to a lower singularity (for some Poisson values) at the surface itself. This implies a zero value of K at the intersection of the crack front and the free surface. The refined F.E. results seem to support this trend. However, because this drop in K is associated *only* with the surface, this phenomenon should not affect the overall crack growth behavior.

The underestimation (average 22%) of the K calculated on the hole surface in [1] in comparison with the K derived from test, could lead to high error in the analytically predicted crack growth rate. If Paris' formula for crack growth is used with the typical value of m=4 in the equation:

$$\frac{da}{dN} = C(\Delta K)^m \qquad (1)$$

then an underestimation of 22% in K could cause an underestimation of 55% in crack growth rate predictions.

For crack propagation analysis in thin aircraft panels with holes, the crack growth along the skin surface (axis x in the present configuration, Figure 1a) seems to be the important dimension. This is because it will dictate the crack length c. From [1] it appears that the calculated K along the plate surface is in excellent agreement with the test, and it may seem that this is sufficient criteria to calculate the correct crack length in this direction. However, it has to be pointed out that the validity of the predictions (calculation) of crack growth in direction x depends also on the prediction in direction y of the quarter elliptical crack because it may affect the aspect ratio a/c of the crack shape. This could lead to error in the calculation of crack size along the plate surface.

III. EXTRAPOLATION PROCEDURE FOR CALCULATION OF K TO ENHANCE ACCURACY OF FATIGUE CRACK GROWTH ANALYSIS

In view of the previous discussion the following procedure is proposed.

If the K values calculated as in [1] show a trend of sudden reduction near the surface, this surface K value need not be used for fatigue crack growth analysis. It is more realistic to consider the K and the corresponding crack growth calculation at the point, not far removed from the hole surface, which has a local maximum K value. This could either be the second or the third point from the edge of the hole in Figure 1b.

To improve further the correlation between the calculated K and the empirical K at the hole surface, it is proposed in this study to curve fit the K values at a few consecutive points on the crack front approaching the hole surface. The last point to be used for the curve fitting of K as a function of θ will be the closest point to the hole surface that does not exhibit a trend of K reduction. The fitted curve will then be used to extrapolate for a K value on the hole surface, (see Figure 1b), to be utilized for crack growth analysis.

This curve fitting has been performed in the present study; for each crack configuration a parabola in the form of $K(\theta) = A\theta^2 + B\theta + C$ was fitted through three of the K points that were calculated in [1]. The closest point (i) to the hole surface to be used in the curve fitting is the one that does not exhibit a K reduction trend. This point is defined here as the first point from the hole surface that has a maximum slope of (dK)/(dθ), where θ is the parametric angle of the point, measured from the plate surface (the hole edge surface under consideration is at 90 degrees). This implies that at point i:

$$\left(\frac{d^2K}{d\theta^2}\right) = 0 \quad \text{and} \quad \left(\frac{d^3K}{d\theta^3}\right) < 0 \qquad (2)$$

If calculated K values are known numerically at discrete points rather than as analytical function of θ (as in Figure 1b), the point θ(i) can be selected as the closest point to the surface that satisfies:

$$\frac{K(i)-K(i-1)}{\theta(i)-\theta(i-1)} > \frac{K(i+1)-K(i)}{\theta(i+1)-\theta(i)} \qquad (3)$$

where θ(i-1) and θ(i+1) are two neighboring points on either side of θ(i); θ(i+1) being closer to the hole edge surface.

For a totally different K distribution a different criteria may be needed, however, in the present study it appeares that selecting more internal points for curve fitting still gives about the same improvement in the calculated surface K values.

The parabola curve fitting was applied to the ten crack configurations from [1] by utilizing the calculated K values in Table 2 of Ref [1] and considering the slope of the corresponding K plots shown in Fig 1b. The θ(i) selection for each crack using the criteria of equation (3) is shown in Table 3. The K values selected for the curve fitting and the K factors extrapolated from the curve fitting at the hole surface (θ=90°) are shown in Table 1. Also shown in Table 1 are the empirical and calculated hole surface K factors from [1] and their percentage differences. The percentage differences of the present curve fitting K results with respect to the corresponding empirical values are also listed (both as error %).

Note that the second and the third lines of K values in Table 1 represent the same crack but with different points chosen for curve fitting, nevertheless, both selections give almost the same hole surface K results.

The last four lines in Table 1 respresent the last (largest) crack but with various curve fitting selections. The first and second lines for this crack represent two parabola curve fitting but with different sets of 3 fitting points. The resulting K values on the hole surface from these two curves differ only by 2%. The third line represents a cubic polynomial curve fitted through 4 points along the crack front. The resulting K on the hole surface is a half percent higher than that of the first parabola fitted to this crack. The last of these four lines represents a least square parabola fit to this last crack giving a surface K value 0.6% lower than the first parabola fit to this crack.

Overall, the curve fitting for the last crack reduced the 21 percent difference between the calculated K in [1] and the empirical K to about 7 percent. The four different curve fitting selections for this crack gave surface K results that vary only within 2 percent of one another, thus indicating a low sensitivity to point selection and method of extrapolation. However, it appears that a direct curve fitting of a parabola or a cubic polynomial, close to the surface, according to the points selection criteria, equation (3), gives a marginally better K correlation on the hole surface.

As shown in Table 1, the parabola curve fitting K results for the ten cracks have a root mean square (R.M.S.) error of 5% with respect to the empirical K factors. This is a significant improvement compared to the R.M.S. error of 23% of the calculated K values in [1].

Since the calculation of K with the present procedure involves extrapolation (rather than interpolation) an additional extrapolation technique has been used to assure that the results obtained are not sensitive to the method of extrapolation.

The cubic spline method, which is normally used for interpolation [10], has been modified for the extrapolation and allowing for the fact that the boundary conditions at the end points are not known. Hence, a set of five consecutive K values at points on the crack front apporaching the hole surface were selected for the curve fitting. One cubic polynomial was fitted through the first three points and another was fitted through the last three points of this set. Equating the first and second derivatives of both polynomials at the mid point (where they meet) gave the remaining two equations required to solve for the coefficients of both cubic polynomials. The cubic polynomial closer to the hole surface was then used to extrapolate the K value on this surface ($\theta = 90°$), see Figure 1b. The resulting K values and the points selected for the curve fitting are shown on the righthand side of Table 2. Note that the last point for the curve fitting for each crack follows the selection criteria from Table 3. It can be seen from Table 2 that the resulting K values on the hole surface are within 0.8% of the previous parabola fit extrapolation results, but the cubic spline K factors are closer in most cases to the corresponding empirical K values from [1].

The root mean square percent error of the cubic spline results is 4.6% versus 5% R.M.S. error of the parabola curve fit results. Hence, the modified cubic spline gives marginally improved results, and at the same time demonstrates the low sensitivity of the proposed procedure for the method of extrapolation.

To assess the accuracy of the modified spline for extrapolation, each of the cubic polynomials is used to extrapolate the K value of an internal point, on the crack front, where the calculated K values in [1] form a smooth curve. These results are shown in the second K column of Table 2 together with the corresponding calculated K values from [1] in the first column. The location θ of each extrapolation is also indicated. It can be seen that the accuracy of the extrapolation is in all cases within 1.5% of the K values calculated in [1].

On the free plate surface ($\theta = 0$) the agreement between the calculated K in [1] and the empirical K is very good (R.M.S. error of 4%) and there is no need for curve fitting. Also, the distribution of K approaching the plate surface, calculated in [1], is smooth and does not show a steep reduction as that approaching the hole edge, which prompted the application of curve fitting. However, to examine the present procedure, the parabola curve fitting and the extrapolation was applied near $\theta = 0$ and 5% R.M.S. error was found. The extrapolated K values are always conservative with respect to the values calculated in [1]. This is shown in Table 4.

IV. DISCUSSION

The large differences between the K calculated in [1] and the test values can be rationalized in some ways. However, the essence of the curve fitting proposed here is that it provides simple means of using existing finite element K results and extrapolation in order to obtain good prediction of crack growth in the present configurations. It allows the assumption that a quarter elliptical shape is maintained, neither having to consider crack front deviations nor variation of crack growth resistance near the surface, and still obtain good approximation in both directions.

It has to be pointed out that the use of extrapolation to determine stress intensity factors is not uncommon. It is used for example in the determination of K from either the calculated displacements or the photoelastic measurements in the area approaching the crack tip.

Note that the experimental da/dN data and thus the empirically derived K show that the K in the plate depth direction (axis y) along the hole surface is larger than the K along the plate surface (axis x). Yet, the calculated stress intensity factors from [1] give lower K along the hole surface than on the plate surface. The present curve fitting gives higher K on the hole surface than on the plate surface and thus eliminates the above discrepancy. This trend of higher K on the hole surface compared to the plate surface is also evident from the experimental study of a very similar corner crack geometry [11] at a hole in 7075-T6511 Aluminum test specimens. This can be deduced qualitatively from the fact that the data in Table 1 of [11] show that, in general, crack growth rate in direction (y) is higher than in direction (x). However, the ratio of measured crack depth (a) to crack length (c) in [11] is lower than the measured a/c in [1]. This can be atributed to the fact that the material crack growth property power $m = 6$ in [1] is higher than that of the aluminum specimen in [11] which ranges between 3 and 5. This trend is consistent with the prediction of the analytical study in [12].

The proposed curve fitting procedure can be applied to other configurations as well as other materials. However, its results will have to be compared first to the corresponding test data for verification of its applicability. Also, it would be important to establish, by fracture testing of a fatigue grown corner crack, how the K values of the proposed procedure compare with Kc of the material.

It is noted that the largest difference between the empirical K and the calculated K in [1] at the hole surface is for the second smallest crack which is within the interpolation range used in [1]. As pointed out in [1] "It is believed that slight deviations from the elliptical shape can cause significant difference." Such K distribution sensitivity to crack front shape can in fact be observed from [13] where K has been derived by photoelastic methods along the front of corner cracks at nozzle to vessel intersection. Their K distributions, which support the above assumptions are presented in Figure 2 (taken from Reference [13]). It also shows the results presented as normalized $k=K/(\sigma\sqrt{\pi a})$. It is seen that a somewhat flattened crack front (vessel 18) has a K distribution which is substantially different from that of a quarter elliptical crack (vessel 8).

It needs to be pointed out that the quarter ellipse in [1] is fitted through two points that are next to the two surfaces. Hence, the "fictitious" crack point at θ=90° on the quarter ellipse, where the K is calculated, is different from the actual crack point on the surface where the empirical K is derived. On the other hand, near the points where the quarter ellipse is fitted through the actual crack front, a local improvement of the agreement between the calculated K from [1] and the empirical K is observed. This correlation is not investigated at this time.

It is recommended to analyse, with the present method, replicated tests with the same material and then to repeat these tests with different size plates and holes and also with relevant structural materials. This will allow to assess the range of applicability of this method, particularly to other materials.

V. CONCLUSIONS

a. A review is presented of a study by Schijve [1], who used published test data for corner crack growth at a hole, to derive empirical K values along the front and then compared them with K values interpolated between F.E. results.

b. A simple curve fitting procedure is rationalized in the present study for improving correlation between calculated and empirical K values at the crack intersection with the hole surface where a maximum underestimation is reported in [1].

c. The proposed procedure is applied to the calculation of K for all ten cracks presented by Schijve. The average underestimation of the calculated K at the hole surface, which is 22 percent in [1], is reduced by the present procedure to 4 percent.

d. The use of the K values obtained on the hole surface, with the present extrapolation procedure, will improve the accuracy of crack growth calculations.

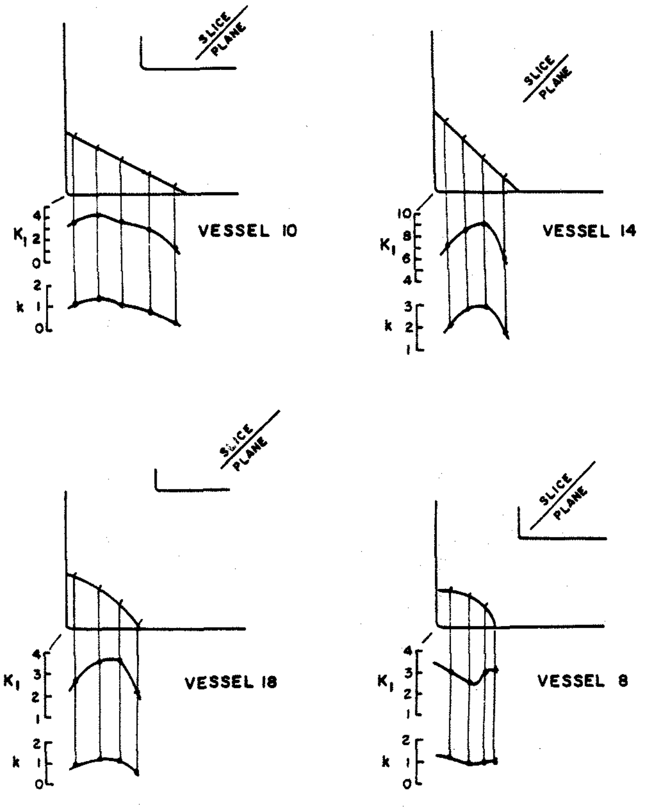

Figure 2 - Variation of K Along Crack Fronts of Various Corner Cracks at Nozzle Junctions

CRACK NO.	Crack Dimensions c (inch)	a (inch)	K VALUES SELECTED FOR CURVE FITTING NEAR HOLE SURFACE (VALUES CALCULATED IN [1]					CALULATED K (REF 1)	CURVE FIT K	EMPIRI-CAL K	ERROR OF CALCULATED K (%) (REF 1)	ERROR OF CURVE FIT K (%)
	θ/π/2 =		.5	.625	.75	.833	.917	1.	1.	1.	1.	1.
1	.115	.143		543	560	574		466	608.7	642	-27	-5.2
2	.130	.183		586	597	607		497	633.6	693	-28	-8.6
2	.130	.183	580	586	597			497	634	693	-28	-8.5
3	.15	.266			632	642	655	534	670.7	693	-23	-3
4	.177	.279			666	679	698	581	722.5	739	-21	-2.2
5	.208	.345			691	709	737	629	774.3	798	-21	-3
6	.236	.413			708	726	758	657	803.2	837	-21.5	-4
7	.265	.471			719	739	776	685	829.1	842	-18.6	-1.6
8	.292	.519			724	747	790	708	852	879	-19.5	-3
9	.316	.567			724	749	796	725	864	919	-21	-6
10	.343	.619			720	749	802	742	877.7	942	-21	-6.8
											RMS=23%	RMS=5%
10				709	720	749		742	860	942	-21	-8.7
10	cubic polynomial curve fit			709	720	749	802	742	883	942	-21	-6.3
10	least square parabola fit			709	720	749	802	742	872	942	-21 (Ref 1)	-7.4

TABLE 1 - PARABOLA CURVE FITTING NEAR HOLE SURFACE

Note: K designates ΔK (psi $\sqrt{\text{inch}}$)

CRACK NO.		CALCULATED K IN [1]		EXTRAPOLATED K		ESTIMATED ERROR (%) OF EXTRA-POLATION PROCEDURE		K FACTORS USED FOR CURVE FITTING (VALUES CALCULATED IN [1])						EXTRA-POLATED K	EMPIRICAL K	ERROR K(%)
	$\frac{\theta}{\pi/2}$ =	.25	.375	.25	.375		$\frac{\theta}{\pi/2}$ =	0.375	0.5	0.625	0.75	0.833	0.917	1.0	1.0	1.0
1		516		521		1.		521	531	543	560	574		607.4	642	-5.4
2		576		574.4		-0.3		577	580	586	597	607		632.6	693	-8.7
3			620		628.6	1.4			620	622	632	642	655	671.1	693	-3.2
4			658		660	0.3			655	656	666	679	698	723	739	-2.2
5			685		683.7	-0.2			679	679	691	709	737	775.5	798	-2.8
6			714		717.8	0.5			703	698	708	726	758	807.9	837	-3.5
7			731		733.3	0.3			716	709	719	739	776	835.2	842	-0.8
8			739		735.7	0.5			722	714	724	747	790	858	879	-2.5
9			742		742.9	0.1			723	713	724	749	796	871	919	-5.2
10			742		733.2	1.2			720	709	720	749	802	882.6	942	-6.3
																RMS=4.6

TABLE 2 - MODIFIED CUBIC SPLINE CURVE FITTING

Crack NO.		$\frac{Ki-K(i-1)}{\theta'i-\theta'(i-1)}$	$\frac{Ki-K(i-1)}{\theta'i-\theta'(i-1)}$	$\frac{Ki-K(i-1)}{\theta'i-\theta'(i-1)}$	$\frac{Ki-K(i-1)}{\theta'i-\theta'(i-1)}$
	$\theta'=\theta/\pi/2=$	.75	.833	.917	.958
1		136	168.6	119	-439
2		88	120.4	119	-439
3			120	154.8	-390.2
4			156	226.2	-268.3
5		96	216.8	333.3	-97.56
6			216	380.9	0.0
7			240.9	440.4	146.3
8			277.1	511.9	268.3
9			269.6	559.5	414.6
10			349.4	630.9	560.9

TABLE 3 - POINTS SELECTION FOR CURVE FITTING NEAR HOLE SURFACE

Note: K designates ΔK (psi $\sqrt{\text{inch}}$)

	K VALUES SELECTED FOR CURVE FITTING NEAR PLATE SURFACE			CALCULATED K (REF 1)	CURVE FIT K	EMPIRICAL K (REF 1)	PERCENT ERROR OF CALCULATED K (REF 1)	PERCENT ERROR OF CURVE FIT K (%)
$\theta/\pi/2$ =	.375	.25	.125	0.0	0.0	0.0	0.0	0.0
Crack No.								
1	521	516	516	509	521	528	-3.6	-1.3
2	577	576	580	575	589	606	-5.0	-2.8
3	620	623	629	627	638	621	1.0	2.7
4	658	663	673	672	688	660	1.8	4.2
5	685	694	707	709	724	694	2	4.3
6	714	730	749	755	771	723	4.4	6.6
7	731	751	776	786	806	782	0.5	3
8	739	763	792	806	826	744	8.3	11
9	742	771	805	822	844	831	-1	1.6
10	742	776	814	835	856	836	-0.1	2.4
							RMS=4%	RMS=5%

TABLE 4 - PARABOLA CURVE FITTING NEAR PLATE SURFACE

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