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ON THE COMPENSATION OF THE PHUGOID MODE INDUCED BY INITIAL CONDITIONS AND WINDSHEARS+

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Abstract

The present communication addresses a sequence of four problems, and justifies the following conclusions: (i) the aerodynamic forces and moments acting upon an aircraft in a perturbed atmosphere, can be calculated from the airspeed vector along each point of the flight path, i.e., the local wind profile does not matter, except for an aircraft flying in the wake of another; (ii) the effects of atmospheric disturbances on flight performance, viz., velocity and incidence changes and vertical accelerations in straight and level flight, can be specified by a single dimensionless parameter, the disturbance intensity, e.g. it indicates if a stall could occur at take-off or landing; (iii) the phugoid mode excited by windshears can be suppressed (not just damped) by a suitable pitch control schedule, which keeps the aircraft on a constant glide slope; (iv) the suppression of the phugoid mode is also relevant to an aircraft starting a dive or climb at initial velocity far removed from the steady flight speeds, in which case the relevant pitch control law is found as a solution of a non-linear stability problem.

I. Introduction

One of the natural motions of an aircraft is the phugoid mode, which is usually countered by introducing damping, e.g. an autothrottle or pitch-stiffening, so as to cause the phugoid motion to decay. In the present paper we consider a different approach, namely, the cancellation of the phugoid mode, by using pitch control to prevent the phugoid motion from occurring. This may designated as the 'inverse phugoid problem', taking Lanchester's [1] famous researches as the reference. In the original phugoid problem [2] the aircraft flies at approximately constant incidence, along a curved flight path, exchanging kinetic and potential energies in an oscillatory manner. In present inverse phugoid problem a pitch control schedule is determined, which keeps the aircraft in a straight flight path, exactly cancelling the phugoid mode. Two kinds of excitation of the phugoid mode should be considered: (V) the case of an aircraft starting a climb (or dive) at a velocity distinct from the steady flight speed, i.e. the phugoid would be excited by initial conditions and cancelled by a suitable pitch control law; (IV) the case of an aircraft in a climb (or dive) at the steady flight speed encountering winds, e.g. the phugoid would be excited by a windshear and cancelled by an appropriate pitch control law. Both of these are response problems in pitch, the former (V) non--linear if the initial velocity is far removed from the steady flight speed. The cancellation of the phugoid mode in a windshear is a linear problem (IV) if the wind velocity is not more than 30% of the groundspeed. As a preliminary to this problem of aircraft response to atmospheric disturbances, we compare aircraft performance in a still and a perturbed atmosphere (III). Both the performance (IV) and response (V) problems take as an input the aerodynamic forces on an aircraft flying in a perturbed atmosphere, and a brief discussion of these is presented first (II).

II. Effects of non-uniform wind on aerodynamic forces

An aircraft flying in still air is subject to [3] an aerodynamic force:

$$\vec{F} = \frac{1}{2}\rho S V^2 \{-C_D \vec{e}_x + C_Y \vec{e}_y + C_L \vec{e}_z\}, \qquad (1)$$

where ρ, S, V have the usual meaning (mass density, reference area and velocity), and C_D is the drag coefficient if the x-axis lies tangent to the flight path, C_L is the lift coefficient if the z-axis lies upward in the plane of symmetry of the aircraft, and C_Y is the sideforce coefficient. In the presence of an uniform wind, the aerodynamic force is still given by (1), provided that $\vec{V}$ be interpreted as the airspeed, i.e. the sum of groundspeed $\vec{U}$ and wind velocity $\vec{v}$. If the incident flow is not uniform, then it is characterized [4] by the dilatation $\nabla.\vec{V}$ and vorticity $\nabla \wedge \vec{V}$. for flight at low-speed, when windshears are of most concern, we can neglect compressibility $\nabla.\vec{V} = 0$, and need consider only the vorticity:

$$\nabla \wedge \vec{V} = (\partial V_z/\partial y - \partial V_y/\partial z)\vec{e}_x + (\partial V_x/\partial z - \partial V_z/\partial x)\vec{e}_y + (\partial V_y/\partial x - \partial V_x/\partial y)\,\vec{e}_z, \qquad (2)$$

which gives rise [5] to a vortical force, scaling as:

$$\vec{H} \sim \rho\, \vec{V} \wedge (\nabla \wedge \vec{V}) = \rho\{V_y(\partial V_y/\partial x - \partial V_x/\partial y) - V_z(\partial V_x/\partial z - \partial V_z/\partial x)\}\,\vec{e}_x + \rho\{V_z(\partial V_z/\partial y - \partial V_y/\partial z) - V_x(\partial V_y/\partial x - \partial V_x/\partial y)\}\,\vec{e}_y + \rho\{V_x(\partial V_x/\partial z - \partial V_z/\partial x) - V_y(\partial V_z/\partial y - \partial V_y/\partial z)\}\,\vec{e}_z. \qquad (3)$$

The fundamental question which arises [6] is: do we need to consider the vortical force (3) in addition to the usual aerodynamic force (1), when considering aircraft in a non-uniform wind?

We address this question in the case of lift, which is given by (1) in an uniform wind [7]:

$$L = \frac{1}{2}\rho\, C_L\, V^2\, S, \qquad (4)$$

and in the presence of vertical shear *(Figure 1)* is modified by a vortical contributions [8]:

$$\Delta L = \frac{1}{2}\rho\, C_S\, S V\, \Omega/c, \qquad (5)$$

where $\Omega \equiv dV/dz$ is the vorticity, c the chord and C_S the shear coefficient. In the case of an uniform wind $dV/dz = 0$, there is no vortical contribution to lift $\Delta L = 0$ in (5), and the lift force (4) is determined by the dynamic pressure acting on the reference area. In the case of a wind increasing upwards *(Figure 1, top)* the airspeed is greater above than below the airfoil, implying a reduction in pressure above and a increase in pressure below, which causes extra lift $\Delta L > 0$; this agrees with the expression (5) for vortical lift, since $\Omega = dV/dz > 0$ in this case. Conversely, if the wind increases downwards *(Figure 1, bottom)* there is a lift loss. The magnitude of the lift loss or gain due to vorticity in the incident stream (5), relative to the aerodynamic lift in an uniform wind (4), is given by:

$$(\Delta L)/L = N\ C_S/C_L, \tag{6}$$

where:

$$N \equiv c\Omega/V, \tag{7}$$

is a dimensionless shear parameter, comparing the airspeed V to the velocity change due to vorticity Ω over a chord length c. Having shown qualitatively that vorticity in the incident stream can affect lift, we should now check whether that effect is quantitatively important.

In order to estimate the shear parameter (7) we note that the vorticity $\Omega \sim u/\ell$ scales on the wind speed divided by a scale ℓ, so that:

$$N \sim (u/V)\ \ (c/\ell), \tag{8}$$

the shear parameter (8) scales on the wind speed u divided by airspeed, and the number of flow scales in a chord length, i.e., is greater for a large aircraft flying at low speed, e.g. a jet transport in take-off or landing configuration. Atmospheric measurements at airports [9] suggest that wind speed changes of $u \sim 5\text{-}75$ m/s occur over scales $\ell \sim 30$ m, leading to vorticity values in the range $\Omega \sim u/\ell \sim 0.2\text{-}2.5\ s^{-1}$; for a large transport aircraft, with a wing chord $c = 3$m, flying at low-speed $V = 60$ m/s, this corresponds to a small value $N = \Omega c/V = 1/100\text{-}1/8$ of the shear parameter. In the wakes of propellers, the velocity profile may be given by:

$$u(z) = V\{1 + N(z/c)\}, \tag{9}$$

where $N = (c/V)\ du/dz = c\Omega/V$ is the shear parameter, which may take values $N \sim 5$ much larger than for the case of atmospheric disturbances. The preliminary conclusion is that the non-uniformity of incident wind is a small effect, unless an aircraft is flying in the wake of another.

The latter conclusion was based on the shear parameter (8) alone, and should be checked by considering the relative lift change due to non-uniformity of the incident stream (6), which involves both the lift C_L and shear C_S coefficients. The lift coefficient depends on body shape and attitude, and is calculated for an uniform incident stream; in flight away from the stall, it is a linear function of effective incidence $\bar{\alpha} = \alpha - \alpha_0$, which is the incidence α relative to the angle of zero lift α_0; as an order of magnitude, we note that the lift slope $\partial C_L/\partial\bar{\alpha}$ is about 2π for a Joukowski airfoil [11]. The shear coefficient C_S depends not only on body shape and attitude, but also on the profile of the incident wind [12]. For example, for a wing in a linearly sheared wind (9), the vortical lift is given [8] by:

$$\Delta L = \pi\ \rho\ c\ \ V\Omega, \tag{10}$$

corresponding by (5) to a shear coefficient:

$$C_S = 2c\ \Delta L/\rho\, S\, V\,\Omega = 2\pi c^2/S = 2\pi/\Lambda, \tag{11}$$

where $\Lambda = S/c^2 = b/c$ is the aspect ratio, with $b = S/c$ denoting the span; in the case of parabolic or other velocity profiles of the incident stream, the shear coefficient would be modified. Also, the shear coefficient for a two-dimensional body, e.g. $C_S = 2\pi = 6.28$ for a cylinder, is larger than that for a corresponding three-dimensional body, e.g. $C_S = 4.4\pi/3 = 4.61$ for a sphere [13]; the reason is that in a vortical flow past a two-dimensional body the vortex lines must be stretched, whereas they can be partially deflected past a three-dimensional body, reducing the deformation of vortex lines.

The comparison of the lift $C_L \sim 2\pi\bar{\alpha}$ and shear $C_S \sim 2\pi/\Lambda$ coefficients suggests that their ratio $C_L/C_S \sim \Lambda\bar{\alpha}$ generally **is** of order unity, and thus the conclusion concerning shear number (8) extends to vortical lift (6); similar arguments could be used to estimate the effects of a non-uniform incident stream, on other components of the aerodynamic force, or on the aerodynamic moments. The overall conclusion is: for an aircraft flying in a perturbed atmosphere, the aerodynamic forces and moments may be calculated from the airspeed, including the wind velocity at each point of the trajectory, and disregarding the profile of the wind; the latter can become important for an aircraft flying in the wake of another.

III. Comparison of flight performance in still and perturbed atmospheres

The preceding conclusion implies that an aircraft flying with groundspeed U in an atmosphere with longitudinal wind u and vertical wind w, has an airspeed V and incidence θ given respectively by:

$$V = \sqrt{(U+u)^2 + w^2}, \quad \theta = \alpha + \arctan\{w/(U+u)\}, \tag{12a,b}$$

and hence experiences a lift:

$$L^* = \frac{1}{2}\, C_L(\theta)\ \rho\, S\, V^2. \tag{13}$$

The ratio of lifts in the perturbed (13) and still (4) atmosphere is:

$$L^*/L = (\theta/\alpha)\ \ (V/U)^2, \tag{14}$$

where we have assumed that the lift coefficient is a linear function of incidence for flight away from the stall. If the wind is 'moderate', in the sense that it does not exceed 30% of the groundspeed $u, w \leq 0.3\ U$, we can neglect squares $u^2, w^2 \leq 0.09 \ll U^2$, and (14) simplifies to:

$$L^*/L = (1 + w/\alpha U)\ \ (1 + 2u/U). \tag{15}$$

If we define [14] the disturbance intensity G as the relative lift change:

$$G \equiv L^*/L - 1 = 2u/U + w/\alpha U, \tag{16}$$

it is clear that lift increases $G > 0$ in a headwind $u > 0$ and upflow $w > 0$, and decreases in a tailwind $u < 0$ and downflow $w < 0$ (for a given groundspeed U and positive incidence).

The simplest application is an aircraft flying straight and level in still air, for which the lift (4) balances the weight:

$$W = \frac{1}{2} \rho S V^2 C_L(\bar{\alpha}), \tag{17}$$

in still air. In the presence of atmospheric disturbances the lift L changes to L* given by:

$$L^* = (1+G)\ L = \frac{1}{2} \rho S V^2\ C_L(1+G), \tag{18}$$

i.e., this is equivalent to replacing the lift coefficient C_L in still air by a value

$$C_L \to C_L\ (1+G), \tag{19}$$

corrected by the disturbance intensity (16) due to wind. The atmospheric disturbances generally change the aircraft velocity and incidence respectively to V^* and α^*, and may cause a vertical acceleration A, so that the balance of transverse forces (17) is replaced by:

$$W + m A = \frac{1}{2} \rho S V_*^2\ C_L(\alpha_*)\ \ (1+G), \tag{20}$$

where $m = W/g$ is the aircraft mass. The ratio of (20) to (17), with lift a linear function of incidence, yields the formula:

$$1 + A/g = (V_*/V)^2\ \ (\alpha_*/\bar{\alpha})\ \ (1+G), \tag{21}$$

relating aircraft flight performance to the atmospheric disturbance intensity G. For example, if the aircraft flies at constant velocity $V = V_*$ and incidence $\alpha = \alpha_*$ through the atmospheric disturbance, it experiences a vertical acceleration $A = G g$, expressed in g's, equal to the disturbance intensity (or relative lift change). This gives a flight dynamical interpretation of the disturbance intensity, which had been defined (16) in aerodynamic terms.

There are other alternative interpretations, e.g., if the aircraft remains on a straight and level flight path $A = 0$, at constant incidence $\alpha_* = \alpha$, in an atmospheric disturbance of intensity G, the velocity changes from V to V_* given by:

$$1 = (V_*/V)^2\ \ (1+G), \tag{22}$$

viz., a headwind or upflow $G > 0$ allows straight and level flight at a given incidence at a lower speed $V_* < V$, whereas a tailwind or downflow $G < 0$ must be compensated by an increase in velocity $V_* > V$. The result also applies to stalling speeds:

$$V_S^* = V_S/\sqrt{1+G}, \tag{23}$$

and shows that a positive disturbance (headwind or upflow) reduces the stalling speed, whereas a negative disturbance increases the stalling speed, as shown in *Figure 2*. A disturbance of intensity $G_1 = -0.17$ raises the stall speed by 10%, to the unstick speed $V_1 = 1.1\ V_S$, causing an aircraft to stall on take-off. A disturbance of intensity $G_2 = -0.42$ raises the stall speed by 30%, to the approach speed $V_2 = 1.3\ V_S$, causing an aircraft to stall prior to landing. The values of the atmospheric winds which, if uncompensated, can cause stall on take-off (or landing), can be calculated from (16), for the critical disturbance intensities $G_1 = -0.17$ ($G_2 = -0.42$); for example, for an aircraft taking off at $U = 60$ m/s in still air at an incidence $\alpha = 10^0$, a downflow $w = -1$ m/s and an uncompensated tailwind of $u = -2.5$ m/s, leads to a disturbance intensity $G = -0.178$, which will cause a stall at unstick speed.

The atmospheric disturbance may give rise to an incidence change, at constant velocity in straight and level flight:

$$\alpha_*/\bar{\alpha} - 1 = (1+G)^{-1} - 1 \simeq -G, \tag{24}$$

which is equal to -G for small values of the disturbance intensity $G^2 \ll 1$. Thus the disturbance intensity equals minus the relative change in incidence, in straight and level flight, at constant velocity, through the disturbance, i.e., a positive disturbance (headwind or upflow) requires a reduction in incidence, and a negative disturbance (tailwind or downflow) is compensated by an increase in incidence. The change in incidence $\Delta\alpha = \alpha^* - \alpha$ required to compensate a disturbance of intensity G, is indicated in *Figure 3*, for three values of the initial effective incidence. We have considered the cases where the atmospheric disturbance is compensated by (i) vertical acceleration $A = Gg$, (ii) incidence (24) or (iii) velocity (23) changes in isolation. Generally all three occur simultaneously, so that no simplification of (21) is possible; it can be re-written:

$$(V/V_*)^2\ \ (1 + A/g) = 1 + h, \tag{25}$$

where

$$h \equiv (1+G)\,(\alpha_*/\bar{\alpha}) - 1 \simeq G + \Delta\alpha/\bar{\alpha}, \tag{26}$$

is the disturbance intensity corrected for incidence change. The ratio of velocities in the presence V_* and absence V of atmospheric disturbance, and the vertical acceleration in g's, are plotted in *Figure 4*, for several values of the corrected disturbance intensity.

The velocity V is the groundspeed if the disturbance intensity (16) includes both the longitudinal and transverse wind; if only the second term, i.e. the transverse wind is included, then V should be interpreted as the airspeed. As an example, consider an aircraft on approach to land at a groundspeed $V = 60$ m/s and effective incidence $\bar{\alpha} = 10^0$; suppose it encounters a tailwind $u = -5.5$ m/s and downflow $w = -2$ m/s, corresponding (16) to a disturbance intensity $G = -0.38$. Assume that incidence is raised to the stall limit $\alpha_* = 12^0$, and the groundspeed has dropped to $V_* = 50$ m/s, as a consequence of pilot reaction to the earlier, headwind phase of windshear. The vertical acceleration (20) is $A = -0.48$ g $= -4$ m/s^2, and if the disturbance acts for two seconds $t = 2$ s, the sink rate will have increased by $\Delta V_z = At = 9.4$ m/s, which is more than an undercarriage is designed for; the height loss $Z = -At^2/2 = -9.4$ m, for a glide slope of 3^0, corresponds to an error of $X = -9.4 \cot 3^0$ m $= 310$ m in touch down point, i.e., the runway could be missed.

IV. Flight on a constant glide slope through a windshear

The preceding performance calculations may give an indication of the order of magnitude of the effect of atmospheric disturbances on aircraft flight, but they do not replace a response calculation, which is appropriate to an unsteady phenomenon like a windshear. Research on this topic has concerned mainly the prediction or simulation of flight path deviations of an aircraft as it flies through a windshear; we would like to consider the inverse problem, of finding the pitch control schedule which would keep *(Figure 5)* the aircraft on a constant glide slope through the atmospheric disturbance. If, and only if, this control scheme is achievable, can the disturbance be fully compensated; in fact, we will give only a partial answer to this problem, since we will consider the compensation of the phugoid mode alone, leaving the short period mode to future work. For an aircraft flying on a constant glide slope γ, the inertia force is equal to thrust, plus the component of weight along the flight path minus drag:

$$m\, dU/dt = W \sin\gamma + T(U) - D, \qquad (27)$$

where the equation is written in terms of the groundspeed U, for flight in still air. The component of weight transverse to the flight path is balanced by the lift $L = W\cos\gamma$, and dividing (27) by W we obtain:

$$g^{-1}\, dU/dt = \sin\gamma - T(U)/W - \cos\gamma\;\; D/L, \qquad (28)$$

where $T(U)/W$ is the thrust-to-weight ratio at velocity U, and the drag D to lift L ratio is given by:

$$D/L = C_D/C_L = C_{Df}/C_L(U) + k\; C_L(U) + K, \qquad (29)$$

where C_{Df} is the form drag coefficient, and k relates the induced drag coefficient to the square of lift coefficient and K applies to a non-symmetric polar. The constancy of lift implies that the lift coefficient varies inversely to the square of velocity:

$$C_L(\bar\alpha)\; U^2 = C_L(\bar\alpha_o)\; U_o^2 \cos\gamma, \qquad (30)$$

with the constant calculated for a reference level flight condition at the same weight.

Substitution of (29,30) into (28) leads to:

$$g^{-1}\, dU/dt = F(U) = f - aU^2 - b/U^2, \qquad (31)$$

where F(U) is a dimensionless force, and the coefficients are given by:

$$f = f_o + \sin\gamma - K\cos\gamma,\quad a = f_1 + \{C_{Df}/C_L(\bar\alpha_o)\}\; U_o^{-2}$$

$$b = f_2 + k\; C_L(\overline{\alpha_o})\; U_o^2 \cos^2\gamma, \qquad (32a,b,c)$$

assuming that the thrust characteristic is:

$$T(U) = \{f_o - f_1\; U^2 - f_2\; U^{-2}\}\, W\,. \qquad (33)$$

For flight in a perturbed atmosphere the groundspeed U_* satisfies (31):

$$g^{-1}\; dU_*/dt = F(V), \qquad (34)$$

with the total force calculated from the airspeed V, and thus including wind effects. The perturbation in groundspeed $q = U_* - U$ due to wind, satisfies:

$$g^{-1}\; dq/dt = g^{-1}\; d(U_* - U)/dt = F(V) - F(U)\,; \qquad (35)$$

for a moderate wind $u^2, w^2 << U^2$ the groundspeed perturbation is also moderate $q^2 << U^2$, and (35) may be linearized:

$$U^{-1}\; dq/dt = \lambda(V - U) \qquad (36)$$

where the coefficient λ may be calculated from the unperturbed velocity:

$$\lambda \equiv g\; U^{-1}\; dF/dU = -2(a - b/U^4)\; g. \qquad (37)$$

We may also use:

$$V - U = (V - U_*) + (U_* - U) = q + u, \qquad (38)$$

since for a moderate wind, the difference between airspeed and groundspeed is the longitudinal wind velocity.

Thus the groundspeed perturbation q(t) satisfies the differential equation:

$$dq/d\xi = \lambda(q + u), \qquad (39)$$

where $U = d\xi/dt$ with ξ the coordinate along the flight path. The constancy of lift:

$$C_L(\bar\alpha)\; U^2 = C_L(\bar\alpha_* + w/U_*)\;\; \{U_*^2 + 2uU_*\}, \qquad (40)$$

for lift linear on the effective incidence $\bar\alpha$, specifies the change of incidence $\bar\alpha_* - \bar\alpha$ due to the windshear:

$$\bar\alpha_* - \bar\alpha = -\{w + 2\bar\alpha(u + q)\}/U, \qquad (41)$$

where the q is the solution of (39) and the longitudinal u and vertical w wind are assumed known. Typical simplified profiles for a windshear [15-18] are: (i) a longitudinal wind changing from a head- to a tailwind:

$$u(t) = A\; U\; \sin(2\pi\xi/\ell), \qquad (42)$$

over a scale ℓ; (ii) a downflow, peaking at the transition from headwind to tailwind:

$$w(t) = -B\; U\bar\alpha\; \sin(\pi\xi/\ell). \qquad (43)$$

In (42,43) the constant $A, B\bar\alpha$ are the amplitude of the longitudinal, transverse wind relative to the unperturbed velocity U.

If the perturbation of groundspeed $q(0) = 0$ starts at $x = 0$, with the windshear, the solution [19] of (39) with u given by (42), is:

$$P(X) = \{A/(1 + \mu^2)\}\{\mu(1 - \cos(2\pi X)\} - \sin(2\pi X)\}, \qquad (44)$$

where the groundspeed is made dimensionless by dividing to the unperturbed velocity (45a), and the distance along the flight path is divided by the shear length:

$$P \equiv q(\xi)/U, \quad X \equiv \xi/\ell; \qquad (45a,b)$$

the aerodynamics of the aircraft appears through the parameter μ defined by:

$$\mu \equiv 2\pi/\lambda\ell = -\pi/\{g\ell(a - b/U_o^4)\} = \pi U_o^2/kg\ell\;\; C_L(\bar\alpha_o)\cos^2\gamma, \qquad (46)$$

where we have used (32b,c), with the simplification of thrust independent of velocity $f_1 = 0 = f_2$ in (33) and form drag much smaller than induced drag, both of which apply approximately for an aircraft in the landing configuration, when γ is small and the lift coefficient is about unity; using $k = 1/\pi\Lambda$ for the polar coefficient, (46) simplifies to $\mu = \pi^2 U^2 \Lambda / / g\ell \sim U^2\Lambda/\ell$. Taking a windshear length $\ell = 2000$ m, the susceptibility parameter μ takes typical values: (i) $\mu = 1.2$ for a light aircraft (approach speed $U = 20$ m/s, wing aspect ratio $\Lambda = 5$); (ii) $\mu = 5.4$ for a jet fighter ($U=60$ m/s, $\Lambda = 3$); (iii) $\mu = 10$ for a large jet transport ($U = 50$ m/s, $\Lambda = 8$). The susceptibility parameter also appears in the dimensionless airspeed perturbation:

$$Q(X) \equiv \{q(\xi) + u(\xi)\}/U = P(X) + A \sin(2\pi X), \quad (47)$$

which is the sum of the groundspeed (44) and longitudinal wind (42); the transverse wind (43) appears in:

$$R(X) = \bar{\alpha}_*(\xi)/\bar{\alpha} - 1 = -2\, Q(X) - B \sin(\pi X), \quad (48)$$

which is the relative change in incidence required to exactly cancel the phugoid mode induced by the windshear.

We have plotted in *Figure 6* the phugoid mode compensation curves, for a windshear with longitudinal wind amplitude $A = 0.3$ of the approach speed, and transverse wind amplitude corresponding to a decrease of incidence by a factor $B = 0.1$. The curves show that, for a large jet transport $\mu \simeq 10$, incidence (bottom) must be decreased in the headwind phase and increased in the tailwind phase, giving (middle) a large initial increase in airspeed followed by a decrease in airspeed, leading (top) to small groundspeed change overall. As the susceptibility parameter becomes smaller, for a jet fighter $\mu \sim 5$ or a light aircraft $\mu \sim 1$, the exact compensation of the phugoid mode involves (top) an increase in groundspeed in the tailwind phase of the windshear, implying (middle) an increase in airspeed in the headwind phase, and (bottom) a delayed change from pitch-down to pitch-up.

V. Straight flight path from an arbitrary initial velocity

Cancelling the phugoid mode, so as to keep on a straight flight path, is an objective not only in the case of an approach to land through a windshear, but also in other situations of interest in aviation. For example, an aircraft which starts a dive towards a target of opportunity, should remain on a constant glide slope for accurate tracking; in the case of emergency landing in a clearing in otherwise cluttered ground, it is important not only to keep on a straight approach path but also to stabilize the velocity. In both cases, if the dive is started at a velocity distinct from the steady flight speeds, the compensation of the phugoid mode will require a suitable pitch control schedule, with the incidence changing as velocity approaches the steady flight speed along the constant slope. If the initial velocity is far removed from the steady flight velocity, this is a non-linear stability problem, which requires an exact solution of the longitudinal equation of motion [20-22], in the present case (31). The linearization of the problem is possible only if the initial velocity is close to the steady flight speed. In order to overcome this limitation, we solve first the non-linear problem, and then consider the linear approximation as a particular case.

The steady flight speed(s) in a dive correspond to zero acceleration or total force in (31), i.e., are roots of the bi-quadratic expression:

$$0 = aU^4 - fU^2 + b; \quad (49)$$

the roots are real and positive:

$$U_\pm^2 = (f \pm \Delta)/2a, \quad \Delta^2 \equiv f^2 - 4ab, \quad (50a,b)$$

if the discriminant (50b) is non-negative. The implication is that steady flight is possible only if (50b) is non-negative, e.g. the minimum thrust for straight and level flight is given by $f^2 = 4ab$, which implies, from (32a,b,c) with $f_1 = 0 = f_2$:

$$(f_o + \sin\gamma - K \cos\gamma)^2 = 4k\, C_{Df} \cos^2\gamma. \quad (51)$$

In the case of level flight $\gamma = 0$ and a symmetrical lift-drag polar $K = 0$, this leads to the usual expression $f_o \equiv T/W > 2\sqrt{C_{Df}\, k}$ for the minimum thrust-to-weight ratio required to sustain steady flight [23]; the formula (51) extends this result to a non-symmetric polar $K \neq 0$ and a dive $\gamma > 0$ or climb $\gamma < 0$. The condition $\Delta = 0$ specifies the minimum drag velocity:

$$U_{md}^2 = f/2a = U_o^2 (f_o + \sin\gamma - K \cos\gamma) \quad C_L(\bar{\alpha}_o)/2C_{Df}, \quad (52)$$

in a dive; in level flight $\gamma = 0$ for a symmetric polar $K = 0$, we have $U_{md} = U_O$ since at the minimum drag condition the form and induced drags are equal $C_{Df} = k\, C_L^2$. In general:

$$U_{md}^2 = (U_+^2 + U_-^2)/2, \quad (53)$$

the minimum drag speed squared is the arithmetic mean of the two steady flight speeds, implying that the higher steady speed is closer to the minimum drag speed than the lower steady speed $U_+ - U_{md} < U_{md} - U_-$.

The total force (31) is given in terms of the steady flight speeds by:

$$F(U) = -a(U^2 - U_+^2)(U^2 - U_-^2)/U^2, \quad (54)$$

and causes an acceleration $F > 0$ for velocities between the steady speeds $U_- < U < U_+$, and a decceleration $F < 0$ outside $U < U_-$, or $U > U_+$. The implication is *(Figure 7)* that the higher steady flight speed is stable, because a velocity perturbation below (above) it is followed by acceleration (decceleration) towards it; it is seen in the same way that the lower steady flight speed is unstable. The acceleration (31) can be puted into the form:

$$dU/dt = (dU/d\xi)(d\xi/dt) = U\, dU/d\xi, \quad (55)$$

where ξ is the coordinate along the flight path. The equation of motion (31), in the form:

$$d(U^2)/d\xi = -2ga(U^2 - U_+^2)(U^2 - U_-^2)/U^2, \quad (56)$$

is integrated readily [24] for the variable U^2, viz.:

$$\exp(-2ga\xi) = \left[\frac{U(\xi)^2-U_+^2}{U_o^2-U_+^2}\right]^{1/(1-U_-^2/U_+^2)} \times$$

$$\times \left[\frac{U(\xi)^2-U_-^2}{U_o^2-U_-^2}\right]^{1/(U_+^2/U_-^2-1)} \quad (57)$$

where $U_o = U(0)$ is the initial velocity. The formula (57) gives velocity as a function of distance, e.g. at a long distance $\xi \to \infty$, either $U \to U_+$ or $U - U_- \to \infty$, i.e. the velocity either approaches the upper U_+ steady flight speed, which is stable, or diverges from the lower steady flight speed U_-, which is unstable.

We could determine similarly [24] velocity as a function of time $U(t)$, or the effective angle of incidence as a function of time $\theta(t)$ or distance $\theta(\xi)$. The latter is the pitch control law which exactly balances the phugoid mode induced by initial conditions. All these profiles are interrelated, e.g. for flight away from the stall the lift coefficient is a linear function of effective incidence $\bar{\alpha} \sim C_L(\theta) \sim U^{-2}$ and varies (30) like the inverse square of velocity thus (57) implies the following pitch control law:

$$\exp(-\theta/\ell) = \left[\frac{1/\theta(\xi) - 1/\theta_+}{1/\theta_o - 1/\theta_+}\right]^{1/(1-\theta_+/\theta_-)} \times$$

$$\times \left[\frac{1/\theta(\xi) - 1/\theta_-}{1/\theta_o - 1/\theta_-}\right]^{1/(\theta_-/\theta_+ - 1)}, \quad (58)$$

where distance along the flight path is measured on the scale $\ell = 2ga$. In the initial stages of the perturbation, i.e., when the velocity and incidence are still close to the initial value and far from the steady value:

$$(U^2 - U_o^2) << (U_\pm^2 - U_o^2)^2, \quad (\theta - \theta_o)^2 << (\theta_\pm - \theta_o)^2, \quad (59a,b)$$

the velocity (57) and pitch (58) control laws simplify respectively to:

$$\{U(\xi)\}^2 = U_o^2\{1 + (U_+^2/U_o^2 - 1)(1 - U_-^2/U_o^2)(1 - e^{-\xi/\ell})\}, \quad (60a)$$

$$\theta(\xi) = \theta_o\{1 + (1 - \theta_o/\theta_-)(\theta_o/\theta_+ - 1)(1 - e^{-\xi/\ell})\}; \quad (60b)$$

for $\xi = 0$ we have $U(0) = U_o$ and $\theta(0) = \theta_o$, but for $\xi \to \infty$ we do not have $U(\xi) \to U_\pm$, $\theta(\xi) \to \theta_\pm$ because the approximation (59a,b) does not hold. It is clear from (60a,b) that $\ell = 2ga$ is the lengthscale of initial convergence towards stable equilibrium, or divergence from unstable equilibrium. In the thrust law (33) with $f_2 = 0$, the term $f_1 U^2$ is the thrust loss relative to the static thrust f_o, due to increasing a velocity. It cannot exceed the static thrust-to-weight ratio, which ranges from 0.2 for a large jet transport to over unity for a high-performance fighter. Taking $f_1 U_o^2 \sim 0.1 - 1.0$, the first term is usually much larger than the second in (32b), so that the lenghtscale $1/\ell = 2ga \sim 2af_1 \sim (0.2 - 2.0) \times g/U_o^2$, is $\ell = 5$ km for a large jet transport at $U_o = 100$ m/s, and $\ell = 2$ km for a jet fighter at $U_o = 200$ m/s.

The evolution of velocity and incidence normalized to the minimum drag values $\bar{U} = U/U_{md}$, $\bar{\theta} = \theta/\theta_{md}$ are plotted as function of distance normalized to the lengthscale $X \equiv \xi/\ell$ in *Figure 8*. Since the higher steady flight velocity cannot by (53) exceed $\sqrt{2} = 1.41$ times the minimum drag speed, we consider the scale $\bar{U}_+ \equiv U_+/U_{md} = 1.2$ which implies a lower steady flight speed $\bar{U}_- \equiv U_-/U_{md} = 0.75$; the initial velocity is given twelve values, in 0.05 steps from 0.65 to 1.35; the incidence normalized to the minimum drag value is the inverse square $\theta = 1/\bar{U}^2$ of the velocity so normalized. The curves show that in order to compensate the phugoid motion exactly, and keep on a constant glide slope; (i) there is a very fast divergence towards the stall below the unstable steady flight speed; (ii) there is a relatively rapid convergence towards the stable steady speed, if starting close above or below it; (iii) the convergence to the stable steady speed is slowest if the initial velocity is close above the unstable steady speed, with the response curve having an inflexion as the minimum drag speed is crossed.

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Legends for the figures

Figure 1: A wind increasing (decreasing) upwards increases (decreases) lift relative to an uniform wind with the same mean velocity.

Figure 2: Ratio of stalling speed V_S^* in the presence of atmospheric disturbances to the stalling speed in still air V_S, as a function of the disturbance intensity G.

Figure 3: Change of incidence $\Delta\alpha$ due to an atmospheric disturbance, as a function of the disturbance intensity G, for three values of the initial effective incidence.

Figure 4: Ratio of airspeed in a perturbed V^* and still V atmosphere, as a function of vertical acceleration, for three values of disturbance intensity corrected for incidence change.

Figure 5: Aircraft flying on a constant glide slope, in the presence of longitudinal and transverse winds, by using pitch control to compensate the phugoid mode.

Figure 6: Groundspeed (top) and airspeed (middle), normalized to velocity in still air, and relative incidence change (bottom) between windshear conditions and still air, as a function of distance along flight path divided by the length of the windshear. The curves specify the phugoid motion compensation required to keep a constant glide slope, for aircraft with susceptibility parameters ranging from 1 to 10.

Figure 7: Drag curve as a function of velocity, indicating the minimum drag speed, and the stable and unstable steady speeds.

Figure 8: Variation of velocity (left) and effective incidence (right) normalized to the values at the minimum drag speed, as a function of distance along the flight path divided by length-scale. Convergence to the stable condition (+ subscript) and divergence from the unstable condition (- subscript) are represented for twelve initial conditions, assuming flight on a constant glide slope with exact compensation of phugoid mode.

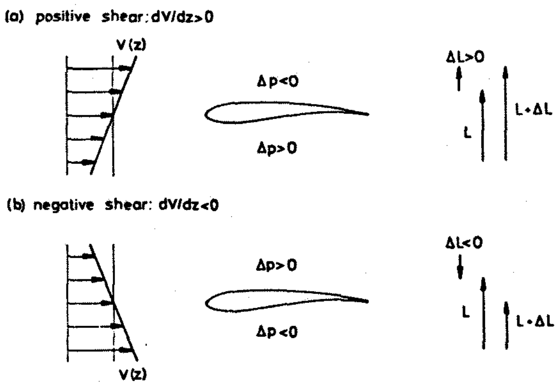

FIGURE 1 - EFFECT OF SHEAR ON LIFT

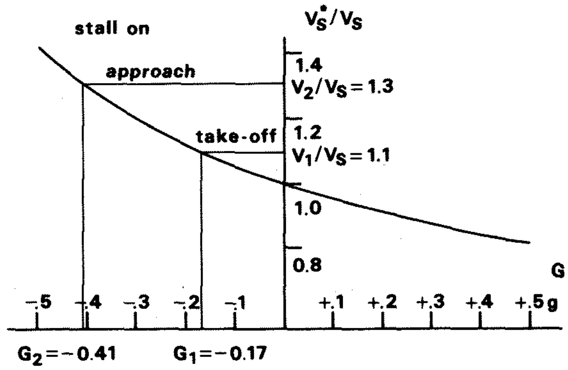

FIGURE 2 - STALLING SPEED vs DISTURBANCE INTENSITY

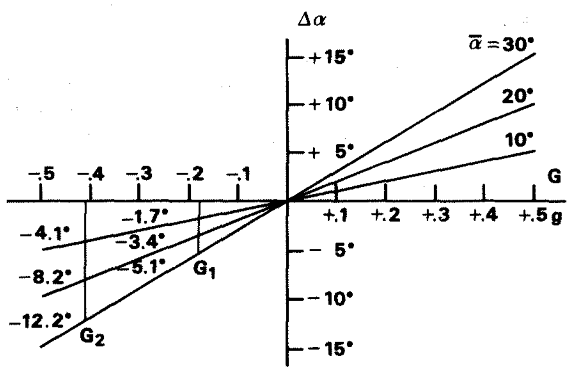

FIGURE 3 - INCIDENCE CHANGE vs DISTURBANCE INTENSITY

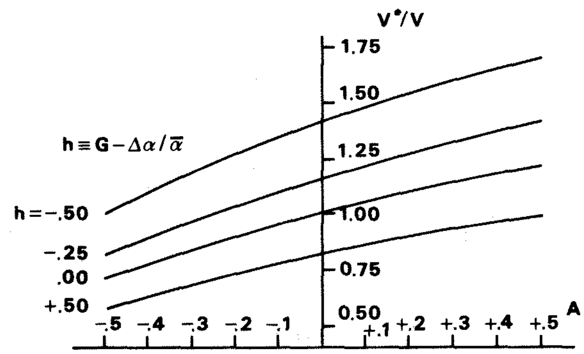

FIGURE 4 - STRAIGHT, HORIZONTAL FLIGHT IN DISTURBANCES

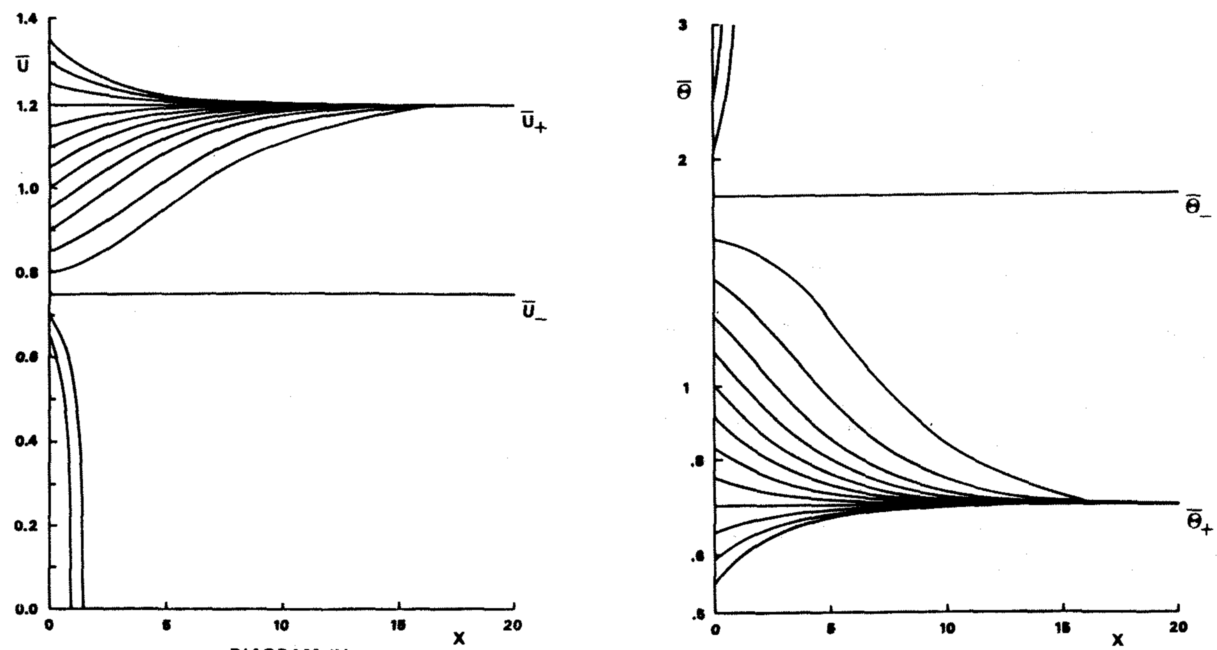

DIAGRAM IVa

DIAGRAM IVb

FIGURE 8

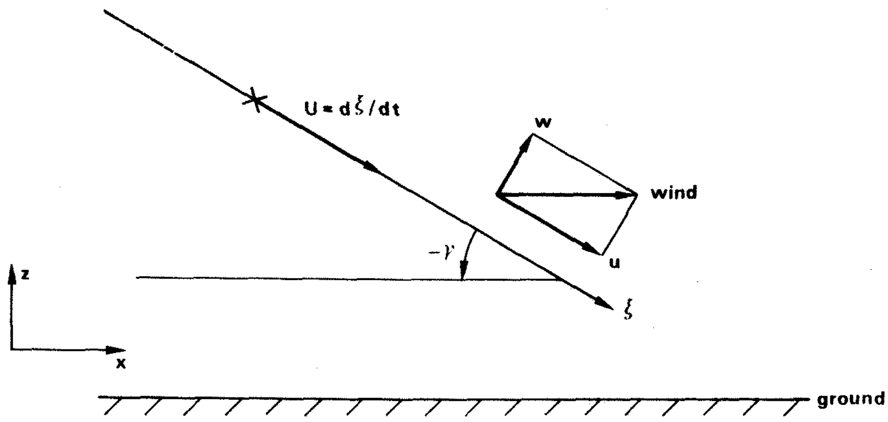

FIGURE 5

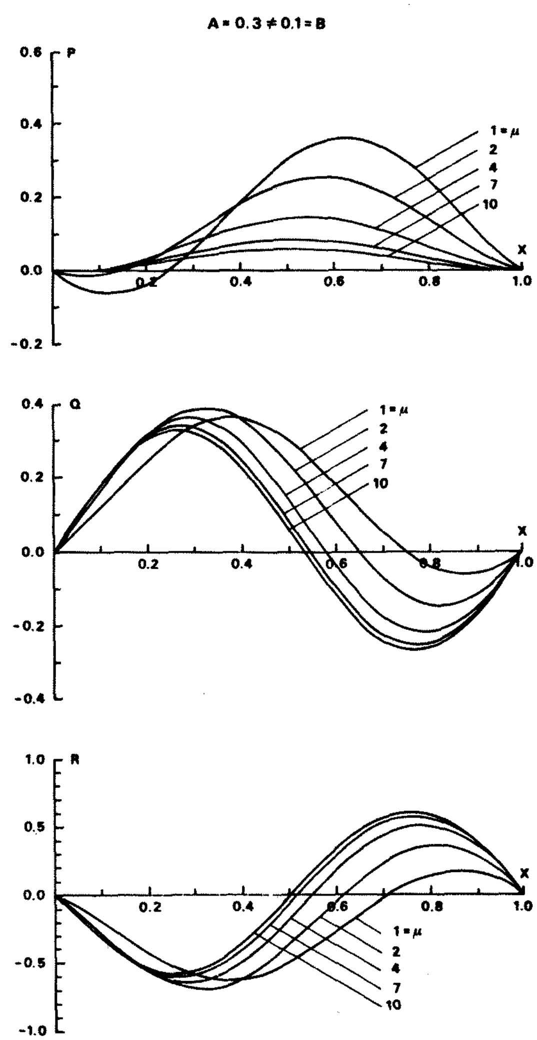

FIGURE 6

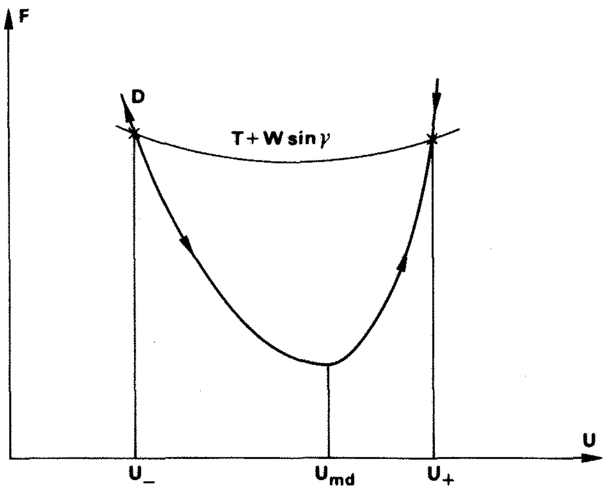

FIGURE 7