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Multivariable Control System Design For an Unstable Canard Aircraft

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Abstract

The control of a longitudinally unstable canard aircraft with high order feedback loops is examined, using multiloop analysis methods. Simple control structures which reflect current industrial practice are successfully implemented on two system designs using the eigenstructure assignment method together with a robustness and sensitivity optimisation scheme. A true multiloop design which incorporates incidence feedback exhibits better robustness and sensitivity characteristics than the more conventional system which uses only pitch rate feedback.

1 Introduction

In recent years the practical application of electronically signalled flying controls has made possible the design of longitudinally statically unstable aircraft with active stabilising control systems. The implementation of such systems is not trivial since they are required to operate full time with high integrity throughout the flight envelope. Furthermore, in practical systems the feedback loops incorporate many dynamic elements such as actuators, and anti-aliasing and structural notch filters. The resulting control system is inevitably of high order.

Severe problems have been encountered in designing control systems of such high order, which are also able to provide good low order characteristics such as pilot handling qualities(1,2). A further problem is that the use of more manageable, simplified low order models to design control systems, has led to stability margin problems when the resulting schemes have been implemented(3). Indeed even with an accurate representation of the system, the simultaneous achievement of good handling qualities and stability margins throughout the flight envelope is a challenging requirement.

Recent European high performance combat aircraft designs have been canard configurations. In these designs the aircraft's pitch response is controlled by two surfaces, the foreplane and flap. Despite this, their use as independent motivators for control in normal flight is not well developed. One aim of this study is to examine the way in which the freedom offered by multiple control inputs can be utilised to the benefit of the overall system design.

Many current flight control systems are basically single loop proportional plus integral pitch rate feedback systems. This type of structure results in the pilot's commands being interpreted as pitch rate demands and has proved to be the most successful in producing good pilot handling qualities(4). Furthermore, it is amenable to analysis using classical single input techniques such as root locus and Nyquist stability margin tests. Modern control techniques such as optimal control with full state feedback or observer systems have not been seen to be beneficial in Industry. One problem with the resulting multiloop designs is that the robustness analysis is more complex than the single loop system, and techniques are not generally accessible to Industry. A further aim of this study is to investigate simple output feedback structures that, together with the multiple control inputs, produce multiloop systems.

2 System Description

The aircraft which formed the basis of this study was modeled using the data set of a representative longitudinally unstable high performance canard aircraft flying at Mach 0.5, 5,000 feet, derived from wind tunnel tests. The work presented here is limited to consideration of longtitudinal motions. The canard aircraft configuration is illustrated in fig 1 and is capable of independent flap and foreplane movement as depicted. The aerodynamic forces included in the data exhibit typical unstable close coupled canard and delta wing aircraft properties[5]. The model was limited to rigid body motions, with no aerodynamic lags or transport delays. The engine dynamics were described by a simple first order lag from the throttle demand.

A feature of this work is the inclusion of high order sensor and actuation system models. The schematic of these is shown in fig 2. Clearly inclusion of all these elements considerably increases the order of the model. The extra complexity is justified because the total phase lag in each path can be significant. Indeed several problems encountered in the X-29 programme through omitting filter and actuator dynamics[3].

This study was limited to the continuous time domain. Thus to model the effects of a digital system sampling and delay, two 1st order Padé approximations were used. Sample rates of 80Hz and delays of 12.5m sec were assumed, since these are typical of flight control systems.

The actuator model was a 3rd order system with rate and travel limits. The size of the model was again justified again by phase considerations. Care was taken to realistically model the rate limits to obtain the correct phase lag effects[6].

This model was used for all time response simulations. All linear analyses were carried out by linearising this model using the built in features of TSIM2, the modeling package used for this study.

For canard configuration aircraft it is possible to trim over a range of incidence angles. For studies at higher normal acceleration conditions, it was required to have a schedule for nominal control deflections with incidence. This was calculated based on a range of g values at the bottom of a pull-up. The schedule was determined so that to achieve a smooth increase in incidence as vertical acceleration is increased

3 Pilot Handling Qualities

The main performance criteria for a flight control system of the type discussed here are the pilot handling qualities. These criteria define acceptable time response boundaries and input/output frequency response characteristics. There are many separate criteria, the most important of which are reviewed in (7). The criteria used in this study are those described in (4 and 8). These place boundaries on features of the time responses following the application of a step input from the pilot. The stick force to pitch attitude frequency response rate of change of phase, at the frequency where the phase angle is -180°, is also limited to a maximum of 100°/Hz.

3.1 Time response parameters

Inspection of the response of a high order system in fig 3 in detail, with the θ and γ responses overlaid, enables the important features to be identified. These are:-

Time at pitch acceleration peak	$t_{\dot{q}}$	sec
Ratio of peak to steady pitch rate	$\frac{q_m}{q_{ss}}$	sec
Time at peak pitch rate	t_{qm}	sec
Effective time delay of pitch rate	t_{qd}	sec
Pitch attitude dropback	$\frac{DB}{q_{ss}}$	sec
Flight path angle delay	t_γ	sec
Response time to steady g	t_n	sec

4 Eigenstructure Assignment

The approach taken to the design of the feedback loop in this study is to use simple output feedback structures with proportional plus integral compensation where required. The resulting system requires a gain matrix connecting the compensated measurements to the actuator inputs.

Determination of the gain matrix requires a systematic approach in which all elements are allowed to take values necessary to meet objectives. An approach which identifies the target modes is appealing because engineers are familiar with the to visualisation of modes and the main elements of the associated responses. One method which can provide a direct solution to this problem in terms of mode frequencies is known as eigenstructure assignment[9].

Eigenstructure assignment provides a means whereby, with certain restrictions, as many eigenvalues as there are independent measurements can be assigned exactly, and as many elements in the corresponding eigenvectors as there are independent controls can also be specified exactly.

The process whereby the gain matrix is calculated involves two stages. First, the total vector assignment, resulting from the specification of m elements, is calculated for each of the r eigenvalues assigned. The gains can then be calculated using the complete eigenstructure specification.

4.1 System Mathematical Description.

Let the system be described in the usual state space form :-

$$\mathbf{x} = \mathbf{Ax} + \mathbf{Bu} \qquad (1)$$

$$\mathbf{y} = \mathbf{Cx} \qquad (2)$$

Where $\mathbf{A}$ is of order n×n, $\mathbf{x}$, n×1, $\mathbf{u}$ m×1, $\mathbf{y}$ r×1, and $\mathbf{B}$, $\mathbf{C}$ are conformably dimensioned.

For output feedback with zero input,

$$\mathbf{u} = \mathbf{Fy} \qquad (3)$$

Hence,

$$\mathbf{x} = (\mathbf{A}+\mathbf{BFC})\mathbf{x} \qquad (4)$$

Putting $\mathbf{x} = \mathbf{x}_0 e^{\lambda t}$ in this equation yields the standard expression

$$(\mathbf{A}+\mathbf{BFC})\mathbf{v}_j = \lambda_j \mathbf{v}_j \quad \text{for } j = 1 \text{ to } n \qquad (5)$$

Where the λ_j are the eigenvalues of the closed loop system and $\mathbf{v}_j$ are the associated eigenvectors.

4.2 Vector assignment

The complete vector assignment, for the case where the number of specified eigenvector elements equals the number of independent controls, can be found as follows.

Let $\mathbf{v}_i$ be the total eigenvector corresponding to the i^{th} eigenvalue assigned and let it be partitioned so that;

$$\mathbf{v}_i = \begin{bmatrix} \mathbf{l}_i \\ \hline \mathbf{d}_i \end{bmatrix} \qquad (6)$$

where $\mathbf{l}_i$ is a vector of specified elements and $\mathbf{d}_i$ contain the unspecified elements. ($\mathbf{l}_i$ is therefore dimension m×1).

From (5),

$$\mathbf{v}_i = (\lambda_i \mathbf{I}-\mathbf{A})^{-1}\mathbf{BFC}\mathbf{v}_i \qquad (7)$$

and thus

$$(\lambda_i \mathbf{I}-\mathbf{A})^{-1}\mathbf{B} = \begin{bmatrix} \mathbf{L}_i \\ \hline \mathbf{D}_i \end{bmatrix} \qquad (8)$$

where (8) is row conformable with (6), so that $\mathbf{L}_i$ is of order m×m.

Clearly, from (6), (7) and (8)

$$\mathbf{l}_i = \mathbf{L}_i \mathbf{FC}\mathbf{v}_i$$

If $\mathbf{L}_i$ nonsingular, this can be solved to obtain

$$\mathbf{FC}\mathbf{v}_i = \mathbf{L}_i^{-1}\mathbf{l}_i$$

and using (7) we can reconstruct the entire eigenvector, viz

$$\mathbf{v}_i = (\lambda_i \mathbf{I}-\mathbf{A})^{-1}\mathbf{B}\mathbf{L}_i^{-1}\mathbf{l}_i \qquad (9)$$

4.3 Gain Calculation

To find the feedback gain matrix, $\mathbf{F}$, may be determined by repeating the above process to find the eigenvectors corresponding to the eigenvalues specified, thus

$$(\mathbf{A}+\mathbf{BFC})\mathbf{V}_r = \mathbf{V}_r \Lambda_r$$

where $\mathbf{V}_r$ denotes the n×r matrix of r assigned eigenvectors, $\mathbf{V}_r = [\mathbf{v}_1, \mathbf{v}_2 \ldots \mathbf{v}_r]$, and Λ_r is the diagonal, r×r, matrix of assigned eigenvalues. Rearranging the above equation gives

$$BFCV_r = V_r\Lambda_r - AV_r$$

and so $BF = (V_r\Lambda_r - AV_r)(CV_r)^{-1}$

Partitioning and making appropriate row interchanges gives

$$\begin{bmatrix} B_1 \\ B_2 \end{bmatrix} F = (V_r\Lambda_r - AV_r)(CV_r)^{-1} \quad (10)$$

Where B_1 is nonsingular (a nonsingular B_1 must be possible since the are m independent controls). Thus

$$F = B_1^{-1}[I_m, 0](V_r\Lambda_r - AV_r)(CV_r)^{-1} \quad (11)$$

where I_m is the identity matrix of dimension m×m. Rather than forming a non-singular B_1 equation (11) can be premultiplied by a matrix T^{-1}. Where T is defined by

$$T^{-1}B = \begin{bmatrix} I_m \\ 0 \end{bmatrix}$$

This leads directly to;

$$F = [I_m, 0]T^{-1}(V_r\Lambda_r - AV_r)(CV_r)^{-1} \quad (12)$$

4.4 Assignment Limitations

Various workers have suggested methods whereby more than m eigenvector elements can be approximately assigned in some optimal way[9,10]. These methods result in an inexact achievement of the desired vector and were found to present practical problems. For this reason exact assignment of r eigenvalues and m eigenvector elements is discussed in this paper. Clearly all feasible assignments can be achieved by manipulating this specification.

Apart from the limitations on the assignment arising from the number of feedbacks and inputs there are important limitations on the achievable assignments. Firstly, in order to guarantee that the gain matrix F is real, it is necessary to specify a conjugate eigenvalue and vectors elements whenever a complex eigenvalue is specified. Secondly, unobservable and uncontrollable modes cannot be affected by any assignment. In addition, as might be expected, unobservable states cannot be assigned values in the desired eigenvectors.

Finally, an important limitation of eigenstructure assignment using output feedback, when some states are not measured, is that closed loop stability cannot be guaranteed by an a-priori assignment.

4.5 Input System

An important aspect of the application of the eigenstructure assignment procedure is the incorporation of an input filter system. The system used in this study is show in block diagram form in fig 4. This provides both feedforward inputs and a filter on the reference input. The feedforward input utilises scheduled gains to set the nominal control surface deflections to the desired operating point as described earlier. This filter is used later to tune the pilot handling qualities. The reference input filter is designed to provide compatible incidence and pitch rate demands. This is achieved by noting that the realationship between incidence and pitch rate given by the approximate short period transfer functions is a first order lag[11]. Thus this filter is a simple first order lag on the incidence input and a gain of 1 on the pitch rate demand. Trim offset inputs are also provided into the feedforward and incidence inputs.

With this system the overall transfer function can be shown to be

$$y = (I+PG_cH)^{-1}P(G_cG_i+G_f)\delta_p \quad (13)$$

4.6 Short Period Mode and Vector Assignment Procedure

The prime objective of the control system is to stabilise the aircraft, and thus the two eigenvalues chosen for assignment were the short period frequency and damping. An undamped natural frequency of 1.1Hz and a damping ratio of 0.9 were chosen to meet the time and frequency response criteria.

The eigenvector assignment, in which only two elements can be fixed, may seem very limited, since even with the simplified models, there are many states of significance to the response. However, it was desired to have some influence over the control surface demands since, as incidence increases, the flap becomes less effective and the foreplane must be used in preference to avoid saturation. This leads to the choice of the actuator deflections as the two vector elements to be assigned. In this way the ratio of their responses was clearly visible and physically meaningful. It also turns out, that varying this assignment has a negligible effect on handling, while having a profound effect on system gains and robustness(11). Furthermore, development of analysis methods for this type of assignment can readily be extended to systems with more inputs.

For steady 1g flight, a foreplane to flap deflection ratio (δ_f/η) of -1.0 is a reasonable a-priori assignment, since this will cause the moments produced to be added together. Conversely a ratio of 1.0 might be expected to generate high gains since the moments produced by the control surfaces will in opposite directions. An interesting feature of this method of assignment is that when the ratio of control surface responses is assigned to be real and equal for all the modes specified, the resulting gain matrix imposes a simple gearing between them. In this situation the rank of the gain matrix is 1, and the resulting loop transfer function matrix has a rank of 1. Thus the system becomes effectively a single loop, albeit with several outputs being mixed. For systems with a simple proportional plus integral q feedback, the loop reduces in any case to rank 1, though the gain matrix may have higher rank.

With the above in mind 2 design cases are presented here, design 1 is a conventional proportional plus integral pitch rate feedback system. Design 2 incorporates the feedback of incidence in addition to proportional plus integral pitch rate.

5 Robustness and Sensitivity Optimisation

So far the discussion has assumed that the eigenstructure assignment procedure can be used for design of the control loop. However, in practical designs, some allowance must be made for both nonlinearities inherent in the system, and the approximations made in formulating the model. Since the system incorporating incidence feedback is a multiloop system, classical single loop methods cannot be used to analyse its robustness and sensitivity. To permit comparisons between designs 1 and 2 and to provide a unified design process, a more general approach to robustness and sensitivity analysis is required.

The general approach has been formulated in terms of the singular values of the return difference and inverse return difference matrices(12,13,14). In this approach the system approximations and nonlinearities are described by a perturbation transfer function which modifies the nominal system. For the system shown in fig 4, described by equation (13) the perturbations can be represented by a multiplicative perturbation to the actuation system inputs. Thus if the perturbation to the system makes $\mathbf{P}(s)$ become $\mathbf{P}'(s)$ then the following relationship can be written

$$\mathbf{P}'(s) = \mathbf{P}(s)\Delta_m \qquad (14)$$

where Δ_m is tne perturbation. For the system show in fig 4 the return difference and inverse return difference matrices appropriate to the above perturbation are

$$(\mathbf{I}+\mathbf{G}_c\mathbf{HP})$$

and $$(\mathbf{I}+[\mathbf{G}_c\mathbf{HP}]^{-1})$$

respectively.

The singular vales of a matrix are the positive non zero square roots of the eigenvalues of $\mathbf{A}^H\mathbf{A}$, where $\mathbf{A}$ is the subject matrix. Singular values are a measure of the size of a matrix(15). The maximum and minimum singular values of a matrix $\mathbf{A}$ are denoted $\bar{\sigma}(\mathbf{A})$ and $\underline{\sigma}(\mathbf{A})$ respectively.

5.1 Robustness

It can be shown(12,14) that for the perturbation described above, the system will remain stable if

$$\bar{\sigma}(\Delta_m-\mathbf{I}) > \underline{\sigma}(\mathbf{I}+\{\mathbf{G}_c\mathbf{HP}\}^{-1}) \quad \text{for all } s = j\omega \qquad (15)$$

or $$\bar{\sigma}(\Delta_m{}^{-1}-\mathbf{I}) < \underline{\sigma}(\mathbf{I}+\mathbf{G}_c\mathbf{HP}) \quad \text{for all } s = j\omega \qquad (16)$$

Inequalities (15) and (16) represent two sufficient conditions for stability. Clearly, therefore, it is necessary to satisfy only one to guarantee robustness against a perturbation defined by Δ_m. The description of the purturbation and the properties of the singular value as a norm are quite general and assume no structure, thus some conservatism could be implied by the inequalities

5.2 Sensitivity

Although the most important requirement is for stability to be maintained in the face of system perturbations, it is necessary to minimise the performance degradation as the system changes. This can be achieved by ensuring the closed loop transfer function remains close to its nominal value.A measure of the closeness of the perturbed systems transfer function to the nominal one is therefore required. A method of achieving this can be found by considering the closed loop transfer function given in equation (13). This equation can be rearrange as

$$\mathbf{y} = \mathbf{P}(\mathbf{I}+\mathbf{G_c H P})^{-1}\mathbf{r}$$

where $\mathbf{r} = (\mathbf{G_c G_i}+\mathbf{G_f})\delta_p$

Since the perturbations considered are multiplicative in nature, let $\mathbf{P}' = \mathbf{P}\Delta_m$ as before, so that for the perturbed system

$$\mathbf{y} = \mathbf{P}\Delta_m(\mathbf{I}+\mathbf{G_c H P}\Delta_m)^{-1}\mathbf{r}$$

$$\therefore \quad \mathbf{y} = \mathbf{P}(\Delta_m^{-1}+\mathbf{G_c H P})^{-1}\mathbf{r}$$

$$\text{or} \quad \mathbf{y} = \mathbf{P S}[(\Delta_m^{-1}-\mathbf{I})\mathbf{S}+\mathbf{I}]^{-1}\mathbf{r} \qquad (17)$$

where $\mathbf{S} = (\mathbf{I}+\mathbf{G_c H P})^{-1}$

From this it can be seen that the perturbation from the nominal is governed by the size of the matrix $(\Delta_m^{-1}-\mathbf{I})\mathbf{S}$, ie. it is desirable for

$$\bar{\sigma}(\Delta_m^{-1}-\mathbf{I})\bar{\sigma}(\mathbf{S}) < \delta$$

or inverting, for

$$\frac{1}{\delta}\bar{\sigma}(\Delta_m^{-1}-\mathbf{I}) < \underline{\sigma}(\mathbf{I}+\mathbf{G_c H P}) \qquad (18)$$

where δ is some small number defined to give the required performance tolerance.

It can be seen that for $\delta < 1$ as required, equation (18) is a stricter condition on $\underline{\sigma}(\mathbf{I}+\mathbf{G_c H P})$ than the stability and robustness criteria given in inequality (16).

It is useful to consider the trends of the above parameters as frequency increases since, as $\omega \to \infty$, the response of each of the transfer functions considered here tends to zero (ie they are strictly proper rational functions). Clearly therefore, as $\omega \to \infty$

$$\Delta\mathbf{P}_m \to \mathbf{0}$$

$$\therefore \quad \Delta_m = (\mathbf{I}+\Delta\mathbf{P}_m) \to \mathbf{I}$$

Similarly

$$\mathbf{G_c H P} \to \mathbf{0}$$

$$\therefore \quad \underline{\sigma}(\mathbf{I}+\mathbf{G_c H P}) \to 1$$

$$\text{and} \quad \underline{\sigma}(\mathbf{I}+[\mathbf{G_c H P}]^{-1}) \to \infty$$

Also, since $\Delta_m = \mathbf{I}+\Delta\mathbf{P}_m$, it is quite possible for $\underline{\sigma}(\Delta_m)$ to be less than 1 at frequencies where the perturbations reduce gains. Thus since $\bar{\sigma}(\Delta_m^{-1}) = 1/\underline{\sigma}(\Delta_m)$, $\bar{\sigma}(\Delta_m^{-1})$ and hence $\bar{\sigma}(\Delta_m^{-1}-\mathbf{I})$ can be much larger than 1. Generally the perturbations due to unmodelled dynamics will be more significant at high frequencies where $\underline{\sigma}(\mathbf{I}+\mathbf{G_c H P}) \approx 1$. Therefore, it might be expected that at high frequencies $\bar{\sigma}(\Delta^{-1}+\mathbf{I}) > \underline{\sigma}(\mathbf{I}+\mathbf{G_c H P})$. In these circumstances robustness must be satisfied by ensuring inequality (15) holds.

To view the system behaviour, two plots are required as shown in fig 5. Clearly, since good performance sensitivity is required over low frequencies, condition (18) needs to be satisfied; thus stability robustness will also be guaranteed from (16). This is illustrated in fig 5a. At higher frequencies above the bandwidth, it is more appropriate to consider the condition implied by (15) shown in fig 5b.

6 Optimisation

Using the above reasoning it is possible to develop singular value objectives, based on likely system perturbations[11]. These objectives lead naturally to the idea of utilising an optimisation scheme to enable to best design to be identified, given the system structure imposed. Indeed a similar idea has been pursued by others [16,17]. However, in these studies the controller was modelled in state space form and every element of the matrices considered to be an independent variable in the optimisation process. The optimisation formulated in this way allows the system roots, and hence performance, to vary, focussing on robustness rather than overall system objectives.

In this study the eigenstructure assignment algorithm is used to obtain the elements of the gain matrix. This approach allows certain eigenvalues and their associated eigenvector elements to specified. Clearly if the assigned eigenvector elements are chosen as independent variables in the optimisation, the assigned eigenvalues will be held where required during the optimisation.

The objectives of the optimisation are, to keep the "size" of the gain matrix as low as possible, and to maximise the low frequency performance sensitivity margin shown in fig 5a. In addition it is required to constrain the solution such that there are no unstable roots and the high frequency stability margin shown in fig 5b is positive.

The resulting cost function is therefore

$$J_A = w_1\sigma(F) - w_2\Delta\sigma_L + w_3[\min(0,\Delta\sigma_H)]^2 + w_4[\min(0,10^{-7}-\mathrm{Re}(\lambda)_{max})]^2 \quad (19)$$

where w_{1-4} are weightings, $\Delta\sigma_L$ and $\Delta\sigma_H$ are the low and high frequency singular value margins respectively. The first 2 terms of the cost function are objectives, whereas the final 2 are penalty functions which constrain the solution as required above. The mimimum of this function can be found using, a quasi-Newton algorithm employing a forward difference approximation of the gradients[18].

7 Results

For the cases where the same, or where appropriate conjugate, vector assignment is specified for all modes, contour plots of the cost function values against the real and imaginary part of the ratio of flap to forplane deflection (δ_f/η), can be drawn. Fig 6 shows contour plots of this kind, for design 1 at 1g and 4g. Fig 7 shows similar results for design 2. In both cases unacceptable values are reached in the upper right hand quadrant where no contours are shown. Indeed since at some assignment it is possible to have the foreplane moment canceling the flap moment, there exists a singularity in the value of the gain matrix within this region.

A list of optimised assignments is given in table 1, for a range of g values at the bottom of a pull-up. It is interesting to note that at 1g the optimisation of design 2 gives an assignment which implies a rank 1 gain matrix. Evidently in this case a single loop design albeit with several outputs mixed would provide the best result. However, this is not in general the case, it is notable for example that different flight conditions have different optimised assignments. Interestingly the results for the conventional design 1 indicate that a phase difference between flap and foreplane responses would be beneficial in improving robustness and sensitivity. In addition, values of $\Delta\sigma_L$ and $\Delta\sigma_H$ given in table 1 suggest that design 2, incorporating incidence feedback, has a better stability margin and lower sensitivity to system changes than the more usual pitch rate feedback system.

Figs 8 and 9 show the time and frequency responses for designs 1 and 2 with the optimised assignment. Some shortcomings exist in the response time to the peak pitch rate. This can be improved by the use of lead lag filters on the input[4]. Fig 10 shows the result of this refinement for design 2. In this case the lead lag filter was applied only to the feedforward input as described earlier. These results demonstrate good precision tracking characteristics in accordance with the requirements of (4).

8 Conclusions

A practical multiloop design method has been demonstrated using, eigenstructure assignment together with a robustness and sensitivity optimisation scheme. Good handling qualities have been demonstrated on the high order system examples presented.

Results for the classical proportional and integral pitch rate feedback system, indicate that robustness can be improved, and gains reduced, if a phase difference is specified between foreplane and flap response in the short period mode. These results also apply to a system which includes incidence feedback and, in addition, indicate that benefits will accrue from the use of a multiloop approach to the design problem. In particular, the feedback loop robustness and sensitivity parameters, based on the singular values of the return difference and inverse return difference matrices, achieve better minimum values.

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Design	g pull up/α	vector assignment: ω_{sp}=1.1Hz, ζ_{sp}=.9 mode δ_f/η	vector assignment: 3rd mode δ_f/η	$\bar{\sigma}(F)$	$\Delta\sigma_L$	$\Delta\sigma_H$	J_A
1	1g/2.74°	-1.52-.256j		1.8	-.2	4.5	.22
"	2g/5.82°	-1.57-.224j		1.7	-.2	4.0	.22
"	4g/11.94°	-1.26-.52j		1.9	-.7	3.6	.33
2*	1g/2.74°	-1.44-.02j	-1.44-.02j	.87	.8	6.5	-.1
2	1g/2.74°	-1.53-.05j	-1.44-.01j	.87	.8	6.5	-.1
"	2g/5.82°	-1.50-.11j	-1.48-1.0j	1.2	.5	6	.01
"	4g/11.94°	-1.34-.25j	-1.49-1.0j	.57	.74	6.4	-.1

* indicates optimisation constrained to equal vector assignments in non conjugate modes

Table 1 Summary of Optimisation Results

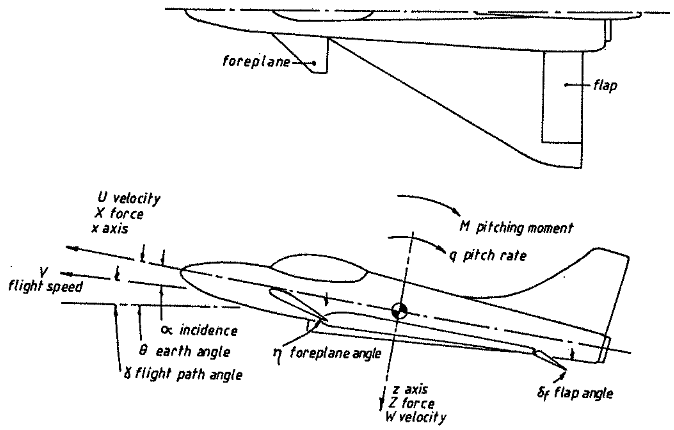


Fig 1 Canard Aircraft Arrangement

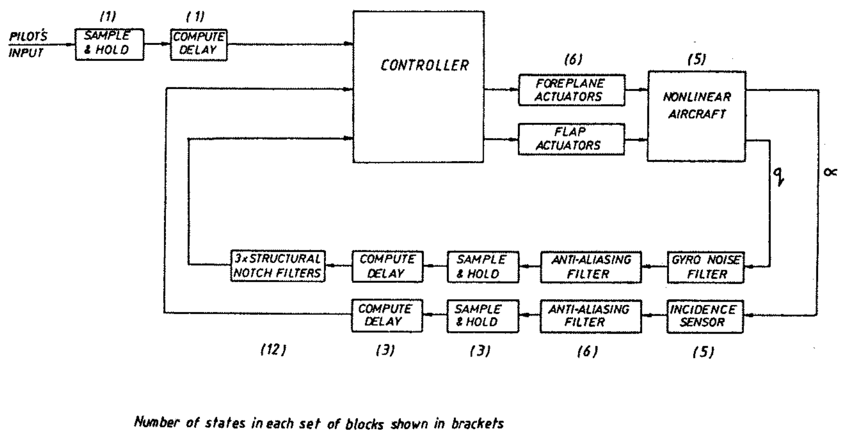


Fig 2 Schematic of Measurement and Actuation Systems

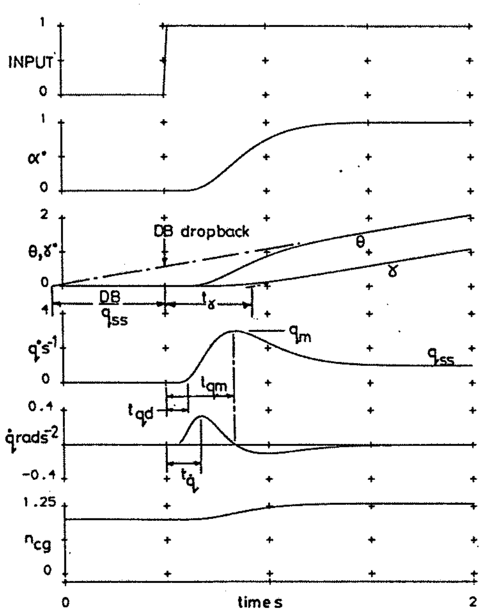


Fig 3 Pilot Handling Qualities Time Response Parameters

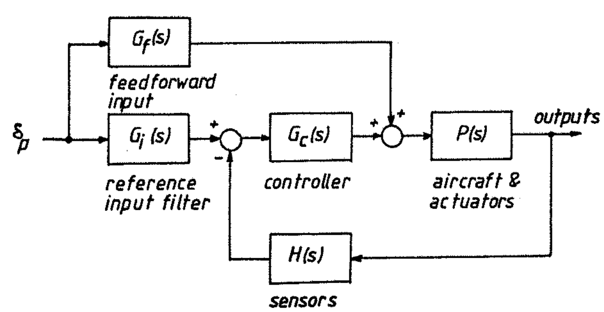


Fig 4 Overall System Structure

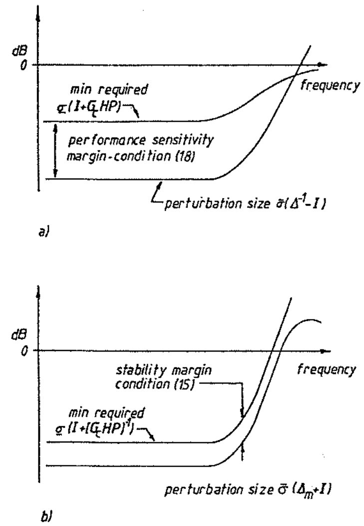


Fig 5 Robustness and Sensitivity Characteristics

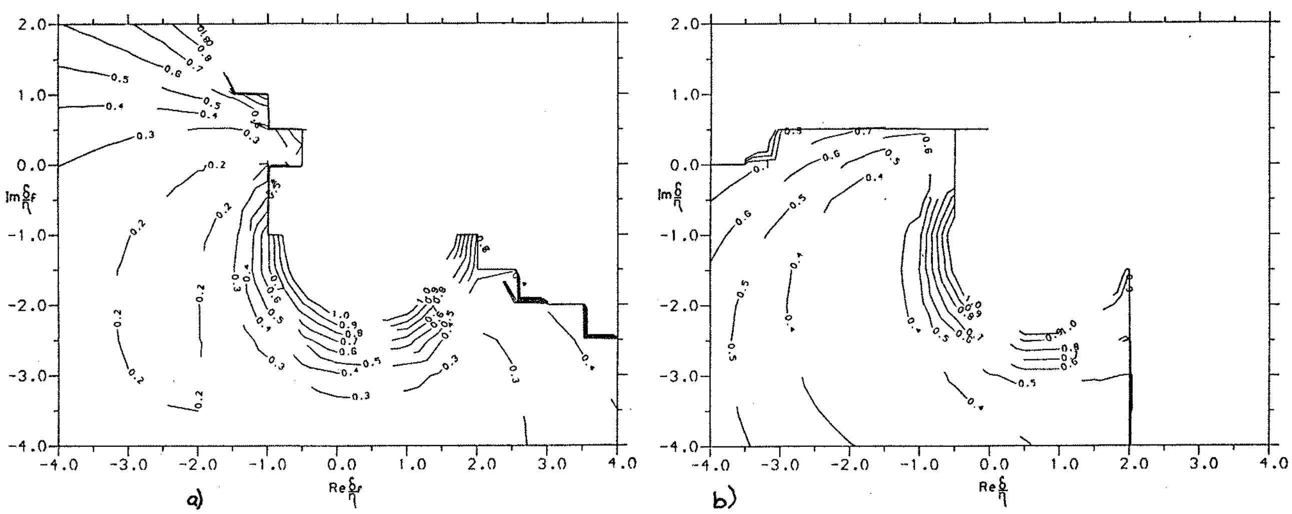


Fig 6 Design 1 Variation of Cost Function With Vector Assignment

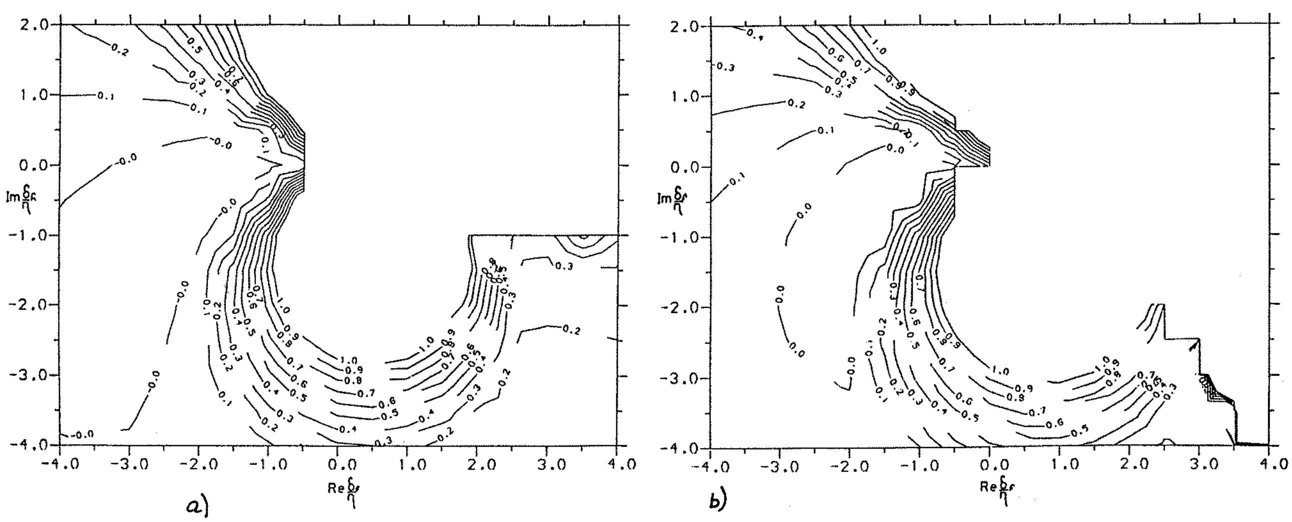
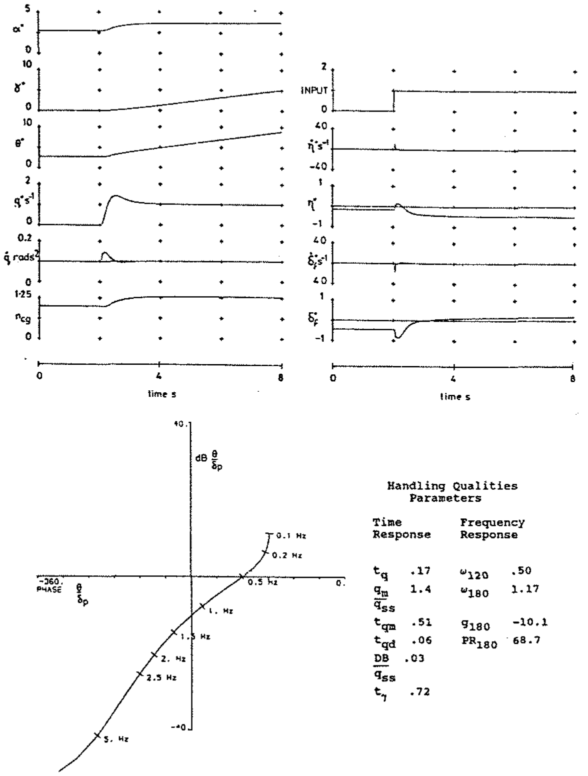


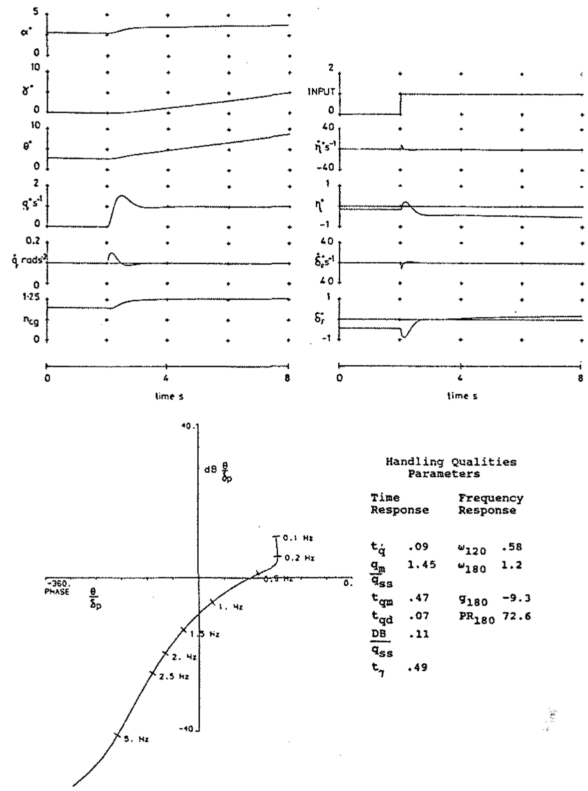
Fig 7 Design 2 Variation of Cost Function With Vector Assignment



Handling Qualities Parameters

Time Response		Frequency Response	
t_q	.17	ω_{120}	.50
$\frac{q_m}{q_{ss}}$	1.4	ω_{180}	1.17
t_{qm}	.51	g_{180}	-10.1
t_{qd}	.06	PR_{180}	68.7
$\frac{DB}{q_{ss}}$	.03		
t_η	.72		

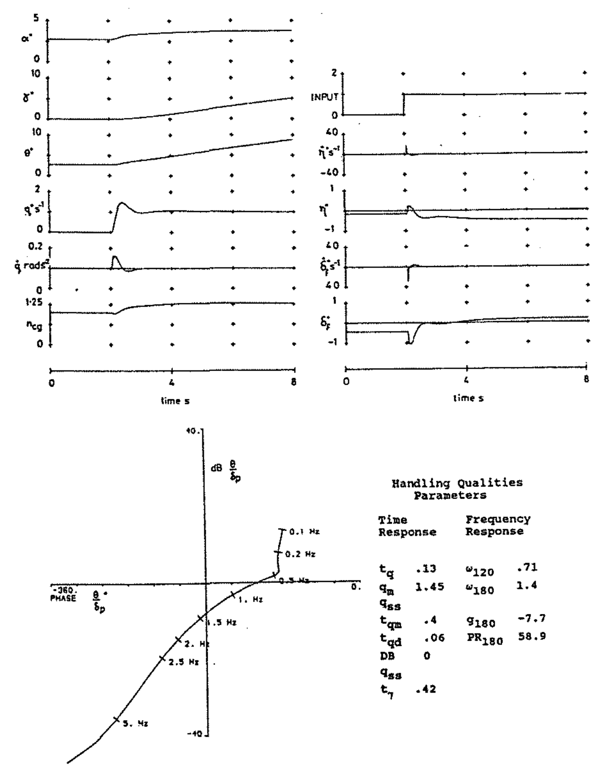
Fig 8 Design 1 Time and Frequency Response with optimised Vector Assignment



Handling Qualities Parameters

Time Response		Frequency Response	
$t_{\dot{q}}$	.09	ω_{120}	.58
$\frac{q_m}{q_{ss}}$	1.45	ω_{180}	1.2
t_{qm}	.47	g_{180}	-9.3
t_{qd}	.07	PR_{180}	72.6
$\frac{DB}{q_{ss}}$	.11		
t_η	.49		

Fig 9 Design 2 Time and Frequency Response with optimised Vector Assignment



Handling Qualities Parameters

Time Response		Frequency Response	
t_q	.13	ω_{120}	.71
q_m / q_{ss}	1.45	ω_{180}	1.4
t_{qm}	.4	g_{180}	-7.7
t_{qd}	.06	PR_{180}	58.9
DB / q_{ss}	0		
t_η	.42		

Fig 10 Design 2 with Improved Pilot Handling Qualities