OPTIMAL DESIGN OF LARGE LAMINATED STRUCTURES

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Abstract

A technique for optimising large laminated composite structures using a multilevel optimisation scheme is described. The multilevel approach allows both ply thicknesses and orientations to be used as design variables since it serves to reduce the number of variables and constraints that need be considered at any given stage in the computational process. The lower level of optimisation, used for element or "super-element" optimisation, uses a multi-criteria objective function with weight and strain energy change as its components to be minimised. The strain energy change in the element is minimised at this level to ensure load path continuity in the overall structure when switching between upper and lower levels, and thereby decouples the problems at the two levels. This system has been used to optimise various aircraft type structures using strain, displacement, buckling and gauge constraints.

1. Introduction

Modern, mathematically based, structural optimisation methods have been the subject of intensive research and development work over the past two decades. The mathematical bases and numerical implementation of these methods have matured to such an extent that automated structural optimisation is now being used in numerous industrial applications. Problems are, however, still encountered in design situations where a very large number of design variables needs to be considered and particularly so if the design constraints are highly non-linear functions of the variables. This is the situation which is inevitably encountered when attempting to optimise a large composite structure.

Laminated composite structural elements are generally constructed from multiple layers (plies) of orthotropic materials of different thicknesses and orientated at various angles. In order to truly optimise the use of these materials the orthotropic properties of the individual layers should be fully exploited, implying that the thickness and orientation of each layer should be used as the design variables. Even a relatively simple laminated composite plate therefore has many more design variables associated with it (2 per layer) than a comparable isotropic plate which has total thickness as its only variable. This dramatic increase in the size of the synthesis problem (relative to that associated with a comparable isotropic structure) is exacerbated by the highly nonlinear manner in which strains and deflections vary with ply orientation.

The result is that large laminated composite structures cannot readily be optimised using existing structural optimisation algorithms and software (developed with isotropic structures in mind). These difficulties are the most probable reasons for relatively few researchers having been attracted to the problem of composite structure optimisation. The few large scale composite optimisation systems that have been described in the open literature suffer from limited design flexibility and capability in that full design variable freedom is not allowed (only layer thickness and not orientation is varied [1]-[3]) or the interaction of all the design variables and constraints is not properly taken into account[4]. Limited computational resources and algorithm restrictions are probably at least partially responsible for these limitations.

A multilevel optimisation scheme which overcomes these problems and allows full design variable freedom (ply thickness and orientation as variables) is described in this paper. An additional benefit of this technique is that it potentially allows specific detail design problems (such as edge effects or interlaminar stresses) to be included in the design optimisation process.

2. Multilevel optimisation formulation

The multilevel optimisation approach was adopted so as to be able to address only limited aspects of the structure at any given stage in the optimisation process, while still maintaining a link with the global constraints. In this manner the number of design variables and (nonlinear) constraints under consideration at any specific stage can be significantly reduced and hence the associated computational problems can be minimised.

In this work only two levels of optimisation were considered, although the formulation of the method is such that it could be extended to a greater number of levels. The two levels used are simply referred to as the upper and lower levels in the remainder of this paper.

The upper level of optimisation is performed considering the entire structure and uses only the ply thicknesses of the laminated structural components as the design variables ie. the ply orientation and number of plies is kept fixed. The exclusion of ply orientations as variables at this level ensures that the highly nonlinear constraint functions are not considered at this level, and thus relatively simple constraint approximations can be effectively used.

The lower level of optimisation considers only the individual elements of the structure with the major constraint being that the change in stiffness of the element should be kept to a minimum as weight is reduced. This ensures that the stiffness, and hence the load paths, in the overall structure do not change substantially, so preserving the continuity when switching back to the upper level. In this manner the global constraints satisfied at the upper level are also not violated when altering the variable values at the lower level. This constraint together with strain, buckling and gauge constraints lead to a well defined, but comparatively small, optimisation problem at the lower level where the highly nonlinear constraints are more readily dealt with, and hence full design variable freedom (ply thickness and orientation) can be allowed.

Although the individual elements in the finite element model of the structure are considered at the lower level in this work, it should be noted that larger "super-elements", such as a complete stiffened panel, could also be used at this level. In such a case a different composite optimisation program could even be used, if required, to optimise the stiffened panel (eg. something like PASCO) as long as the constraint requiring the stiffness change of the "super-element" to be minimised can be enforced.

Results are obtained by iteratively first optimising at the upper level followed by successive optimisation of all the elements at the lower level. Any suitable convergence criteria can then be used at the upper level to terminate the process. A schematic description of the two level method used in this work is given in Figure 1.

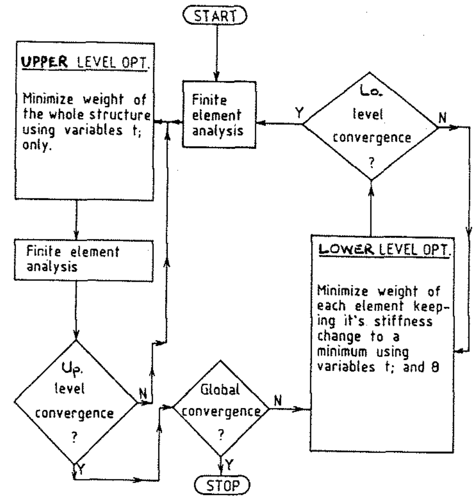

Figure 1. Multilevel design logic

The general composite structure optimisation problem can be written as:

Minimise $W(t)$

Subject to: i) $c(\theta,t) \le 0$
ii) $\theta^l \le \theta \le \theta^u$
iii) $t^l \le t \le t^u$

where $W(t)$ is the total structural weight, $c(\theta,t)$ are the constraints, θ,t are the ply-angle and thickness variables respectively and l,u are the superscripts indicating the lower and upper bounds respectively imposed on the variables.

This problem can be re-written as below, expressing it in the multilevel form.

Upper level:
minimise $W(t)$
subject to: i) $G(t) \le 0$
ii) $t^l \le t \le t^u$

Lower level:
minimise $w(t)$
subject to: i) $\delta k \to 0$
ii) $g(\theta,t) \le 0$
iii) $\theta^l \le \theta \le \theta^u$
iv) $t^l \le t \le t^u$

where $G(t)$ and $g(\theta,t)$ are the constraints applicable at the upper and lower levels respectively, $w(t)$ is the weight of the element being considered at the lower level, and δk is the change in stiffness of that element. The different design variables considered at the two levels are reflected in the problem statement.

The two levels of optimisation are discussed in more detail below.

3. Upper level optimisation

The upper level of optimisation is in fact very similar to some of the simpler composite optimisation techniques in use today, in that the weight of the entire structure is minimised within the given constraints boundaries by varying only the individual ply thicknesses within the elements. The objective function for this level is thus very simply

$$W(t) = \sum_{j}^{NEL} \sum_{i}^{L} (\rho A t)_{ij} \qquad (1)$$

where NEL is the total number of elements in the finite element model, L is the number of layers in element j, A is the surface area of element j, and ρ and t are the density and thickness of layer i in element j respectively.

The constraints considered at this level were strain and displacement limits, primary mode panel buckling and bounds on the lamina thicknesses.

Suffice it to state at this stage that the upper level optimisation produced results that were similar to those obtained by other composite optimisation programs with similar capability.

4. Lower level optimisation

The multilevel nature of the optimisation process is not at all reflected in the formulation of the upper level of optimisation (in terms of additional constraints, penalty functions coupled to the objective function etc) and the continuity between the levels is established only by the formulation used for the lower level optimisation problem.

In order to ensure that the load paths within a structure are not significantly altered (thereby ensuring continued constraint satisfaction at the upper level) when invoking the lower level optimisation, a requirement for keeping the stiffness change of the elements to a minimum in this level's optimisation process is introduced. This requirement can be satisfied by either including the stiffness change (δk) as a constraint or, alternatively, as the objective function to be minimised.

Considering the first of these options, the stiffness change could be included as an equality constraint, $\delta k = 0$, but this would be too restrictive and in many cases may prohibit the finding of an optimum. The alternative would be to introduce it as an inequality constraint with relatively tight limits. The establishment of these bounds would, however, be extremely difficult and would in all probability have to be varied not only for different problems but also for different parts of a given structure.

The use of the stiffness change as the function to be minimised thus has more appeal. To use this alone as the objective function will, however, not generally be sufficient to drive the design to an optimum. This is particularly well illustrated in the case where the element satisfies all the constraints, as there is then no incentive to change the design, since the stiffness change would then obviously be zero ie. the required minimum. Since the primary objective of the optimisation process is to minimise weight (while satisfying all the constraints) this is combined with the stiffness change to form a multi-criteria objective function that is to be minimised. The inclusion of weight as part of the objective function ensures that both layer thickness and lay-up angles will be used to obtain a design that achieves a good compromise of minimum weight and stiffness change, while satisfying all the constraints.

The multi-criteria objective function was formed using the basic, weighted objectives method[5]. The components of the objective function are added together using different weighting coefficients thereby transforming the multi-criteria problem to a scalar optimisation problem of the form

$$f(x) = \sum_{i} q_i \, f_i(x) \, c_i \qquad (2)$$

where the $f_i(x)$ are the original objective functions, x are the variables, q_i are the weighting coefficients representing the relative importance of the criteria ($q_i \geq 0$ and $\sum q_i = 1$) and c_i are constant multipliers.

The weighting coefficients q_i do not reflect proportionally the relative importance of the objectives but are only factors, which when varied, would locate different points in the design space (Pareto optima). The q_i can, however, be made to reflect more closely the relative importance of the objective functions if the factors c_i are used to ensure that the functions have numerical values of the same order. The best results are usually obtained if $c_i = 1/f_i^{\circ}$[5], where f_i° is the ideal optimum of the objective function f_i within the bounds of the prescribed constraints.

In order to avoid the additional computational effort required to evaluate the f_i° value for the weight function and the problem of $c_i = \infty$ (since $f_i^{\circ} = 0$) for the stiffness change part of the objective function, the following assumptions were made:

- the f_i° value for the weight function is taken to be the element weight obtained at the upper level immediately prior to the lower level optimisation
- the f_i° value for the stiffness change function is taken to be the stiffness of the element upon entry to that level of optimisation.

The objective function can thus be written as

$$f(x) = q_1 f_1(x)/W + q_2 f_2(x)/k \qquad (3)$$

where W and k represent the f_i^* values for the weight and stiffness change components as described above. This form of the objective function ensures that unless vastly different weighting coefficients are assigned to the two parts, neither part of the objective function will dominate the solution. The effectiveness of this is illustrated in the next section.

Due to the layered nature of composite plates (and the associated form of the stiffness matrix) quantifying the "stiffness change" in a given element is not a simple matter. A number of ways were evaluated[3],[6] but using the change in strain energy seemed to produce the most satisfactory results. This was also thought to provide a more accurate gauge of the load continuity at the upper optimisation level, since both element strain and stiffness are taken into account. The element strain energy is evaluated as

$$U = \{\epsilon_{pl}\}^T[R]\{\epsilon_{pl}\} \qquad (4)$$

and hence the change in strain energy is given by

$$\delta U = \{\epsilon_{pl}\}^T[R]\{\epsilon_{pl}\} - U^* \qquad (5)$$

where $\{\epsilon_{pl}\}$ and $[R]$ are the element strains and laminate rigidity matrix respectively (as defined in classical lamination theory), and U^* is the strain energy of the element on entry to this level of optimisation.

The objective function at the lower level can thus be written as follows (from equation (3)):

$$f(x) = q_1 f_1(x)/W + q_2 f_2/\delta U \qquad (6)$$

This function was minimised subject to higher mode buckling, strain, and ply thickness and orientation constraints.

5. Lower level optimisation results

The example given below demonstrates how effectively the stiffness change of the structure can be controlled by minimising the strain energy change in the element, and also shows the ability of the weighting coefficients (in equation (6)) to represent the relative importance of the two parts of the multi-criteria objective function.

The cantilevered, rectangular box section (which can be considered to be representative of a simple wing box) shown in figure 2 below, was optimised using various weighting coefficients for the two loading cases given in table 1. The entire bottom skin of the box was defined to be of laminate type 1 (ie design variable linking was used), the upper skin of laminate type 2 and the webs and ribs of laminate type 3. The various laminate types are defined in table 2, with only the upper half of the symmetric layup being given (layer 1 being outermost)

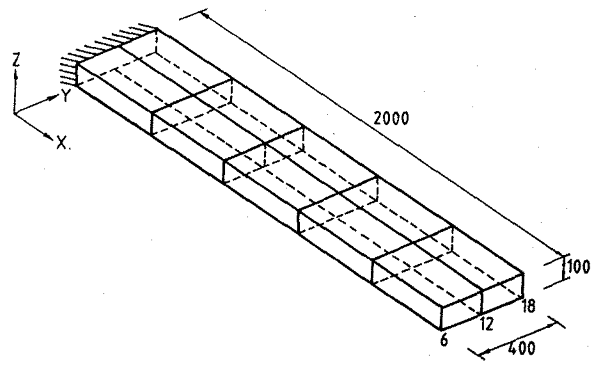

Figure 2. Rectangular box beam

	Load 1 (kN)			Load 2 (kN)		
Node no.	6	12	18	6	12	18
Force	10	20	10	10	15	20

Table 1. Load cases

Laminate type	Layer no.	Thickness (mm)	Angle (deg)
1	1	2,5	0,0
	2	1,25	-45,0
	3	1,25	45,0
2	1	2,0	0,0
	2	2,0	-45,0
	3	2,0	45,0
3	1	1,0	-45,0
	2	1,0	45,0

Table 2. Laminate types

All layers were assigned the following material properties:- E_L = 130 Gpa, E_T = 9 Gpa, G_{LT} = 4,8 Gpa, ν = 0,28 while longitudinal and transverse strains were limited to 0,004 and shear strains to 0.0055.

The results of the optimisation process are shown in figure 3. There is a smooth, progressive transition from the weight minimisation extreme to the strain energy change bias as the weighting coefficients are varied, demonstrating the viability and flexibility of the form of the objective function developed for the lower level.

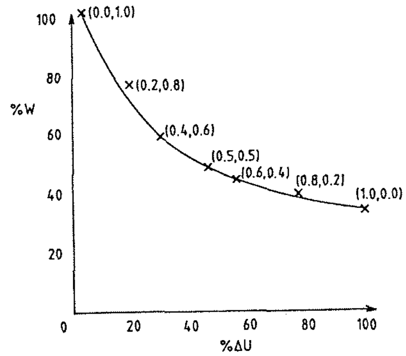

Figure 3. Influence of weighting coefficients (q_1, q_2)

6. Results of multilevel optimisation

Influence of weighting coefficients used at the lower level

Numerous test cases have been run to assess the sensitivity of the results and the convergence speed to the weighting coefficients chosen. These have included tests on structures representative of delta wings, forward swept and straight rectangular wings. The final results proved to be relatively insensitive to the coefficients chosen whereas if they were heavily biased (q_1 or $q_2 > 0.9$ typically) one way or the other, the convergence speed would decrease significantly.

Examples

i) Multilaminate rectangular box beam

This example was chosen as the results are readily explained using simple design logic. The general layout is the same as that shown in figure 2, while the load cases considered, the strain limits and material properties are all the same as those used in the example given to illustrate the lower level optimisation. In this example, however, a greater number of laminate types were considered and these are defined in table 3. Design variable linking was used to define that the four bottom skin, root end elements were of laminate type 1, the four top skin, root end elements were of laminate type 2, and the remaining bottom and top skins of laminate type 3 and 4 respectively. The shear webs (spars) and ribs were all of laminate type 5. Linking was also used to ensure laminate symmetry about the midplane and the layups given in table 3 only reflect the one half of these laminate (layer 1 being outermost). Note that the ply orientations are given relative to the local x axes which all lie parallel to the spanwise lines shown on the finite element grid. Finally a displacement of ±120 mm was placed on the tip nodes and all forms of buckling were constrained.

A convergence criteria of a weight change of less than 2% was used and this was achieved after 3 iterations between the two levels of optimisation. The final results are also given in table 3.

Laminate type	Initial design		Final design	
	t (mm)	θ (°)	t (mm)	θ (°)
1. Ply 1	2,5	0,0	1,98	-4,2
2	1,25	-45,0	0,83	-50,1
3	1,25	45,0	0,02	47,1
2. Ply 1	2,5	0,0	2,07	-1,9
2	1,25	-45,0	1,31	-40,4
3	1,25	45,0	0,05	59,7
3. Ply 1	2,0	0,0	0,60	21,2
2	2,0	-45,0	1,71	-55,0
3	2,0	45,0	0,01	32,2
4. Ply 1	2,0	0,0	0,29	30,9
2	2,0	-45,0	1,95	-53,8
3	2,0	45,0	0,01	27,8
5. Ply 1	1,0	-45,0	1,41	-47,4
2	1,0	45,0	1,30	41,4
Weight	329,7 kg		183,6 kg	

Table 3. Multilaminate rectangular box results

Laminates 1 and 2 (root end bottom and top skins respectively) show similar tendencies in their material distribution having the largest component in approximately the 0° direction to offer the necessary bending stiffness. Both have significant components in the region of -45° offering resistance to the torsional component of the second load case. However, the top skin, which is in compression, has distinctly more material in this region than the bottom skin so that the panels do not buckle (45° material offers optimum resistance to buckling for square plates).

Laminates 3 and 4 (tip elements in bottom and top skin respectively) both have large components of material in approximately the 54° direction, with proportionately more being found in the top skin (under compression). Although not quite at the optimum angle, this material provides near optimal torsional stiffness to resist the torsion load component of the second load case, and also provides very good resistance to buckling. The material in layer 1 in both laminates is orientated at 20° - 30° where it provides a good compromise between additional torsional rigidity and bending stiffness.

ii) Forward swept wing

This example is a representative of a forward swept wing box type structure, and the general arrangement is shown in figure 4.

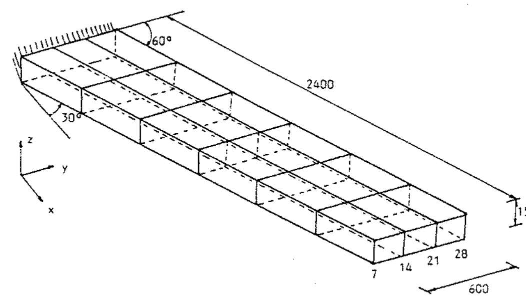

Figure 4. Forward swept wing

For this problem the elements were defined to be of several different laminate types as follows:- the 9 root end elements of the lower skin were of laminate type 1, the 9 root end elements of the top skin of laminate type 2, the remaining bottom and top skins of laminate types 3 and 4 respectively and the spar webs and ribs of laminate type 5. As in the previous example the laminates were constrained to remain symmetric about their midplane. The initial designs are defined in table 4 with only the upper half of the laminates being given (layer 1 being the outermost). The loading considered was 60 000N distributed over the lower skin tip nodes and a displacement limit of 80mm was placed on the tip nodes of the structure. The strain limits imposed were, as before, 0.004 on the allowable longitudinal and transverse strains and 0.0055 on the allowable shear strain.

The design converged in 4 iterations (with a weight change of less than 2%) and the results are given below in table 4.

Laminate type	Initial design		Final design	
	t (mm)	θ (°)	t (mm)	θ (°)
1. Ply 1	2,5	0,0	2,41	3,1
2	1,25	-45,0	0,15	83,1
3	1,25	45,0	3,08	35,4
2. Ply 1	2,5	0,0	6,14	13,6
2	1,25	-45,0	1,26	-24,5
3	1,25	45,0	0,90	53,0
3. Ply 1	2,0	0,0	0,77	0,4
2	2,0	-45,0	0,04	-44,4
3	2,0	45,0	0,12	45,6
4. Ply 1	2,0	0,0	0,76	0,2
2	2,0	-45,0	0,05	-44,0
3	2,0	45,0	0,14	44,0
5. Ply 1	1,0	-45,0	2,40	-32,7
2	1,0	45,0	1,64	40,0
Weight	556,4 kg		460,2 kg	
δ_z node 7	87,7		80,6	
14	94.1		84,4	
21	99,4		87,5	
28	105,4		90,0	

Table 4. Forward swept wing results

Note that the ply orientations are again given with respect to the element local x-axes which all lie parallel to the spanwise lines shown in the finite element grid in figure 4.

The final design is feasible in that all the constraints have been satisfied with respect to the linearised problem (a sequential LP is used as the optimisation algorithm). The displacements at the tip nodes are, however, greater than the prescribed limits due to the inaccuracies involved in making linear approximations to the non-linear constrains, and thus the design is not truly feasible with respect to the non-linear design space.

The final values assigned to the various design variables are quite realistic with the exception of the unduly heavy spar webs and ribs (laminate type 5). The reason for this phenomenon was poor element behaviour in the finite element model (artificially high shear stiffness in the elements used).

Notwithstanding this, the final values of the other design variables can be quite easily explained.

Considering first laminate type 2 (root end elements of the top skin), a very large proportion of the material has been orientated at an angle of 13,6° ahead of the spanwise lines.

This would produce a favourable shear coupling effect when the laminate is loaded in compression (as it is under the applied bending load) which would tend to twist the wing leading edge down. These effects can be used to achieve a minimum weight design whilst satisfying the displacement constraints, i.e. under a given bending load the wing leading edge will twist up less than for the 0° or isotropic material case. This characteristic has already been applied to aircraft with forward swept wings to avoid divergence problems, eg. the Grumman X-29 wing. The other components in this laminate can be seen to be at just about ±40° on either side of the major material direction (13,6°) which is close to the optimal orientation for torsional resistance (bending induced torsion is found in swept wings) and for relief of the shear strains induced in the 13,6° material component.

The solution given for laminate type 1 (root end elements of the bottom skin) is not quite as easily explained. Layers 1 and 3 contain nearly all the material in this laminate - layer 2 can almost be ignored except for the small contribution it makes to reducing shear and transverse stresses in the other layers. They are both angled ahead of the spanwise lines and hence would produce some advantageous shear coupling effects. While each of these layers assists in this role, layer 1, being near to 0° offers substantial bending stiffness and layer 3, being close to the 45° position offers good torsional resistance. The angle between them is sufficient to ensure that they provide some measure of relief to each other in terms of shear and transverse strains.

The reason for laminate types 1 and 2 being so different is not apparent, but can perhaps be explained by there being two or more local optima near to each other, to which the designs may be driven. The exact one that is found may be dependent on the loads imposed on and the stress state induced in the various laminates.

In contrast to laminates 1 and 2, the final designs for laminate type 3 and 4 (outer skin elements of the bottom and top skins respectively) are very similar indeed - so similar in fact that a discussion of one set of results will suffice for the other. This similarity between top and bottom skins could be expected since the allowable strains in tension and compression are the same and no buckling constraints have been included. There is sufficient material in layer 1 of these laminates orientated at 0° (or just about 0°) to resist the tension/compression induced in them by the bending load on the wing. A lesser quantity of material has been placed near a 45° orientation (layer 3) to resist the bending induced torsion in the wing. This 0°/45° also provides a reasonably favourable shear coupling effect. The remaining plys at -44° are negligably small.

The final design of the forward swept wing is thus realistic and readily explained using simple design logic.

7. Conclusions

The results presented here show that the multilevel optimisation system provides an effective method for optimising large scale laminated structures while allowing full design variable freedom (ie varying ply thickness and orientation). The multi-criteria objective function used at the lower level of optimisation helps to ensure rapid and stable convergence of the design procedure.

Due to a lack of published information of results obtained from similar synthesis systems (if any do in fact exist) the efficiency of this multilevel method is difficult to assess on a relative basis. The fact that all the results obtained are readily explained using simple design logic indicates that the system is indeed efficient in terms of producing realistic, minimum weight designs that satisfy all the imposed constraints. The method generally converges within 3-4 iterations between the two levels, with only a few iterations required at each level. While this may not be considered as being particularly efficient it should be viewed in the light of the optimisation algorithm used at the different levels. A sequential linear programming method was used throughout, with sequentially decreasing move limits. This technique, while useful in the development stages, is not regarded as being a very satisfactory, or efficient, method for use beyond the development phase. A more sophisticated nonlinear mathematical programming method is likely to produce somewhat better convergence charateristics.

The optimisation technique presented here provides an effective method for optimising large multi-element laminated composite structures, by reducing the number of design variables and constraints under consideration at any given stage in the process. The method also has the appeal of being able to add many more constraints types at the element level to cater for effects such as interlaminar stresses and edge effects if suitable mathematical expressions (even if empirically derived) can be found to quantify them. This potential for growth is one of the systems most attractive features.

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