VALIDATION OF NONSTATIONARY AERODYNAMICS MODELS FOR LONGITUDINAL AEROPLANE MOTION ON THE BASIS OF FLIGHT MEASUREMENTS

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Abstract

On the basis of the physical analysis, basic and simplified models of nondimensional aerodynamic frequency transfers and related nondimensional aerodynamic step admittances have been derived. They comprise separated quasi-stationary and nonstationary parts. The analysis is useful for didactic purposes in flight mechanics and aerodynamics. There was further suggested an objective way for verification of validity of aerodynamics models used at the aeroplane design. It is based on flight measurements on a real aeroplane. For this purpose the loss functions of motion equations for the short period mode of the aeroplane longitudinal motion were arranged in a special way. They were employed also for verification of the significancy of different components of aerodynamics models.

1. Introduction

In results of flight measurements carried out with a A 145 light transport aeroplane at its longitudinal motion which were described in [1], some differencies have been stated in the range of low airspeeds.

In [2] it was qualitatively shown by means of spectrally weighted aerodynamic derivatives that the reason of the differencies might be the using of a non appropriate model of quasi-stationary aerodynamics in which the effect of Strouhal number has not been considered. The analogical conclusion was made also in [5].

Therefore in [3] the author has dealt with an estimation of nonstationary aerodynamics parameters from flight measurements. The basic aerodynamic derivatives values were taken from measurements at steady flights.

In the presented paper further simplified models of nonstationary aerodynamics are derived which are compared with quasi-stationary models. The loss function is suggested for the both motion equations describing the short-period longitudinal motion of an aeroplane in the form appropriate for the verification validity of aerodynamic models which are given by tables of values or by analytical expressions. This form is also very convenient for the objective verifying effects of different components of an aerodynamic model from the flight mechanics point of view.

The validity of aerodynamics models has been verified in the frequency domain. Advantages and disadvantages of this approach for parameters estimation and for models identification when using constant derivatives were discussed in [4]. When using nonstationary aerodynamics models in the frequency domain, one must thus advance carefully and employ an appropriately great range of angular frequencies.

The paper gives instructions for the objective verifying the aerodynamic data employed at designing an aeroplane which is done on the basis of results of flight measurements carried out with a real aeroplane and it has a didactic significance too.

2. Short-period logitudinal motion equations

It is supposed that an unsteady motion of a rigid aeroplane with a zero thrust has been excited by an elevator deflection of a triangular pulse form at a straight steady gliding flight in the calm atmosphere. In the observed time interval further the airspeed and state quantities of the atmosphere are supposed to be constant. Maximum values of the elevator deflection are considered to be as small as the controlled dynamic system of the aeroplane motion may be treated to be linear. The researched motion has two degrees of freedom and therefore it is characterized by four state quantities. For a given purpose it is sufficient to consider two generalized coordinates: angle of attack of the aeroplane and its rate of pitch.

The motion equations are related to the air-path axis system and have the form of deviation equations from an initial steady flight condition. As the nonstationary aerodynamics effects are studied, the forces and moments in motion equations are divided into two groups: those of the aerodynamic origin, and those of the other origin (as mass and inertial forces). For this reason the aerodynamic forces and moments are not divided into the control and response ones and the equations are not given as state equations.

If considering real time the motion equations are of the form:

$$\Delta C_A[\Delta\gamma(t), \Delta\Theta(t), \Delta\eta(t); t] = \mu\tau_A \Delta\dot{\gamma}(t) + C_{W0}\cdot\Delta\gamma(t) \quad (1)$$

$$\Delta C_m[\Delta\gamma(t), \Delta O(t), \Delta\gamma(t); t] = \mu \tilde{i}_y^2 \tau_A^2 \cdot \Delta\ddot{\Theta}(t), \quad (2)$$

where $\Delta\Theta = \Delta\gamma + \Delta\alpha$ and $\Delta\dot{\Theta} = \omega_y(t)$

By the integral Fourier transformation and by dividing them by $\overline{\Delta\eta}$, eq. (1) and (2) are converted into the form:

$$\left[\frac{\overline{\Delta C_A}}{\overline{\Delta\eta}}(i\omega)\right]_T = \left(\mu\tau_A + \frac{1}{i\omega}C_{W0}\right)\cdot\left[\frac{\overline{\Delta\omega_y}}{\overline{\Delta\eta}}(i\omega) + - i\omega\cdot\frac{\overline{\Delta\alpha}}{\overline{\Delta\eta}}(i\omega)\right] \quad (3)$$

$$\left[\frac{\overline{\Delta C_m}}{\overline{\Delta\eta}}(i\omega)\right]_T = \mu \tilde{i}_y^2 \tau_A^2 \cdot i\omega\cdot\frac{\overline{\omega_y}}{\overline{\Delta\eta}}(i\omega) \quad (4)$$

With respect to the assumption of constant airspeed, the equations (3) and (4) are correct beginning of the angular frequency values which are substancially greater than those of angular

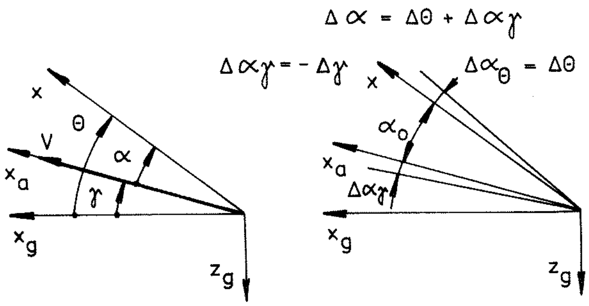


FIGURE 1 - SCHEME DIAGRAM OF ANGLES

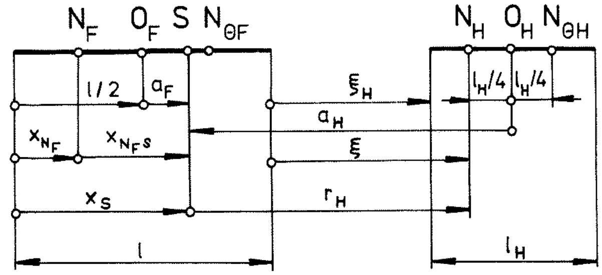


FIGURE 2 - SCHEME DIAGRAM OF LENGTHS

frequency of the phugoid oscillations with airspeed varying in time.

3. Nonstationary aerodynamics models

3.1 Aerodynamic frequency transfer function

At deriving nonstationary aerodynamics models for longitudinal motion of an aeroplane, it is convenient to employ in terms of generalized coordinates the quantities $\Delta\alpha_\gamma$ and $\Delta\Theta$ as this makes for physically instructive expressions, see fig. 1. At solving aeroelastic problems the quantities Δz_a and $\Delta\Theta$ are used, for which a transforming formula

$$\Delta\alpha_\gamma = -\Delta\gamma = \Delta\dot{z}_a / V \qquad \text{resp.} \qquad (5)$$

$$\overline{\Delta\alpha_\gamma} = -\overline{\Delta\gamma} = i\omega \cdot \overline{\Delta z}_a / V \qquad \text{is valid.}$$

The basic nonstationary aerodynamics model has been derived in [3] on the basis of a physical analysis, and it comprises components of circulation and inertial origins.

Changes of lift coefficient ΔC_A and pitching moment coefficient ΔC_m, which are denoted together by ΔC_K (K = A, m), in nonstationary aerodynamics phenomena are functions of time explicitely, and besides it also implicitely. Aerodynamic coefficients changes may be therefore expressed by means of nondimensional aerodynamic step admittances at using convolutory integrals in the form:*

$$\Delta C_K(t^x) = \int_0^{t^x} A_{C_K \cdot \eta}(t^x - \tau^x) \cdot \dot{\eta}^x(\tau^x) . d\tau^x + \qquad (6)$$
$$+ \int_0^{t^x} A_{C_K \cdot \alpha_\gamma}(t^x - \tau^x) \cdot \dot{\alpha}^x_\gamma(\tau^x) \cdot d\tau^x + \int_0^{t^x} A_{C_K \cdot \Theta}(t^x - \tau^x) \cdot \dot{\Theta}^x(\tau^x) \cdot d\tau^x$$

where K = A, m.

As they are deviations with zero initial conditions, the $\Delta C_K(t^x)$ changes may be expressed after the Fourier transformation in the form:

$$\Delta C_K(i\omega) = F_{C_K \eta}(i\omega^x) + F_{C_K \cdot \alpha_\gamma}(i\omega^x) \cdot \overline{\Delta\alpha_\gamma}(i\omega^x) + \qquad (7)$$
$$+ F_{C_K \cdot \Theta}(i\omega^x) \cdot \overline{\Delta\Theta}(i\omega^x)$$

where K = A, m and $\omega^x = \omega \cdot \tau_A$ [1].

For to express aerodynamic nondimensional frequency transfer functions, expressions form [3] are employed. As a common reference point for the wing and tailplanes, the gravity center S of the aeroplane is considered, see fig. 2. From the analysis, it follows an important relation:

$$F_{C_K \cdot \Theta}(i\omega^x) = F_{C_K \cdot \alpha_\gamma}(i\omega^x) + i\omega^x \cdot F_{C_K \cdot \dot{\Theta}^x}(i\omega^x) \qquad (8)$$

* In [7] the term "indicial admittances" is used.

When using (8) and the relation $\overline{\Delta\Theta} + \overline{\Delta\alpha_\gamma} = \overline{\Delta\alpha}$, from (7) after dividing by $\overline{\Delta\eta}(i\omega^x)$ the formula for total aerodynamic frequency transfer function follows in the form:

$$\left[\frac{\overline{\Delta C}_K}{\overline{\Delta\eta}}(i\omega^x)\right]_T = F_{C_K \cdot \eta}(i\omega^x) + F_{C_K \cdot \alpha_\gamma}(i\omega^x) \frac{\overline{\Delta\alpha_\gamma}}{\overline{\Delta\eta}}(i\omega^x) + $$
$$+ F_{C_K \cdot \dot{\Theta}^x}(i\omega^x) \cdot \frac{\overline{\omega^x_y}}{\overline{\Delta\eta}}(i\omega^x) \qquad (9)$$

where $\omega^x_y = i\omega^x \cdot \Theta = \omega_y \cdot \tau_A$ [1].

The aerodynamic step admittance and the aerodynamic frequency transfer function are related one to the other by the important formula:

$$\mathcal{F}\left\{A_{C_K \cdot x}(t^x)\right\} = F_{C_K \cdot x}(i\omega^x) \cdot \frac{1}{i\omega^x} \qquad (10)$$

where K = A, m and $x = \eta, \alpha, \dot{\Theta}^x$.

The frequency transfer functions for the longitudinal motion for an aeroplane in (9), $F_{\omega^x_y \cdot \eta}$ and $F_{\alpha \cdot \eta}$, may be determined by measurements in flight. In (9) they represent the degree of utilization of the aerodynamic frequency transfer functions $F_{C_K \cdot \alpha_\gamma}$ and $F_{C_K \cdot \dot{\Theta}^x}$ at the given motion that was excited by $\overline{\Delta\eta}(i\omega^x)$. An example of their graphic representation is shown on fig. 9 and 10.

3.1.1. Basic models

Aerodynamic frequency transfer functions $F_{C_K \cdot x}(i\omega^x)$, where K = A, m and $x = \eta, \alpha, \dot{\Theta}^x$, may be calculated by numerical methods on computing machines or from analytical expressions which make possible to study in a closed form the effects of various physical parameters. Analytical expressions are suggested in the form in which aerodynamic derivatives of quasi-stationary aerodynamics are brought separately from normalized nondimensional aerodynamic transfer functions $C_F(i\omega^x)$, $C_H(i\omega^x)$ and $C_{\alpha H}(i\omega^x)$. The first two functions of the Theodorsen type express phenomena of nonstationary aerodynamics on the wing and tailplanes and the third one then expresses the interaction between the wing and tailplanes. A survey of parameters of the aircraft geometry and of aerodynamic derivatives of quasi-stationary aerodynamics is given in table 1.

Analytical expressions are of the following form:

a) for the lift coefficient:

$$F_{C_A \cdot \eta}(i\omega^x) = a_{H1} \cdot (a_2/a_1) \cdot C_H(i\omega^x) \qquad (11)$$

$$F_{C_A \cdot \alpha_\gamma}(i\omega^x) = \left\{a_{F1} \cdot C_F(i\omega^x) + a_{H1} \cdot [C_H(i\omega^x) + \right.$$
$$\left. - (d\alpha_a/d\alpha) \cdot C_{\alpha H}(i\omega^x)]\right\}_C + [i\omega^x \cdot K_{A\dot{\alpha}^x}]_i \qquad (12)$$

$$F_{C_A\cdot\dot{\theta}^x}(i\omega^x) = [a_{F3}\cdot C_F(i\omega^x) + a_{H3}\cdot C_H(i\omega^x)]_c + [i\omega^x\cdot K_{A\dot{\theta}^x}]_i \quad (13)$$

$$K_{A\dot{\alpha}^x} = a_{F2}\cdot K_{\lambda F} + a_{H2}\cdot K_{\lambda H} \quad (14)$$

$$K_{A\dot{\theta}^x} = -a_{F4}\cdot K_{\lambda F} + a_{H4}\cdot K_{\lambda H} \quad (15)$$

b) for the pitching moment coefficient:

$$F_{C_m\cdot\eta}(i\omega^x) = m_{H1}\cdot(a_2/a_1)\cdot C_H(i\omega^x) \quad (16)$$

$$F_{C_m\cdot\alpha_\gamma}(i\omega^x) = \{m_{F1}\cdot C_F(i\omega^x) + m_{H1}\cdot[C_H(i\omega^x) + -(d\alpha_a/d\alpha)\cdot C_{\alpha H}(i\omega^x)]\}_c + [i\omega^x\cdot K_{m\dot{\alpha}^x}]_i \quad (17)$$

$$F_{C_m\cdot\dot{\theta}^x}(i\omega^x) = [m_{F3}\cdot C_F(i\omega^x) + m_{H3}\cdot C_H(i\omega^x)]_c + [K_{m\dot{\theta}^x} + i\omega^x\cdot K_{m\ddot{\theta}^x}]_i \quad (18)$$

$$K_{m\dot{\alpha}^x} = m_{F2}\cdot K_{\lambda F} - m_{H2}\cdot K_{\lambda H} \quad (19)$$

$$K_{m\dot{\theta}^x} = -(m_{F4}\cdot K_{\lambda F} + m_{H4}\cdot K_{\lambda H}) \quad (20)$$

$$K_{m\ddot{\theta}^x} = K'_{m\ddot{\theta}^x} + K''_{m\ddot{\theta}^x} \quad (21a)$$

$$K'_{m\ddot{\theta}^x} = -(m_{F5}\cdot K_{\lambda F} + m_{H5}\cdot K_{\lambda H}) \quad (21b)$$

$$K''_{m\ddot{\theta}^x} = -(m_{F6} + m_{H6}) \quad (21c)$$

The index c denotes the circulations components and the index i denotes the inertial ones. The approximate expressions for C_F, C_H and $C_{\alpha H}$ according to [3] are brought in the Appendix. Values of their parameters for the A 145 aeroplane are given in tab. 9.

3.1.2. Simplified models

The starting point is the basic model determined by (11) to (13) and (16) to (18). It can be simplified in two ways.

a) When taking into account the fig. 3, the following approximations are possible:

$$C_F \doteq C_H \doteq h_H = C_X(i\omega^x)$$
$$C_{\alpha H}(i\omega^x) = C_{\alpha a}\cdot h_H \doteq C_{\alpha a}(i\omega^x)\cdot C_X(i\omega^x)$$
$$K_{\lambda F} \doteq K_{\lambda H} = K_{\lambda X} \quad (22a,b,c)$$

where X = F, H, L, LH represent various shapes of lifting surface which is characterized especially by the aspect ratio λ.

b) The normalized frequency transfer function $C_{\alpha a}$ can be simplified for $T_1 = 0$ to a mere transport lag of the trailing edge vortex flowing downstreams from the trailing edge of the wing. The transport lag is described by the relations:

$$\alpha)\quad C_{\alpha a}(i\omega^x) = \cos\omega^x\tau_W^x - i\sin\omega^x\tau_W^x \quad (23)$$

where $\tau_W^x = \xi_H/V\cdot\tau_A$ resp. $\xi_H/V_H\cdot\tau_A$

$$\beta)\quad C_{\alpha a}(i\omega^x) \doteq 1 - i\omega^x\cdot\tau_W^x \quad (24)$$

where $\tau_W^x = \tau_H^x = r_H/V_H\cdot\tau_A$

Simplified transfer functions $F_{C_A\cdot x}(i\omega^x)$:

$$F_{C_A\cdot\eta}(i\omega^x) = C_{A\cdot\eta}\cdot C_X(i\omega^x) \quad (25)$$

$$F_{C_A\cdot\alpha_\gamma}(i\omega^x) = \{[A_1 - a_{H1}\cdot(d\alpha_a/d\alpha)\cdot C_{\alpha a}(i\omega^x)]\cdot C_X(i\omega^x)\}_c + [i\omega^x\cdot A_2\cdot K_{\lambda X}]_i \quad (26)$$

$$F_{C_A\cdot\dot{\theta}^x}(i\omega^x) = [A_3\cdot C_X(i\omega^x)]_c + [i\omega^x\cdot A_4\cdot K_{\lambda X}]_i \quad (27)$$

	PARAMETER	WING: GEOMETRY x)	WING: AERODYNAMICS xx)	PARAMETER	TAILPLANE: GEOMETRY x)	TAILPLANE: AERODYNAMICS xx)
LIFT EQ. (11, 12, 13)	a_{F1}	1	$a = \partial C_{AF}/\partial\alpha_F$	a_{H1}	$\tilde{S}_H$	$a_1 k_H = k_H\,\partial C_{AH}/\partial\alpha_H$
	a_{F2}	1	a	a_{H2}	$\tilde{S}_H\,\tilde{\tilde{l}}_H$	$a_1\,k_H$
	a_{F3}	$\tilde{x}_{SN\theta} = (0{,}75 - \tilde{x}_S)$	a	a_{H3}	$\tilde{S}_H\,(\tilde{\tilde{r}}_H + 0{,}50)\cdot\tilde{\tilde{l}}_H$	$a_1\,k_H$
	a_{F4}	$\tilde{x}_S - 0{,}50$	a	a_{H4}	$\tilde{S}_H\,(\tilde{\tilde{r}}_H + 0{,}25)\cdot\tilde{\tilde{l}}_H^2$	$a_1\,k_H$
MOMENT EQ. (16, 17, 18)	m_{F1}	$\tilde{x}_{NFS}$	a	m_{H1}	$-\tilde{r}_H\,\tilde{S}_H$	$a_1\,k_H$
	m_{F2}	$\tilde{x}_S - 0{,}50$	a	m_{H2}	$+(\tilde{\tilde{r}}_H + 0{,}25)\,\tilde{S}_H\tilde{l}_H\tilde{\tilde{l}}_H$	$a_1\,k_H$
	m_{F3}	$\tilde{x}_{NFS}\cdot(0{,}75 - \tilde{x}_S)$	a	m_{H3}	$-\tilde{r}_H\,\tilde{S}_H(\tilde{\tilde{r}}_H + 0{,}50)\tilde{\tilde{l}}_H$	$a_1\,k_H$
	m_{F4}	0,25	a	m_{H4}	$+0{,}25\,\tilde{S}_H\tilde{\tilde{l}}_H$	$a_1\,k_H$
	m_{F5}	$m'_{F5} = (\tilde{x}_S - 0{,}50)^2$	a	m_{H5}	$m'_{H5} = \tilde{S}_H\cdot\tilde{l}_H[\tilde{\tilde{r}}_H + 0{,}25]^2\cdot\tilde{\tilde{l}}_H^2$	$a_1\,k_H$
	m_{F6}	$m'_{F6} = 1/128$	a	m_{H6}	$m'_{H6} = \tilde{S}_H\cdot\tilde{l}_H\cdot\tilde{\tilde{l}}_H^2\cdot 1/128$	$a_1\,k_H$

x) a'_{Xi}, m'_{Xi}; xx) a^A_{Xi} m^A_{Xi}; X = F, H; i = 1,2,..,6 — $a_{Xi} = a'_{Xi}\cdot a^A_{Xi}$ [1]; $m_{Xi} = m'_{Xi}\,m^A_{Xi}$ [1]

$a_{H1\eta} = a_{H1}\cdot(a_2/a_1)$ — $a_{H1\alpha} = a_{H1}\cdot(d\alpha_a/d\alpha)$ — $m_{H1\eta} = m_{H1}\cdot(a_2/a_1)$ — $m_{H1\alpha} = m_{H1}\cdot(d\alpha_a/d\alpha)$

$\tilde{l}_H = l_H/l$, $\tilde{\tilde{l}}_H = \tilde{l}_H\sqrt{k_H}$ [1] — $\tilde{r}_H = r_H/l$, $\tilde{\tilde{r}}_H = \tilde{r}_H/\tilde{l}_H$ [1] — $\tilde{S}_H = S_H/S$ [1] — $\tilde{x}_S$, $\tilde{x}_{NFS}$, $\tilde{x}_{SN\theta}$ see Fig. 2

TABLE 1 - PARAMETERS OF AERODYNAMIC FREQUENCY TRANSFERS

where $A_1 = a_{F1} + a_{H1}$, $A_2 = a_{F2} + a_{H2}$ (28a,b)

$A_3 = a_{F3} + a_{H3}$, $A_4 = -a_{F4} + a_{H4}$ (28c,d)

Simplified transfer functions $F_{C_m \cdot x}(i\omega^*)$:

$$F_{C_m \cdot \eta}(i\omega^*) = C_{m \cdot \eta} \cdot C_H(i\omega^*) \quad (29)$$

$$F_{C_m \cdot \alpha_\gamma}(i\omega^*) = \left\{[m_1 - m_{H1} \cdot (d\alpha_a/d\alpha) \cdot C_{\alpha_a}(i\omega^*)] \cdot C_X(i\omega^*)\right\}_C + [i\omega^* \cdot m_2 \cdot K_{\lambda X}]_i \quad (30)$$

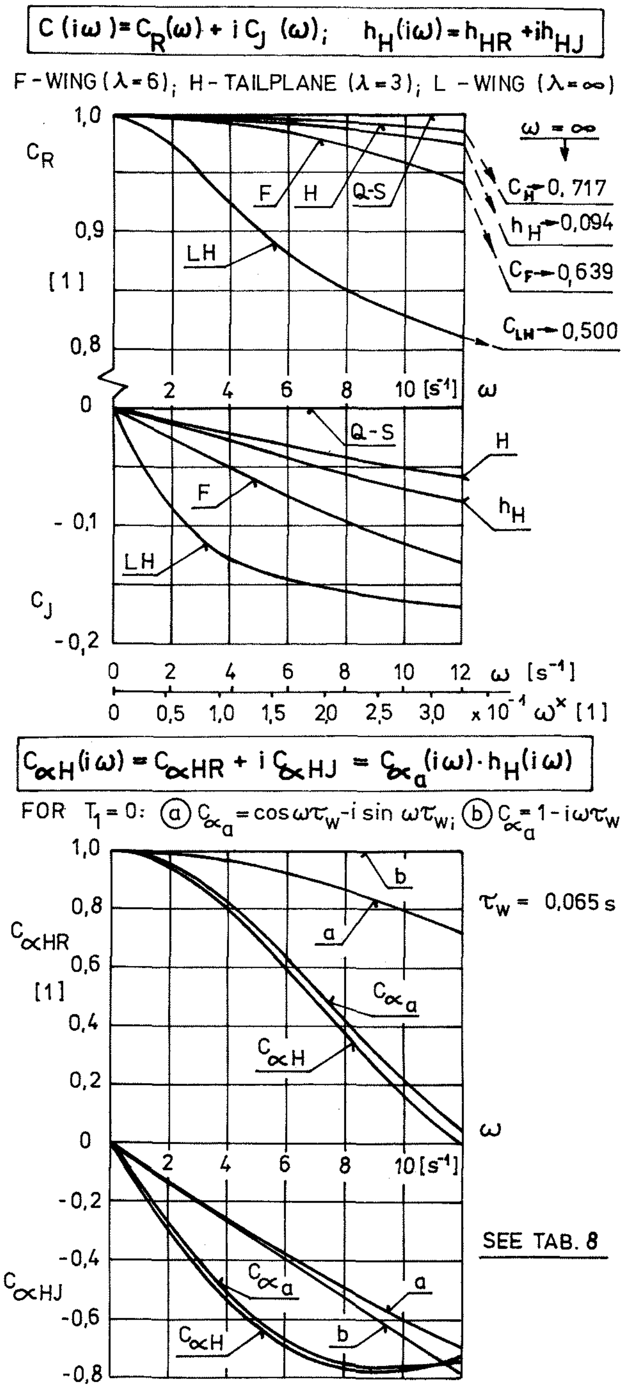


FIGURE 3 - NORMALIZED NONDIMENSIONAL FREQUENCY TRANSFERS FOR LIFT AND DOWN-WASH ANGLE

$$F_{C_m \cdot \dot\theta^*}(i\omega^*) = [m_3 \cdot C_X(i\omega^*)]_C + [-m_4 \cdot K_{\lambda X} + - i\omega^*(m_5 \cdot K_{\lambda X} + m_6)]_i \quad (31)$$

where

$m_1 = m_{F1} + m_{H1}$, $m_2 = m_{F2} + m_{H2}$ (32a,b)

$m_3 = m_{F3} + m_{H3}$, $m_4 = m_{F4} + m_{H4}$ (32c,d)

$m_5 = m_{F5} + m_{H5}$, $m_6 = m_{F6} + m_{H6}$ (32e,f)

When $C_X(i\omega^*) \equiv 1$ and $K_{\lambda X} \equiv 0$ and when the transfer function $C_{\alpha_a}(i\omega^*)$ is given by (23) or by the more often used (24), then the models (25) to (27) and (29) to (31), arranged in this way, represent the usual quasi-stationary models.

By using (24), one gets for the lift and pitching moment coefficient: $(K = A, m)$

$$F_{C_K \cdot \eta}(i\omega^*) = C_{K \cdot \eta}, \quad F_{C_K \cdot \dot\theta^*}(i\omega^*) = C_{K \cdot \dot\theta^*} \quad (35a,b)$$

$$F_{C_K \cdot \alpha_\gamma}(i\omega^*) = C_{K \cdot \alpha} + i\omega^* \cdot C_{K \cdot \dot\alpha^*} \quad (36)$$

3.2. Aerodynamic responses to step inputs

Aerodynamic step admittances can be derived from the aerodynamic frenquency transfer functions, which are given in chap. 3.1, by means of inverse Fourier transfromation and of the realation (10):

$$A_{C_K \cdot x}(t^*) = \mathcal{F}^{-1}\left\{(1/i\omega^*) \cdot F_{C_K \cdot x}(i\omega^*)\right\} \quad (37)$$

where $K = A$, m and $x = \eta, \alpha, \dot\theta^*$.

In the relations (11) to (13) and (16) to (18), it is sufficient to replace the normalized frequency transfer functions $C_F(i\omega^*)$, $C_H(i\omega^*)$ and $C_{\alpha H}(i\omega^*)$ by the normalized step admittances, which are of the form:

a) Step admittances of the Wagner type:

$$k_1(t^*)_c = \mathcal{F}^{-1}\left\{(1/i\omega^*) \cdot C(i\omega^*)\right\} = 1(t^*) - \sum_i {}^0c_i \cdot e^{-t^*/T_i^*} \quad (38)$$

b) Step admittances of the Küssner type:

$$k_2(t^*) = \mathcal{F}^{-1}\left\{(1/i\omega^*) \cdot H_H(i\omega^*)\right\} = 1(t^*) - \sum_i {}^0h_i \cdot e^{-t^*/T_{Hi}^*} \quad (39)$$

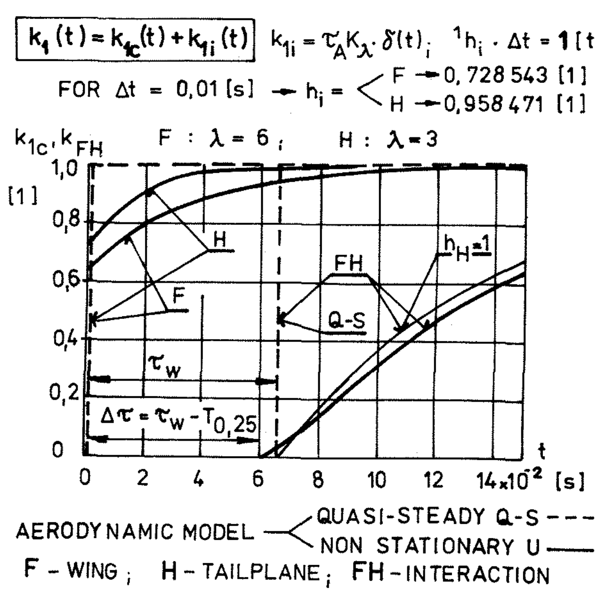


FIGURE 4 - NORMALIZED NONDIMENSIONAL STEP ADMITTANCES FOR LIFT OF THE A 145 AEROPLANE

c) Step admittances for the interaction wing-tailplanes:

$$k_{FH}(t^*) = \mathcal{F}^{-1}\left\{(1/i\omega^*)\cdot C_{\alpha_d}(i\omega^*)\cdot h_H(i\omega^*)\right\} = \tag{40}$$

$$= 1(t^*-\Delta\tau^*) - (1+K_T^*)\cdot e^{-(t^*-\Delta\tau^*)/T_1^*} + K_T^*\cdot e^{-(t^*-\Delta\tau^*)/T_H^*}$$

where

$$K_T^* = {}^0h\cdot T_H^*/(T_1^*-T_H^*), \quad \Delta\tau^* = \tau_W^* - T_{H\,0,25}$$

For $t^* < \Delta\tau^*$ there is $k_{FH} = 0$,
for $t^* \geq \Delta\tau^*$ there is $1 > k_{FH} > 0$.

d) In the frequency transfer functions (12), (13), (17) and (18), to the terms of the type $i\omega^*\cdot K_{Kz}$, when K = A, m and $z = \dot{\alpha}^*, \dot{\theta}^*, \ddot{\theta}^*$, step admittances correspond that are of the type:

$$\mathcal{F}^{-1}\left\{(1/i\omega^*)\cdot i\omega^*\cdot K_{Kz}\right\} = K_{Kz}\cdot\delta(t^*) \tag{41}$$

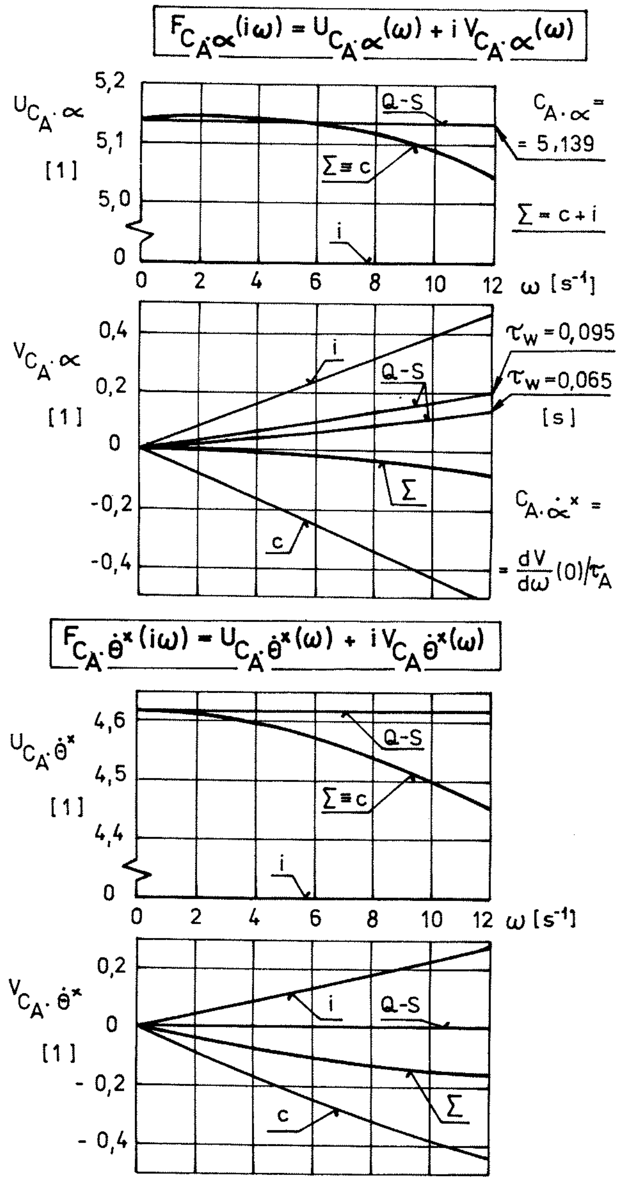
where $\delta(t^*) = 0$ for $t^* \gtrless 0$ and $\delta(t^*) \neq 0$ for $t^* = 0$, and at the same time $\int_{-\infty}^{+\infty}\delta(t^*)\,dt^* = 1$

In the case that $h_H(i\omega^*) = 1$ and $T_1^* = 0$, the formulae (23) and (24) are transformed to:

$$k_{FH}(t^*) = \mathcal{F}^{-1}\left\{(1/i\omega^*)\cdot e^{-i\omega^*\tau_W^*}\right\} = 1(t^*-\tau_W^*) \tag{42}$$

$$k_{FH}(t^*) \cong \mathcal{F}^{-1}\left\{(1/i\omega^*)\cdot(1-i\omega^*\tau_W^*)\right\} = 1(t^*) - \tau_W^*\cdot\delta(t^*) \tag{43}$$

These two expressions serve as an example, how a mathematically correct simplification of (23) into (24) for small values $\omega^*\tau_W^*$ modifies a



COMPONENT: CIRCULATION - c ; INERTIAL - i

FIGURE 5 - LIFT FREQUENCY TRANSFERS OF THE A 145 AEROPLANE

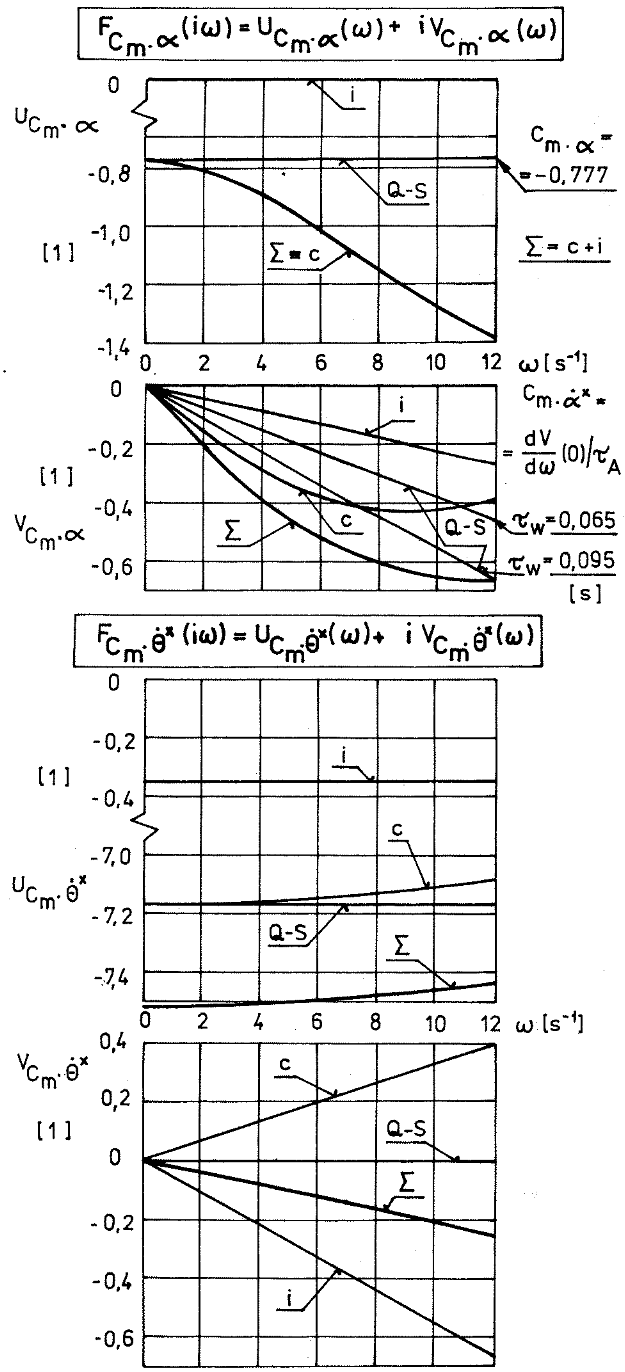


FIGURE 6 - PITCHING MOMENT FREQUENCY TRANSFERS OF THE A 145 AEROPLANE

description of the physical substance of a phenomenon. A step change with a transport lag (42) is modified in the equation (43) formally into a step change without a transport lag and a pulse, see fig. 7 B and 8 B.

3.3. Example for the A 145 aeroplane

For getting an idea on significancy of various components of nonstationary aerodynamics, some examples of aerodynamic frequency transfer functions and aerodynamic transient functions have been calculated. Necessary data were taken from [2] and are given in the appendix in tab. 6 to 10. In the examples the dimensional arguments $\omega = \omega^x/\tau_A$ and $t = t^x \tau_A$ are used. For these arguments, parameters in normalized functions are given in tab. 8.

An idea about the share of the wing and tailplanes in a lift and pitching moment of the whole aeroplane inclusive of nonstationary aerodynamics can be gained by means of tab. 9. A surprising fact is the same share of the wing and tailplanes in the case of a_{F3} and a_{H3} parameters of the circulation component of the lift in (13) for $F_{CA} \cdot \dot{\theta}_C^x$.

In the upper part of fig. 3 the effect of the aspect ratio of the lifting surface is shown. In the lower part of fig. 3, different models for $C_{\alpha_a}(i\omega^x)$ are illustrated. The effect of $h_H(i\omega^x)$ on $C_{\alpha H}(i\omega^x)$ is relatively small one, which fact is confirmed also in fig. 4.

Aerodynamic frequency transfer functions for the lift and pitching moment coefficients are shown in fig. 5 and 6. In the imaginary parts of the both transfers the inclinations of the curves tangents at the beginning are proportional to the "dotted derivatives". In the both transfers for the lift coefficient on fig. 5, the opposite effect is seen of the circulation and inertial components. In the frequency transfers for the pitching moment coefficient on fig. 6, the circulation and inertial components in the imaginary parts for the $\Delta\alpha$ input are in the same direction while for damping they are in the opposite directions.

In the figures 7 and 8, time histories of the lift and pitching moment coefficients after a step change of $\Delta\alpha$ and $\Delta\dot{\theta}^x$ are shown. The inertial components effect appears by

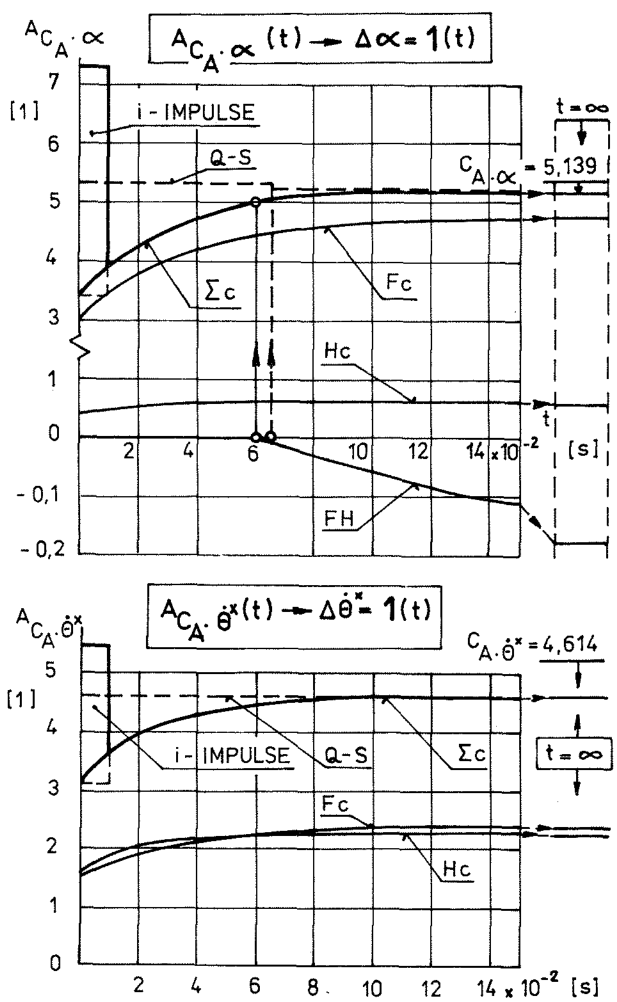


FIGURE 7 - NONDIMENSIONAL STEP ADMITTANCES FOR LIFT OF THE A 145 AEROPLANE : (SEE FIGURE 4) A) NONSTATIONARY MODEL

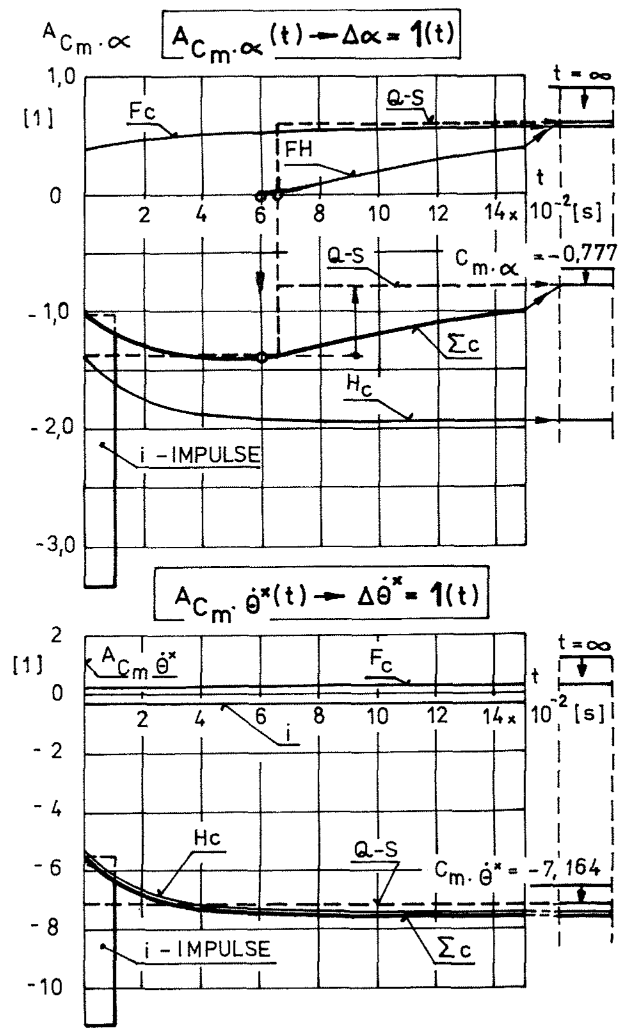


FIGURE 8 - NONDIMENSIONAL STEP ADMITTANCES FOR PITCHING MOMENT OF THE A 145 AEROPLANE: (SEE FIG. 4) A) NONSTATIONARY MODEL

$A_{C_A \cdot \alpha}(t) \rightarrow \Delta\alpha = 1(t)$

$k_{1F} = k_{1H} = 1(t)$; $k_{FH} = \mathcal{F}^{-1}\left\{\frac{1}{i\omega} C_{\alpha H}(i\omega)\right\}$ [1]

a) $C_{\alpha H} = \cos\omega\tau_w - i\sin\omega\tau_w$ b) $C_{\alpha H} = 1 - i\omega\tau_w$

$k_{FH} = 1(t - \tau_w)$ $k_{FH} = 1(t) - \tau_w \cdot \delta(t)$

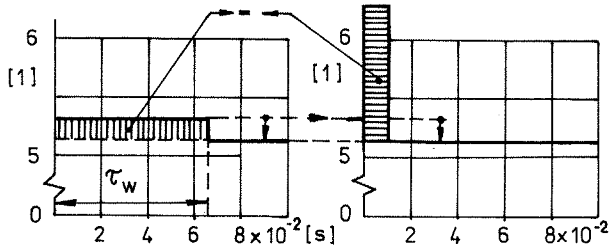


FIGURE 7 - CONTINUATION: (SEE FIG.4)
B) Q-S MODELS

rectangular pulses in the time t = 0. The pitching moment coefficient, after a pulse having been applied in the stabilizing direction, grows due to circulation component from a higher value as is the value corresponding to t = ∞ , and only after a transport lag time $\Delta\tau = \Delta\tau_W - T_{H\,0,25}$ it decreases exponentially to the value of the quasi-stationary derivative.

A comparison of values of aerodynamic derivatives when using nonstationary and quasi--stationary (Q-S) aerodynamics models is given in tab. 2. Remarkable differencies are seen at "dotted derivatives". They follow from different substances of models.

Notions worth seeing follow from fig. 7 B and 8 B, where step admittance of a quasi--stationary model are illustrated for lift and pitching moment coefficients. In this case the admittances do not embrace the pulses that correspond to inertial components of nonstationary aerodynamics. From the comparison of the cases a) and b) one can see that in the both cases the matter are pulse changes in the same direction but with different time bases at the same surface of the pulses. A longer basis ad a) is physically justified, a shorter basis ad b) has not a physical justification, but it

$F_{y.x}$		$dV/d\omega\ (0)$		$C_{y.x}\ (0)^{x)}$		$C_{y.\dot{x}^x}\ (0)^{x)}$	
y	x	c	i	$U_{y.x}(0)$	Q-S	xx)	Q-S
C_A	η	-0,00177	—	0,341	0,341	-0,065	—
	α	-0,03831	+0,03854	+5,139	+5,139	+0,008	+0,615
	$\dot{\theta}^x$	-0,04239	+0,02352	+4,614	+4,614	-0,691	—
C_m	η	+0,00591	—	-1,138	-1,138	+0,216	—
	α	-0,08390	-0,02292	-0,777	-0,777	-3,912	-2,051
	$\dot{\theta}^x$	+0,03497	-0,05522	-7,517	-7,164	-0,742	—

x) $y = A, m$, $x = \eta, \alpha, \dot{\theta}^x$ xx) $c + i \rightarrow dV/d\omega\,(0)/\tau_A$

TABLE 2 - AERODYNAMIC DERIVATIVES OF THE A 145 AEROPLANE

$A_{C_m \cdot \alpha}(t) \rightarrow \Delta\alpha = 1(t)$

$k_{1F} = k_{1H} = 1(t)$ $k_{FH} = \mathcal{F}^{-1}\left\{\frac{1}{i\omega} C_{\alpha H}(i\omega)\right\}$ [1]

a) $C_{\alpha H} = \cos\omega\tau_w - i\sin\omega\tau_w$ b) $C_{\alpha H} = 1 - i\omega\tau_w$

$k_{FH} = 1(t - \tau_w)$ $k_{FH} = 1(t) - \tau_w \cdot \delta(t)$

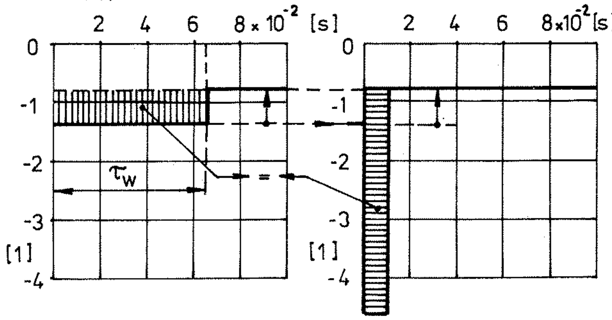


FIGURE 8 - CONTINUATION: (SEE FIG.4)
B) Q-S MODELS

recomponses in some manner for the effect of a missing inertial pulse.

4. Loss functions for the short-period longitudinal aeroplane motion

A theoretical analysis of basic relations was given in [3]. In the submitted paper, a loss function for the both equations of motion (3) and (4) is proposed in a form suitable for proving the validity of aerodynamic models which are given by tables of values or by analytical expressions. For a dimensional argument, the loss function is defined by the relation:

$$S_K = \sum_{j=2}^{k} \left[e^2_{KR}(\omega j) + e^2_{KJ}(\omega j) \right] \qquad (44)$$

where K = A, m; R is the real part and I denotes the imaginary part.
Further there is:

$$e_K(i\omega_j) = Y_{KE}(i\omega_j) - Y_{KT}(i\omega_j) \qquad (45)$$

where Y_{KE} are the experimental frequency transfers, determined by right hand sides of equations (3) and (4), and Y_{KT} are the total aerodynamic frequency transfers determined by the equation (9).

As an example of utilization, the loss functions for the A 145 aeroplane have been calculated with parameters values from tab. 8 and 9. The results for 6 cases of frequency transfers of lift and pitching moment coefficients are given in tab. 3. For the lift coefficient the least value $S_{A\,min}$ = 1,97 % for the case 3 without the inertial component. For the pitching moment coefficient $S_{m\,min}$ = 2,75 % for the case 5 with the inertial component but without the circulation one.

For the A 145 aeroplane loss functions for simplified models of nonstationary aerodynamics were further calculated by (25) to (32) and for quasi-stationary aerodynamics models by (33) to (36) with parameters values given in tab. 8 and 10. The results for 7 cases are seen in tab. 4 and 5. For the frequency transfer of the lift coefficient in tab. 4, the least value $S_{A\,min}$ = = 1,47 % for the cases 4 and 5 without the inertial component. The effect of the approximate form $C_{\alpha H}$ and of the τ_W values is practically the same. For the frequency transfer of the pitching moment coefficient in tab. 5, the minimum values are $S_{m\,min}$ = 1,87 % and 1,77 % for the case 4. Unlike at the lift coefficient, the inertial component shows here very favourably. Also the form $C_{\alpha H}$ according to (24) is more convenient then according to (23), as well as the greater value of τ_W, especially for the quasi-statio- nary aerodynamics model.

5. Concluding remarks

The longitudinal motion of a light transport aeroplane is considered as a response to a pulse form deflection of elevator at a slow straight steady flight.

On the basis of a physical analysis, basic and simplified models were proposed for nondimensional aerodynamic frequency transfers. They enclose the separated quasi-stationary and nonstationary parts. The utilization of aerodynamic frequency transfers in a model of the longitudinal aeroplane motion, however, depends on the frequency transfers of aeroplane responses on elevator deflections, see (3), (4) and (9).

A solution in the frequency domain has enabled by means of inverse Fourier transformation to derive from the aerodynamic frequency transfer for the down-wash angle at tailplanes a step admittance for the down-wash angle. If it is simplified to a mere transport lag according to (42), then it is possible to explain the consequences of a mathematically correct simplification of the expression for transport lag to (43), which is usually used in the quasi-stationary model. The step change with a transport lag is formally modified to a step change without a transport lag but with a pulse, see fig. 7 B and 8 B.

The calculated examples for the A 145 aeroplane have proved that the proposed simplified expressions for the nonstationary aerodynamics model are applicable in a low flight-velocity interval and that they are even better then the quasi-stationary aerodynamics model used generally till now. In frequency transfers of the fift coefficient, the inertial component effect is not significant, as being diminished by the imaginary part of the circulation origin. In frequency transfers of the pitching moment coefficient this effect is significant as the both components reinforce one the other.

The paper gives instructions for objectively quantitative checking of the correctness of aerodynamic data used by the aeroplane design, which is based on the results of flight measurements with a real aeroplane prototype. Besides it has also a didactical importance.

		CASE 1	2	3	4	5	6
C_{Fc}	o_{c_F} [1]	0, 361	X	X	0	0	0
	T_F [s]	0,03583	X	X	0	0	0
C_{Hc}	o_{c_H} [1]	0, 283	X	X	0	0	0
	T_H [s]	0,01835	X	X	0	0	0
$C_{\alpha H}$	T_1 [s]	0,07501	X	0	X	0	X
	τ_W [s]	0, 065	X	X	X	X	X
	o_{h_H} [1]	0, 679	0	0	X	0	0
	T_H [s]	0, 01775	0	0	X	0	0
	$T_{0,25}$ [s]	0, 00495	0	0	X	0	0
C_A	$\tau_A K_{A\dot{\alpha}^x}$ xx)	0,038537	X	0	0	X	0
	$\tau_A K_{A\dot{\Theta}^x}$ xx)	0,023522	X	0	0	X	0
	S_A % x)	2, 53	2,51	<u>1,97</u>	2,99	3,39	2,97
C_m	$\tau_A K_{m\dot{\alpha}^x}$ xx)	-0,02292	X	0	0	X	0
	$K_{m\dot{\Theta}^x}$ [1]	-0, 35283	0	0	0	X	0
	$\tau_A K_{m\ddot{\Theta}^x}$ xx)	-0, 05522	X	0	0	X	0
	S_m % x)	5,47	5,63	3,43	5,96	<u>2,75</u>	5,63

x) $(S_A)_{max}$ = 254,482 [1]; $(S_m)_{max}$ = 19, 069 [1] xx) [1]

FOR OPTIMAL PARAMETER VALUES: $S_{A\,opt.}$ = <u>2,58 %</u>; $S_{m\,opt.}$ = <u>1, 77 %</u> SEE [3], TAB.5

TABLE 3 - PARAMETER INFLUENCE ON LOSS FUNCTION OF THE A 145 AEROPLANE: NONSTATIONARY AERODYNAMIC MODEL

	CASE 1	2	3	4	5	6	7
	F (λ=6) x)	LH ($\lambda=\infty$) x)				Q-S	
o_{c_1} [1]	0, 361	0,165				0	0
T_1 [s]	0,03583	0, 217 72				0	0
o_{c_2} [1]	0	0, 335				0	0
T_2 [s]	0	0, 03302				0	0
τ_W [s]	0, 065		0,095		0,065		0,095
K_λ [1]	0, 26681	0,25	0, 25	0	0	0	0
$C_{\alpha H} \equiv C_{\alpha_a}(i\omega) = \cos \omega\tau_W - i \sin \omega\tau_W$							
S_A %	2, 27	1,56	1,57	<u>1,47</u>	<u>1,47</u>	2,79	2,87
$C_{\alpha H} \equiv C_{\alpha_a}(i\omega) = 1 - i\omega\tau_W$							
S_A %	2, 26	1,56	1,58	<u>1,48</u>	<u>1,48</u>	2,78	2,85
x) SEE TABLE 8			$(S_A)_{max}$ = 19, 069 [1]				

TABLE 4 - PARAMETER INFLUENCE ON LOSS FUNCTION OF THE A145 AEROPLANE: SIMPLIFIED LIFT AERODYNAMIC MODELS

	CASE						
	1	2	3	4	5	6	7
	$F(\lambda=6)$ x)	$H(\lambda=3)$ x)	$LH(\lambda=\infty)$ x)			Q - S	
${}^{o}c_1$ [1]	0,351	0,283	0,165			0	0
T_1 [s]	0,03583	0,01835	0,217 72			0	0
${}^{o}c_2$ [1]	0	0	0,335			0	0
T_2 [s]	0	0	0,03302			0	0
τ_W [s]	0,065			0,095	0,065		0,095
K_λ [1]	0,266 81	0,351 01	0,25	0,25	0	0	0
$C_{\alpha H} \equiv C_{\alpha a}(i\omega) = \cos\omega\tau_W - i\sin\omega\tau_W$							
S_m %	2,44	2,46	2,34	1,87	3,96	3,41	3,80
$C_{\alpha H} \equiv C_{\alpha a}(i\omega) = 1 - i\omega\tau_W$							
S_m %	2,05	2,00	2,33	1,77	3,87	2,84	2,55
x) SEE TABLE 8			$(S_m)_{max} = 254{,}482$ [1]				

TABLE 5 - PARAMETER INFLUENCE ON LOSS FUNCTION OF THE A145 AEROPLANE: SIMPLIFIED PITCHING MOMENT AERODYNAMIC MODELS

Acknowlegement

The author would like to express his thanks to the Director of the ARTI for the permission to publish this paper as well to Mr. K. Jansa for his suggestions and translation into English and to Mr. J. Kučera for reviewing the English text as well to other colleages for their help.

6. Appendix: The A 145 aeroplane data

The used A 145 aeroplane data are taken over from [1] and are given in tab. 6. The results of flight measurements of aerodynamic coefficients and derivatives from steady flights are given in tab. 7.

Aerodynamic normalized nondimensional transfer functions C_X and H_H are according to [3] given by relations:

$$C_X(i\omega^x) = C_{Xc} + C_{Xi} = 1 - \sum_i {}^{o}c_i \frac{i\omega^x T^x_{Xi}}{1 + i\omega^x T^x_{Xi}} + i\omega^x \cdot K_{\lambda X} \qquad (46)$$

where X = F, H, L, LH; F - wing, $\lambda = 6$; H - tailplanes, $\lambda = 3$; L - aeroplane, $\lambda = \infty$; LH - L after a recalculation to $\omega^x_H = \omega l_H / V_H$.

$$H_H(i\omega^x) = 1 - \sum_i {}^{o}h_i \frac{i\omega^x T^x_{Hi}}{1 + i\omega^x T^x_{Hi}} \qquad (47)$$

$$h_H(i\omega^x) = H_H(i\omega^x) \cdot e^{+i\omega^x T^x_{H\,0,25}} \qquad (48)$$

where $T^x_{H\,0,25} = 0{,}25 \cdot \tilde{l}_H / \sqrt{k_H}$

The normalized nondimensional transfer function for the down-wash angle may be according to [3] expressed approximately by:

$$C_{\alpha a}(i\omega^x) = \frac{1}{1 + i\omega^x T_1^x} \cdot e^{-i\omega^x \tau^x_W} \qquad (49)$$

and

$$C_{\alpha H}(i\omega^x) = C_{\alpha a}(i\omega^x) \cdot h_H(i\omega^x) \qquad (50)$$

The parameters values of the functions C_X, C_H, $C_{\alpha a}$ for the A 145 aeroplane are seen from tab. 8.* The parameters values from tab. 1 are in the tab. 9 and for the simplified model in tab. 10.

The frequency transfer functions of the aeroplane responses $F_{\omega_y \cdot \eta}(i\omega)$ and $F_{\alpha \cdot \eta}(i\omega)$ from flight measurements are also taken over from [1] and are drawn on fig. 9 and 10. Their mean standard deviations are: $(s_E)_{\omega_y \cdot \eta} = \pm 0{,}1225\ [s^{-1}]$ $(s_E)_{\alpha \cdot \eta} = \pm 0{,}0675\ [1]$

7. Symbols

A, A_F, A_H	Lift of aeroplane, wing or tailplane respectively
$a = \partial C_{AF}/\partial\alpha_F$; $a_1 = \partial C_{AH}/\partial\alpha_H$; $a_2 = \partial C_{AH}/\partial\eta$	
$A_{y.x}(t^x)$	Unit step admittance - response of the y quantity to a unit step change of the x quantity
$C_A = A/q \cdot S$	Lift coefficient of aeroplane
$C_m = M/q \cdot S \cdot l$	Pitching moment coefficient
$C_{y.x} = \frac{\partial C_y}{\partial x}$	Aerodynamic derivative; y = A, m, AF, AH; x = α, α_r, θ, η; $\dot\alpha^x$, $\dot\theta^x$, $\dot\eta^x$, α_F, α_H.
C_W	Drag coefficient
$C(i\omega^x)$	Aerodynamic normalized nondimensional transfer function of the Theodorsen type
$C_{\alpha a}(i\omega^x)$	Normalized nondimensional transfer function for the downwash angle
$C_{\alpha H}(i\omega^x) = C_{\alpha a}(i\omega^x) \cdot h_H(i\omega^x)$	
$F_{y.x}(i\omega^x) = U_{y.x}(\omega^x) + iV_{y.x}(\omega^x)$	Frequency transfer function of the response y on the input x
$H(i\omega^x), h(i\omega^x)$	Aerodynamic normalized nondimensional transfer functions of the Sears type, related to the leading edge or to 0,25 l_H respectively
$\tilde{i}_y = \sqrt{J_y/ml^2}$	Nondimensional moment of inertia around the y - axis
$k_H = q_H/q$	
$k_1(t^x), k_2(t^x)$	Normalized nondimensional lift step admittances of the Wagner or Küssner type
K_λ	Coefficient of aerodynamic inertial component
l	Lenght of aerodynamic mean chord, [m]
m	Aeroplane mass, [kg]
$q = \rho V^2/2$	Kinetic pressure, [N/m]
r_H	see fig. 2, [m]
$t^x = Vt/l = t/\tau_A$	Strouhal number in time domain

* For the wing the parameters for the aspect ratio 6 were used, whereas the real wing is of the aspect ratio 8.78.

$S = \sum_j e_j^2$	Loss function, j = 2,..., k
S, S_H	Wing or tailplane area, [m]
V	True velocity of an aeroplane, [m/s]
α, α_F, α_H	Angle of attack of aeroplane, wing, tailplane respectively
α_a	Downwash angle-positive in opposite sign of α
$\Delta\alpha_\gamma$, $\Delta\alpha_\theta$	"Path" or "attitude" change of angle of attack
γ	Flight path inclination angle
Θ	Aeroplane inclination angle
η	Elevator angle
λ	Aspect ratio
$\mu = 2m/\rho S l$	Aeroplane normalized mass
ρ	Air density, [kg/m^3]
$\tau_A = l/V$	Aerodynamic unit of time, [s]
ω	Circular frequency, [s^{-1}]
$\omega^* = \omega \cdot \tau_A$	Strouhal number - reduced frequency

Denominations

$\bar{x}(i\omega^*)$	Fourier transform of the x(t*)

$\tilde{x} = x/x_{ref}$; $\dot{x}^* = \dot{x} \cdot \tau_A$; $\dot{x} = d/dt$; $^{o}x = x/x(0)$

m	kg	1530	V	m/s	54,20
$\tilde{i}_H^2$	1	0,859	H	m	1390
$\tilde{x}_S$	1	0,247	ρ	kg/m^3	1,070
μ	1	112,96	τ_A	s	0, 027 306
$\tau = \mu\tau_A$	1	3,084	$1/\tau_A$	s^{-1}	36,622
$\tilde{x}_{N_F}$	1	0,126	$\tilde{x}_{SN_\Theta}$	1	0,503
$\tilde{x}_{N_F S}$	1	0,121	$\tilde{\xi}_H$	1	2,373
l	m	1,480	l_H	m	1,030
b	m	12,25	b_H	m	3,39
λ	1	8,78	λ_H	1	3,47
S	m^2	17,09	S_H	m^2	3,31
r	m	5,119	r_H	m	4,940
ξ	m	3,770	ξ_H	m	3,512
$\tilde{i}_H$	1	0,696	$\tilde{\tilde{i}}_H$	1	0,726
$\tilde{r}_H$	1	3,338	$\tilde{\tilde{r}}_H$	1	4,796
$\tilde{S}_H$	1	0,194	$\tilde{\tilde{r}}_H + 0,25$	1	5,046
$\tilde{S}_H \tilde{r}_H$	1	0,646	$\tilde{\tilde{r}}_H + 0,5$	1	5,296

TABLE 6 -CHARACTERISTICS OF THE A 145 SMALL TWIN ENGINED AEROPLANE AND OF STEADY FLIGHTS - REF.[2]

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(Comma denotes in the presented tables decimal points.)

	λ	i	$^{o}c_i$ [1]	T_i^* [1]	$T_i = T_i^* \cdot \tau_A$ [s]
C_F	6	1	0, 361	1,312 336	0,035 835
C_F	3	1	0, 283	0,925 926	0,025 283
C_H	3	1	0, 283	0, 671 828	0, 018 345
C_L	∞	2	0,165	10, 989 011	0, 300 066
			0,335	1, 666 667	0,045 510
C_{LH}	∞	2	0, 165	7, 973 341	0, 217 720
			0,335	1,209 290	0, 033 021
H_H	3	2	0, 679	0, 650 156	0, 01 7 753
			0,227	0, 113 371	0, 003 096
h_H	$T^*_{H\,0,25}$ = 0,181 394 [1]			$T_{H\,0,25}$ = 0,004 953 [s]	
C_{α_a}	$T_1^* = T_1/\tau_A$ = 2,747 015 [1]			T_1 = 0, 075 01 [s]	
	$\tau_W^* = \tau_W/\tau_A$ = 2,380 429			τ_W = 0, 0650 [s]	
$K_{\lambda F}$ = 0,266 807		$K_{\lambda H}$ = 0, 351 011 [1]			$K_{\lambda\infty}$ = 0,25 [1]

TABLE 8 - PARAMETERS OF NORMALIZED AERODYNAMIC FREQUENCY TRANSFERS

DERIVATIVE	VALUE	COEFFICIENT	VALUE
$a = \partial C_{AF}/\partial \alpha_F$	4,735	C_{A0}	0,546
$a_1 = \partial C_{AH}/\partial \alpha_H$	3,261	$k_H = q_H / q$	0,920
a_2 / a_1	0,587	$\sqrt{k_H}$	0,959
$d\alpha_a / d\alpha$	0,304	C_{W0}	0,0302

TABLE 7 - DATA FROM STEADY FLIGHT MEASUREMENTS OF A 145 AEROPLANE

		VALUE			VALUE
a_{F1}	1	+ 4,735	a_{H1}	1	+ 0,581 043
a_{F2}	1	+ 4,735	a_{H2}	1	+ 0,421 590
a_{F3}	1	+ 2,381 705	a_{H3}	1	+ 2,232 788
a_{F4}	1	- 1,197 956	a_{H4}	1	+ 1,543 577
–		–	$a_{H1\eta}$	1	+ 0,340 879
–		–	$a_{H1\alpha}$	1	+ 0,176 637
$\tau_A K_{A\dot{\alpha}}{}^{x}$	s	+ 0,038 537	$\tau_A K_{A\dot{\theta}}{}^{x}$	s	+ 0,023 522
m_{F1}	1	+ 0,572 935	m_{H1}	1	- 1,939 427
m_{F2}	1	- 1,197 956	m_{H2}	1	+ 1,480 552
m_{F3}	1	+ 0,288 186	m_{H3}	1	- 7,452 683
m_{F4}	1	+ 1,183 751	m_{H4}	1	+ 0,105 398
m_{F5}	1	+ 0,303 083	m_{H5}	1	+ 5,420 751
m_{F6}	1	+ 0,036 992	m_{H6}	1	+ 0,001 663
m_6 x)	1	+ 0,038 653	$m_{H1\eta}$	1	- 1,137 804
–		–	$m_{H1\alpha}$	1	- 0,589 58
$\tau_A K_{m\dot{\alpha}}{}^{x}$	s	- 0,022 92	$K_{m\dot{\theta}}{}^{x}$	1	- 0,352 83
m_E xx)	s^2	+ 0,072 386	$\tau_A K_{m\ddot{\theta}}{}^{x}$	s	- 0,055 22
$K_{\lambda F}$	1	+ 0,266 807	$K_{\lambda H}$	1	+ 0,351 011
x) $m_6 = m_{F6} + m_{H6}$			xx) $m_E = \mu \tau_A^2 \tilde{i}_y^2$		

TABLE 9 - VALUES OF TABLE 1 PARAMETERS OF THE A 145 AEROPLANE

		VALUE			VALUE
A_1	1	+ 5,316 043	m_1	1	- 1,366 492
A_2	1	+ 5,156 590	m_2	1	- 2,678 508
A_3	1	+ 4,614 493	m_3	1	- 7,164 497
A_4	1	+ 2,741 532	m_4	1	+ 1,289 149
–		–	m_5	1	+ 5,723 834
$K_{\lambda\infty}$	1	+ 0,25	m_6	1	+ 0,038 653

TABLE 10 - VALUES OF SIMPLIFIED AERODYNAMIC MODEL PARAMETERS OF THE A 145 AEROPLANE

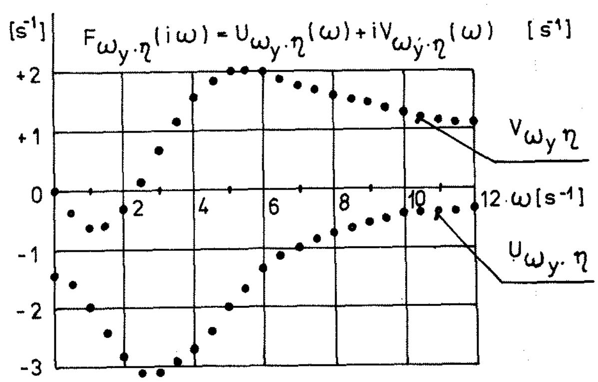

FIGURE 9 - FREQUENCY TRANSFERS OF THE A 145 AEROPLANE - $\bar{\omega}_y / \bar{\eta}\,(i\omega)\,s^{-1}$

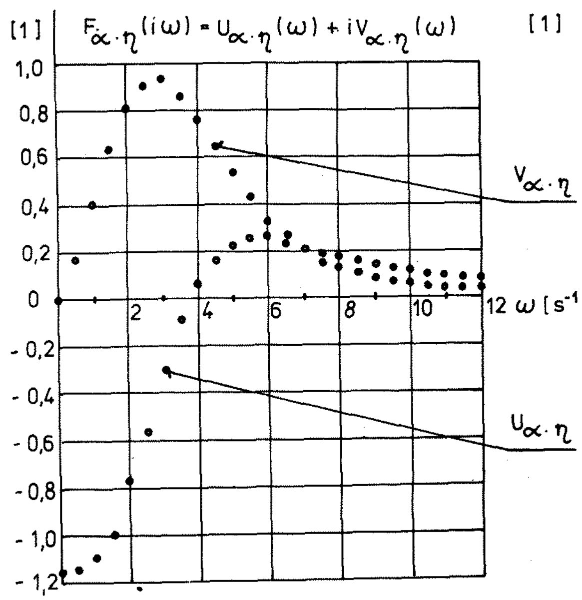

FIGURE 10 - FREQUENCY TRANSFERS OF THE A 145 AEROPLANE - $\Delta\bar{\alpha} / \bar{\eta}\,(i\omega)\,[1]$