PRECISE SOLUTION FOR RATIONAL TRANSFER PARAMETERS OF FLIGHT VEHICLE

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Abstract

The deviations of some design parameters always exist in adjusting aerodynamic configuration, arrangement and control loop. In this paper, a mathematic model, which is used for calculating precisely transfer parameters of vehicles, is established in the matrix form involving dynamic factors, transform factors, structure parameters and their deviations. It not only simplifies computer aided design programming, but also offer an intuitive sense.

Taking the dynamic factors for the function of aerodynamic and flight parameters and their deviations, the calculation method presented here is simplified. Since the transform factors being used, this method is suitable to normal, canard, control wing and ballistic vehicles.

I. Introduction

In order to increase the qualities of whole flight system, it is repeatedly necessary to demonstrate aerodynamic parameters, structure parameters and flight parameters, so that the deviations of these parameters occur. In this paper, making use of dynamic factors and transform factors, a mathematic model of transfer parameters of vehicle is derived involving these parameters and their deviations. The results obtained give an easy method for calculating each transfer parameter precisely and analyzing dynamic characteristic. For the normal, canard, control-wing and ballistic vehicle, the dynamic factors and transform factors are presented. Therefore, this method is proved to be available for all kinds of flight vehicles.

The transfer parameters involving flight, aerodynamic and structure parameters and their deviations are expressed in the matrix form. It not only simplifies computer aided design programming, but also has intuitional effect. An example of calculation for a flight vehicle is given in the end.

ω_V = natural angular frequency of the vehicle
T_V = time constant of the vehicle
K_V = transfer coefficient of the vehicle
K_α = transfer coefficient on angle of attack
ξ_V = relative damping coefficient
D = dynamic factor
ζ = transform factor
a = dynamic coefficient
S = acting area of aerodynamic force
x_g = center position of gravity
x_p = center of pressure
m = mass
J = moment of inertia
V = flying velocity

Δ = sign of the deviations

i = subscript (i=t,w,b, i.e. tail, wing, body)

II. Precise Solution

State equation of simplified longitudinal perturbation motion of flight vehicles is (See Ref.1, 2)

$$[\dot{\omega}_z, \dot{\alpha}, \dot{\vartheta}]^T = L[\omega_z, \alpha, \vartheta]^T + N\delta_z + N'\delta_z \quad (1)$$

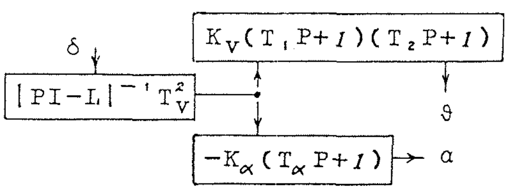
where

$$L = \begin{bmatrix} -a_1 - a'_1 & a'_1 a_4 + a'_1 a''_4 - a_2 & -a'_1 a''_4 \\ 1 & -a_4 - a''_4 & a''_4 \\ 1 & 0 & 0 \end{bmatrix}$$

$$N = [-a_2 + a_1 a_5), -a_5, 0]^T$$

$$N' = [-a_5, 0, 0]^T$$

According to the state equation, the transfer function can be expressed as



When $a''_4 = 0$, we can write

$$|PI-L|^{-1} T_V^2 = \omega_V^{-2} P^2 + 2\xi_V \omega_V^{-1} P + 1 \quad (2)$$

Transfer parameters ω_V, T_V, K_V, K_α, ξ_V, T_1, T_2 and T_α are functions of structure parameters (S, x_g, J, m, ...), aerodynamic parameters (C_{yi}^α, x_p, K_i, ε^α, ...) and flying parameters (v, h, ...), etc.

The matrix on structure parameters can be expressed as

$$X_{g-p} = \begin{bmatrix} x_g - x_{pt} & & & 0 \\ & x_g - x_{pw} & & \\ & & x_g - x_{pb} & \\ 0 & & & \ddots \end{bmatrix} \quad (3)$$

$$S = [s_t, s_w, s_b, \cdots] \quad (4)$$

$$\Delta S = [\Delta s_t, \Delta s_w, \Delta s_b, \cdots] \quad (5)$$

$$\Delta X_{g-p} = \begin{bmatrix} \Delta x_g - \Delta x_{pt} & & & 0 \\ & \Delta x_g - \Delta x_{pw} & & \\ & & \Delta x_g - \Delta x_{pb} & \\ 0 & & & \ddots \end{bmatrix} \quad (6)$$

Using dynamic factors D_i and their deviations ΔD_i, we can get following matrix of dynamic factors (the expressions of dynamic factors see Appendix 1).

$$D_i = D_i(V, h, C_{yi}^\alpha, K_i, \varepsilon_i^\alpha \cdots) \quad (7)$$

$$\Delta D_i = \frac{\partial D_i}{\partial V}\Delta V + \frac{\partial D_i}{\partial h}\Delta h + \frac{\partial D_i}{\partial C_{yi}^\alpha}\Delta C_{yi}^\alpha + \cdots \quad (8)$$

$$D + \Delta D = \begin{bmatrix} D_t + \Delta D_t & & & 0 \\ & D_w + \Delta D_w & & \\ & & D_b + \Delta D_b & \\ 0 & & & \ddots \end{bmatrix} \quad (9)$$

Above symmetrical matrixes are all interchangeable.

At a time, using transform factors (see Appendix 2), we have following matrixes

$$\zeta_1 = [\zeta_{1t}, \zeta_{1w}, \zeta_{1b}, \cdots]^T$$

$$\zeta_2 = [1, 1, 1, \cdots]^T$$

$$\zeta_3 = [\zeta_{3t}, \zeta_{3w}, \zeta_{3b}, \cdots]^T$$

For a real vehicle, the row of above vectors should equal to row of the symmetrical matrixs.

Because changing size of wings

and control surfaces or installing positions of each part of the vehicle, it is the mass, center of gravity and moment of inertia that will change. Their deivtaions can be expressed respectively (see Ref. 6)

$$\Delta m = \langle (\overline{\frac{m\eta}{S}})_i , \overline{\Delta S}_i \rangle \quad (10)$$

$$\Delta x_g = \frac{1}{m+\Delta m} (\langle \overline{m}_i , \overline{\Delta x}_{gi} \rangle - \langle \overline{(x_g - x_{gi})}, (\overline{\frac{m\eta}{S}})_i \overline{\Delta S}_i \rangle + \langle \overline{\Delta x}_{gi}, (\overline{\frac{m\eta}{S}})_i \overline{\Delta S}_i \rangle) \quad (11)$$

$$\Delta J = -2 \langle \overline{m_i (x_g - x_{gi})}, \overline{\Delta x}_{gi} \rangle + \left(\frac{m}{m+\Delta m}\right)^2 \langle (\overline{x_g - x_{gi}})^2, \overline{(\frac{m\eta}{S})_i \Delta S_i} \rangle + m \Delta x_g^2 \quad (12)$$

(η -- correction coefficients of mass distribution)

As mentioned above, we can obtain

$$\left[\begin{Bmatrix} SX_{g-p} \\ S\Delta X_{g-p} \\ \Delta SX_{g-p} \\ \Delta S\Delta X_{g-p} \end{Bmatrix} \times \left(\frac{D+\Delta D}{J+\Delta J}\right)\zeta_3 \right]^T = \Pi_3$$

$$\left[\begin{Bmatrix} SX_{g-p} \\ S\Delta X_{g-p} \\ \Delta SX_{g-p} \\ \Delta S\Delta X_{g-p} \end{Bmatrix} \times \left(\frac{D+\Delta D}{J+\Delta J}\right)\zeta_2 \right]^T = \Pi_2 \quad (13)$$

$$\left[\begin{Bmatrix} SX^2_{g-p} \\ 2SX_{g-p}\Delta X_{g-p} \\ 2\Delta SX_{g-p}\Delta X_{g-p} \\ \Delta S\Delta X^2_{g-p} \end{Bmatrix} \times \left(\frac{D+\Delta D}{J+\Delta J}\right)\zeta_1 \right]^T = \Pi_1$$

$$\left[\begin{Bmatrix} S \\ \Delta S \end{Bmatrix} \times \left(-\frac{D+\Delta D}{mV+\Delta mV}\right)\zeta_2 \right]^T = \Lambda_4$$

$$\left[\begin{Bmatrix} S \\ \Delta S \end{Bmatrix} \times \left(-\frac{D+\Delta D}{mV+\Delta mV}\right)\zeta_3 \right]^T = \Lambda_5 \quad (14)$$

$$\frac{F}{mV+\Delta mV} = \Lambda_6 \quad (15)$$

(F--Thrust)

and use unit vectors

$$e_1 = [1,1,1,1]^T \ ; \ e_2 = [1,1]^T$$

By aid of the equations (13) to (15), the natural angular frequency can be written

$$\omega_V^2 = \overline{T}_V^{\,2} = \Pi_2 e_1 + (\Pi_1 e_1 + (S\Delta X^2_{g-p} + \Delta SX^2_{g-p})\left(\frac{D+\Delta D}{J+\Delta J}\right)\zeta_1)(\Lambda_4 e_2 + \Lambda_6) \quad (16)$$

The transfer coefficient K_V of the vehicle and the tranfer coefficient K_α on angle of attack can be expressed as

$$K_V = \overline{\omega}_V^{\,2} (\Pi_3 e_1 (\Lambda_4 e_2 + \Lambda_6) - \Pi_2 e_1 (\Lambda_5 e_2)) \quad (17)$$

$$K_\alpha = \overline{\omega}_V^{\,2} (\Pi_3 e_1 + (\Pi_1 e_1 + (S\Delta X^2_{g-p} + \Delta SX^2_{g-p})\left(\frac{D+\Delta D}{J+\Delta J}\right)\zeta_1)(\Lambda_5 e_2)) \quad (18)$$

The relative damping coefficient ξ_V is

$$\xi_V = 0.5\omega_V^{-1}(\Pi_1 e_1 + (\Delta S X^2_{g-p} + S\Delta X^2_{g-p})$$

$$\cdot(\frac{D+\Delta D}{J+\Delta J})\zeta_1 + (\Lambda_4 e_2 + \Lambda_6)$$

$$+(S'_1+\Delta S'_1)(x'_1+\Delta x'_1)^2$$

$$\cdot(\frac{D'_1+\Delta D'_1}{J+\Delta J})\zeta'_1) \qquad (19)$$

These transform parameters are most important ones for study dynamic characteristics of the flight vehicle. The formulas are also available to axis symmetrical vehicles.

The precise calculating formulas of T_1, T_2 and T_α are

$$T_1 T_2 = -(K_V\omega_V^2)^{-1}(S'_3+\Delta S'_3)$$

$$\cdot(x'_3+\Delta x'_3)^2(\frac{D'_3+\Delta D'_3}{J+\Delta J})\zeta'_3 \qquad (20)$$

$$T_1 - T_2 = (K_V\omega_V^2)^{-2}(\Pi_1 e_1 + (S'_3+\Delta S'_3)$$

$$\cdot(x'_3+\Delta x'_3)^2(\frac{D'_3+\Delta D'_3}{J+\Delta J})\zeta'_3(\Lambda_4 e_2$$

$$+\Lambda_6) - (S'_1+\Delta S'_1)(x'_1+\Delta x'_1)^2$$

$$\cdot(\frac{D'_1+\Delta D'_1}{J+\Delta J})\zeta'_1(\Lambda_3 e_2)) \qquad (21)$$

$$T_\alpha = (K_V\omega_V^2)^{-1}((S'_3+\Delta S'_3)(x'_3+\Delta x'_3)^2$$

$$\cdot(\frac{D'_3+\Delta D'_3}{J+\Delta J})\zeta'_3 + \Lambda_3 e_2) \qquad (22)$$

III. Rational Parameters

In overall design stage of the vehicle, provided areas of wing and control surface no change, i.e. ΔS equals zero, the relative positions of wing and control surface only are adjusted to get national transfer parameters. Therefore, when $\Delta X_{g-p} \neq 0$ the equations (16) to (22) can be simplified, and formula for calculating natural anqular frequency is

$$\omega_V^2 = S(X+\Delta X)_{g-p}(\frac{D+\Delta D}{J+\Delta J})\zeta_2$$

$$+S(X^2+2X\Delta X+\Delta X^2)_{g-p}$$

$$\cdot(\frac{D+\Delta D}{J+\Delta J})\zeta_1(\Lambda_4 e_2+\Lambda_6) \qquad (23)$$

$$((X+\Delta X+\cdots)_{g-p} = X_{g-p} + \Delta X_{g-p} + \cdots)$$

Thus by adjustment of installing position of wing and control surface, the extreme condition of must satisfy

$$S\Delta X_{g-p}(\frac{D+\Delta D}{J+\Delta J})(\zeta_2+(2X+\Delta X)_{g-p}$$

$$\cdot\zeta_1(\Lambda_4 e_2+\Lambda_6) > -SX_{g-p}(\frac{D+\Delta D}{J+\Delta J})$$

$$\cdot(\zeta_2+X_{g-p}\zeta_1(\Lambda_4 e_2+\Lambda_6)) \qquad (24)$$

When the $\Delta S=0$, the expression of transfer coefficient, equation (17), become

$$K_V = \omega_V^{-2} S(X+\Delta X)_{g-p}(\frac{D+\Delta D}{J+\Delta J})$$

$$\cdot(\zeta_3(\Lambda_4 e_2+\Lambda_6)-\zeta_2(\Lambda_3 e_2)) \quad (25)$$

As seen from the formula, if increasing transfer coefficient K_V, we must decrease natural angular frequency ω_V . So selected installing positions of wing and control surface, for normal configuration we hope the following inequality exist

$$S\Delta X_{g-p}(\frac{D+\Delta D}{J+\Delta J})(\zeta_3(\Lambda_4 e_2+\Lambda_6)$$

$$-\zeta_2(\Lambda_3 e_2)) > 0 \qquad (26)$$

For canard confiquration

$$S\Delta X_{g-p}(\frac{D+\Delta D}{J+\Delta J})(\zeta_3(\Lambda_4 e_2+\Lambda_6)$$

$$-\zeta_2(\Lambda_3 e_2)) < 0 \qquad (27)$$

If the wing and control surface is fixed and only their size can be changed ,in order to make the dynamic qualities of the flight vehicle better,the following derived formulas can give the precise calculation of transfer parameters.

When we assume $\Delta x_{pt}=\Delta x_{pw}=\Delta x_{pb}=0$ the natural angular frequency can be calculated precisely by following equation

$$\omega_V^2=(S+\Delta S)X_{g-p}\left(\frac{D+\Delta D}{J+\Delta J}\right)(\zeta_2+X_{g-p}\zeta_1\cdot(\Lambda_4 e_2+\Lambda_6))+S\Delta X_{g-p}\left(\frac{D+\Delta D}{J+\Delta J}\right)\cdot(\zeta_2+2X_{g-p}\zeta_1(\Lambda_4 e_2+\Lambda_6)) \qquad (28)$$

In the case of $\Delta x_{pi}=0$, the transfer coefficient can be obtained by equation (17)

$$K_V=\omega_V^{-2}((S+\Delta S)X_{g-p}+S\Delta X_{g-p})\cdot\left(\frac{D+\Delta D}{J+\Delta J}\right)(\zeta_3(\Lambda_4 e_2+\Lambda_6)-\zeta_2(\Lambda_5 e_2)) \qquad (29)$$

If we want to increase K_V, for normal vehicles we hope

$$\Delta S_i((x_g-x_{pi})-(\dot m\eta)_i(x_g-x_{gi}))\cdot(\zeta_{3i}(\Lambda_4 e_2+\Lambda_6)-\zeta_{2i}(\Lambda_5 e_2))<0 \qquad (30)$$

Canard vehicles requires above inequality greater than zero.

At last, it must be pointed out that the equation of relative damping coefficient ξ_V and T_1, T_2, T_α also be simplified respectively.

IV. Conclusion

As an example, after altering the relative position of wing and control surface in a canard vehicle, the value of ω_V and K_V K_α is shown in Fig.1 and Fig.2. If the control surface moves forewards 10%, ω_V only decrease 1.7% (dashed line). It can be seen that the position of rudder is rational.

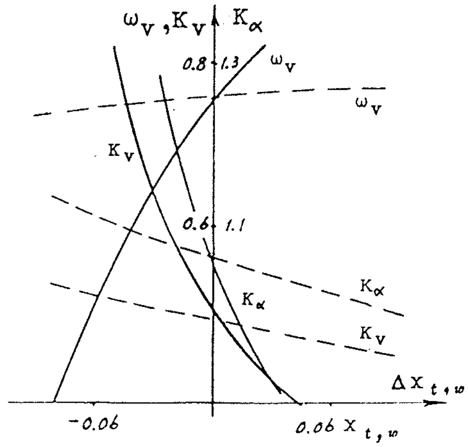


Fig.1 Calculating example 1

It is obvious that when the wing moves with equal to ±1.5%, the transfer coefficient K_V will decrease drastically, showed by solid lines in Fig.1. As shown in Fig.2, increasing rudder area by 4% will improve the dynamic characteristics (dashed line) if the rudder system is of surplus power.

In a word, applying the mathematic model mentioned above, several aerodynamic, structure and flight parameters or any one of them can be modified at the same time, and the precise transfer parameters can be obtained correspondently.

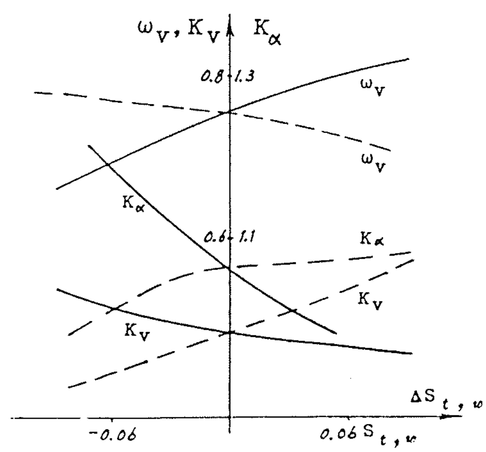


Fig.2 Calculating example 2

Appendix 1

(C Canard vehicle; W Control-wing; N -- Normal; B -- ballistics vehicle. See Ref. 6)

	C	W
D_w	$-C_{yw}^{\alpha}(K_{wb}+K_{bw})\cdot(1-\varepsilon^{\alpha})k_{\varphi}k_q q$	$-C_{yw}^{\alpha}(K_{wb}+K_{bw})k_{\varphi}q$
D_t	$-C_{yt}^{\alpha}(K_{tb}+K_{bt})k_{\varphi}q$	$-C_{yt}^{\alpha}(K_{tb}+K_{bt})(1-\varepsilon^{\alpha})k_{\varphi}k_q q$
D_b	$-(C_{yb}^{\alpha}-0.035\,\delta)q$	$-C_{yb}^{\alpha}q$
D_1'	D_w	D_t
D_3'	D_w	D_t

	N	B
D_w	$-C_{yw}^{\alpha}(K_{wb}+K_{bw})k_{\varphi}q$	0
D_t	$-C_{yt}^{\alpha}(K_{tb}+K_{bt})\cdot(1-\varepsilon^{\alpha})k_{\varphi}k_q q$	$-C_{yt}^{\alpha}(K_{tb}+K_{bt})k_{\varphi}q$
D_b	$-C_{yb}^{\alpha}q$	$-C_{yb}^{\alpha}q$
D_1'	D_t	0
D_3'	0	0

Appendix 2

	C	W
ζ_{1w}	$-57.3(1-\varepsilon^{\alpha}\frac{x_g-x_{pt}}{x_g-x_{pw}})/V(1-\varepsilon^{\alpha})\sqrt{k_q}$	$-57.3\ v^{-1}$
ζ_{1t}	$-57.3\ v^{-1}$	$-57.3(v\sqrt{k_q})^{-1}$
ζ_{1b}	$-57.3\ v^{-1}$	$-57.3\ v^{-1}$
ζ_1'	$57.3\varepsilon^{\alpha}\frac{x_{pw}-x_{pt}}{x_g-x_{pw}}/v(1-\varepsilon^{\alpha})$	$-57.3\ \varepsilon^{\alpha}/v(1-\varepsilon^{\alpha})\sqrt{k_q}$
s_1'	s_w	s_t
x_{p1}'	x_{pw}	x_{pt}
ζ_{3w}	$\frac{-\varepsilon^{\delta}}{(1-\varepsilon^{\alpha})}$	$\frac{(k_{wb}+k_{bw})n}{(K_{wb}+K_{bw})\cos\varphi}$
ζ_{3t}	$\frac{(k_{tb}+k_{bt})n}{(K_{tb}+K_{bt})\cos\varphi}$	$\frac{-(k_{tb}+k_{bt})\varepsilon^{\delta}}{(K_{tb}+K_{bt})(1-\varepsilon^{\alpha}}$
ζ_{3b}	0	0

	N	B
ζ_{1w}	$-57.3\ v^{-1}$	
ζ_{1t}	$-57.3(v\sqrt{k_q})^{-1}$	$-57.3\ v^{-1}$
ζ_{1b}	$-57.3\ v^{-1}$	$-57.3\ v^{-1}$
ζ_1'	$-57.3\ \varepsilon^{\alpha}/v(1-\varepsilon^{\alpha})\sqrt{k_q}$	0
s_1'	s_t	0
x_{p1}'	x_{pt}	0
ζ_{3w}	0	0
ζ_{3t}	$\frac{(k_{tb}+k_{bt})n}{(K_{tb}+K_{bt})(1-\varepsilon^{\alpha})\cos\varphi}$	$\frac{(k_{tb}+k_{bt})n}{(K_{tb}+K_{bt})}$
ζ_{3b}	0	0

($s_1'=s_3'$ $x_1'=x_3'$)

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