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Abstract

Model homing missile flight control system is required to be adaptable to the wide variation of altitude and Mach number in order to obtain optimal performance, good generalization and design flexibility for different flight trajectories. Two types of adaptive flight control system schemes based on hyperstability theory are presented in this paper, and applied to a given homing missile flight control system which has highly time-varying, nonminimum phase and nonlinear airframe. A simple adaptive controller based on a conventional controller which is easily implemented by microprocessors is obtained from simulations and may prove to be a better scheme. The optimization design of the adaptive flight control system parameters is also briefly presented.

I. Introduction

After the determination of aerodynamic configuration for a given homing missile, the airframe transfer function will be very damped and its parameters vary a great deal with flight altitude and Mach number, in fact from negative value to positive value for flight trajectory of medium altitude or high altitude. Thus the flight control system is required not only to get good dynamic performance but also to be adaptable to the variation of altitude and Mach number. The latter requirement is usually called "Robust".

The investigation and comparison of three types of flight control system using classical compensation theory were made in reference (1). The best system among them is Accelerometer Flight Control System consisting of rate gyro loop, synthetic stability loop and acceleration loop, as shown in Fig.1. There are three selectable parameters, K_R , ω_1 , and K_A which can independently be determined by using trial and error classical control theory to obtain desired cross-over frequency, damping ratio and system time constant.

The miniaturization, availability

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and low cost of microprocessors has made possible a digital autopilot for an antitank homing missile as reported in refs.(2) and (3) and allowed the implementation of various complex controller algorithms such as model reference adaptive systems (MRAS), which were previously difficult to implement.

This paper focuses on the study of the adaptive control scheme applied to a particular flight control system in order to obtain better performance, generalization and design flexibility than those of the conventional flight control systems. Two kinds of adaptive flight control schemes based on hyperstability theory are first presented and applied to a given airframe dynamics with three time-varying parameters.

The design procedure of both schemes consists of three steps. The first step chooses a reference model according to the desired performance; the second step derives a parameter adaptation law based on hyperstability used to compensate for the variation of airframe dynamics; the final step is to prove whether or not the adaptive flight control system can meet necessary and sufficient condition of asymptotic hyperstability as proven in Appendix. Comparison of performance for two adaptive schemes is made by using digital simulation in the final step.

II. Adaptive flight control system

As shown in Fig.2, the combination of servo system with airframe dynamics without instrumentation is used as a generalized plant.

Hydraulic servo system dynamics being neglected, the generalized plant becomes bare airframe transfer function. Its parameters such as k_n , a_1 , and a_0 vary sharply with time as shown in Fig.3.

Globally stable MRAS can be designed using either Liapunov Second Method or Hyperstability Approach as developed by Landau in reference (6). However, both methods require that $n-1$ derivatives of n th order plant be available of the state vector is not directly measurable.

A scheme for designing MRAS using only the filtered input and the filtered output signals of the plant has been proposed in reference (7) to avoid the requirement of pure derivatives of the

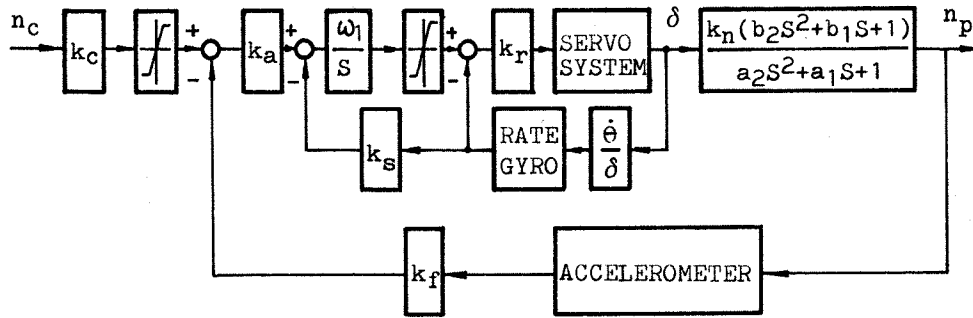


Fig.1 Accelerometer flight control system

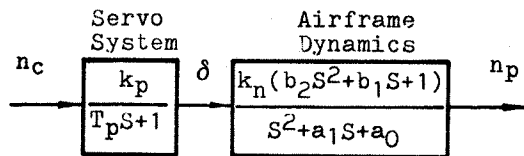


Fig.2 Generalized plant

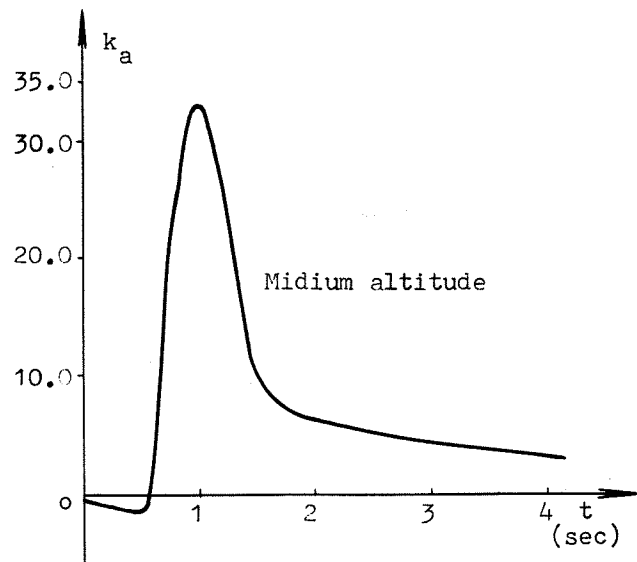


Fig.3. Airframe gain versus time plot

plant output signals. It is also indicated that the combination of Proportional plus Integral (P+I) and Relay adaptive laws produces good tracking performance even though the plant is unstable.

As shown in Fig.4, the whole system consists of a generalized plant with five time-varying parameters, a reference model which ensures desired performance, A P+I and Relay adaptive combined law and state variable filters. The reference input signal is filtered by a prefilter, the prefiltered output signal is then added to the adaptive compensation signal U_2 . The input and output signals of the plant are filtered to generate U_{1f} and n_{pf} and their derivatives.

The design objective of adaptive

control system is the synthesis of the signal U_2 which incorporates all the necessary parameter adaptations to drive the error to zero without derivatives of the plant output signal.

The generalized plant, reference model, filters and the error shown in Fig. 4 are described by the following differential equations:

$$\ddot{n}_m + c_1 \dot{n}_m + c_0 n_m = k_m R \quad (1)$$

where $b_2 = b_1 = 0$, because of its small.

$$\ddot{n}_p + a_1 \dot{n}_p + a_0 n_p = k_n k_p U_1 \quad (2)$$

$$U_1 = \frac{k_m}{1 + T_f s} R + U_2 \quad (3)$$

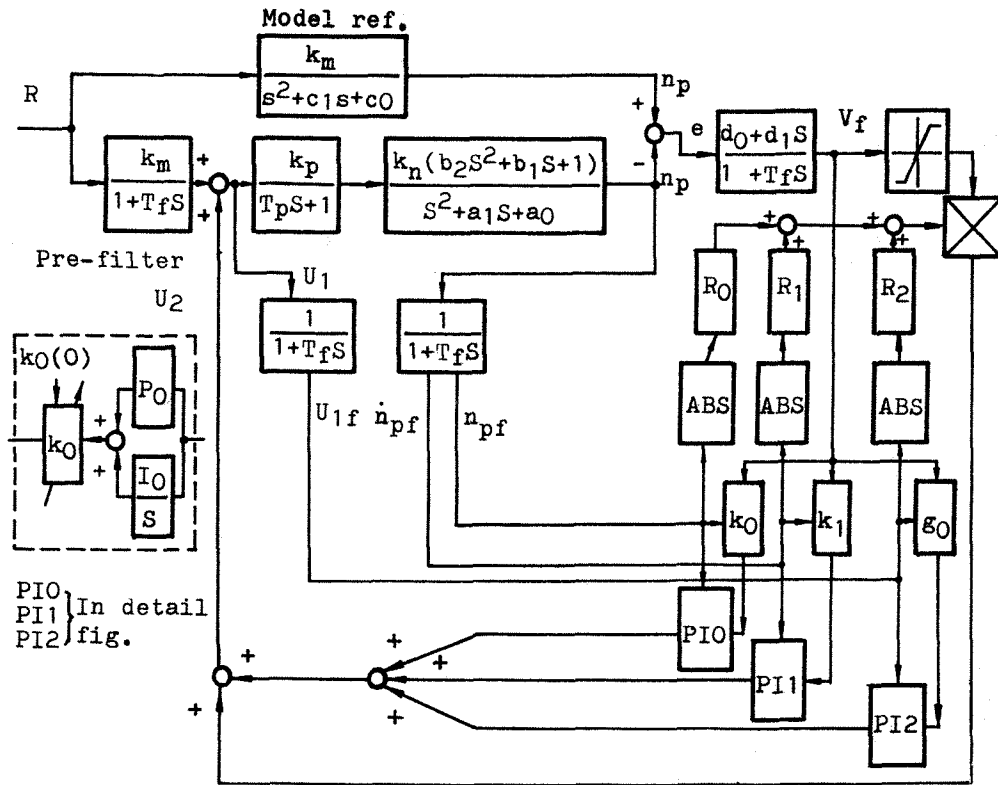


Fig.4. Simulation diagram of an adaptive flight control system

$$e(t) = n_m(t) - n_p(t) \quad (4)$$

$$U_{1f} = \frac{1}{1 + T_f s} U_1 \quad (5)$$

$$n_{pf} = \frac{1}{1 + T_f s} n_p \quad (6)$$

$$e_f = \frac{1}{1 + T_f s} e \quad (7)$$

where n_{pf} is the normal acceleration of the airframe and T_f is the time constant of the filters.

Following the hyperstability design procedure, the filtered error signal is processed by a series compensation $D_c(s)$ of first order to ensure that the feed-forward transfer function be strictly positive real. Therefore, using equation (1) to (7) obtains the following filtered error equation

$$V_f = D_c(s)e_f = (d_0 + d_1s)e_f$$

$$= \frac{d_0 + d_1s}{s^2 + c_1s + c_0} \left\{ \left((a_1 - c_1)s + (a_0 - c_0) \right) n_{pf} + (k_m - k_p k_n) U_{1f} - U_2 \right\} \quad (8)$$

where $d_0 = d_1 = 1$

The parameter adaptation can be incorporated into U_2 by selecting the negative feedback signal, thus

$$U_2 = (k_1 s + k_0) n_{pf} + g_0 U_{1f} \quad (9)$$

The combined adaptive law we proposed in reference (7) is chosen to satisfy the Popov Integral Inequality:

. P + I adaptive law

$$k_0(t) = I_0 \int_0^t V_f n_{pf} dt + P_0 V_f n_{pf} + k_0(0)$$

$$k_1(t) = I_1 \int_0^t V_f \dot{n}_{pf} dt + P_1 V_f \dot{n}_{pf} + k_1(0)$$

$$g_0(t) = I_2 \int_0^t V_f U_{1f} dt + P_2 V_f U_{1f} + g_0(0)$$

where $k_0(0)$, $k_1(0)$ and $g_0(0)$ are initial values

$$u_1(t) = (R_0 | \dot{n}_{pf} | + R_1 | \ddot{n}_{pf} | + R_2 | U_{1f} |) \cdot Z$$

$$Z = \text{Sat} (kV_f)$$

where $I_i (i=0,1,2)$ are integral adaptation coefficients

$P_i (i=0,1,2)$ are proportional adaptation coefficients

$R_i (i=0,1,2)$ are relay adaptation coefficients

k is the slope of saturation characteristics

Simulation result

According to the desired behavior of rise time, overshoot and static gain for a particular flight control system, model reference is chosen as

$$G_m(s) = \frac{n_m(s)}{R(s)} = \frac{176.25}{s^2 + 12s + 75}$$

Time-varying parameters a_1, a_0, k_n are implemented by using polynomial or linear interpolation program. Optimum values of P_i and I_i obtained by simulation are $P_i = I_i = 2.0$, $R_i = 2.5$ and

$T_f = 0.5$. Square wave with amplitude equal to 2.0 voltage is chosen as input signal.

Simulation result shows that P + I only adaptive law applied to the flight control system produces unstable response while P + I and Relay combined law gives stable response with acceptable tracking performance, as shown in Fig. 5.

III. Adaptive flight control system with conventional controller

Although the stable flight control system is obtained for the unstable, time-varying airframe using P + I and Relay law in the above scheme, the microprocessor implementation is complex. It is sometimes advantageous to combine a simple adaptive controller with non-adaptive controller when implementing the final control law (e.g., by implementing an adaptive controller around an existing conventional controller) (See reference 8).

It is obviously seen in Fig.6 that three adjustable adaptation parameters replace three constant parameters in Fig. 1. The gain K_R in the rate gyro loop determines the cross-over frequency; the gain ω_1 in the synthetic loop determines damping ratio and the gain K_A determines the whole system time constant.

Therefore, for desired cross-over frequency, damping ratio and time constant to be ensured by model reference, the three adjustable parameters can automatically be changed to compensate for three time varying parameters of the airframe in any application.

The derivation of parameter adapta-

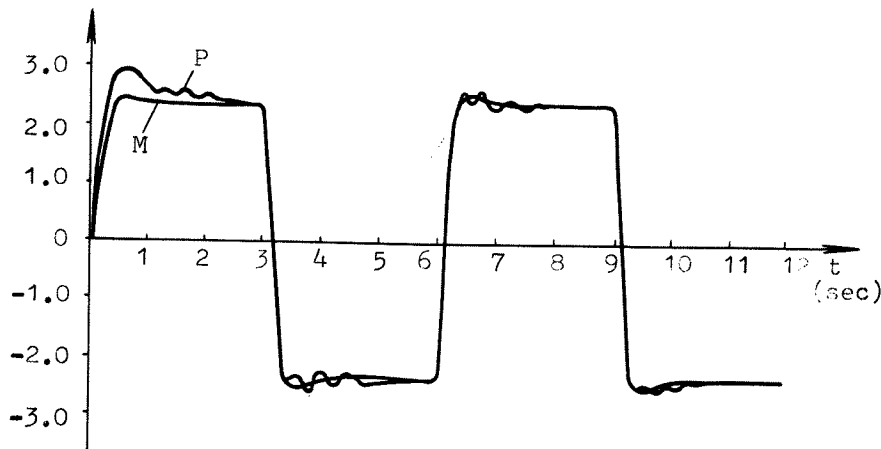


Fig.5. Simulation result using P + I and Relay law for unstable airframe

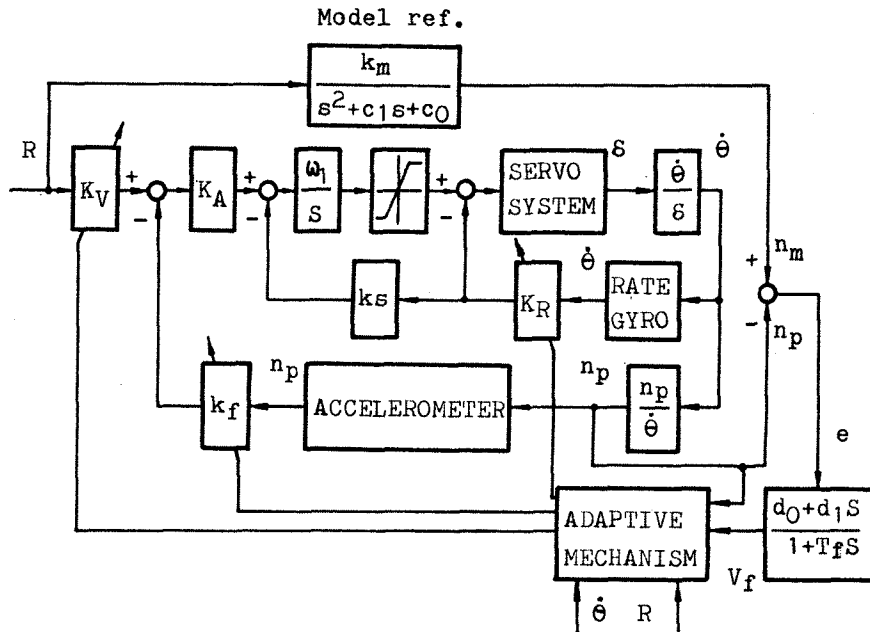


Fig.6 Simulation diagram of an adaptive flight control system based on an existing conventional controller

tion law based on hyperstability theory is the same as that of the above scheme. The error equation shown in Fig.6 can be directly derived as

$$\ddot{e} = c_1 \dot{e} + c_0 e + (a_1 - c_1) \dot{n}_p + (a_0 - c_0) n_p + (k_m - k_a) R + U_2$$

where k_a is the gain of feedforward loop in Fig.6. As before, in order to drive error to zero, the adaptive compensation signal U_2 should be selected as follows.

$$U_2 = K_R \dot{n}_p + k_f n_p + K_V R$$

The normal acceleration derivative \dot{n}_p is a nonmeasurable variable which can be replaced by $\dot{\theta}$ according to the relation

$$\dot{n}_p + r(t) n_p = s(t) \dot{\theta}$$

Since $r(t)$ is far less than 1.0, the $r(t) n_p$ can be neglected. Thus, n_p is proportional to $\dot{\theta}$. The linear compensator is used to make linear block of feedforward loop strictly positive real and the P + I adaptive law is used to make the nonlinear feedback block meet Popov's Integral in equality. The P + I adaptive law is given by

$$K_R = P_1 \dot{\theta} V_f + I_1 \int_0^t \dot{\theta} V_f dt + K_R(0)$$

$$k_f = P_2 n_p V_f + I_2 \int_0^t n_p V_f dt + k_f(0)$$

$$K_V = P_3 R V_f + I_3 \int_0^t R V_f dt + K_V(0)$$

where $K_R(0)$, $k_f(0)$, $K_V(0)$ are values which are optimum for a given existing conventional flight control system.

Simulation runs were made for two cases. The only difference between this two cases is that one is to choose the second order reference model and the other is to use the third order reference model.

Simulation results show that the tracking performance with the adaptive controller around the existing conventional controller in the flight control system, as shown in Fig.7 is better than that of conventional flight control system without adaptive controller.

IV. Optimization design of adaptive control system

It is well known that after the determination of the mathematical model of the plant and of the requirements of the control system, the optimum values for the controller parameters are general-

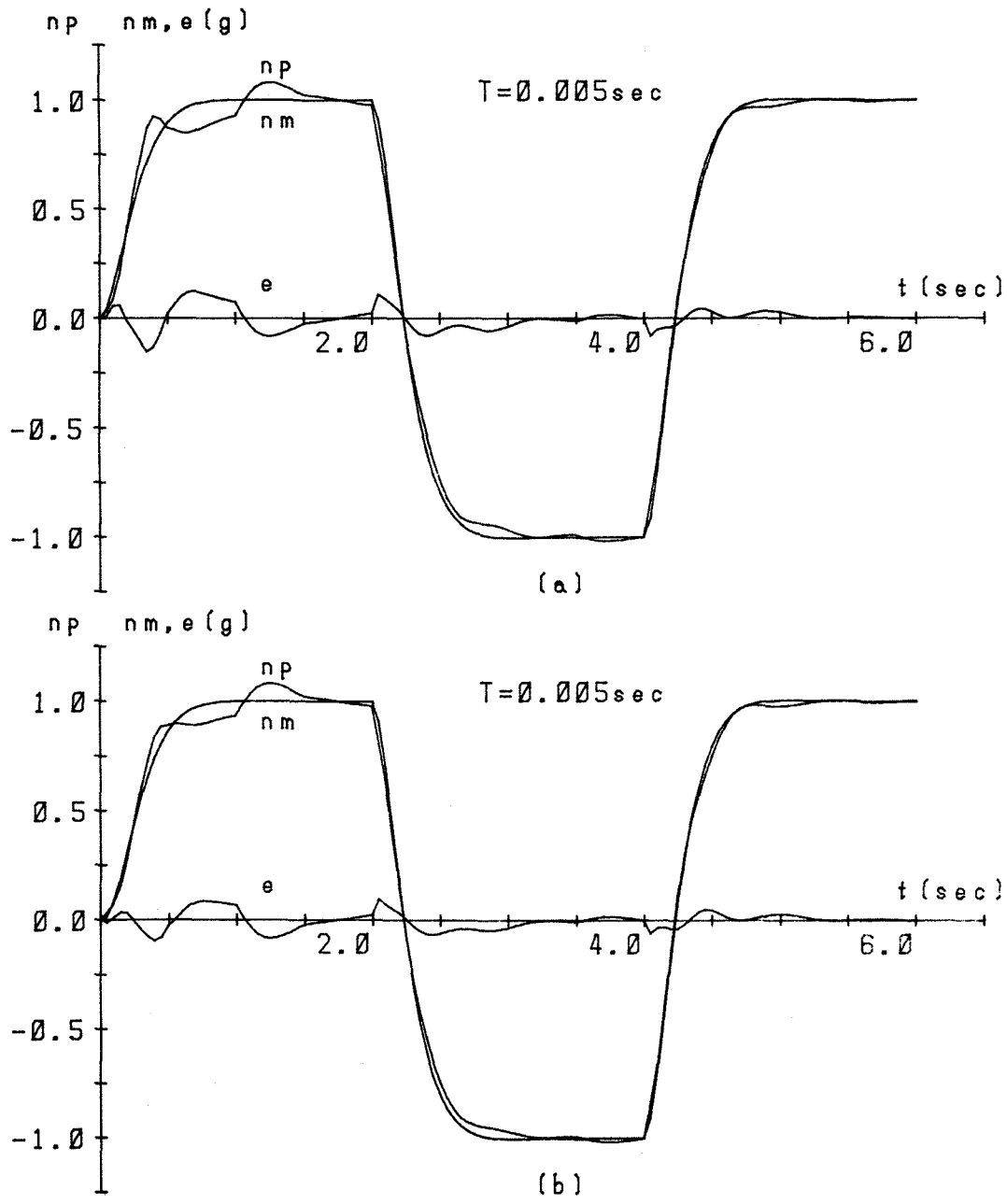


Fig.7 Simulation result using an adaptive controller around a conventional controller of flight control system
 (a) Second order model reference with third order plant
 (b) Third order model reference with third order plant

ly determined by trial and error method. The optimization design method is an automatic design method which searches for the optimum values of the parameters using digital computer.

There are a lot of objective functions which can be applied to the optimization design of control systems. Among them, a simple and effective one in reference (5) is known as the integral of time multiplied by the absolute value of error (ITAE).

$$OBJ = \int_0^{t_f} t|e|dt$$

where e is the difference between the input and output of the control system.

When this OBJ is a minimum, the system performance is said to be optimal. The optimization design for the adaptive flight control system is similar to that for the control system in references (4) and (5). But the error of objective function refers to the difference between the outputs of the model reference and the plant. The purpose of parameter optimization for the adaptive controller is to drive the error to zero in order to obtain the optimal tracking performance.

It is shown from simulations that, for P + I adaptive controller increasing the integral adaptation coefficients I_i improves system performance and reduces the static error, while increasing adaptation coefficients P_i can make the error rapidly to approach zero. But large values of P_i will result in instability. Thus, there exists a set of the optimum values for P_i and I_i . It is reasonable to select P_i and I_i as design variables.

When the P + I and Relay combined con-

troller, shown in Fig.4, is used, the discontinuous sign function is approximately replaced by the continuous saturation function in the digital simulation. As before, increasing the slope of the saturation function k can get faster convergence of the error. But large values of k will result in instability. Similarly the large values for R_i is suitable for the satisfaction of Liapunov negative condition. K and R_i

are also selected as the design variables in the optimization design of the adaptive flight control system.

The procedure for carrying out the whole program is shown in Fig.8.

V. Conclusions

1. Since microprocessor is becoming small, cheaper and faster, digital flight control system has considerable potential for replacing analog system now used in the homing missile, complex adaptive control configuration, easily implemented with microprocessor is promising in the future generation of the flight control system.
2. The P + I and Relay controller law, using filtered input and output of the plant and applied to the adaptive flight control system, produces a good performance when the plant is a nonminimum phase system. This means that the result reported by reference (7) can be extended to the present case.
3. The adaptive controller based on the existing conventional flight control system is simpler and better than the adaptive controller without conventional controller in the flight control system. It can be predicted that this scheme will be used in the homing missile.

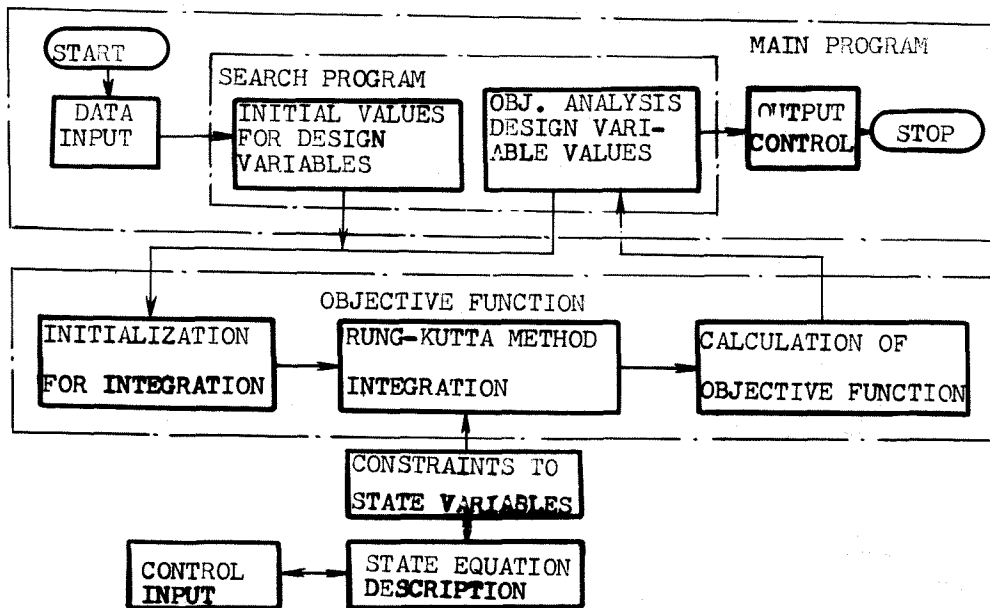


Fig.8. Flow chart of optimization design

Appendix

Hyperstability proof for the combined P + I and Relay adaptive control law

The system shown in Fig.9 is asymptotically hyperstable if the following conditions are satisfied:

- (1) the transfer function Z(s) be strictly positive real;
- (2) the non-linear, time-varying block satisfies the following inequality for all t

$$\int_0^t v^T w dt \geq -r_0^2$$

The adaptive flight control system can be rearranged as the feedback system of Fig.9. For the case of the adaptive system described by equation

$$V_f(s) = \frac{d_0 + d_1 s}{s^2 + c_1 s + c_0} \left\{ \left[(a_1 - c_1 - k_1) s + (a_0 - c_0 - k_0) \right] n_{pf} + (k_m - k_p k_n - g_0) U_{1f} \right\} = -Z(s)W(s) \tag{10}$$

Hence the linear block is

$$Z(s) = \frac{d_0 + d_1 s}{s^2 + c_1 s + c_0}$$

which satisfies strictly positive real only if d_0 is less than $c_1 d_1$. If $d_1 = 1$, then c_1 must be great than d_0 .

The nonlinear block with adjustable parameters k_1, k_0 , and g_0 in equation (10) which are nonlinear function of V_f . It is easy to prove according to reference (6) that P + I and Relay adaptive laws are required to satisfy the Popov Integral Inequality.

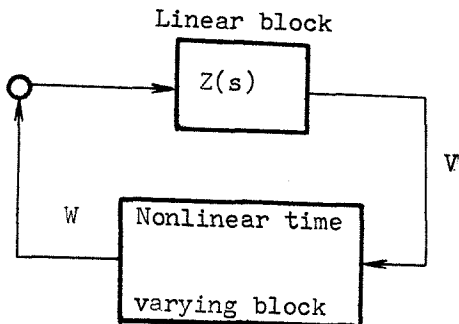


Fig.9. Nonlinear time-varying feedback system satisfying the Popov hyperstability condition

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