ON THE OPTIMIZATION OF FLUTTER CHARACTERISTICS OF LAMINATED ANISOTROPIC CYLINDRICAL SHELLS

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Abstract

The axisymmetrical supersonic flutter of a simply supported circular cylindrical laminated shell with orthotropic or isotropic layers is investigated. A comparative analysis to demonstrate the influence of geometrical and mechanical characteristics of the shell on its critical flutter speed is performed.

The purpose of this investigation is to perform the optimization of the flutter characteristics of cylindrical multilayered shells, - important components of aeronautical and space structures, - by varying their geometrical, elastical and mechanical parameters.

The solution of the problem is obtained on the base of the methodology developed in the papers of Movchan [2], Krumhaar [1] and Stepanov [3], extended on the multilayered and orthotropic cylindrical shells. This methodology concerns with the application of the linearized Timoshenko shell equations and linear piston theory, which lead to a non-selfadjoint eigenvalue problem, solved without futher approximations.

Complete results of this investigation may be found in papers [7], [9].

1. Geometrical and mechanical relationships

A circular cylindrical shell of finite length L, simply-supported on its frontal edges, made up from an odd number (2l-1); l=1,2... of orthotropic or isotropic layers, situated symmetric in respect with the middle surface of the radius R, is taken into account (Fig.1.1).

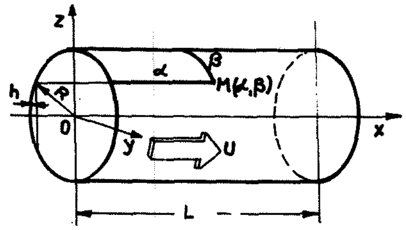

Fig.1.1 Geometry of the shell and coordinate system

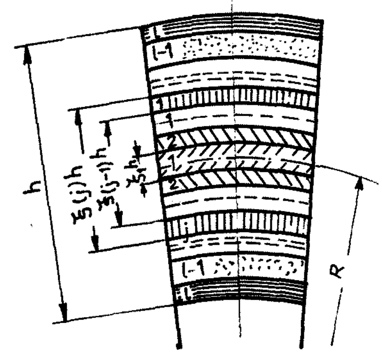

Fig.1.2 An element cut from a multilayered shell

An element cut from the shell is shown in Fig.1.2. On the middle surface of the shell is considered the system of curvilinear coordinates α - β, corresponding to its main curvature lines. The principal stress directions of every layer correspond with the directions of coordinate lines α and β.

The elastical characteristics of the j-th layer of the shell are:

$$E_\alpha^{(j)},\ E_\beta^{(j)},\ \mu_\alpha^{(j)},\ \mu_\beta^{(j)}$$

and its main mass density is $\rho_m^{(j)}$.

The index (j) of the above symbols and of the following ones show the current number of the layer. At the internal layer cut by the middle surface j=1, and at the external layers j= l.

Following non-dimensional elastical, mechanical and geometrical parameters are used in the computing process:

$$q_{(j)}=\frac{E_\alpha^{(j)}}{E_\alpha^{(l)}};\ \bar{q}_{(j)}=\frac{\mu_\alpha^{(j)}}{\mu_\alpha^{(l)}};\ \tilde{q}_{(j)}=\frac{\rho_m^{(j)}}{\rho_m^{(l)}};$$

$$k_{(j)}=\frac{E_\beta^{(j)}}{E_\alpha^{(j)}};\ L^*=\frac{L}{R};\ h^*=\frac{h}{R}. \tag{1.1}$$

The thickness of the j-th layer Δh_j is expressed in terms of the non-dimensional parameters ξ_{j+1} and ξ_j by the formula:

$$\Delta h = \frac{\xi_{(j+1)} - \xi_{(j)}}{2} h \,. \qquad (1.2)$$

The specific weight on the unit length of the shell has the following expression:

$$p = \pi g R^2 h^* \varsigma_m^{(\ell)} \sum_{j=1}^{\ell} \tilde{q}_{(j)} (\xi_{(j)} - \xi_{(j-1)}) \quad [\ell b/_{in}] \qquad (1.3)$$

In order to characterize the relationship between the specific weight of the shell to the critical Mach number ($\mathcal{M}_{cr}$), necessary to show the influence of different parameters to obtain the best quality after the criterion weight/critical flutter velocity, the following parameters (with dimension [lb/in^3]) will be adopted:

$$\bar{p} = \frac{p}{R^2} \,; \qquad \psi = \frac{\bar{p}}{\mathcal{M}_{cr}} \qquad (1.4)$$

2. Formulation of the Problem. Basical Equations

The axisymmetric vibrations with exponential time factor of the considered shell submitted externally to a supersonic uniform stream with the velocity U, and internally to a constant pressure, are investigated.

The governing equation of the aeroelastic dynamic equilibrium of the shell is obtained on the base of the Donnell--Vlasov's engineering theory, from the system of general equations for laminated anisotropic shells. This equation, with corresponding boundary conditions on the edges of the shell, forms the following basical boundary-value problem for the radial deflections w of the shell:

$$-D_{11}\frac{\partial^4 w}{\partial \alpha^4} + T_{11}\frac{\partial^2 w}{\partial \alpha^2} - \frac{1}{R^2 A_{22}} w - \Delta p(\alpha,t) - m_0 \frac{\partial^2 w}{\partial t^2} + b\frac{\partial w}{\partial t} = 0 \qquad (2.1)$$

$$w(0,t) = w(L,t) = 0, \left[\frac{\partial^2 w}{\partial \alpha^2}\right]_{\alpha=0} = \left[\frac{\partial^2 w}{\partial \alpha^2}\right]_{\alpha=L} = 0 \qquad (2.2)$$

The aerodynamic pressure of the surface of the shell (Δp) which enters in the equation (2.1) is appreciated from the piston theory:

$$\Delta p(\alpha,t) = -a_H \varsigma_H \left(U\frac{\partial w}{\partial \alpha} + \frac{\partial w}{\partial t}\right), \qquad (2.3)$$

where a_H and ς_H are the velocity of sound and the mass density of the undistrubed airstream at the altitude H;

t - is the time

Anotner notations used in the relationships (2.1) are presented in Appendix 1 (see also [7], [9]).

By introducing new parameters:

$$A = a_H^2 \varsigma_H \frac{\mathcal{M} L^2}{D_{11}} \qquad (2.4)$$

called 'generalized velocity',

$$B = \frac{L^4}{D_{11} R^2 A_{22}} \,; \qquad (2.5)$$

$$\lambda^* = \frac{L^4}{D_{11}}\left(m_0 \omega^2 - i\omega \langle a_H \varsigma_H + b\rangle\right), \qquad (2.6)$$

$$\lambda = \lambda^* - B \,, \qquad (2.7)$$

where $$\mathcal{M} = \frac{U}{a_H} \qquad (2.8)$$

is the Mach number, and adopting the nondimensional variable

$$\eta = \frac{\alpha}{L} \,, \qquad (2.9)$$

we search for the solution of the basical problem (2.1) in the form:

$$w(\eta,t) = \bar{w}(\eta) e^{i\omega t} \qquad (2.10)$$

which is the product of a real function of η [$\bar{w}(\eta)$], and of a factor variable with the time, where ω is the complex frequency of the variable movement (App A2).

The function $w(\alpha, t)$ is the solution of the initial problem only when $\bar{w}(\eta)$ is the eigenfunction of the following boundary-value problem;

$$\frac{d^4 \bar{w}}{d\eta^4} + A\frac{dw}{d\eta} = \lambda \bar{w} \quad (0 \leqslant \eta \leqslant 1)$$

$$\bar{w}(0) = \bar{w}(1) = \left(\frac{d^2\bar{w}}{d\eta^2}\right)_{\eta=0} = \left(\frac{d^2\bar{w}}{d\eta^2}\right)_{\eta=1} = 0 \qquad (2.11)$$

derived from the initial one; the parameter λ (or λ^*) is the eigenvalue of the same problem.

For fixed A and λ parameters the solution of the equation (2.11) is:

$$\bar{w} = \sum_{i=1}^{4} C_i \bar{w}_i(A,\lambda,\eta) \,, \qquad (2.12)$$

where C_i are integration constants.

Substituting in the equation $(2.11)_1$:

$$\bar{w} = e^{-z\eta} \qquad (2.13)$$

we obtain the characteristic equation

$$p(z,A,\lambda) \equiv z^4 - Az - \lambda = 0 \quad (A>0) \qquad (2.14)$$

whose roots $z_1 \dots z_4$ satisfy the equation:

$$F(z_1 \dots z_4) = 0 \qquad (2.15)$$

(see Appendix A2)

only when λ is the eigenvalue of the boundary problem (2.11).

By adopting new parameters:

$$\bar{\alpha} = \frac{1}{2}(z_1 + z_2); \quad \bar{\beta} = \frac{1}{2i}(z_1 - z_2) \qquad (2.16)$$

(z_3 and z_4 being expressed in terms of z_1 and z_2), we obtain the characteristic system

$$F(\bar{\alpha},\bar{\beta}) = 0 \qquad (2.17)$$
$$A = 4\bar{\alpha}(\bar{\alpha}^2-\bar{\beta}^2),$$

which posesses the property that, for fixed parameters A and variable $\bar{\alpha}$ and $\bar{\beta}$, to every solution corresponds an eigenvalue λ and to every λ at a certain A corresponds at least one solution ($\bar{\alpha}$, $\bar{\beta}$) of the system.

By eliminating the parameter $\bar{\beta}$ the system reduces to the equation:

$$F^*(\bar{\alpha},A) = 0 \qquad (2.18)$$

Complete expressions of equations (2.15), (2.17) and (2.18) are presented in Appendix A2.

The eigenvalues λ may be expressed in terms of parameters A and $\bar{\alpha}$ by the equation:

$$\lambda = \frac{A^2}{16\,\bar{\alpha}^2} - 4\bar{\alpha}^2. \qquad (2.19)$$

3. The method to determine the characteristics of the flutter vibration

The characteristics of the self vibrational movement of the shell - i.e. the critical flutter velocity and frequency may be determined by the examination of the eigenvalues of the boundary-value problem (2.11) which are situated in the external or internal part of the parabola of stability (Fig.3.1), represented in the complex plane $Re(\lambda)$-$Im(\lambda)$ by the equation:

$$Re(\lambda)+B = C\left[Im(\lambda)\right]^2, \qquad (3.1)$$

where

$$C = \frac{m_0 D_{11}}{(a_H\rho_H+b)^2 L^4}. \qquad (3.2)$$

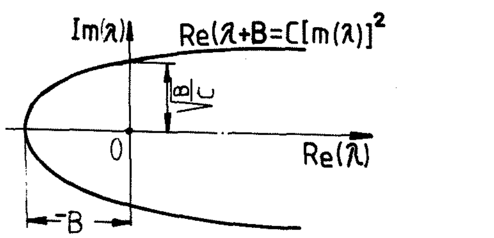

Fig.3.1 The parabola of stability

In the internal part of the parabola there are situated all eigenvalues λ_n corresponding to the frequencies ω_n, which produce the stable vibrational movement, whereas in its external part - the unstable vibrational movement, growing with the time.

The dependence of λ_n(A) with the parameter A is investigated, in order to determine the critical generalized velocity A_{cr} for which the eigenvalues $\lambda(A_{cr})$ are situated on the stability parabola.

All real λ_n(A) are positive and are situated in the internal part of the parabola; only the complex λ_n (A) may be on its external part.

The equations of the characteristic system (2.17) are represented in the real plane $\bar{\alpha}$, $\bar{\beta}$ as a network of curves. The intersections of these curves correspond to real λ_n. The smaller value of the critical speed A_{cr} corresponds to the first complex eigenvalue λ_1(A) and only this value presents importance for our investigation (see Fig. A1-1 from Appendix A1).

4. Determination of the critical flutter velocity

The critical flutter velocity may be determined by solving the equation (2.15) for different fixed values of the parameter A; we determine the complex values of $\bar{\alpha}$ and from them - the complex eigenvalues. It is necessary to determine the intersection points of the curves λ_1(A), drawn in the complex plane $Re(\lambda)$-$Im(\lambda)$ with the stability parabola (Fig.3.2).

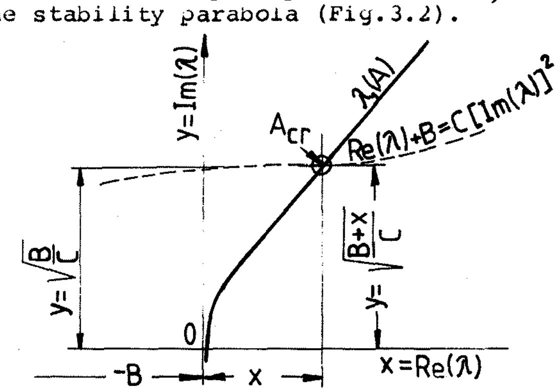

Fig.3.2 The intersection of λ_1 (A) curve with stability parabola

The ordonates of the intersection points are determined from the equation:

$$y = Im[\lambda(A)] = \sqrt{\frac{B+Re(\lambda)}{C}} \cong \sqrt{\frac{B}{C}} \qquad (3.3)$$

After substituting the parameters (1.1) and (1.2) in expressions (2.4), (2.5) and (3.2) we obtain following working relationships:

$$A = \frac{12\,a_H^2\,\rho_H\,m_0}{E_\alpha}\left(\frac{L^*}{h^*}\right)^3 \boxed{H}_{m_0}, \qquad (3.4)$$

$$B = 12\,\frac{L^{*4}}{h^{*2}}\,\boxed{H}_B, \qquad (3.5)$$

$$C = \frac{E_\alpha\,\rho_m}{12(a_H\rho_H+b)^2}\left(\frac{h^*}{L^*}\right)^3 \boxed{H}_C, \qquad (3.6)$$

where $\boxed{H}_{m_0}$, $\boxed{H}_B$ and $\boxed{H}_C$ are structural parameters (see Appendix A3) containing all the characteristics of the multilayered shell in non-dimensional form.

The sequence of computational operations

The computational process is lead to the following succesion:

1. The intersection point of the curve λ_1(A) with the stability parabola is determined graphically.

2. The value of the critical generalized velocity A_{cr} of the intersection point, is established.

3. The critical Mach number $\mathcal{M}_{cr}$ is determined from its dependence relationship with A_{cr} (3.1).

In Appendix A2 are presented in a nondimensional form the expressions of the coefficients of the stability parabola ($\boxed{H}$), which contain all characteristics of the multilayered shell.

5. Numerical computation

Numerical computations were performed in order to establish the influence of the nonuniformity (heterogeneity), orthotropy and geometrical paramters of a multilayered cylindrical shell on the values of its critical Mach.

Simply (one-layered) orthotropic and three layered cylindrical shells with orthotropic and/or isotropic layers, were taken into account. Three layered shells were taken with the parameter $\tilde{q}$ =0,1 (representing the ratio between the mass densities of internal and external layers) and variable coefficient ξ ($0 < \xi < 1$) - (representing the relative thickness of the layers). (See Fig.5.1)

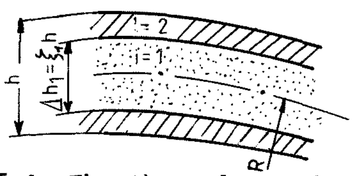

Fig.5.1 The three-layered shell

The influence of all parameters (1.1) - (1.4) was investigated.

Complete results of this investigation are described and presented in Fig.6.1 - 6.10 of ref. [7] and in Fig. 7.1 - 7.2; 8.1 - 8.5 and 9.1 - 9.2 of ref. [9].

6. The discussion of results

6.1. The influence of the parameter h^*

In every conditions the parameter h^* has the greatest influence on the critical flutter velocity.

This influence is evidenced in Fig.6.1, where it is shown the dependence $\mathcal{M}_{cr} = f(h^*)$ for three-layered isotropic shell with variable parameter ξ ($0<\xi<1$).

$\mathcal{M}_{cr}$ is fast growing with h^*, especially for shells with thick external 'strong' layers.

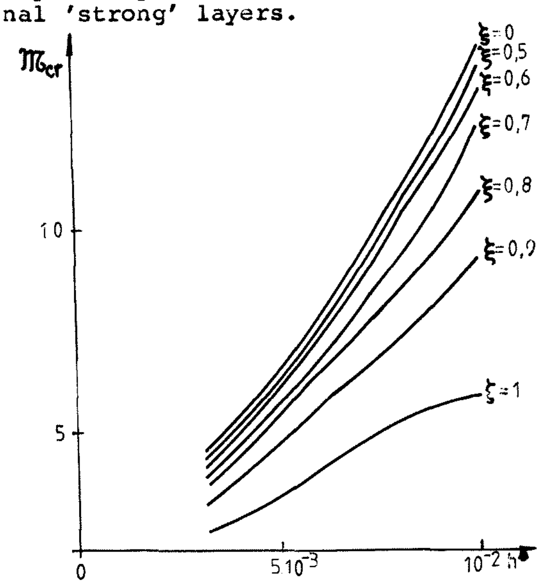

Fig.6.1 Dependence $\mathcal{M}_{cr}=f(h^*)$ for three-layered isotropic shell

6.2. The influence of orthotropy of the layers

The ratio between the elastical moduli E_α and E_β of the layers is characterized by the factors k .

The influence of this parameter is not so clearly evidenced in the case of axisymmetrical flutter vibrations because E_β doesn't enter in the governing equation; its influence appears in the terms containing the expression $1-k\mu_\beta$.

Though, the influence of this factor is significant as it can be seen in the Fig.6.2, where it is shown the dependence $\mathcal{M}_{cr} = f(k)$ for a one-layered orthotropic shell.

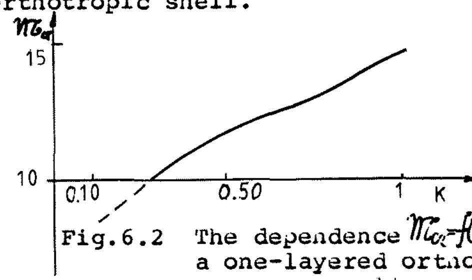

Fig.6.2 The dependence $\mathcal{M}_{cr}=f(k)$ for a one-layered orthotropic shell

6.3. The influence of the relative thickness of the layers of the shell

Three layered (sandwich) shells with orthotropic or isotropic layers, characterised by the variable thickness of the layers ($0 < \xi < 1$) and the parameter $\tilde{q}$ =0.1, were investigated.

The curves $\mathfrak{M}_{cr} = f(\xi)$ show (see Fig.6.3) that (when other parameters are fixed) $\mathfrak{M}_{cr}$ shell has a minimal value for $\xi = 1$ (the shell consists only of one internal 'weak' core), and has a fast growing at $\xi = 0.9 - 0.7$ (with the appearence of thin external 'strong' layers); when these layers become thick enough ($\xi < 0.7$) the growth of $\mathfrak{M}_{cr}$ is much slower and practically stops at $\xi \leqslant 0.5$.

This particularity shows that from the point of view of the specific weight the most advantageous are the shells with external 'strong' very thin layers.

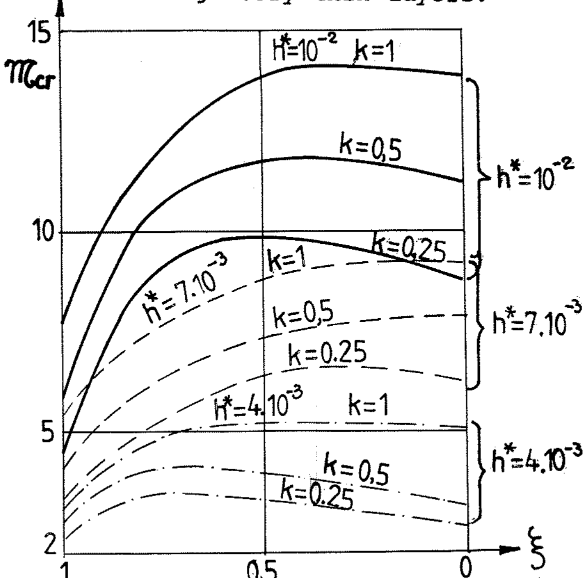

Fig.6.3 The dependence $\mathfrak{M}_{cr} = f(\xi)$ for a three-layered isotropic shell

6.4. The influence of the parameter L^* in connection with the parameters h^*, k and ξ on $\mathfrak{M}_{cr}$.

The curves in graphs Fig.6.4 represent the dependence $\mathfrak{M}_{cr} = f(L^*)$ for a one-layered orthotropic shell at different values of parameters h^* and k, and in Fig.6.5 - for a three-layered shell (with isotropic layers) at different h^* and ξ.

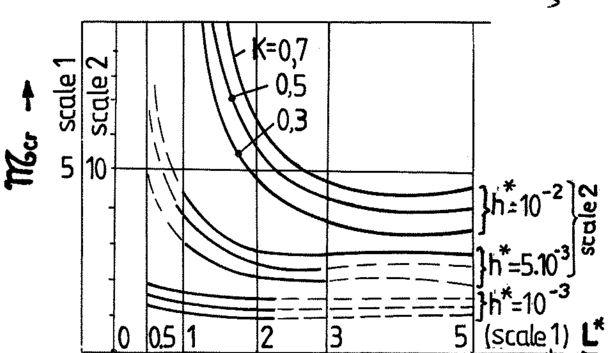

Fig.6.4 The dependence $\mathfrak{M}_{cr} = f(L^*, h^*, k)$ for one-layered orthotropic shell

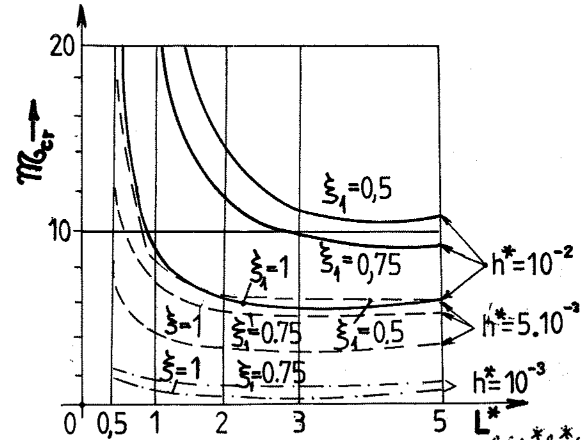

Fig.6.5 The dependence $\mathfrak{M}_{cr} = f(L^*, h^*, \xi)$ for three-layered isotropic shell

The variation of $\mathfrak{M}_{cr}$ is extremely fast at certain values of L^* and h^*. An important growth of $\mathfrak{M}_{cr}$ may be obtained by the variation of L^* towards its diminuation (at $L^* < 2$). For $L^* \geqslant 3$ this variation quite vanishes.

6.5. The reciprocal influence of the parameters L^*, h^* and ξ on the parameter ψ.

The parameter ψ was adopted in order to illustrate the influence of different parameters to obtain the shell of optimal quality i.e. having the lightest structure in connection with $\mathfrak{M}_{cr}$ as great as possible.

In order to satisfy this criterion ψ must be as small as possible.

Further the influence of the parameters L^*, h^* and ξ on this parameter is analysed.

In Fig.6.6 a family of curves $\psi = f(L^*)$ for a three-layered shell characterised by different values of h^* ($h^* = 10^{-3}$; $5 \cdot 10^{-3}$; 10^{-2}) and ξ ($\xi = 1$; 0.75; 0.5; 0.25; 0), are represented.

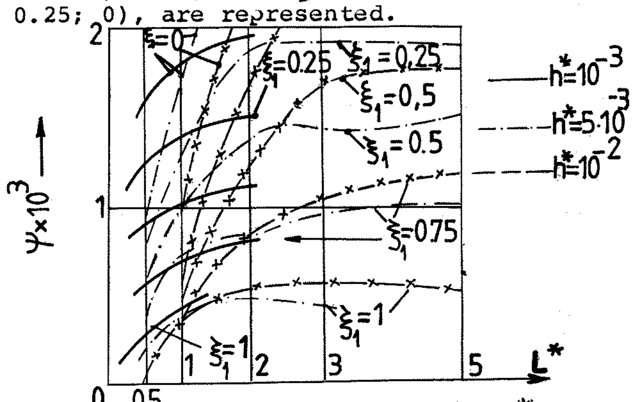

Fig.6.5 Dependences $\psi = f(L^*, h^*, \xi)$ for a three-layered shell

Analysing these dependences we arrive to the following conclusions, regarding the influence of different parameters on the parameter ψ :

a) Shells with the parameter ξ appropriate to 1 (with very thin external layers and with internal light core predominant in the overall thickness) are optimal.

b) Studying the reciprocal influence of h^* and L^* on ψ (for a certain value of ξ) we conclude that at small values of the parameter L^* ($L^* = 0.5 - 1$) the minimal values of ψ result for greater h^* ($h^* = 10^{-2}$), whereas at $L^* \geqslant 2$ this situation inverses: the optimal ψ results for smaller h^*.

7. Conclusions

The analysis of previous results allows us to draw some conclusions about the behaviour of the investigated class of shells, subjected to the action of a supersonic aerstream, from the point of view of the appearance of flutter vibrations. The final purpose of the study is to determine the criteria for obtaining the lightest structure with the critical flutter limit as great as possible.

The separate and reciprocal influence of different geometrical and mechanical parameters of the shell on its flutter vibrational characteristics was outstended.

This analysis allowed us to determine the optimal shape of the geometrical and structural characteristics of cylindrical multilayered orthotropic shells used in aeronautical and aerospace constructions.

So, the most advantageous three-(or multilayered) shells result when the external 'strong' layers are very thin in comparison with the overall thickness of the shell. With the growing of the thicknesses of these layers grow both the critical Mach and the specific weight of the shell, but the growing of the last is faster, so the overall quality of the shell becomes worst.

The parameter h^* is the most important factor to obtain the growth of the $\mathcal{M}_{cr}$. It influences directly the aeroelastic characteristics and always when it is necessary to obtain the increase of the flutter velocity we must act this parameter firstly.

The parameter L^* is an important shape-characteristic of the shell which besides h^*, has a great influence on the $\mathcal{M}_{cr}$.

In designing the aerospace cylindrical structures it is necessary to choose this parameter so, that the aeroelastic quality of the structure should be best.

The critical Mach is strongly affected by the parameter L^*, being advantageous that the shell should be relative as short as possible.

The parameter ψ allows us to appreciate in which way the shell must be designed in order to obtain the optimum flutter quality (the lightest structure with the flutter limit as great as possible). To meet these conditions the shell must be characterised by the parameters: ξ appropriate to value of 1.0, h^* as great and L^* as small as possible. If, from constructive considerations, L^* must be big enough ($L^* = 2 - 3$) the role of the parameter h^* becomes opposite: it must be as small as possible. This interaction of the parameter L^* and h^* is of the greatest importance to obtain the optimal values of the flutter quality of the shell.

So, the optimal geometrical and elasto-mechanical characteristics of the cylindrical shells, entering as component parts of aeronautical and space structures may be determined by design of these structures.

Appendix A1

Notations used in the Basical Boundary-Value Problem (Ch.2)

$$D_{11}=\frac{1}{12}\sum_{j=1}^{\ell} B_{11}^{(j)} h^3\left(\xi_{(j)}^3-\xi_{(j-1)}^3\right) \qquad \text{(A1-1)}$$

$$A_{22}=\left(C_{11}C_{66}-C_{16}^2\right)/\Omega \qquad \text{(A1-2)}$$

$$\Omega=\left(C_{11}C_{22}-C_{12}^2\right)C_{66}+2C_{12}C_{16}C_{26}-C_{11}C_{26}^2-C_{22}C_{16}^2 \qquad \text{(A1-3)}$$

$$C_{ik}=\sum_{j=1}^{\ell} B_{ik}^{(j)} h\left(\xi_{(j)}-\xi_{(j-1)}\right)\quad \binom{i=1,2}{k=1,2} \qquad \text{(A1-4)}$$

The quantities B_{ik} are 'engineering constants' [4], which for orthotropic layers keep expressions (2.9) presented in [7].

$$m_0=\sum_{j=1}^{\ell}\rho_m^{(j)} h\left(\xi_{(j)}-\xi_{(j-1)}\right)= h\rho_m^{(\ell)}\sum_{j=1}^{\ell}\tilde{q}_{(j)}\left(\xi_{(j)}-\xi_{(j-1)}\right) \qquad \text{(A1-5)}$$

$T_{11}=0$ is the axial normal stress of the shell (considered equal to zero in the case of axi-symmetrical mode)

b is the damping factor of the material

'Engineering constants' of a multilayered orthotropic shell

$$B_{11}^{(j)} = \frac{E_\alpha^{(j)} q_{(j)}}{1-\mu_\alpha^{(j)}\mu_\beta^{(j)}} = \frac{E_\alpha^{(\ell)} q_{(j)}}{1-k_{(j)}\bar{q}_{(j)}^2 \mu_\alpha^{2(\ell)}}$$

$$B_{22}^{(j)} = \frac{E_\beta^{(j)}}{1-\mu_\alpha^{(j)}\mu_\beta^{(j)}} = \frac{E_\alpha^{(\ell)} k_{(j)} q_{(j)}}{1-k_{(j)}\bar{q}_{(j)}^2 \mu_\alpha^{2(\ell)}}$$

$$B_{12}^{(j)} = \frac{E_\alpha^{(j)} \mu_\beta^{(j)}}{1-\mu_\alpha^{(j)}\mu_\beta^{(j)}} = \frac{E_\alpha^{(\ell)} \mu_\alpha^{(\ell)} k_{(j)} q_{(j)} \bar{q}_{(j)}}{1-k_{(j)}\bar{q}_{(j)}^2 \mu_\alpha^{2(\ell)}}$$

$$B_{66}^{(j)} = G^{(j)} = \frac{\varkappa^{(j)}\sqrt{E_\alpha^{(j)} E_\beta^{(j)}}}{2\left(1+\sqrt{\mu_\alpha^{(j)}\mu_\beta^{(j)}}\right)} =$$

$$= \frac{\varkappa^{(j)}\sqrt{k_{(j)}}\, q_{(j)} E_\alpha^{(\ell)}}{2\left(1+\sqrt{k_{(j)}}\,\bar{q}_{(j)}\mu_\alpha^{(\ell)}\right)}$$

$$B_{16} = B_{26} = 0. \qquad \text{(A1-6)}$$

$$\varkappa^{(j)} \cong 1.$$

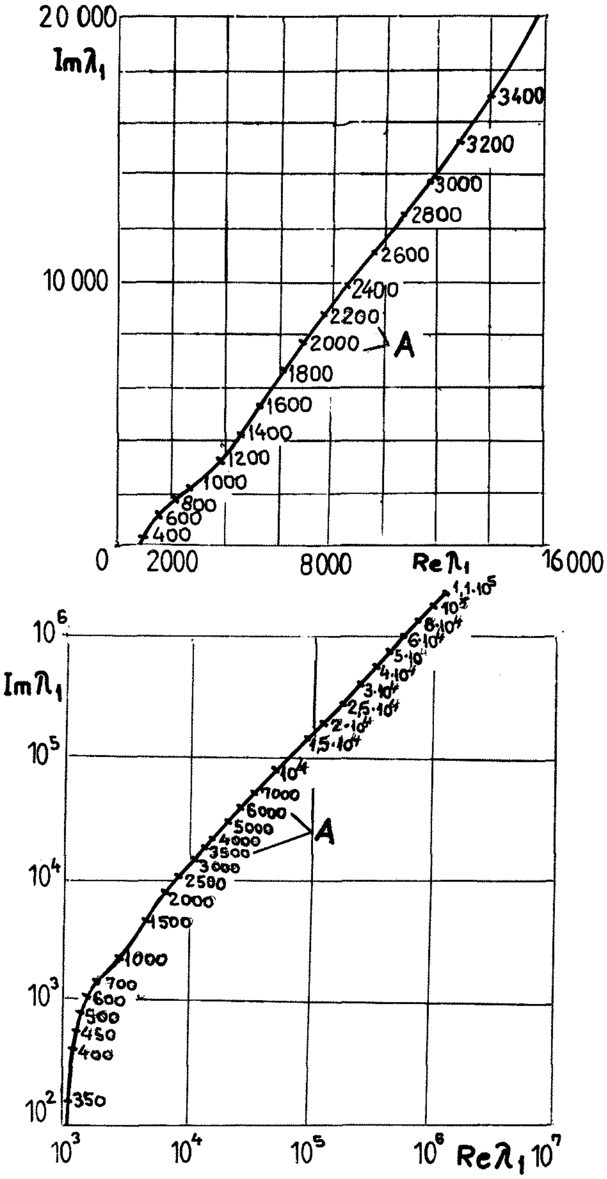

Fig.A1-1 Eigenvalues (A) :
a) for A 3400
b) for A 110000

$$\frac{1}{A_{22}} = C_{22} - \frac{C_{12}^2}{C_{11}} = E_\alpha h \left[\sum_{j=1}^{\ell} \frac{k_{(j)} q_{(j)}}{1-k_{(j)}\bar{q}_{(j)}^2 \mu_\alpha^2} \left(\xi_{(j)} - \xi_{(j-1)}\right) - \right.$$

$$\left. - \frac{\left(\sum_{j=1}^{\ell} \frac{k_{(j)} q_{(j)} \bar{q}_{(j)} \mu_\alpha^{(\ell)}}{1-k_{(j)}\bar{q}_{(j)}^2 \mu_\alpha^{2(\ell)}} \left\langle \xi_{(j)} - \xi_{(j-1)} \right\rangle \right)^2}{\sum_{j=1}^{\ell} \frac{q_{(j)}}{1-k_{(j)}\bar{q}_{(j)}\mu_\alpha^{2(\ell)}} \left(\xi_{(j)} - \xi_{(j-1)}\right)} \right] \qquad \text{(A1-7)}$$

Appendix A2

Main expressions used in the derived Boundary-Value Problem (2.11)

A2.1 The complex frequency of the vibrational movement

$$\omega_{n_{1,2}} = i\,\frac{a_H \rho_H + b}{2m_o}\left[1 \pm \sqrt{1 - \frac{4m_o \lambda_1^* D_{11}}{(a_H \rho_H + b)^2 L^4}}\right] \quad \text{(A2-1)}$$

A2.2 The equation (2.15)

$$F(z_1 \ldots z_4) \equiv \frac{1}{g}\begin{vmatrix} 1 & 1 & 1 & 1 \\ e^{-z_1} & e^{-z_2} & e^{-z_3} & e^{-z_4} \\ z_1^2 & z_2^2 & z_3^2 & z_4^2 \\ z_1^2 e^{-z_1} & z_2^2 e^{-z_2} & z_3^2 e^{-z_3} & z_4^2 e^{-z_4} \end{vmatrix} \quad \text{(A2-2)}$$

where:

$$g(z_1 \ldots z_4) = (z_1 - z_2)(z_1 - z_3)(z_1 - z_4) \times$$
$$\times (z_2 - z_3)(z_2 - z_4)(z_3 - z_4) \quad \text{(A2-3)}$$

A2.3 The equation (2.17)

$$F(\bar\alpha, \bar\beta) \equiv \frac{(3\bar\alpha^4 - \bar\beta^4 - 2\bar\alpha^2\bar\beta^2)\,\frac{\sin\bar\beta}{\bar\beta}\cdot\frac{\operatorname{sh}\sqrt{\bar\beta^2 - 2\bar\alpha^2}}{\sqrt{\bar\beta^2 - 2\bar\alpha^2}}}{(3\bar\alpha^2 - \bar\beta^2)^2 + 4\bar\alpha^2\bar\beta^2} +$$

$$+ \frac{-2\bar\alpha^2 \cos\bar\beta\,\operatorname{ch}\sqrt{\bar\beta^2 - 2\bar\alpha^2} + 2\bar\alpha\,\operatorname{ch}(2\bar\alpha)}{(3\bar\alpha^2 - \bar\beta^2)^2 + 4\bar\alpha^2\bar\beta^2} = 0 \quad \text{(A2-4)}$$

A2.4 The equation (2.18)

$$F^*(A, \bar\alpha) \equiv \frac{1}{7\bar\alpha^4 + \left(\bar\alpha^2 - \frac{A}{4\bar\alpha}\right)^2}\left[2\bar\alpha^4 + \frac{A^2}{16\bar\alpha^2}\,\frac{\sin\sqrt{\left(\bar\alpha^2 - \frac{A}{4\bar\alpha}\right)}\,\sin\sqrt{\left(\bar\alpha^2 + \frac{A}{4\bar\alpha}\right)}}{\sqrt{\left(\bar\alpha^2 - \frac{A}{4\bar\alpha}\right)}\,\sqrt{\left(\bar\alpha^2 + \frac{A}{4\bar\alpha}\right)}} -\right.$$

$$-2\bar\alpha^2 \cos\sqrt{\left(\bar\alpha^2 - \frac{A}{4\bar\alpha}\right)}\cos\sqrt{\left(\bar\alpha^2 + \frac{A}{4\bar\alpha}\right)} + \quad \text{(A2-5)}$$

$$\left. + 2\bar\alpha^2 \operatorname{ch}(2\bar\alpha)\right] = 0$$

Appendix A3

Expressions of structural parameters of the (2l-1) layered, orthotropic shell [in (3.4), (3.5), (3.6)]

$$\Xi_B = \frac{\sum_{j=1}^{l} \bar a_{(j)}(\xi_{(j)} - \xi_{(j-1)}) \times \sum_{j=1}^{l} \bar a_{(j)} k_{(j)} (\xi_{(j)} - \xi_{(j-1)})}{\sum_{j=1}^{l} \bar a_{(j)}(\xi_{(j)}^3 - \xi_{(j-1)}^3) \times \sum_{j=1}^{l} \bar a_{(j)}(\xi_{(j)} - \xi_{(j-1)})} - \quad \text{(A3-1)}$$

$$- \frac{\mu_\alpha^{(l)2}\left(\sum_{j=1}^{l} \bar a_{(j)} k_{(j)} \bar q_{(j)} \langle \xi_{(j)} - \xi_{(j-1)} \rangle\right)^2}{\sum_{j=1}^{l} \bar a_{(j)}(\xi_{(j)}^3 - \xi_{(j-1)}^3) \times \sum_{j=1}^{l} \bar a_{(j)}(\xi_{(j)} - \xi_{(j-1)})},$$

$$\Xi_C = \sum_{j=1}^{l} \bar a_{(j)}(\xi_{(j)}^3 - \xi_{(j-1)}^3) \times \sum_{j=1}^{l} \tilde q_{(j)} (\xi_{(j)} - \xi_{(j-1)}) \quad \text{(A3-2)}$$

$$\Xi_{\pi\sigma} = \sum_{j=1}^{l} \bar a_{(j)} (\xi_{(j)}^3 - \xi_{(j-1)}^3), \quad \text{(A3-3)}$$

where:

$$\bar a_{(j)} = \frac{q_{(j)}}{1 - k_{(j)} \bar q_{(j)}^2 \mu_\alpha^{(l)2}}. \quad \text{(A3-4)}$$

Structural parameters of the three layered (l = 2) orthotropic shell

$$q_2 = \bar q_2 = 1; \quad \xi_2 = 1; \; \xi_1 = 0$$

$$\bar a_{(1)} = \frac{q_1}{1 - k_1 \bar q_1^2 \mu_\alpha^{(l)2}}; \quad \text{(A3-5)}$$

$$\bar a_{(2)} = \frac{1}{1 - k_2 \mu_\alpha^{(l)2}} \quad \text{(A3-6)}$$

$$\xi_2 - \xi_1 = 1 - \xi_1 = \bar\xi_1 \quad \text{(A3-7)}$$

$$\Xi_B = \frac{(\bar{a}_1\xi_1 + \bar{a}_2\bar{\xi}_1)(k_1\bar{a}_1\xi_1 + k_2\bar{a}_2\bar{\xi}_1) - \mu_\alpha^{(2)^2}(k_1\bar{q}_1\bar{a}_1\xi_1 + k_2\bar{a}_2\bar{\xi}_1)^2}{(\bar{a}_1\xi_1^3 + \bar{a}_2\langle 1-\xi_1^3\rangle)(\bar{a}_1\xi_1 + \bar{a}_2\bar{\xi}_1)} \quad \text{(A3-8)}$$

$$\Xi_c = [\bar{a}_1\xi_1^3 - \bar{a}_2(1-\xi_1^3)][1-\xi_1(1-\tilde{q}_1)] \quad \text{(A3-9)}$$

$$\Xi_{m\sigma} = \bar{a}_1\xi_1^3 + \bar{a}_2(1-\xi_1^3) \quad \text{(A3-10)}$$

Structural parameters of the three layered isotropic shell

$$k_1 = k_2 = 1\ ;\ \bar{a}_1 = q_1\bar{a}_2\ ;\ \bar{q} \cong 1$$

$$\Xi_B = (1-\mu^{(2)^2})\frac{\xi_1 q_1 + \bar{\xi}_1}{1-\xi_1^3(1-q_1)}; \quad \text{(A3-11)}$$

$$\Xi_c = \bar{a}_2[1-\xi_1^3(1-q_1)][1-\xi_1(1-\tilde{q})]; \quad \text{(A3-12)}$$

$$\Xi_{m\sigma} = \bar{a}_2[1-\xi_1^3(1-q_1)]. \quad \text{(A3-13)}$$

Structural parameters of the one-layered (l = 1) orthotropic shell

$$q_1 = \bar{q}_1 = 1;\ \xi_1 = 0;\ \bar{\xi}_1 = 1;\ k_{(j)} = k.$$

$$\bar{a}_1 = \bar{a}_2 = \frac{1}{1-k\mu_\alpha^2} = \bar{a} \quad \text{(A3-14)}$$

$$\Xi_B = k(1-k\mu_\alpha^2); \quad \text{(A3-15)}$$

$$\Xi_c = \Xi_{m\sigma} = \bar{a} = \frac{1}{1-k\mu_\alpha^2} \quad \text{(A3-16)}$$

Structural parameters of the one-layered isotropic shell

$$k = 1;\ \mu_\alpha = \mu$$

$$\Xi_B = 1-\mu^2 \quad \text{(A3-17)}$$

$$\Xi_c = \Xi_{m\sigma} = \frac{1}{1-\mu^2} \quad \text{(A3-18)}$$

The parameter ζ

$$\zeta = \sqrt{\frac{\Xi_B}{\Xi_c}} \quad \text{(A3-19)}$$

The parameter ζ of the three-layered orthotropic shell

$$\zeta = \frac{1}{\Xi_{m\sigma}}\sqrt{\frac{\xi_1 q_1 + \bar{\xi}_1}{1-\xi_1(1-\bar{q}_1)}} \quad \text{(A3-20)}$$

if $q_1 = \bar{q}_1$

$$\zeta = \frac{1}{\Xi_{m\sigma}} \quad \text{(A3-21)}$$

The parameter ζ of the one-layered orthotropic shell

$$\zeta = \sqrt{k}(1-k\mu_\alpha^2) \quad \text{(A3-22)}$$

The parameter ζ of the one-layered isotropic shell

$$\zeta = \Xi_B = 1-\mu^2. \quad \text{(A3-23)}$$

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