# THEORETICAL CONSIDERATIONS OF THE SPECIFIC IMPULSE OF RAMJET ENGINES

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## Abstract

Evaluation of ramjet engines and other air-breathing and rocket propulsion systems is often done by their fuel specific impulse (or the inverse property, the thrust specific fuel consumption). The specific impulse, which represents the energetic performance of the system, is desired to be as high as possible. However, the desire for the highest specific impulse may contradict practical system requirements such as certain thrust level. In order to investigate the peak specific impulse conditions of a ramjet engine and the main effects and trends when changing operation factors, theoretical analysis of an ideal ramjet was conducted. The study revealed interesting results about the influence of fuel to air ratio, fuel energy, combustion gas properties and flight Mach number on the ramjet motor performance expressed by its specific impulse. For ideal ramjet maximum theoretical specific impulse is always achieved when the fuel to air ratio goes to zero. Interestingly, this maximum is linearly proportional to the heat of combustion per unit mass of fuel. Furthermore, maximum specific impulse implies distinct value of flight Mach number, which solely depends on the gas properties. The analysis also demonstrates the optimal flight Mach numbers for any thrust level and fuel to air ratio. It is shown that optimal flight Mach number increases with increasing fuel to air ratio and fuel energy. This study can serve as a useful tool for ramjet engine design and for mission and system analysis.


## 1. Introduction

The objective of this study is to conduct a theoretical analysis of the specific impulse of an ideal ramjet as a measure of the engine performance under different conditions, and to point out the peak performance situations, the main trends when varying important operation parameters, and the specific practical aspects and effects on the system, resulting from the modes under consideration.

Figure 1a shows the main characteristics of a ramjet engine. Basic description and definitions as appear in this introduction are common[1]. Air at ambient conditions $T_a$, $P_a$, and a speed $u_a$ relative to the engine (point a) flows into the engine at mass flow rate $\dot{m}$ and is compressed in a diffuser to stagnation conditions $T_{o2}, P_{o2}$. It is then introduced into a combustion chamber, where $\dot{m}_f$ mass-flow-rate of fuel is added, and the flow reaches stagnation conditions $T_c, P_c$ after the combustion process. Following this stage the gas expands through a nozzle. The nozzle exit conditions are $T_e, P_e$ and the gas speed of the exhaust jet is $u_e$. In the ideal cycle analysis it is assumed that the compression and expansion processes are isentropic, that there are no pressure losses in the combustion chamber, and that the exhaust nozzle is adapted to the ambient pressure, i.e., $P_e = P_a$ (Fig. 1b). In addition, steady, one-dimensional, constant properties (equal to those of air), ideal gas flow is assumed. Although practical engine cycles present some deviations from the ideal cycle, the main trends and characteristics should be preserved, hence the ideal ramjet analysis should be indicative of practical ramjet systems and can serve as a useful tool for motor design and for mission evaluation.

The most commonly used criterion to characterize and evaluate the energetic performance of a ramjet engine, as well as that of other air breathing and rocket propulsion devices, is the specific impulse, $I_{sp}$, which is defined as the thrust developed, F, per unit weight-flow-rate of fuel

$$I_{sp} = \frac{F}{\dot{m}_f g_o} \tag{1}$$

The specific impulse is desired to be as high as possible. Another measure, the thrust specific fuel consumption, TSFC, gives an equivalent information

$$TSFC = \frac{\dot{m}_f}{F} = \frac{1}{I_{sp} g_o} \tag{2}$$

For TSFC as low as possible a value is desired.

Absolute maximum of $I_{sp}$ (or minimum of TSFC) is not always a realistic requirement, as it may contradict other system constraints, e.g. thrust level. It can, however, serve as a basis for evaluating system performance in different conditions. Furthermore, both $I_{sp}$ and TSFC depend on the ambient conditions, flight characteristics, motor design, fuel energy, combustion efficiency, and other parameters. This is in contrast to a rocket motor, where the performance depends almost solely on the propellant energy.

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## 2. Analysis

### 2.1 General Background

The thrust developed by a ramjet engine where the exit pressure is equal to the ambient pressure (adapted nozzle) is

$$F = \dot{m}_e u_e - \dot{m}_a u_a$$
$$= \dot{m}_a [(1+f)\ u_e - u_a] \qquad (3)$$

where $f \equiv \dot{m}_f/\dot{m}_a$ is the fuel to air (mass) ratio.

Substituting (3) in (1) yields:

$$I_{sp} = \frac{1}{f g_o} [(1+f)\ u_e - u_a]$$
$$= \frac{u_a}{f g_o} [(1+f)\ \frac{u_e}{u_a} - 1] \qquad (4)$$

The exit velocity, $u_e$, can be expressed in terms of the combustion chamber stagnation temperature, $T_c$, and the pressure ratio in the exit nozzle

$$u_e = \sqrt{2 C_p T_c [1 - (\frac{P_e}{P_c})^{\frac{\gamma-1}{\gamma}}]} \qquad (5)$$

For an ideal gas the specific heat $C_p = \frac{\gamma}{\gamma-1} \frac{R_o}{W}$, where $R_o$ is the Universal Gas Constant, and W is the average molecular weight of the combustion gases.

The velocities can also be expressed in terms of local gas properties and Mach number

$$u_a = M_a \sqrt{\gamma_a R_a T_a} \qquad (6)$$

$$u_e = M_e \sqrt{\gamma_e R_e T_e} \qquad (7)$$

where $R = R_o/W$. Eq. (8) expresses the constant gas property assumption:

$$\gamma_e = \gamma_a \ ; \quad W_e = W_a \ ; \ (\text{hence, } R_e = R_a) \qquad (8)$$

In addition, in an ideal ramjet motor the exit and flight Mach numbers are the same

$$M_e = M_a = M \qquad (9)$$

This yields:

$$\frac{u_e}{u_a} = \sqrt{\frac{T_e}{T_a}} = \sqrt{\frac{T_{oe}}{T_{oa}}} = \sqrt{\frac{T_c}{T_{oa}}} \qquad (10)$$

These equalities result from the expression for stagnation temperature, e.g.,

$$T_{oa} = T_a (1 + \frac{\gamma-1}{2} M_a^2) \qquad (11)$$

when the inlet and exit Mach numbers and $\gamma$ are the same.

Energy balance for the combustion chamber gives the combustion temperature dependence on fuel energy (heat of combustion, $q_R$, i.e. the heat release per unit mass of fuel) and other parameters:

$$T_c = \frac{C_p T_{oa} + f q_R}{(1+f) C_p} = \frac{T_{oa}}{(1+f)} + \frac{f}{1+f} \cdot \frac{q_R}{C_p} \qquad (12)$$

The ideal engine analysis assumes that the energy release is linearly proportional to the amount of fuel injected into the combustion chamber (or to the fuel to air ratio, f). In practice it is a reasonable assumption for low f values. However, one should bear in mind, that in practical systems for high values of f or for very energetic fuels or very high combustion temperatures, chemical equilibrium considerations may reveal deviations from the ideal case towards a reduction in the energy release and the combustion temperature.

Let us now examine the expression for $I_{sp}$. The expression is derived by substituting Eq. (10) and (12) in Eq. (4):

$$I_{sp} = \frac{u_a}{g_o f} \{[(1+f)(1 + \frac{f q_R}{C_p T_{oa}})]^{\frac{1}{2}} - 1\} \qquad (13)$$

### 2.2 Conditions for peak specific impulse

#### Effect of fuel/air ratio, f

Equation (13) will be analyzed in order to look for the fuel-to-air ratio which yields the maximum specific impulse. The derivative of Eq. (13) with f reveals that regardless of the flight, ambient or fuel parameters, the maximum $I_{sp}$ for an ideal ramjet will always be achieved at $f \to 0$, i.e., when the fuel/air ratio goes to zero. The value of $I_{sp}$ at $f \to 0$ is

$$\lim_{f \to 0} I_{sp} = \frac{1}{2} \frac{u_a}{g_o} \frac{q_R}{C_p T_{oa}} \qquad (14)$$

This value is directly proportional to the heating value of the fuel, $q_R$.

#### Effect of Mach number, M

Ambient and flight conditions affect the specific impulse through Mach number, ambient temperature, and $\gamma$. Substituting Eqs. (6) and (11) for $u_a$ and $T_{oa}$ yields at $f \to 0$

$$I_{sp} = \frac{1}{2} \frac{\sqrt{\gamma R}}{g_o \sqrt{T_a}} \cdot \frac{q_R}{C_p} \cdot \frac{M}{1 + \frac{\gamma-1}{2} M^2}, \quad f \to 0 \qquad (15)$$

The dependence on Mach number (Eq. (15)) reveals, that when $f \to 0$, the specific impulse, $I_{sp}$, always gets its *maximum* for the following Mach number:

$$M = \sqrt{\frac{2}{\gamma - 1}} \tag{16}$$

Interestingly, the inlet stagnation temperature, $T_{oa}$, is then exactly twice the ambient temperature. For the specific case of $\gamma$=1.4, (corresponding to ambient air), the value of M from Eq. (16) is

$$M = \sqrt{5} = 2.236, \quad \gamma = 1.4 \tag{17}$$

Interestingly, the lower the $\gamma$ of the gases, the higher the Mach number is at which the maximum of $I_{sp}$ will be achieved (Fig. 2). This Mach number approaches infinity for the theoretical value of $\gamma$=1.

The effect of Mach number on the specific impulse can be demonstrated by the function f(M), which is defined as

$$f(M) = \frac{M}{1 + \frac{\gamma-1}{2} M^2} \tag{18}$$

The values of f(M) in Eq. (18) correspond to the relative values of $I_{sp}$ in different Mach numbers for a fixed value of $\gamma$ (Fig. 3). Nevertheless, they cannot be used as a measure of the relative values of the specific impulse for different $\gamma$.

The *maximum* of f(M) at each $\gamma$, which is achieved at Mach number as is defined in Eq. (16), just like the maximum of $I_{sp}$, is

$$\max\ f(M) = \frac{M}{2} = \frac{1}{2}\sqrt{\frac{2}{\gamma-1}}\ , \quad M = \sqrt{\frac{2}{\gamma-1}} \tag{19}$$

Effect of specific heat ratio, $\gamma$

The function f(M) in Eq. (18) shows the effect of Mach number on the specific impulse at a given particular value of $\gamma$. It should be noted, however, that the entire effect of $\gamma$ is neither revealed in Eq. (18) nor in any of Eqs. (15) through (19), since the specific heat, $C_p$, is also coupled with $\gamma$. Substituting the expression of $C_p$ for an ideal gas, $C_p = \gamma R/(\gamma-1)$, results in rearrangement of Eq. (15) for the specific impulse at f→0:

$$I_{sp} = \frac{1}{2g_o} \cdot \frac{q_R\sqrt{W}}{\sqrt{R_oT_a}} \cdot \frac{\gamma-1}{\gamma} \cdot \frac{M}{1 + \frac{\gamma-1}{2}M^2}$$

$$= \frac{1}{2g_o} \cdot \frac{q_R\sqrt{W}}{\sqrt{R_oT_a}} \cdot \Phi(\gamma, M) \quad , \quad f \to 0 \tag{20}$$

where

$$\Phi(\gamma, M) = \frac{\gamma-1}{\sqrt{\gamma}} \cdot \frac{M}{1 + \frac{\gamma-1}{2}M^2} \tag{21}$$

In contrast to f(M), $\Phi(\gamma,M)$ increases monotonically with increasing $\gamma$, while for each particular $\gamma$ it gets a maximum for $M = \sqrt{\frac{2}{\gamma-1}}$, just like f(M). The value of this *maximum* for a given $\gamma$ is

$$\max\ \Phi(\gamma, M) = \sqrt{\frac{\gamma-1}{2\gamma}}\ , \quad M = \sqrt{\frac{2}{\gamma-1}} \tag{22}$$

Figure 4 shows the variation of the maxima of $\Phi(\gamma,M)$ with $\gamma$. The figure gives a relative measure of the dependence of the peak value of $I_{sp}$ on $\gamma$.

Effect of ambient temperature, $T_a$

Equation (15) shows that the value of the peak specific impulse for a given fuel (i.e. given $q_R$ and $\gamma$), which implies certain Mach number M = 2/($\gamma$-1) and fuel/air ratio f→0, is proportional to $T_a^{-\frac{1}{2}}$. Hence, regarding $T_a$ the highest theoretical $I_{sp}$ will be achieved at the lowest ambient temperature possible. For atmospheric flight it means $T_{ac}$= - 56.2°C = 216.8K, which is the temperature at the altitude between 11,000 m and 22,500 m (approximately 36,000 to 74,000 ft.).

Effect of fuel energy, $q_R$

It should be noted again that for the specific conditions of peak $I_{sp}$ (i.e., f→0), the value of this peak is linearly proportional to the fuel energy, namely to the heat of combustion of the fuel per unit mass of fuel, $q_R$.

Reference Calculations

The peak theoretical values of the specific impulse of an ideal ramjet employing hydrocarbon (HC) fuel at sea level ($T_a$=288K) and at high altitude (lowest ambient temperature, $T_a$=216.8K) are presented as a reference example:

Common properties and conditions:

$q_R$ = 10,000 kcal/kgfuel = $4.185.10^7$J/kgfuel
$\gamma$ = 1.40
W = 29 kg/kgmol
f → 0
Hence, M = $\sqrt{5}$ = 2.236

Sea level performance:

max $I_{sp}$ = 2823 sec

min TSFC = 1.278 $\frac{\text{kgfuel/hr}}{\text{kgforce}}$

High altitude performance:

max $I_{sp}$ = 3246 sec

min TSFC = 1.109 $\frac{\text{kgfuel/hr}}{\text{kgforce}}$

## 2.3 Specific impulse in variable conditions.

Practical operation requirements would always deviate from the theoretical conditions of the absolute maximum specific impulse, as this maximum implies fuel/air ratio $f \to 0$, which means zero thrust. Producing higher thrust from given motor flight conditions requires higher fuel/air ratio. Figure 5 shows the variations of $I_{sp}$ and TSFC of an ideal ramjet employing hydrocarbon (HC) fuel with f at Mach number M=2.236, (which is optimal for $f \to 0$). The figure shows that both at sea level and at high altitude, the higher the value of f, the lower the energetic performance is (i.e., lower $I_{sp}$ and higher TSFC).

The effect of Mach number on $I_{sp}$ is demonstrated in Fig. 6 for different values of f. The figure reveals the following: (a) For any fixed mach number, the specific impulse is always higher for smaller values of fuel/air ratios. (b) The variation of $I_{sp}$ with M at constant f obtains a maximum at certain Mach number which is characteristic to the level of f. (c) When operating at $f \neq 0$, maximum $I_{sp}$ is achieved at higher Mach number than that of $f \to 0$. The higher the f, the higher M is for maximum $I_{sp}$. For HC fuel it is changed from M=2.236 at $f \to 0$ to M=3.45 at f=0.067, which is the stoichiometric fuel/air ratio. Nevertheless, the peak values of $I_{sp}$ (at optimal M) still decrease with increasing the fuel/air ratio, e.g. from 2823 sec at $f \to 0$ and M=2.236 to 1880 sec at f=0.067 (stochiometric ratio) and M=3.45, a decrease to about 67%.

The effect of fuel energy is demonstrated by the dependence of $I_{sp}$ on the equivalence ratio $\phi \equiv f/f_{st}$ at constant Mach number for four representative ideal fuels having different energy values, $q_R$. In the calculation it was assumed that the working fluid is a gas of constant properties equal to those of air, and that there are no two-phase flow losses. It is also assumed that the heat release per unit mass of air is linearly proportional to f (i.e., that Eq. 12 exists). For demonstration, the fuel energy values used were equal to those of the following four fuels: hydrocarbon (HC), boron (B), aluminum (Al) and magnesium (Mg). The fuel energies and the stoichiometric ratios are summarized in Table 1, and the $I_{sp}$ vs. $\phi$ plots are given in Fig. 7.

Table 1: Heats of combustion and stoichiometric fuel/air ratios of four different fuels.

| Fuel | Heat of combustion $q_R$[kcal/g] | Stoich.fuel/air ratio $f_{st}$ |
|---|---|---|
| HC | 10.00 | 0.067 |
| B | 13.87 | 0.105 |
| Al | 7.41 | 0.262 |
| Mg | 5.91 | 0.354 |

## 2.4 Comparison of energetic performance at similar motor requirements

Flight conditions, thrust level, fuel/air ratio and other parameters affect the energetic performance of the ramjet system (i.e. the $I_{sp}$), hence the comparison between different systems employing different fuels is not always straightforward.

A more meaningful comparison of the energetic performance of different fuels can be done on the basis of equal specific requirements from the system, i.e., the production of the same thrust per unit mass flow rate of air, $F/\dot{m}_a$, at the same air speed and ambient conditions. Given thrust per unit mass flow rate of air means also a given air specific impulse, $I_{sp,a}$:

$$F/\dot{m}_a = I_{sp,a} \cdot g_o \tag{23}$$

Since

$$I_{sp} = I_{sp,a}/f \tag{24}$$

it implies, that at constant air specific impulse

$$I_{sp} \propto \frac{1}{f} \quad , \quad I_{sp,a} = \text{const.} \tag{25}$$

which means that best $I_{sp}$ for constant $I_{sp,a}$ will be achieved at the lowest f that would satisfy the required $I_{sp,a}$. Under these conditions the thrust specific fuel consumption is directly proportional to f

$$\text{TSFC} \propto f \quad , \quad I_{sp,a} = \text{const.} \tag{26}$$

The requirement of constant $I_{sp,a}$ at given air speed implies

$$(1+f)(C_p T_{oa} + f q_R) = \text{const}, I_{sp,a} = \text{const} \tag{27}$$

and for the same ambient conditions and for relatively small fuel/air ratio (as often occurs), Eq. (27) yields

$$f q_R \cong \text{const}, \; I_{sp,a} = \text{const}, \; f \ll 1 \tag{28}$$

Equation (28) reveals that the requirement of fixed $F/\dot{m}_a$ at given flight and ambient conditions is equivalent to the situation of equal heat capacity of the air in the combustion chamber, namely, the same heat release per unit mass of air in the combustion chamber. Combining Eq. (25) with Eq. (28) yields for these conditions:

$$I_{sp} \propto q_R \; , \; f q_R = \text{const} \tag{29}$$

namely, for the same heat release per unit mass of air in the combustion chamber, or for similar thrust per unit air mass flowrate, the specific impulse is linearly proportional to the heat of combustion per unit mass of the fuel at any thrust level.

Due to the difference in heating values and in stoichiometric fuel/air ratios, different maximum values of heat capacity, $fq_R$, can be obtained by using different fuels. Lines of equal heat capacities are drawn in Fig. 7.

## 3. Concluding Remarks

Theoretical considerations of the specific impulse of a ramjet engine should be the basis for evaluation of the energetic performance of this propulsion system. Analysis of an ideal ramjet cycle is best for giving the insight and for understanding the basic phenomena, revealing the fundamental peculiar dependence of the specific impulse on fuel and flight parameters, e.g. heat of combustion, gas properties, ambient conditions and flight Mach number. It is indicative of non-ideal cycles and practical ramjet operation, thus can serve as an important tool in the engine design and mission analysis. The main conclusions and findings of the theoretical study of the ideal ramjet are as follows:

For any given flight or fuel parameters, best theoretical specific impulse always takes place for fuel/air ratio $f \to 0$. (This is, of course, a limiting case which is associated with zero thrust.)

For these peak performance conditions, maximum specific impulse is achieved at a certain value of flight Mach number, $M = \sqrt{2/(\gamma-1)}$. (e.g., for $\gamma=1.4$, $M=\sqrt{5}=2.236$.) Inlet stagnation temperature is then exactly twice the ambient temperature.

This maximum value of specific impulse is directly proportional to the heat of combustion per unit mass of fuel.

The lower the ambient temperature, the higher the peak specific impulse.

For practical operations of non-zero fuel/air ratios, the higher the fuel/air ratio, the lower the specific impulse and the higher is the Mach number at which the best $I_{sp}$ occurs.

For operations at equal thrust per unit mass flow rate of air (i.e., similar heat capacity), the specific impulse is linearly proportional to the heat of combustion per unit mass of fuel (fuel energy).

## References


1. Hill, P.G. and Petersen, C.R., Mechanics and Thermodynamics of Propulsion, Ch. 6, Addison-Wesley Pub. Co. Inc., 1965.


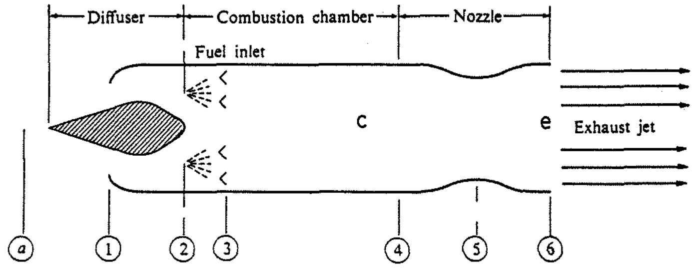


Fig. 1: Schematic description of a ramjet engine.

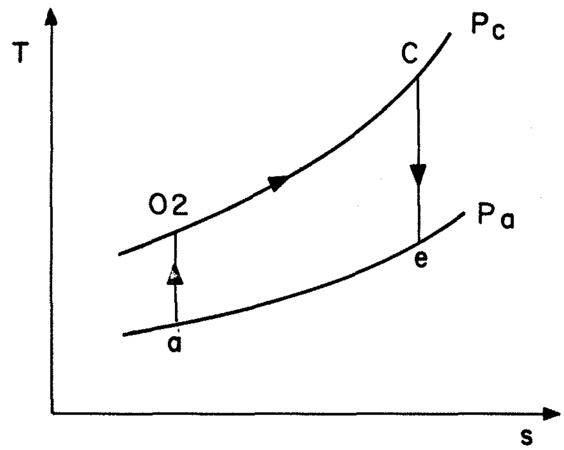


Fig. 1b: Thermodynamic (T-S) diagram of an ideal ramjet cycle.

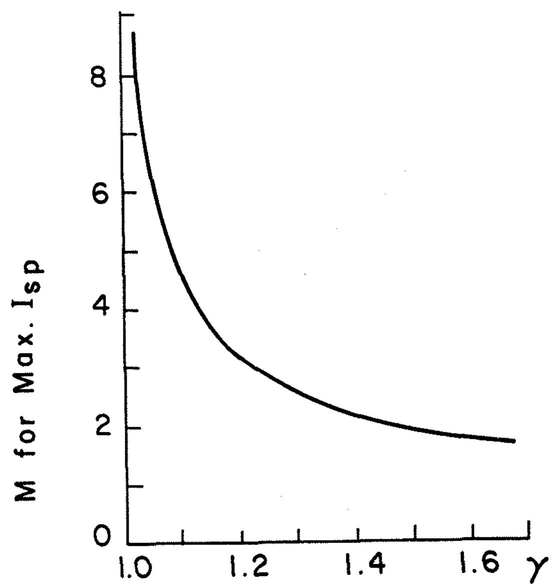


Fig. 2: Variation of Mach number versus $\gamma$ for maximum specific impulse at fuel/air ratio $f \to 0$.

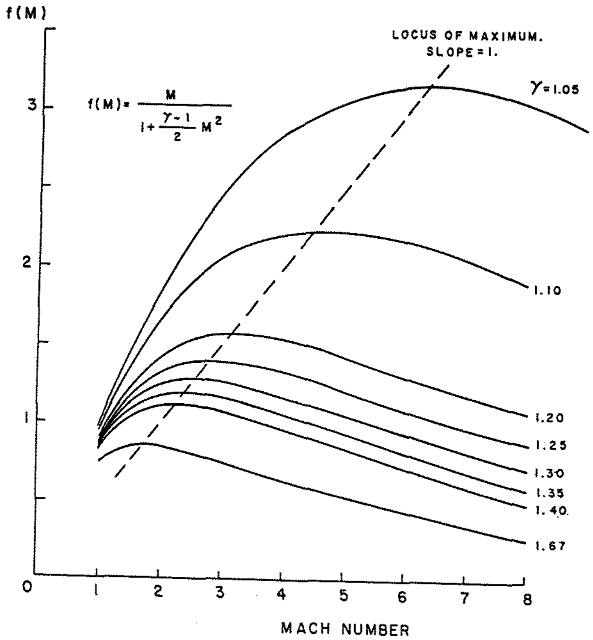


Fig. 3: f(M) vs. M with $\gamma$ as a parameter.

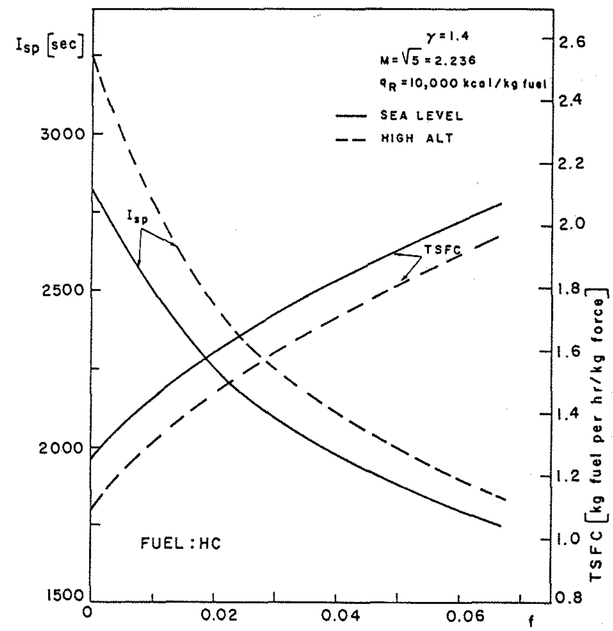


Fig. 5: Dependence of an ideal ramjet performance ($I_{sp}$ and TSFC) on fuel/air ratio at a fixed Mach number.

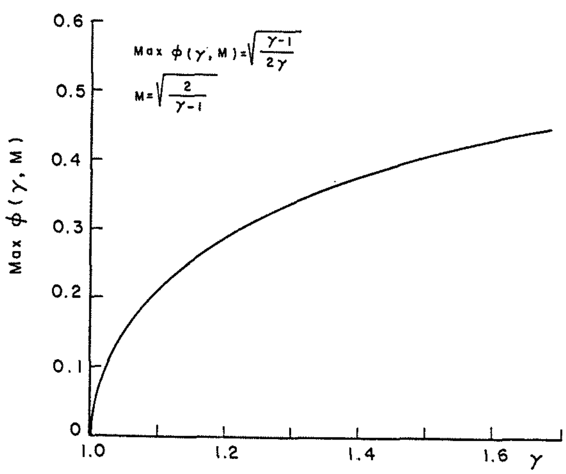


Fig. 4: Variation of max $\Phi(\gamma,M)$ with $\gamma$ gives a relative measure for the dependence of the peak value of $I_{sp}$ on $\gamma$.

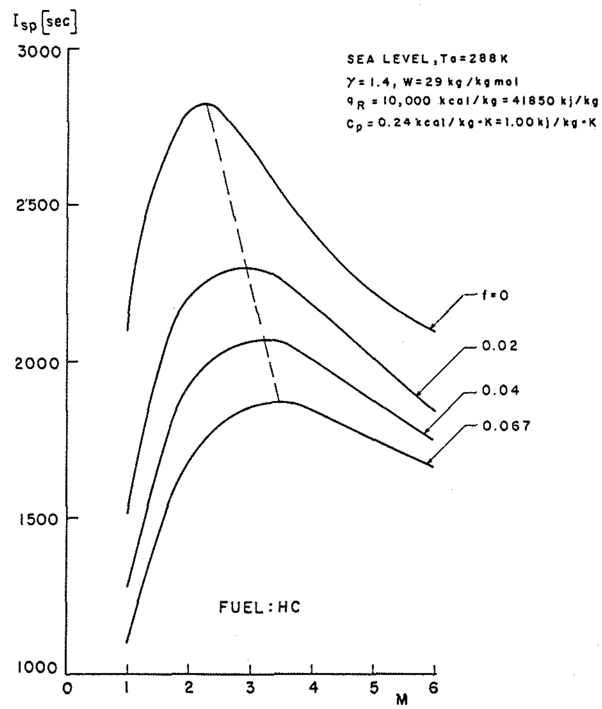


Fig. 6: Dependence of $I_{sp}$ on M at different values of f for an ideal ramjet shows that higher specific impulse values are obtained for lower f, and that optimal Mach number increases with increasing f. Fuel: HC.

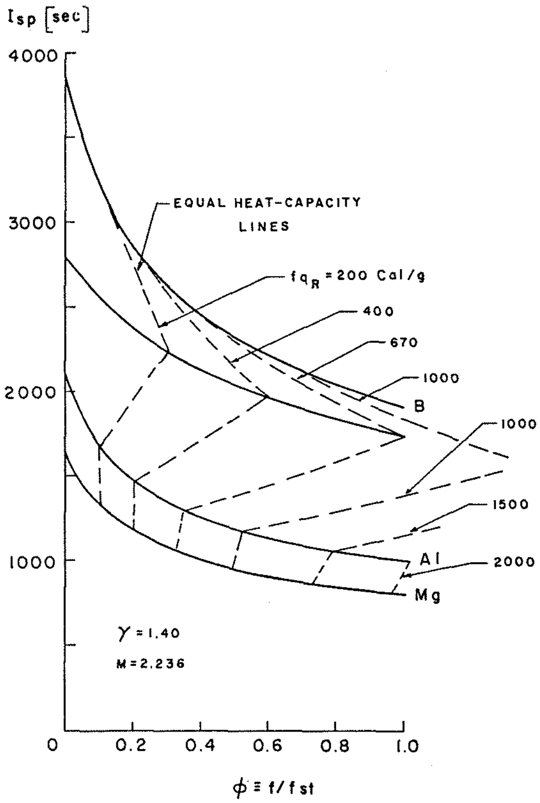


Fig. 7: Ideal theoretical calculations of the variation of $I_{sp}$ with $\varphi$ for four different fuels in an ideal ramjet. Two-phase flow losses are not considered, and a complete forward chemical reaction with no dissociation is assumed.