SPECTRUM FATIGUE MODEL FOR COMPOSITE LAMINATES*

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Abstract

A comprehensive analytical and experimental programme was undertaken to develop a fatigue model capable of estimating the lifetime of composite laminates subject to spectrum loading. The analysis was formulated to consider cumulative fatigue damage as a function of ply orientation and stacking sequence. This necessitated treating each cycle separately and calculating an 'equivalent' damage in terms of all preceding cycles. The basis for the computational procedure, however, required the experimental evaluation of 'fatigue functions' in the principal material property directions. These fatigue functions were first utilized to predict constant amplitude fatigue life of $(0, \pm45, 90)_s$ laminates under compression loading and the results compared to test data. Subsequently, the fatigue analysis was then applied to the same laminate configuration using the FALSTAFF compression spectrum, and the fatigue life predictions compared to test results.

Nomenclature

A_{ij}	$\sum_{k=1}^{\bar{N}} (\bar{Q}_{ij})_k (h_k - h_{k-1})$
B_{ij}	$\frac{1}{2} \sum_{k=1}^{\bar{N}} (\bar{Q}_{ij})_k (h_k^2 - h_{k-1}^2)$
D_{ij}	$\frac{1}{3} \sum_{k=1}^{\bar{N}} (\bar{Q}_{ij})_k (h_k^3 - h_{k-1}^3)$
E_{11}, E_{22}, G_{12}	orthotropic elastic constants
h_k	distance from laminate mid-plane to top of k-th lamina
$\bar{N}$	total number of plies
Q_{11}	$\frac{E_{11}}{(1 - \nu_{12}\nu_{21})}$
Q_{22}	$\frac{E_{22}}{(1 - \nu_{12}\nu_{21})}$
Q_{12}	$\frac{\nu_{21}E_{11}}{(1 - \nu_{12}\nu_{21})} = \frac{\nu_{12}E_{22}}{(1 - \nu_{12}\nu_{21})}$
Q_{66}	G_{12}
$\bar{Q}_{11}$	$Q_{11}C^4 + 2(Q_{12} + 2Q_{66})\tilde{S}^2C^2 + Q_{22}\tilde{S}^4$
$\bar{Q}_{22}$	$Q_{11}\tilde{S}^4 + 2(Q_{12} + 2Q_{66})\tilde{S}^2C^2 + Q_{22}C^4$
$\bar{Q}_{12}$	$(Q_{11} + Q_{22} - 4Q_{66})\tilde{S}^2C^2 + Q_{12}(\tilde{S}^4 + C^4)$
$\bar{Q}_{66}$	$(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\tilde{S}^2C^2 + Q_{66}(\tilde{S}^4 + C^4)$
$\bar{Q}_{16}$	$(Q_{11} - Q_{12} - 2Q_{66})\tilde{S}C^3 + (Q_{12} - Q_{22} + 2Q_{66})\tilde{S}^3C$
$\bar{Q}_{26}$	$(Q_{11} - Q_{12} - 2Q_{66})\tilde{S}^3C + (Q_{12} - Q_{22} + 2Q_{66})\tilde{S}C^3$
R	ratio of algebraic minimum/maximum cyclic stresses
$\tilde{S}$, C	$\sin\theta$, $\cos\theta$, respectively
S	static lamina shear strength measured in 1-2 plane
t	laminate thickness
X, X'	tensile and compressive static lamina strengths measured in the 1-direction, respectively
Y, Y'	tensile and compressive static lamina strengths measured in the 2-direction, respectively

Subscripts

1, 2	lamina material axes parallel and orthogonal to the fibre reinforcement, respectively

Greek Symbols

σ	stress
σ_1, σ_2	normal stresses measured in 1 and 2 directions, respectively
σ_6	shear stress measured in the 1-2 plane
θ	fiber orientation relative to structural reference axis

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I. Introduction

In general, aircraft structures are subjected to random loading associated with flight through gusts and turbulence, as well as during the landing phase due to runway roughness. Consequently, the evaluation of structural fatigue life should in fact be based on random or spectrum loading tests, rather than the traditional constant amplitude sample tests that often form the basis for fatigue predictions. This is particularly true for composite laminates that are seeing widespread applications in aircraft, especially fighters such as Canada's CF-18. For example, flaws in the form of local delaminations, arising from manufacturing defects or impact damage, can lead to loss in structural integrity, particularly under compressive loading that can produce skin buckling and subsequent laminate fracture. The growth of such flaws under cyclic loading and the resulting reductions in stiffness and strength form a major part of damage assessment and fatigue life predictions for composite laminates.

Consequently, realistic evaluation of composite material fatigue strength necessitates the use of spectrum loading both with regard to stress and temperature. This was not feasible in the past due to testing facility limitations, but currently available electro-hydraulic closed loop test control equipment has overcome this difficulty. Note, however, that the use of constant amplitude fatigue testing can still be used in certain aircraft parts such as rotating machinery. Moreover, such tests do provide a measure of fatigue properties which can be used to establish an analytical methodology for assessing damage, residual fatigue life, and residual strength.

As a first step for creating an appropriate spectrum for fighter aircraft, it was found necessary to actually obtain load factor histories from different sources (i.e., various aircraft) subject to various operational circumstances.[1,2] These load histories were then transformed into a mixture of nondimensional stress histories and finally classified into a bivariate statistical summary of stress range. An example of such a stress-range bivariate distribution is the FALSTAFF spectrum (Fighter Aircraft Loading Standard for Fatigue Evaluation) which is described later in this report.

Fatigue damage of composite laminates under constant amplitude loading has been investigated by many researchers. Experimental and theoretical results have been obtained for constant R values ($\sigma_{min}/\sigma_{max}$) for a variety of materials. However, during spectrum loading, R values vary significantly depending on the spectrum profile. Thus the basic problem that one encounters is the lack of experimental and/or theoretical fatigue damage data for a spectrum of R values. To obtain such a data base for commonly used materials would be a rather time consuming and expensive process. Thus an attempt has been made in this report to account for R variations in the fatigue equations. In the following report, both constant amplitude and spectrum fatigue models are developed using constant amplitude, fixed R-ratio unidirectional fatigue data, a strength criterion and a damage formulation. Subsequently these models are then used to predict the fatigue life of $(0, \pm 45, 90)_s$ graphite/epoxy-laminates (AS4/3501-6) subject to compression loading, and the results compared to test data.

II. Development of Fatigue Models

To predict the fatigue life of a composite laminate first requires a lamina failure criterion. Subsequently one must then develop a 'structural' failure model in which failed laminae can be 'removed', or accounted for, to enable one to estimate an ultimate laminate fatigue strength. One method which is phenomenological in nature, and attempts to employ a macro-mechanics approach is to utilize a modified form of a static lamina failure criterion in which the static strength parameters are replaced by 'fatigue functions'. Each strength term then becomes a function of frequency of loading (f), the number of cycles (N) and the stress ratio R. Previous work by other authors using this approach[3-5] for constant amplitude loading has demonstrated reasonable correlation with test data in many instances based on a simple, quadratic form of failure relation. In these cases, only the fatigue strengths under tension-tension and shear loading were required (X, Y, S), although a delamination effect was included. Another constant amplitude model based on a quadratic Tsai-Hill failure criterion has also been presented,[6] again using only the X, Y and S strength parameters. Although reasonable comparisons with test data were reported for S-glass/epoxy (SP-250-SF1), such was not the case for graphite/epoxy (E788/T300) based on limited results published to-date. However, despite this apparent problem, a similar approach was utilized in our fatigue study to estimate the fatigue life of laminates for both constant amplitude and spectrum loading. The essential differences lie in the strength model formulation and damage calculation required to implement the spectrum load procedure. This report presents two different approaches for calculating spectrum fatigue life, one of which (the tensor polynomial) has the capability for predicting residual strength.

All of the above mentioned formulations represent approximations which are encompassed by the general tensor polynomial failure criterion advocated in Refs. 7 and 8 for static strength calculations. The following relation is used to describe the lamina failure surface in stress space;

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j + F_{ijk}\sigma_i\sigma_j\sigma_k + \dots = f(\sigma) \quad \begin{cases} < 1 & \text{no failure} \\ = 1 & \text{failure} \\ > 1 & \text{exceeded failure} \end{cases} \qquad (1)$$

for i, j, k = 1...6. F_i, F_{ij} and F_{ijk} are strength tensors of the 2nd, 4th and 6th rank, respectively.

Plane Stress State

If one restricts the analysis to a plane stress state and considers a quadratic formulation as being a reasonable representation of the failure surface, then Eq. (1) can be reduced to

$$F_1\sigma_1 + F_2\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 + 2F_{12}\sigma_1\sigma_2 = 1 \qquad (2)$$

Simple tension, compression and shear tests will yield the principal strength components which are defined by

$$F_1 = \frac{1}{X} - \frac{1}{X'},\quad F_2 = \frac{1}{Y} - \frac{1}{Y'},\quad F_{11} = \frac{1}{XX'},\quad F_{22} = \frac{1}{YY'},\quad F_{66} = \frac{1}{S^2} \qquad (3)$$

where each of the strength parameters (X, X', Y, Y', S) is regarded as a function of (f, N, R).

Under simple loading conditions when f and R are constants, then the fatigue strength parameters are only a function of the number of cycles, N. As stated earlier, the quadratic formulation provides accurate strength predictions for such load cases as uniaxial tension and compression. Consequently, for this limited set of conditions, which are typical in fatigue tests, then the fatigue strength functions necessary to characterize a lamina are given by Eq. (3).

To determine the quadratic interaction term (F_{12}) would require a biaxial fatigue test. However, for non-biaxial applied loading, it has been found that F_{12} contributes little to the static strength prediction. In any case, fatigue tests must be conducted on 0° and 90° samples for given R values to determine the fatigue functions. This involves tension and compression fatigue tests in both the fiber (1) and transverse (2) directions, as well as pure shear in the 1-2 plane.

Based on an extensive test programme that will be described later, it has been found, for the materials investigated to-date, that the following general relation can be used to describe each of the fatigue functions (X, X', Y, Y', S);

$$\sigma(N) = \sigma_S\{\frac{\sigma_e}{\sigma_s} + (1 - \frac{\sigma_e}{\sigma_s})\ (4N)^{\beta}\} \qquad (4)$$

where σ denotes a particular strength parameter, σ_e and σ_s represent the 'endurance' and 'static' ultimate strengths, while 'β' is an exponent to be determined from the σ-N test data for each loading mode.

Constant Amplitude Fatigue Life Predictions

Consider a given symmetric laminate of $\bar{N}$ plies subject to a cyclic stress of constant amplitude (σ_A). The corresponding principal stresses in the k-th ply are given from laminate theory by,

$$\begin{bmatrix}\sigma_1\\ \sigma_2\\ \sigma_6\end{bmatrix}_k = t[T]_k\ [\bar{Q}_{ij}]_k\ [A]^{-1}\ \begin{bmatrix}\sigma_A\\ 0\\ 0\end{bmatrix} \qquad (5)$$

where

$$[T]_k = \begin{bmatrix} C^2 & \tilde{S}^2 & 2C\tilde{S} \\ \tilde{S}^2 & C^2 & -2C\tilde{S} \\ -\tilde{S}C & \tilde{S}C & C^2-\tilde{S}^2 \end{bmatrix}_k$$

Equation (5) can be used to calculate the stress state for all plies, i.e., for $1 \leq k \leq \bar{N}$. Assuming a data base exists for all strength parameters in the form of σ-N curves, then the number of cycles to failure for each ply can be calculated from the equation,

$$F_1(N)_k\sigma_{1k} + F_2(N)_k\sigma_{2k} + F_{11}(N)_k\sigma_{1k}^2 + F_{22}(N)_k\sigma_{2k}^2 + F_{66}(N)_k\sigma_{6k}^2 = 1 \qquad (6)$$

if $F_{12} = 0$. When first ply (or plies) failure occurs, then the resulting laminate stiffness must be modified and revised ply stresses [Eq. (5)] calculated. One conservative procedure that has been invoked is to remove the ply matrix material first, then the complete ply after the fibres have failed. Thus an iterative analysis can be used to arrive at an overall laminate fatigue life.

It should be noted that the implementation of Eq. (6) is based on the assumption that fatigue function curves exist for given R values. Clearly this constitutes a rather expensive data base to acquire and a modified form of Eq. (4) can be used to account for variations in R values, i.e.,

$$\sigma(N) = \sigma_s\{\frac{\sigma_e}{\sigma_s} + (1 - \frac{\sigma_e}{\sigma_s})\ (4N)^{(\frac{1-R}{1-R_0})\beta} \qquad (7)$$

where R_0 and R represent the stress ratio values associated with the baseline data (R_0) and the applied cyclic load on the laminate (R) being analysed. At present, we have no test data to confirm the validity of Eq. (7) and it serves only as a proposed formula.

Tensor Polynomial Spectrum Fatigue Life Predictions

The spectrum analysis outlined below is based on treating each cycle separately and estimating the amount of 'damage' done to each ply by that cycle. To understand the procedure, one must first define the concepts of 'damage' and 'residual strength' as used in this model. For our purposes, we shall employ the definition,

$$\sigma_r = \sigma_s(1-d) \qquad (8)$$

where d $\equiv$ damage done to lamina,

σ_s = static ultimate strength,

σ_r = residual strength of lamina after a prescribed number of fatigue cycles has been accumulated.

In general, one does not know what the 'residual strength' of a laminate is and recourse to estimating this quantity based on some cumulative damage criterion is necessary.

Consider now the case where 'n' cycles of constant amplitude stress (σ_b) are applied to a lamina. From Fig. 1 it can be seen that the lamina life is given by 'N_b'. However, if the cycling is stopped at (N_b-1) cycles, then it is clear that one more cycle at stress 'σ_b' should cause failure. Hence for this case, the residual strength = applied stress (σ_b). Thus it is proposed that the following damage relation be used to account for 'n' cycles of loading,

$$(d)_k = \left(\frac{n}{N_b}\right)_k \left(\frac{\sigma_s - \sigma_b}{\sigma_s}\right)_k = \left(\frac{n}{N_b}\right)_k \left(1 - \frac{1}{K}\right)_k \quad (9)$$

where $K = \sigma_s/\sigma_b$.

Calculation Procedure

First Cycle at Start-UP (Fig. 2, a-b)

Based on the peak cycle stress σ_b, one first calculates the principal stresses in each ply using Eq. (5). Subsequently, using the static strength parameters, one then calculates the lamina ultimate static strength (σ_s) and the stress ratio K from Eq. (2).

Knowing the ply peak stresses $(\sigma_1, \sigma_2, \sigma_6)_k$, one utilizes the fatigue equations [as given in the general form by Eq. (7)] to calculate the cycles to failure (N_1) from Eq. (6), employing the appropriate R value corresponding to the end point stress levels at 'a' and 'b'.

The next step involves estimating the damage done in each ply due to this first cycle from Eq. (9), i.e.,

$$(d_1)_k = \left(\frac{1}{N_1}\right)_k \left(1 - \frac{1}{K_1}\right)_k \quad (10)$$

Second Cycle (c-d)

The ply stresses are calculated again from Eq. (5) at the new stress level 'σ_d', and the static ultimate lamina strength obtained from Eq. (2). This yields 'K_2'.

With the known R ratio corresponding to levels 'c' and 'd', one can employ the fatigue functions and Eq. (6) to determine the cycles to failure, 'N_2'.

Next, one proceeds to check for each ply, that at the new stress level σ_d, failure will not occur, i.e.,

$$(d_1)_k < \left(1 - \frac{1}{K_2}\right) \quad (11)$$

where $K_2 = \sigma_s/\sigma_d$. If this relation is not satisfied for all laminae, then ply failure has occurred and a modified stiffness matrix is calculated along with new ply stresses.

Because the peak cyclic stresses are changing during spectrum loading one must now determine an 'equivalent number' of cycles that are required (in each lamina) at the new stress level to yield the same damage that has occurred due to all prior stress cycles, i.e.,

$$\left(\frac{(n_e)_d}{N_2}\right)_k \left(1 - \frac{1}{K_2}\right)_k = (d_1)_k \quad (12)$$

Note that by comparing Eqs. (11) and (12), the lamina failure criterion can be re-written in the form

$$(n_e)_d < N_2 \text{ for no failure}$$

i.e., the equivalent number of cycles $(n_e)_d$ at the new stress level must not exceed the cycles to failure value at that stress level (N_2).

Finally, the new damage value is calculated for each ply using the relation

$$(d_2)_k = \left(\frac{(n_e)_d + 1}{N_2}\right)_k \left(1 - \frac{1}{K_2}\right)_k \quad (13)$$

where the total number of cycles at σ_d has been set equal to $[(n_e)_d + 1]$.

This procedure is repeated for each ply and at each cycle throughout the spectrum until a ply failure is observed. As noted earier, a reduced stiffness matrix is necessary before one can iterate to obtain overall laminate failure. The general form of the damage equation for iteration purposes is:

$$(d_i)_k = \left(\frac{(n_e)_i + 1}{N_2}\right)_k \left(1 - \frac{1}{K_i}\right)_k \quad (14)$$

where i = cycle number,
k = ply number.

Cumulative Damage Model for Predicting Fatigue Life

Another approach that can be used to estimate fatigue life is that of a cumulative damage model. The following modified form of Owen's[9] equation was also employed to provide a comparison with the previous fatigue model and test data:

$$\sum_{i=1}^{\tilde{N}} \left\{\left(A_1 \frac{n_i}{N_i^{(1)}} + A_2 \frac{n_i}{N_i^{(2)}} + A_6 \frac{n_i}{N_i^{(6)}}\right) + \left(B_1\left(\frac{n_i}{N_i^{(1)}}\right)^2 + B_2\left(\frac{n_i}{N_i^{(2)}}\right)^2 + B_6\left(\frac{n_i}{N_i^{(6)}}\right)^2\right)\right\}_k = 1 \quad (15)$$

where A_1, A_2, A_6, B_1, B_2, B_6 represent material constants

$\tilde{N}$ = number of load blocks

n_i = number of cycles at the i-th stress level

$N_i^{1,2,6}$ = number of cycles to failure (based on constant amplitude loading tests) corresponding to σ_1, σ_2, σ_6 stresses, respectively

One can readily see that when $B_1 = B_2 = B_6 = 0$, Eq. (15) reduces to the Miner-Palmgren damage model. In the following derivations, for convenience it will be assumed that $A_1 = A_2 = A_6 = A$

and $B_1 = B_2 = B_6 = B$. Note that the special case of constant amplitude loading would then require that $A + B = 1$. However, for fatigue life predictions it is not necessary to evaluate these parameters.

For purposes of computation and modifying lamina stiffness due to fatigue failure of either the matrix or fibre components, it is convenient to re-write Eq. (15) in the following form:

$$\sum_{i=1}^{\tilde{N}} (dm_i + df_i)_k = 1 \qquad (16)$$

where

$$dm_i = \{A(\frac{n_i}{N_i^{(2)}} + \frac{n_i}{N_i^{(6)}}) + B[(\frac{n_i}{N_i^{(2)}})^2 + (\frac{n_i}{N_i^{(6)}})^2]\}$$
$$df_i = A(\frac{n_i}{N_i^{(1)}}) + B(\frac{n_i}{N_i^{(1)}})^2 \qquad (17)$$

Note that Eq. (16) is applied to each ply (k) and when the left hand side attains (or exceeds) the value unity, for any ply j, that j-th ply has been deemed to fail. If, based on the accumulated damage (dm)ascribed to the matrix component, $dm_j >> df_j$, then the matrix is removed from the j-th lamina but the fibres are retained. This is achieved by re-calculating the laminate stiffness matrix, setting $E_{22} = G_{12} = \nu_{12} = 0$ in the j-th ply. Once the fibres fail, the lamina is removed and a revised laminate stiffness matrix is calculated. This procedure is repeated until all plies have failed.

Calculation Procedure

First Cycle at Start-Up (Fig. 2, a-b)

For the given peak stress σ_b, it is again necessary to calculate the principal ply stresses $(\sigma_1, \sigma_2, \sigma_6)_k$ using Eq. (5). Subsequently, the fatigue equations associated with the principal material axes (see Fig. 3) are consulted to determine the cycles to failure, again using the modified form [Eq. (7)] to account for variations in the R value.

Hence the damage from the firt cycle in the k-th ply can be written as,

$$(d_1)_k = \{A(\frac{1}{N_b^{(1)}} + \frac{1}{N_b^{(2)}} + \frac{1}{N_b^{(6)}}) + B[(\frac{1}{N_b^{(1)}})^2 + (\frac{1}{N_b^{(2)}})^2 + (\frac{1}{N_b^{(6)}})^2]\}_k \qquad (18)$$

Second Cycle (c-d)

Before one can commence to calculate $(d_2)_k$, one needs to determine the number of cycles that would cause the equivalent damage in each ply at the new load levels (c and d) due to the first cycle. Note that load level c < load level d. Thus $(n_e)_d$ is calculated in the following manner for each ply:

$$(d_1)_k = A[\frac{(n_e)_d}{N_d^{(1)}} + \frac{(n_e)_d}{N_d^{(2)}} + \frac{(n_e)_d}{N_d^{(6)}}]_k + B\{(\frac{(n_e)_d}{N_d^{(1)}})^2 + (\frac{(n_e)_d}{N_d^{(2)}})^2 + (\frac{(n_e)_d}{N_d^{(6)}})^2\}_k \qquad (19)$$

Note that $N_d^{(1)}$, $N_d^{(2)}$ and $N_d^{(6)}$ are based on the k-th ply principal stresses $(\sigma_1, \sigma_2, \sigma_6)_k$ associated with the end level stress 'σ_d'. Solving for $(n_e)_d$, one can now proceed to calculate the total damage cumulated up to the end of cycle #2. Thus for the k-th ply,

$$(d_2)_k = \{A[\frac{(n_e)_d + 1}{N_d^{(1)}} + \frac{(n_e)_d + 1}{N_d^{(2)}} + \frac{(n_e)_d + 1}{N_d^{(6)}}] + B[(\frac{(n_e)_d + 1}{N_d^{(1)}})^2 + (\frac{(n_e)_d + 1}{N_d^{(2)}})^2 + (\frac{(n_e)_d + 1}{N_d^{(6)}})^2]\}_k \qquad (20)$$

At the completion of each cycle, one must check to determine if $(d_i)_k$ for each ply > 1. When this occurs, the ply is said to hae failed and revised stiffness matrices are calculated. This can be done by removing the matrix material in the ply and then setting the total damage equal to the fibre damage component. Subsequently, damage is then calculated for the partially failed ply until the fibre damage > 1, at which time the complete ply is removed from the laminate.

This procedure is repeated until all plies fail.

III. FALSTAFF - Spectrum Fatigue Loading

For the purpose of the present work, a stress (load) history comprised of repeated blocks of 200 flights was selected. Each of these blocks consisted of 35,966 end levels or 'reversals'. The number of end levels as well as the loading sequence, such as ground-air-ground, taxi, gust, etc., varied from flight to flight. The particular computer code utilized to drive the MTS servo-hydraulic test facility* in this experimental program is described in Ref. 10. The FALSTAFF spectrum is described by 32 end levels with zero stress corresponding to the value of 7.5269, as illustrated in Fig. 4. End levels are assigned real life stress values once the peak stress is set at #32. In this study involving only compressive loading, end levels were selected > 7.5 to prevent load reversal. This was accomplished by clipping the actual FALSTAFF spectrum (Fig. 5) at end level #8 and filtering the acceptable response at end level #19, i.e. only those stress cycles with initial end point stress levels > 8 and final end point stress levels > 19 were applied to the test sample. This is illustrated in Fig. 5. Clipping and filtering

*Located at the National Aeronautical Est., National Research Council of Canada, Ottawa, Canada.

a full range spectrum results in considerable time savings by eliminating the low stress cycles. Thus it was found that the number of end points in a block of loading was reduced from 35,966 to 4,542 — a significant time saving! The flow chart in Fig. 6 outlines the calculation procedures used to estimate the spectrum fatigue life. Either the Owen's or tensor polynomial damage criteria can be utilized, details of which have been presented earlier.

At the beginning, the computer code accepts data including the laminate configuration, material stiffness properties, material fatigue strength properties, stress at maximum end level 32 and FALSTAFF clipping and filtering levels, and the filtered and clipped spectrum.

The code continues with the calculation of stiffness matrices Q and $\bar{Q}$ and the allocation of stresses to end levels from 8 to 32. A complete stress analysis utilizing stress levels from 8 to 32 gives the stresses for every ply within the laminate and finally stores it in an L×K×I matrix, where L = number of end levels (i.e., levels from 8 to 32; for our case, L = 25; K = direction, i.e., 1, 2 and 6. I = ply number, 1-8 for our case).

IV. Experimental Programme

Static Strengths and Fatigue Functions

An experimental programme was undertaken to evaluate the static strengths and fatigue functions for a graphite/epoxy prepreg material — Hercules Magnamite AS4/3501-6. This necessitated generating a set of σ-N curves corresponding to the fatigue functional quantities, X, X', Y, Y' and S required for the implementation of the failure theory. This was accomplished by conducting a series of static and fatigue tests on unidirectional laminates (0° and 90° construction).

Static tests were performed on a Tinius-Olsen testing machine, while a Sonntag SF-1-U universal fatigue machine (operating at 30 Hz, which produced a sample temperature rise of only 10°F) was used in the fatigue portion of the experiment.

Standard tension coupons were employed to obtain the X and Y strength values whereas compression data (X', Y') were obtained from sandwich beam samples subject to four-point bending. Circular cylindrical tubes were used to investigate shear strength (S) by means of a special torsion fixture attached to the Sonntag machine. Details of the test samples and experimental procedures can be obtained from Ref. 11.

Fatigue data for constant amplitude cyclic loading at 30 Hz have been plotted in Fig. 7 as a function of N. The corresponding values of the fatigue strength parameters used in Eq. (7) are summarized in Table 1 based on a 'best fit' curve through the data.

To ascertain the capability of the fatigue failure models to predict the life of an arbitrary laminate, compression fatigue tests (at R = 20) were also conducted at 30 Hz on a quasi-isotropic laminate of $(0°, \pm45°, 90°)_s$ construction, the results of which are plotted in Fig. 8.

Spectrum Fatigue Loading Tests

Figure 9 contains a view of the four-point bending fixture with a sandwich beam sample mounted in a MTS fatigue machine. This configuration provides essentially 'pure' compression loading on the graphite/epoxy face sheet, details of which can be found in Refs. 11 and 12. Note that 'pure tension' can also be obtained simply by inverting the sandwich beam. However, in this investigation, interest was centered on the compressive fatigue life of the $(0, \pm45, 90)_s$ laminate subject to the modified FALSTAFF spectrum loading profile previously described.

Figure 10 presents a view of the test fixture with a sample in the MTS load frame which is driven by the control system shown. Sensitivity was increased by using a 5.5 Kip load cell and the gain was set to optimum permissible levels. The test was performed under load control to ensure that the loads applied would not be affected when the sample stiffness changed. Constant loading rate was selected over constant amplitude. The advantage of the former was that the important peak cycles (i.e., 19 ≤ end level ≤ 32) would be attained while the prescribed filtered lower cycles (8 ≤ end level < 19) would be speeded up, thus reducing test time. Emergency limit stops were set in both load and stroke control to terminate the test when the specimen failed. A summary of the test results for 15 beams is listed in Table 2. The 'run-out' number of blocks was set to 150, which represents the design condition of 30,000 flight-hours for fighter aircraft.

V. Application of Fatigue Failure Analysis to Laminates

To demonstrate the application of the fatigue failure equations using the data presented, let us consider the quasi-isotropic laminate $(0°, \pm45°, 90°)_s$ of graphite/epoxy material. Lamina failure envelopes were constructed for static loading and as a function of the number of load cycles N, as shown in Figs. 11 to 13, based on the fatigue function data in Table 1.

These results were then used in both the tensor polynomial and a modified Owen's formulation (i.e., B = 0, A = 1 due to lack of material property data) to predict the constant amplitude compression fatigue life for the above laminate (see Fig. 8). Two sets of curves are shown, based on the fatigue functions corresponding to the 'best fit' and 'minimum' values derived from the available test data (see Table 1). Very little difference exists between the two model predictions, and it is evident that the 'minimum' value curves show the best agreement with the limited test data shown.

The tensor polynomial formulation was also used to predict tension fatigue life for the same laminate, as shown in Fig. 14. It is interesting to compare the tension and compression results.

Whereas overall compression fatigue was initiated at first ply failure, the ultimate tension fatigue strength followed after first and second ply failures.

Finally, both models were used to estimate the fatigue life of the $(0, \pm 45, 90)_S$ laminate under compression loading using the FALSTAFF spectrum. These results are compared with the test data in Fig. 15, again including predictions based on 'best fit' and 'minimum' fatigue function curves. Reasonable agreement with test results can be seen over most of the block range, although some discrepancy exists near the run-out limit.

VI. Conclusions

A computational procedure has been developed for estimating the fatigue life of laminates subject to constant amplitude and spectrum loading. Various damage models have been formulated based on the Miner-Palmgren and Owen's cumulative damage relations, including a form of the tensor polynomial whose strength parameters were determined as fatigue functions. It has been demonstrated that these models yield reasonably good fatigue life predictions using a clipped and filtered form of the FALSTAFF spectrum which was constrained to provide compressive loading. However, although test data provided correlation with these model predictions over most of the lifetime range studied, serious discrepancies did occur near the 'run-out' design lifetime ($\geqslant$ 100 blocks). This aspect requires further attention both experimentally, to confirm 'run-out', and analytically, to improve strength reduction estimates at the lower peak stress levels. In this latter case, work needs to be done particularly on the evaluation of R effects since insufficient evidence exists confirming the validity of the 'correction' procedure employed. Finally it should be noted that the tensor polynomial formulation presented in this report does provide a means for estimating laminate residual strength. Clearly much more experimental data are required to substantiate the model proposed.

References

1. G. M. Van Dijk, J. B. deJonge, "Introduction to a Fighter Aircraft Loading Standard for Fatigue Evaluation 'FALSTAFF'", Part I of "A Fighter Aircraft Loading Standard for Fatigue Evaluation" published in Problems With Fatigue in Aircraft: Proc. 8th Symposium and Colloquium, Lausanne, Switzerland, June 1975.

2. M. Hück and W. Schütz, "Generating the FALSTAFF Load History by Digital Minicomputers", Part II: J. Branger, "Influence of Differences Between Genuine and Generated Load Sequences of Ground-Load Variations of FALSTAFF-Like Programs", Part III; D. Schutz and H. Lowak, "The Application of the Standardized Test Program for the Fatigue Life Estimation of Fighter Wing Components", Part IV; Proc. 8th Symposium and Colloquium, Lausanne, Switzerland, June 1975.

3. Z. Hashin and A. Rotem, "A Fatigue Failure Criterion for Fiber Reinforced Materials", J. Composite Materials, Vol. 7, Oct. 1973.

4. A. Rotem and Z. Hashin, "Fatigue Failure of Angle Ply Laminates", AIAA J., Vol. 14, July 1976.

5. A. Rotem, "Fatigue Failure of Multidirectional Laminate", AIAA J., Vol. 17, March 1979.

6. D. F. Sims and V. H. Brogdon, "A Comparison of the Fatigue Behavior of Composites Under Different Loading Modes", Proc. ASTM Symposium on "Fatigue of Filamentary Composite Materials", Denver, Colorado, Nov. 1976.

7. S. W. Tsai and E. M. Wu, "A General Theory of Strength for Anisotroic Materials", J. Composite Materials, Vol. 5, 1971.

8. R. C. Tennyson, D. MacDonald and A. P. Nanyaro, "Evaluation of the Tensor Polynomial Failure Criterion for Composite Materials", J. Composite Materials, Vol. 12, 1978, p. 63.

9. M. J. Owen, "Fatigue Damage in Glass-Fiber-Reinforced Plastics", Vol. 5, Fracture and Fatigue, Academic Press, 1974, p. 313.

10. M. D. Raizenne, "FALSTF: A Computer Program to Run FALSTAFF on MTS Automated Servo-hydraulic Test Equipment", NRC Report LTR-ST-1502, June 1984.

11. R. C. Tennyson, J. S. Hansen, G. E. Mabson, G. E. Wharram and G. Elliott, "Development of General Failure Model to Predict Static Failure and Fatigue Life of Unflawed and Flawed Graphite/Epoxy Laminates", Canadian Dept. National Defence Report, Contract No. 06SB.97708-2-2721, Defence Research Est. Pacific, Victoria, B.C., Canada, Dec. 1983.

12. G. E. Mabson, J. S. Hansen, R. C. Tennyson and G. E. Wharram, "Investigation of the Strength and Fatigue Properties of Graphite/Epoxy Laminates", Canadian Dept. National Defence, Defence Research Est. Pacific Contractor's Report Series 83-32, June 1982.

Table 1

Summary of Principal Material Properties of AS4/3501-6 Graphite/Epoxy

(Ambient Conditions)

Loading Mode	Best Fit Values				Minimum	
	R	σ_S (ksi)	σ_e (ksi)	β	σ_S (ksi)	σ_e (ksi)
X	.05	324.5	160.0	-.1044	310.5	113.0
X'	20	233.4	115.0	-.0952	229.1	66.3
Y	.05	7.767	4.12	-.3084	6.76	3.64
Y'	20	44.9	20.0	-.1659	42.6	12.85
S	.05	12.6	6.89	-.2257	11.5	6.51

Table 2

Summary of Experimental Results for AS4/3501-6 Graphite/Epoxy Laminates

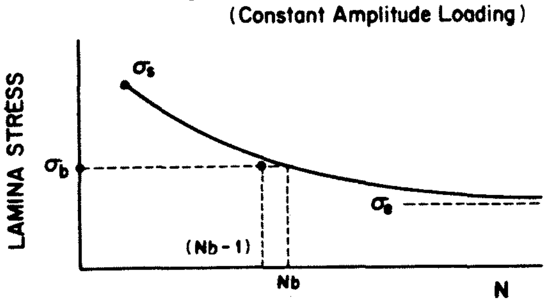
$(0, \pm 45, 90)_s$

Subject to FALSTAFF Compressive Spectrum* Loading

Beam Sample No.	Applied Stress (ksi)	Blocks	Flights	End Levels	Failure Blocks**
16	75	22	32	397	22.16
03	75	15	32	161	15.16
07	60	120	173	91	120.87
09	65	77	173	91	77.87
15	65	67	173	91	67.87
13	70	40	32	397	40.16
01	75	12	32	397	12.16
04	70	57	173	91	57.87
11	70	62	173	91	62.87
10	60	>150			>150
8	60	125	32	397	125.16
5	70	43	173	91	43.87
6	60	102	32	397	102.16
18	65	105	32	397	105.16
2	65	89	32	397	89.16

*Clipped at End Level 8 and filtered at End Level 19
**End levels not included

Fig. 1 Lamina Fatigue Curve (Constant Amplitude Loading)



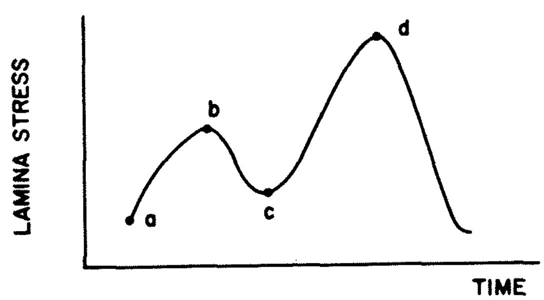
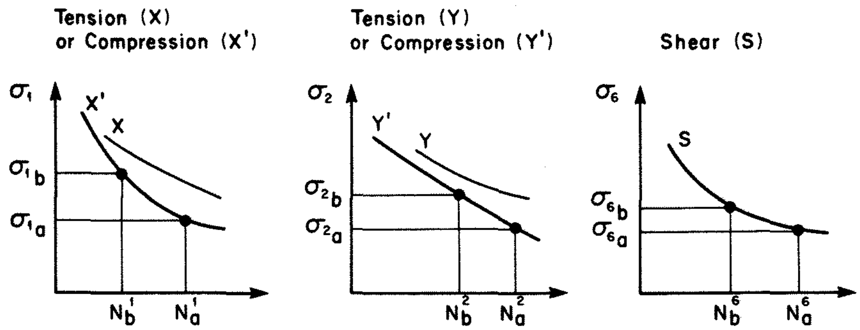
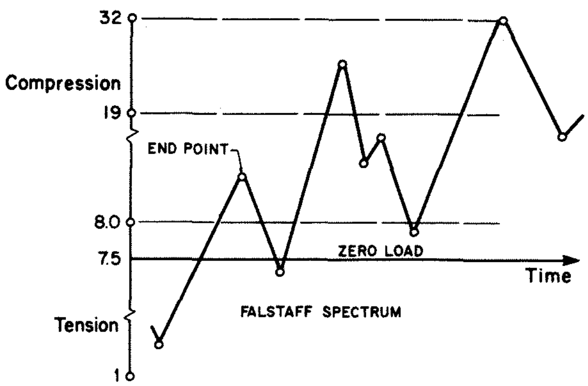


Fig. 2 Spectrum Load Applied to Lamina

Fig. 3 Stresses / Cycles to Failure for j th Ply





Basic Spectrum Block of Loading :

One Block = 200 flights
= 200 hrs
= 35,966 End Points

Design Condition : 30,000 hrs
= 150 Blocks (repeated)
x 200 hrs / Block

Fig. 4

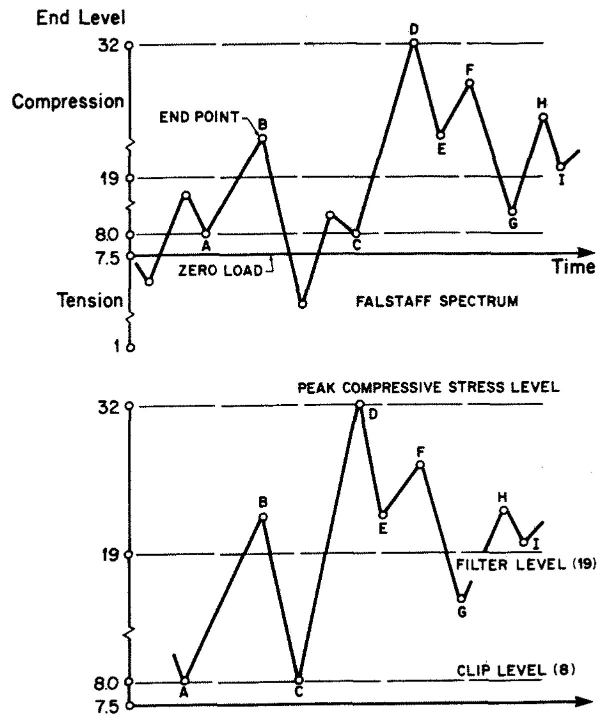


Fig. 5 Clipped and Filtered Compressive Load Test Spectrum
(Modified FALSTAFF)

FIG. 6 FLOW CHART FOR CALCULATING FATIGUE DAMAGE USING FALSTAFF SPECTRUM LOADING

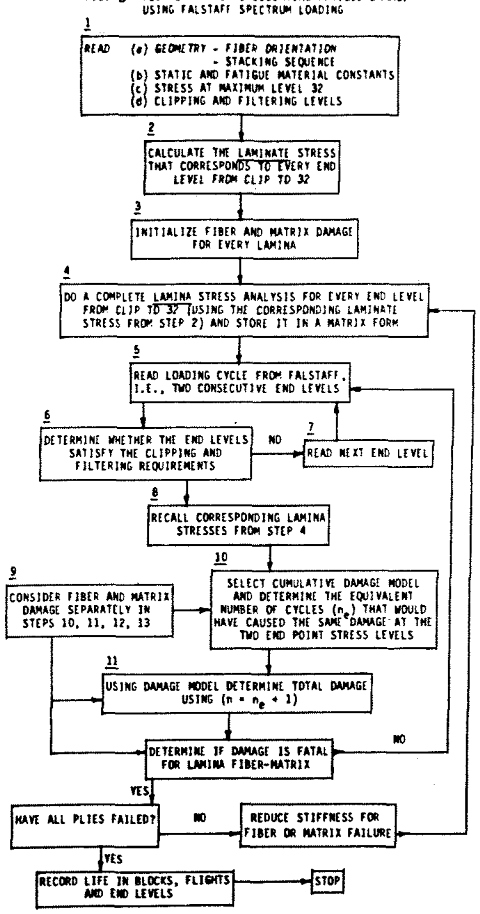


Fig. 7b

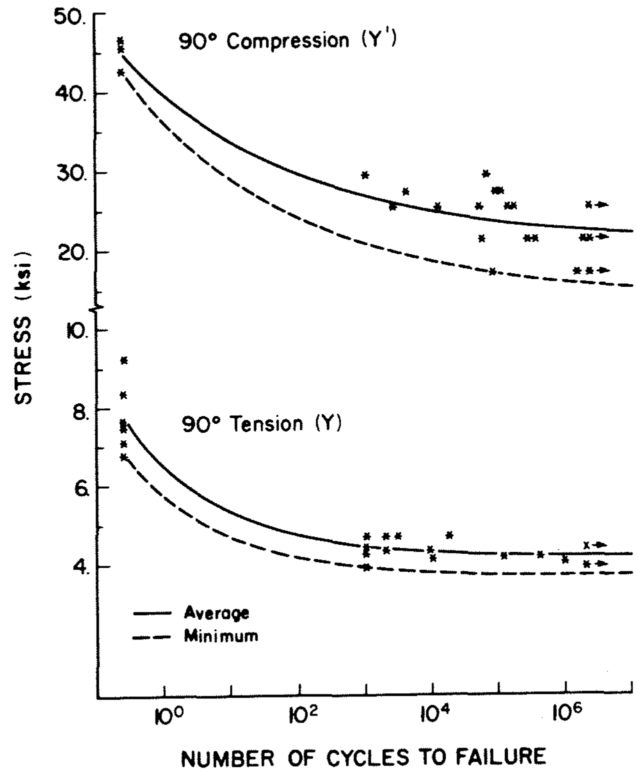


Fig. 7a Principal Material Property Fatigue Curves for Graphite/Epoxy AS4 3501-6

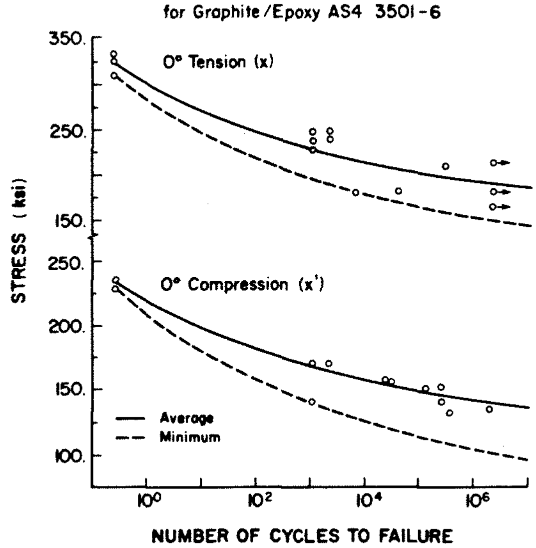


Fig. 7c

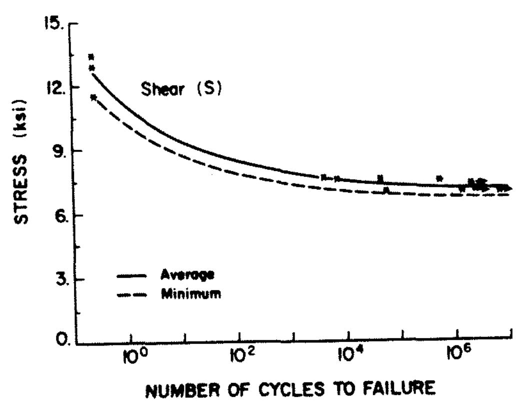
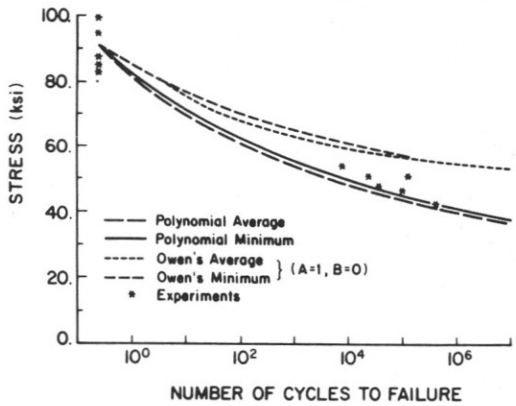


Fig. 8 Constant Amplitude Compressive Fatigue Life of Laminates $(0, \pm 45, 90)_s$ (AS4 / 3501-6)



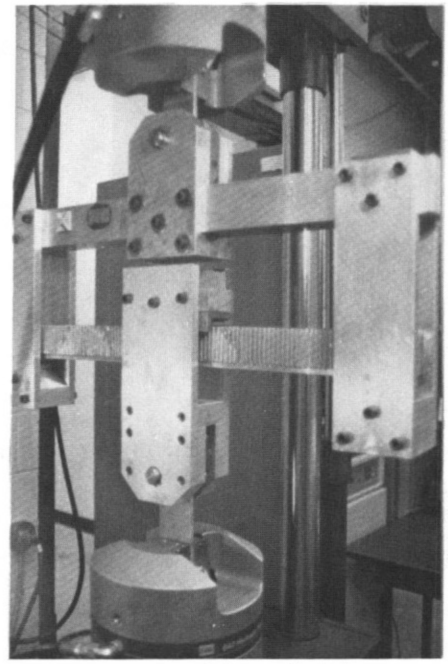


Fig. 9 View of Four-Point Bending Fixture with Sandwich Beam Test Sample

Fig. 10 View of Test Fixture and Beam Sample in MTS Fatigue Machine Providing FALSTAFF Spectrum Loading

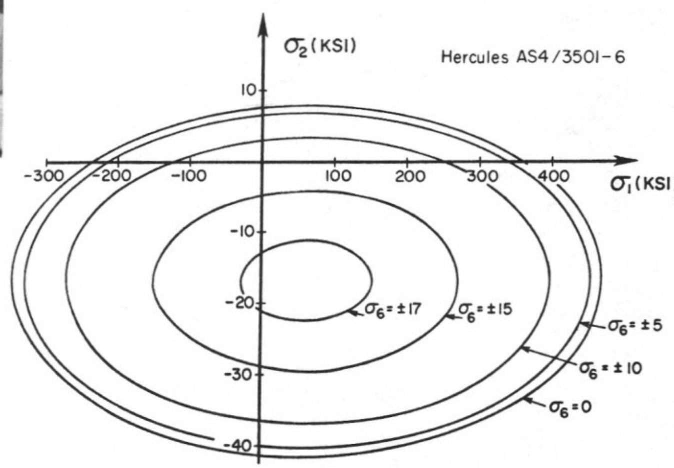


Fig. 11 Lamina Failure Envelope – Static

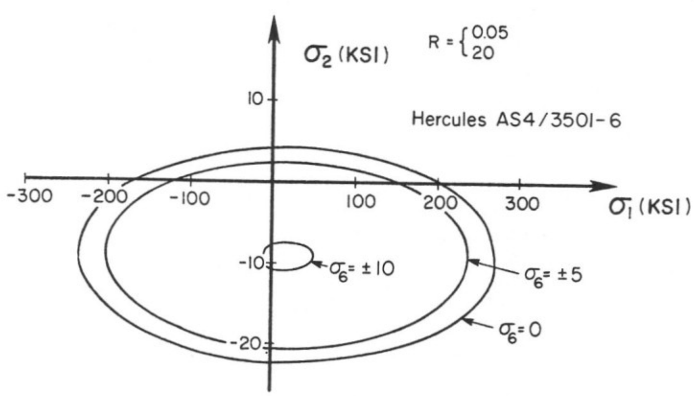


Fig. 12 Lamina Failure Envelope – 10^4 Cycles

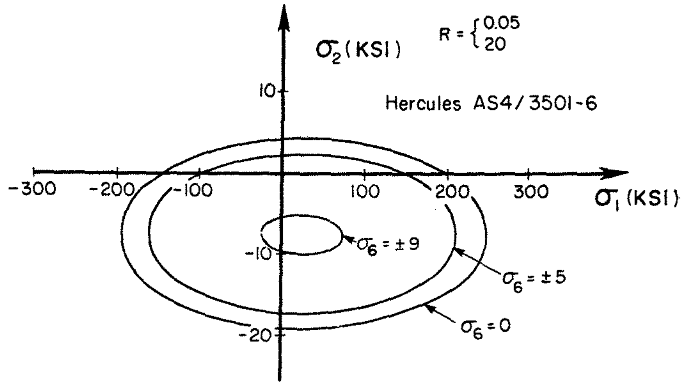


Fig. 13 Lamina Failure Envelope – 10^6 Cycles

Fig. 14 TENSILE FATIGUE CURVES $(0, \pm 45, 90)_s$

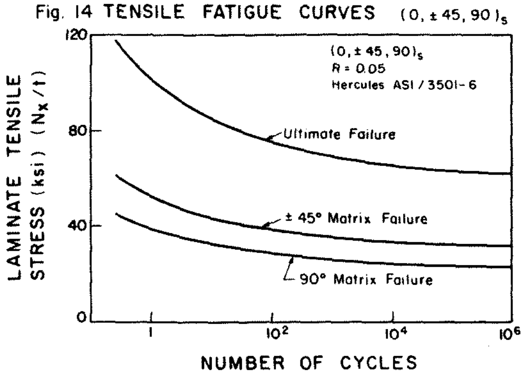


Fig.15 FALSTAFF Spectrum Life Predictions Compared with Test Data for AS4/3501-6 Graphite/Epoxy Laminates $(0, \pm 45, 90)$ s under Compressive Loading