DYNAMIC CONTROL ASPECTS OF DEVELOPMENT OF THREE SHAFT TURBOPROP ENGINE

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Abstract
Three shaft conseption of turboprop make subsequent demand step up of economical power plant especially for commuter and executive aircraft. Significantly higher claims are laid on engine control systems in a field of dynamic parameters and characteristics. Some difficulties are necessary coped with a respect of the demand on engine control in a stage design of proper engine.
The aim of this paper to present some results from investigation dynamic control parameters and characteristics by more perfect, but always aproximate calculation and to call on its reverse formulas for acquiring of necessary bases for turboprop compromises, including control field, too.

List of Symbols and Subscripts
/General Nomenclature/

Q - quantity delivered
n - revolutions
c_P - coefficient of propeller power
α - angular acceleration
I - polar moment of inertia
W - specific work
p - pressure
T - temperature
π - calculated air compression, expansion of gases
η - efficiency factor
P - propeller shaft power
$\varkappa, \varkappa'$ - ratio of specific heat capacities of air, gases
δx - ratio of relative differences and absolute value of x-parameter $/\equiv \bar{x}/$
Δ - relative difference /increment/
t - time
${}^{l}k_{m}$ - dimensionless boosting $\underline{l}$ parameter with respect $\underline{m}$ parameter
τ - time constant, dynamic lag, relative temperature gradient
s - the Laplacian operator
C/≡K/,A,B - coefficients of reciprocal influences

K - compressor
T - turbine
V - propeller
SK - combustion chamber
VST - inlet
VÝST - exhaust
R - reduction gear
RS - control system
2R,3R - concerning two, three shaft turboprop engine
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f - fuel
c - total conditions
r - reduction on ISA
d - differentiating
o - value on curve of equilibrium modes of operation
Σ - summary

$\Rightarrow$ ensure from
$\uparrow$ - value increase
$\downarrow$ - value recede

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1.0 Introduction

Applications of turboprops with free power turbine aimed at asserting three shaft design concept of gas turbine engines in one evolutionary direction. However, three shaft engine has some essential distincts from two shaft design concept if we rate it as controlled and regulated plant. There are the most important differences in a field of dynamic properties, which are limiting for all attainable control modes and actions at operating failures. Meaningfully increasing qualities of many items must be reached, on basic requirements,e.g. minimum weight, specific fuel consumption, final and operating cost, with maximum reliability/maintainability, etc. - /L1/, but good static and dynamic control properties , including easy starting, available and effective thrust reversal, too - /L2/. For engine designeres many of requirements are conflict and they must arrive at best compromise. Some items are more importance than others, depending on the three shaft design concept and its application, as for commuter, utility and executive aircrafts.

Essentially in connection what was writen, is necessary to gain the resultant dynamic control characteristics and properties not only the control system, but with "the contribution" of the engine as controlled and regulated plant. The engine could have certain dynamic control characteristics and properties act upon closely connected with
- non interaction
- invariation

-t-optimal control - /L3/ - with high figure of merit - minimum overshooting, etc. As a matter of course the control system is designed for the engine and no opposite. But is necessary the control standpoint into the engine compromises included. For this activity and largely management the minimal data sets are needfull and their measuring is needed.

2.0 Aproximate Matrix Calculation of Dynamic Characteristics

Basic block diagrams /fig. 2. and 3./ are shown principal differents between three and to the present time two shaft turboprop operated.

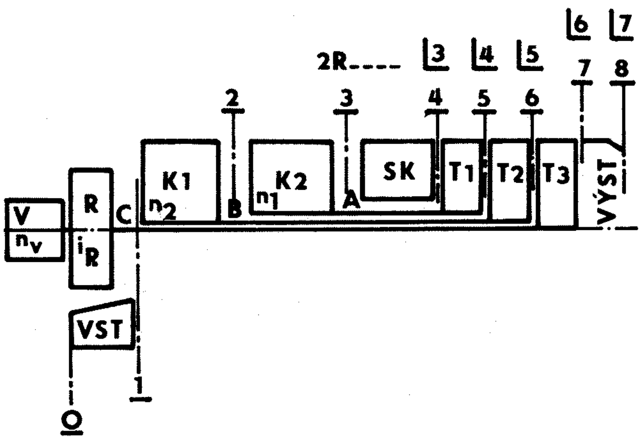

Fig. 1 Schematical Diagram of Three Shaft Turboprop

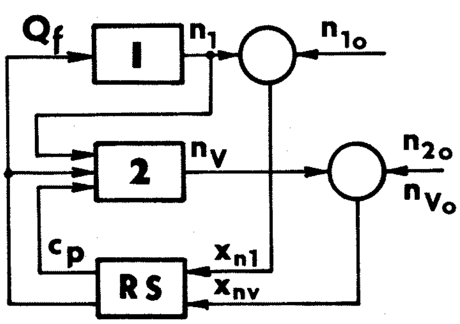

Fig. 2 Basic Block Diagram of 2R Turboprop Control

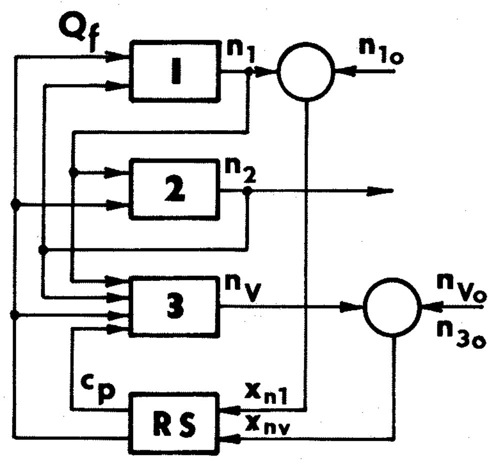

Fig. 3 Basic Block Diagram of 3R Turboprop Control

If the same laws of control by comparison: $Q_f \rightarrow n_1$ and $c_p \rightarrow n_V$, resultant rectangular matrix of linear dynamic coefficients inclusive of three shaft coefficient system can be written as

${}^{3R}\mathbf{M}$ **TABLET**

$\emptyset \backslash \emptyset$	δQ_f	δc_p	δn_1	δn_2	δn_V	δp_{1c}	δT_{1c}
$\delta \pi_{K1c}$	$\emptyset$	$\emptyset$	$\emptyset$	${}^{\pi_{K1c}}k_{n_2}$	$\emptyset$	$\emptyset$	${}^{\pi_{K1c}}k_{T_{1c}}$
δQ_{K1}	$\emptyset$	$\emptyset$	$\emptyset$	${}^{Q_{K1}}k_{n_2}$	$\emptyset$	1	${}^{Q_{K1}}k_{T_{1c}}$
$\delta \pi_{K2c}$	$\emptyset$	$\emptyset$	${}^{\pi_{K2c}}k_{n_1}$	${}^{\pi_{K2c}}k^{*}_{n_2}$	$\emptyset$	$\emptyset$	${}^{\pi_{K2c}}k_{T_{1c}}$
δp_{2c}	$\emptyset$	$\emptyset$	$\emptyset$	${}^{p_{2c}}k^{*}_{n_2}$	$\emptyset$	1	${}^{p_{2c}}k_{T_{1c}}$
δT_{2c}	$\emptyset$	$\emptyset$	$\emptyset$	${}^{T_{2c}}k_{n_2}$	$\emptyset$	$\emptyset$	${}^{T_{2c}}k_{T_{1c}}$
δp_{3c}	$\emptyset$	$\emptyset$	${}^{p_{3c}}k_{n_1}$	${}^{p_{3c}}k^{*}_{n_2}$	$\emptyset$	1	${}^{p_{3c}}k_{T_{1c}}$
δT_{3c}	$\emptyset$	$\emptyset$	${}^{T_{3c}}k_{n_1}$	${}^{T_{3c}}k^{*}_{n_2}$	$\emptyset$	$\emptyset$	${}^{T_{3c}}k_{T_{1c}}$
δT_{4c}	${}^{T_{4c}}k_{Q_f}$	$\emptyset$	${}^{T_{4c}}k_{n_1}$	${}^{T_{4c}}k^{*}_{n_2}$	$\emptyset$	${}^{T_{4c}}k_{p_{1c}}$	${}^{T_{4c}}k_{T_{1c}}$
$\delta \pi_{T3c}$	$\emptyset$	$\emptyset$	${}^{\pi_{T3c}}k^{*}_{n_1}$	${}^{\pi_{T3c}}k^{*}_{n_2}$	$\emptyset$	$\emptyset$	${}^{\pi_{T3c}}k^{*}_{T_{1c}}$
δT_{7c}	${}^{T_{7c}}k_{Q_f}$	$\emptyset$	${}^{T_{7c}}k^{*}_{n_1}$	${}^{T_{7c}}k^{*}_{n_2}$	$\emptyset$	${}^{T_{7c}}k^{*}_{p_{1c}}$	${}^{T_{7c}}k^{*}_{T_{1c}}$
α_1	${}^{\alpha_1}k_{Q_f}$	$\emptyset$	${}^{\alpha_1}k_{n_1}$	${}^{\alpha_1}k^{*}_{n_2}$	$\emptyset$	${}^{\alpha_1}k_{p_{1c}}$	${}^{\alpha_1}k_{T_{1c}}$
α_2	${}^{\alpha_2}k_{Q_f}$	$\emptyset$	${}^{\alpha_2}k^{*}_{n_1}$	${}^{\alpha_2}k^{*}_{n_2}$	$\emptyset$	${}^{\alpha_2}k^{*}_{p_{1c}}$	${}^{\alpha_2}k^{*}_{T_{1c}}$
α_V	${}^{\alpha_V}k_{Q_f}$	${}^{\alpha_V}k_{c_p}$	${}^{\alpha_V}k_{n_1}$	${}^{\alpha_V}k^{*}_{n_2}$	${}^{\alpha_V}k_{n_V}$	${}^{\alpha_V}k_{p_{1c}}$	${}^{\alpha_V}k_{T_{1c}}$
δn_1	${}^{n_1}k_{Q_f}$	$\emptyset$	${}^{n_1}k^{*}_{n_1}$	${}^{n_1}k^{*}_{n_2}$	$\emptyset$	${}^{n_1}k^{*}_{p_{1c}}$	${}^{n_1}k^{*}_{T_{1c}}$

$$
\begin{array}{c|ccccccc}
\delta n_2 & {}^{n_2}k^{*}_{Q_f} & \varnothing & {}^{n_2}k^{*}_{n_1} & {}^{n_2}k^{*}_{n_2} & \varnothing & {}^{n_2}k^{*}_{p_{1c}} & {}^{n_2}k^{*}_{T_{1c}} \\
\hline
\delta n_v & {}^{n_v}k_{Q_f} & {}^{n_v}k_{C_P} & {}^{n_v}k_{n_1} & {}^{n_v}k^{*}_{n_2} & {}^{n_v}k_{n_v} & {}^{n_v}k_{p_{1c}} & {}^{n_v}k_{T_{1c}}
\end{array}
\qquad / \ 2.1 \ /
$$

^{D}**M**-requisiting elements $^{l}k_{m}$

Rectangular matrix ^{3R}M covers up ^{2R}M of two shaft turboprop. Matrix form as the result of the conjecture dynamic characteristics identification, where put all general knowledges from /L4/ and especially /L5/. Start equations have frequented forms:

$$\alpha_{1(2)} = \frac{30}{\pi}\cdot\frac{1}{I_{1(2)}}\cdot\frac{[Q_K]_o\cdot[W_{K2(1)c}]_o}{[n_{1(2)}]_o^2}\left[\delta W_{T1(2)c} - \delta W_{K2(1)c}\right] \qquad / \ 2.2 \ /$$

$$\alpha_{v} = \frac{30}{\pi}\cdot\frac{1}{I_{\Sigma v}}\cdot\frac{[P_v]_o}{[n_v]_o}\left(\delta W_{T3c} - \delta W_{vc}\right) \qquad / \ 2.3 \ /$$

$$\delta W_{T_{ic}} = \delta T_{jc} + K_{3i}\,\delta\pi_{T_{ic}}\ ; \quad i = 1,2,3; \quad j = 3,4,5 \qquad / \ 2.4 \ /$$

$$K_{3i} = \frac{\dfrac{\varkappa'}{\varkappa'-1}}{[\pi_{T_{ic}}]_o^{\frac{\varkappa'}{\varkappa'-1}} - 1} \qquad / \ 2.5 \ /$$

$$\delta W_{K2(1)c} = \delta T_{1(2)c} + A_1(B)_1\,\delta\pi_{K2(1)c} \qquad / \ 2.6 \ /$$

$$A_1(B)_1 = \frac{\dfrac{\varkappa-1}{\varkappa}\,[\pi_{K2(1)c}]_o^{\frac{\varkappa-1}{\varkappa}}}{[\pi_{K2(1)c}]_o^{\frac{\varkappa-1}{\varkappa}} - 1} \qquad / \ 2.7 \ /$$

But

$$\delta\gamma_{T_{ic}} = \delta\gamma_{K2(1)c} = \varnothing \qquad / \ 2.8 \ /$$

$$\delta W_{vc} = \delta P_v - \delta Q_K \qquad / \ 2.9 \ /$$

$$\delta P_v = \delta C_P + 3\delta n_v + \delta p_{1c} - \delta T_{1c} \qquad / \ 2.10 \ /$$

and low pressure compressor K1 is discribing:

$$\delta Q_{K1} = \frac{1}{K_{11}}\left[K_{11}\,\delta p_{1c} - 0{,}5(K_{11}+1)\delta T_{1c} - K_{12}\,\delta\pi_{K1c} + \delta n_2\right] \qquad / \ 2.11 \ /$$

$$\delta\pi_{K1c} = \frac{1}{B_1 K_{14}}\left[K_{13}\,\delta p_{1c} - 0{,}5(K_{13}+1)\delta T_{1c} - K_{13}\,\delta Q_{K1} + \delta n_2\right] \qquad / \ 2.12 \ /$$

$$\delta(\Delta T_{K1c}) = \frac{1}{K_{14}}\left\{K_{13}\,\delta p_{1c} + [K_{14} - 0{,}5(K_{13}+1)]\delta T_{1c} - K_{13}\,\delta Q_{K1} + \delta n_2\right\} \qquad / \ 2.13 \ /$$

$$\delta T_{2c} = \left(1 - \frac{0{,}5 B_2}{K_{14}}\right)\delta T_{1c} + \frac{B_2}{K_{14}}\,\delta n_2 \qquad / \ 2.14 \ /$$

Coefficients:

$$K_{11} = \frac{\Delta n_{2r}}{\Delta Q_{K1r}}\cdot\frac{[Q_{K1r}]_o}{[n_{2r}]_o} \qquad / \ 2.15 \ /$$

$$K_{12} = \frac{\Delta n_{2r}}{\Delta \pi_{K1c}}\cdot\frac{[\pi_{K1c}]_o}{[n_{2r}]_o} \qquad / \ 2.16 \ /$$

are rearranged from transformated function /fig. 4/

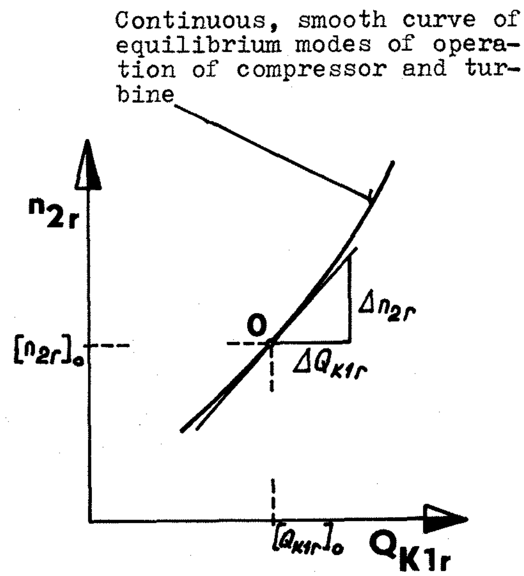

Fig. 4 Diagram $n_{2r} = n_{2r}\left(Q_{K1r}\right) - \pi_{K1c} = const.$

and

$$K_{13} = \left(K_{11}\right)_{\tau_{K1c} = const.} \qquad / \ 2.17 \ /$$

$$K_{14} = \frac{\Delta n_{2r}}{\Delta \tau_{K1c}}\cdot\frac{[\tau_{K1c}]_o}{[n_{2r}]_o} \qquad / \ 2.18 \ /$$

from

$$n_{2r} = n_{2r}\left(\tau_{K1c}\right) - Q_{K1r} = const. \qquad / \ 2.18a \ /$$

Continuos functions of high pressure compressor K2:

$$\delta Q_{K2} = \delta p_{1c} - 0{,}5\left[\left(1 - \frac{0{,}5 B_2}{K_{14}}\right)\left(1 + \frac{1}{K_{21}} - \frac{K_{22}}{A_1\cdot K_{21}\cdot K_{24}}\right) + \frac{1}{B_1\cdot K_{14}}\right]\delta T_{1c} + \frac{1}{K_{21}}\left(1 - \frac{K_{22}}{A_1\cdot K_{24}}\right)\delta n_1 + \frac{1}{K_{14}}\left[\frac{1}{B_1} - 0{,}5 B_2\cdot\left(1 + \frac{1}{K_{21}} - \frac{K_{22}}{A_1\cdot K_{21}\cdot K_{24}}\right)\right]\delta n_2 \qquad / \ 2.19 \ /$$

$$\delta\pi_{K2c} = -\frac{0{,}5}{A_1\cdot K_{24}}\left(1 - \frac{0{,}5 B_2}{K_{14}}\right)\delta T_{1c} + \frac{1}{A_1\cdot K_{24}}\,\delta n_1 - \frac{0{,}5\cdot B_2}{A_1\cdot K_{14}\cdot K_{24}} \qquad / \ 2.20 \ /$$

$$\delta T_{1c} = \left[\left(1 - \frac{0{,}5 B_2}{K_{14}}\right)\left(1 - \frac{0{,}5 A_2}{K_{24}}\right)\delta T_{1c} + \frac{A_2}{K_{24}}\delta n_1 + \frac{B_2}{K_{14}}\cdot\right.$$

$$\left.\cdot\left(1 - \frac{0{,}5 A_2}{K_{24}}\right)\delta n_2\right] \qquad / \ 2.21 \ /$$

$$A_{,}(B)_2 = \frac{[\Delta T_{K2,(1)c}]_o}{[T_{2,(3)c}]_o} \qquad / \ 2.22 \ /$$

The functions, especially defined from / 2.9 / to / 2.21 / meant higher effect into proper identification. Matrix form of the results allows to direct physical interpretation. Each element of ^{3R}M matrix meants concrete boosting of Ø column by Ø row parameters.

If dynamic control characteristics of two shaft turboprop are exspressed in the same way by laws of control equally and respecting fig. 1, the change of three shaft characteristics against two shaft ones is determined $^{\mathcal{D}}M$ - difference matrix from / 2.1 /

$$^{\mathcal{D}}M = {}^{3R}M - {}^{2R}M \qquad / \ 2.23 \ /$$

Unzero elements are fully reserved |forth| column δn_2. From point of control it meants, that δn_2 influences all δ - parameters determined in Ø column. For this case, central shaft system exites very meaningful failure effectes, unfavourable in some modes. In the difference matrix $^{\mathcal{D}}M$ the existence of unzero elements apart from |forth| column reflects aggravated conditions for unchanging of control curcuits if the external failures are affecting /in δp_{1c} and δT_{1c} concentrated/ and removed far more from total self control /government/ by control curcuits of the same transmission responses as ones for two shaft turboprop.

3.0 Comparison of Two and Three Shaft Turboprop Properties

Both design of turboprops represent two and three parameter base system, where the fact the laws of control and regulation are identical. Connecting with the control system, which is securing non interaction and invariation on fig. 5, total invariation of both design is obviously ensured by conditions:

$$^{2R}R^* = \left(^{2R}U\right)^{-1} \cdot {}^{2R}Z \qquad / \ 3.1 \ /$$

$$^{3R}R^* = \left(^{3R}U\right)^{-1} \cdot {}^{3R}Z \qquad / \ 3.2 \ /$$

Conditions for total non interaction: product

$^{2R}U \cdot {}^{2R}R$ and $^{3R}U \cdot {}^{3R}R$

must be "clear" diagonal matrices. After transformation of ^{2R}M and ^{3R}M matrices to

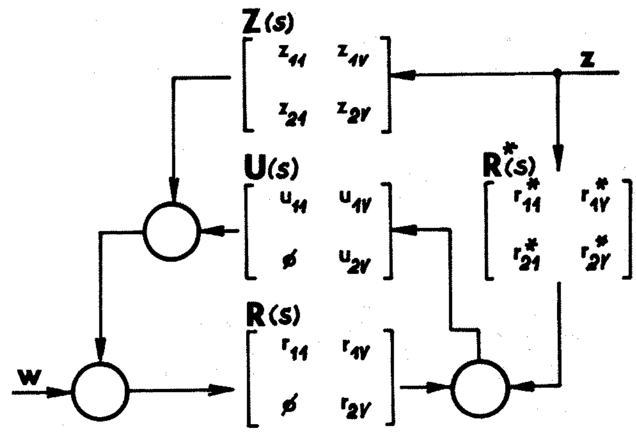

Fig. 5 Common Matrix Block Diagram of 2R and 3R Turboprop Control

transfer express of elements and filling total invariation and non interaction for two shaft turboprop:

$$^{2R}\mathbf{Z} = \begin{bmatrix} -\dfrac{^{n_1}k_{p_{1c}}}{\tau_1 s + 1} & -\dfrac{^{n_1}k_{\Sigma p_{1c}}(\tau_{dp_{1c}} s + 1)}{^{x}\tau_2^2 s^2 + {}^{x}\tau_1 s + 1} \\ \dfrac{^{n_1}k_{T_{1c}}}{\tau_1 s + 1} & -\dfrac{^{n_V}k_{\Sigma T_{1c}}(\tau_{dT_{1c}} s + 1)}{^{x}\tau_2^2 s^2 + {}^{x}\tau_1 s + 1} \end{bmatrix} \qquad / \ 3.3 \ /$$

$$^{2R}\mathbf{U} = \begin{bmatrix} \dfrac{^{n_1}k_{Qf}}{\tau_1 s + 1} & \dfrac{^{n_V}k_{\Sigma Qf}(\tau_{dQf} s + 1)}{^{x}\tau_2^2 s^2 + {}^{x}\tau_1 s + 1} \\ \phi & -\dfrac{^{n_V}k_{cp}}{\tau_V s + 1} \end{bmatrix} \qquad / \ 3.4 \ /$$

$$^{2R}\mathbf{R} = \begin{bmatrix} \dfrac{\tau_1 s + 1}{^{n_1}k_{Qf}} & \dfrac{^{n_V}k_{\Sigma Qf}(\tau_{dQf} s + 1)}{^{n_1}k_{Qf} \cdot {}^{n_V}k_{c\varphi}} \\ \phi & -\dfrac{\tau_V s + 1}{^{n_V}k_{c\varphi}} \end{bmatrix} \qquad / \ 3.5 \ /$$

$$^{2R}\mathbf{R}^* = \left[\begin{matrix} -\dfrac{^{n_1}k_{p_{1c}}}{^{n_1}k_{Qf}} & \dfrac{^{n_V}k_{\Sigma p_{1c}}(\tau_{\Sigma p_{1c}} s + 1)}{^{n_V}k_{c\varphi} \cdot (\tau_1 s + 1)} \end{matrix} \right.$$

$$\begin{bmatrix} \dfrac{^{n_1}k_{T_{1c}}}{^{n_1}k_{Q_f}} & \dfrac{^{n_v}k_{\Sigma T_{1c}}(\tau_{\Sigma T_{1c}} s+1)}{^{n_v}k_{C_p}\cdot(\tau_1 s+1)} \end{bmatrix} \qquad / \ 3.6 \ /$$

and for three shaft turboprop:

$$^{3R}\mathbf{U} = \begin{bmatrix} \dfrac{^{1}k_{\Sigma Q_f}(^{1}\tau_{dQ_f} s+1)}{^{x}\tau_2^2 s^2 + {}^{x}\tau_1 s+1} & \dfrac{^{n_v}k_{\Sigma Q_f}(^{Q_f}\tau_2^2 s^2 + {}^{Q_f}\tau_1 s+1)}{(^{x}\tau_2^2 s^2 + {}^{x}\tau_1 s+1)(\tau_v s+1)} \\ \phi & -\dfrac{^{n_v}k_{C_p}}{\tau_v s+1} \end{bmatrix} \qquad / \ 3.7 \ /$$

$$^{3R}\mathbf{R} = \begin{bmatrix} \dfrac{^{x}\tau_2^2 s^2 + {}^{x}\tau_1 s+1}{^{1}k_{\Sigma Q_f}(^{1}\tau_{dQ_f} s+1)} & \dfrac{^{n_v}k_{\Sigma Q_f}(^{Q_f}\tau_2^2 s^2 + {}^{Q_f}\tau_1 s+1)}{^{n_v}k_{C_p}\cdot {}^{1}k_{\Sigma Q_f}(^{1}\tau_{dQ_f} s+1)} \\ \phi & -\dfrac{\tau_v s+1}{^{n_v}k_{C_p}} \end{bmatrix} \qquad / \ 3.8 \ /$$

$$^{3R}\mathbf{Z} = \begin{bmatrix} \dfrac{^{1}k_{\Sigma p_{1c}}(^{1}\tau_{dp_{1c}} s+1)}{^{x}\tau_2^2 s^2 + {}^{x}\tau_1 s+1} & \dfrac{^{n_v}k_{\Sigma p_{1c}}(^{p_{1c}}\tau_2^2 s^2 + {}^{p_{1c}}\tau_1 s+1)}{(^{x}\tau_2^2 s^2 + {}^{x}\tau_1 s+1)(\tau_v s+1)} \\ \dfrac{^{1}k_{\Sigma T_{1c}}(^{1}\tau_{dT_{1c}} s+1)}{^{x}\tau_2^2 s^2 + {}^{x}\tau_1 s+1} & \dfrac{^{n_v}k_{\Sigma T_{1c}}(^{T_{1c}}\tau_2^2 s^2 + {}^{T_{1c}}\tau_1 s+1)}{(^{x}\tau_2^2 s^2 + {}^{x}\tau_1 s+1)(\tau_v s+1)} \end{bmatrix} \qquad / \ 3.9 \ /$$

$$^{3R}\mathbf{R}^{*} = \begin{bmatrix} \dfrac{^{1}k_{\Sigma p_{1c}}(^{1}\tau_{dp_{1c}} s+1)}{^{1}k_{\Sigma Q_f}(^{1}\tau_{dQ_f} s+1)} & \dfrac{^{n_v}k_{\Sigma p_{1c}}(^{p_{1c}}\tau_2^2 s^2 + {}^{p_{1c}}\tau_1 s+1)}{^{n_v}k_{C_p}(^{x}\tau_2^2 s^2 + {}^{x}\tau_1 s+1)} \\ \dfrac{^{1}k_{\Sigma T_{1c}}(^{1}\tau_{dT_{1c}} s+1)}{^{1}k_{\Sigma Q_f}(^{1}\tau_{dQ_f} s+1)} & \dfrac{^{n_v}k_{\Sigma T_{1c}}(^{T_{1c}}\tau_2^2 s^2 + {}^{T_{1c}}\tau_1 s+1)}{^{n_v}k_{C_p}(^{x}\tau_2^2 s^2 + {}^{x}\tau_1 s+1)} \end{bmatrix} \qquad / \ 3.10 \ /$$

By comparison the elements of the transfer matrices $^{2R}Z/{}^{3R}Z$ and $^{2R}U/{}^{3R}U$ showing a higher qualitative exacting as three shaft from two shaft turboprop. By analogue comparison the transfer matrices $^{2R}R/{}^{3R}R$ and $^{2R}R^{*}/{}^{3R}R^{*}$ inure to a higher qualitative exacting on control and equalization curcuits. Concretely: generator of gases is the first order dynamic plant with the characteristic values τ_1 and $^{n_1}k_{Q_f}$. Free power turbine with propeller and mechanic transmission is also the first order dynamic plant with τ_v and $^{n_v}k_{C_p}$.

Deviation transfer element $^{2R}u_{2v}$:

$$^{2R}u_{2v} = \frac{^{n_v}k_{Q_f}(\tau_{dQ_f} s+1)}{^{x}\tau_2^2 s^2 + {}^{x}\tau_1 s+1} \qquad / \ 3.11 \ /$$

δp_{1c} and δT_{1c} influences are analogous upon both shaft revolutions. Non interaction control n_1 and n_v and elimination of influences Q_f upon n_v is already fully realized by PD type of transfer functions. Total dynamic invariation is managed only with proportional and for n_v PDT^{-1} couplings.

Three shaft turboprop position is meaningfully complicated, in pursuance of all transfer functions are systematically one order higher about. Theoretically non interaction control n_1 does not ensure PD, but by PD^2T^{-1} curcuits only. By PDT^{-1} control curcuits non interaction n_1 and n_v is not always absolute. For dynamic invariation - PDT^{-1}, let us say, PD^2T^2 curcuits are needed. It is all owing to increasing of one degree of control freedom. That is way, for realisation is important: the transfer functions, which are put upon places of the matrix elements $^{3R}r_{11}$, $^{3R}r_{1v}$ and $^{3R}r_{2v}$, closely corresponding with $^{3R}u_{11}$, $^{3R}u_{1v}$ and $^{3R}u_{2v}$.

4.0 Reverse Procedure of Calculation. Deriving of Advisable Trends and Values

Realisation of the transfer functions $^{3R}r_{11}$, $^{3R}r_{1v}$ and $^{3R}r_{2v}$ is simultaneously connected with t-optimal control. Problem t-optimal control is generally described f.e. in /L6/, aplicate papers are too numerous, but without counter control plant contribution. Not only in general, which are the possibilities in this field ?. Conventionally this question have not applicable solution. Considering, in every particular problem is always possible:

- the matrices ^{3R}Z, ^{3R}U, ^{3R}R and $^{3R}R^{*}$ by formulas for / 2.1 /, ..., / 3.7 / to / 3.10 / numerical appointed for full devide of operation modes
- to assess the advisable control trends, like f.e. for more perfect t-optimal control

$$^{1}\tau_{dQ_f},\ {}^{x}\tau_2,\ {}^{x}\tau_1,\ {}^{Q_f}\tau_2,\ {}^{Q_f}\tau_1,\ \tau_v \longrightarrow min. \qquad / \ 4.1 \ /$$

$$^{Q_f}\tau_2 \ll \tau_1 \qquad / \ 4.2 \ /$$

Because these coefficients are inured from the elements of the matrices ${}^{3R}M,\ {}^{D}M$:

$$ {}^{1}\tau_{dQ_f} = {}^{n_1}k_{Q_f}\,\tau_2 \left(1 - {}^{n_1}k_{n_2}\cdot {}^{n_2}k_{n_1}\right)^{-1} \longrightarrow \min. \qquad / \ 4.3 \ / $$

$$ {}^{x}\tau_2^{\,2} = \tau_1\,\tau_2 \left(1 - {}^{n_1}k_{n_2}\cdot {}^{n_2}k_{n_1}\right)^{-1} \longrightarrow \min. \qquad / \ 4.4 \ / $$

$$ {}^{x}\tau_1 = (\tau_1 + \tau_2)\left(1 - {}^{n_1}k_{n_2}\cdot {}^{n_2}k_{n_1}\right)^{-1} \longrightarrow \min. \qquad / \ 4.5 \ / $$

$$ {}^{Q_f}\tau_2^{\,2} = {}^{n_v}k_{Q_f}\cdot {}^{x}\tau_2^{\,2}\left({}^{n_v}k_{Q_f} + {}^{n_v}k_{n_1}\cdot {}^{1}k_{\Sigma Q_f} + {}^{n_v}k_{n_2}\cdot {}^{2}k_{\Sigma Q_f}\right)^{-1} \longrightarrow \min. \qquad / \ 4.6 \ / $$

$$ {}^{Q_f}\tau_1 = \left({}^{n_v}k_{Q_f}\cdot {}^{x}\tau_1 + {}^{n_v}k_{n_1}\cdot {}^{1}k_{\Sigma Q_f}\cdot {}^{1}\tau_{\Sigma Q_f} + {}^{n_v}k_{n_2}\cdot {}^{2}k_{\Sigma Q_f}\cdot {}^{2}\tau_{dQ_f}\right)\cdot $$
$$ \cdot\left({}^{n_v}k_{Q_f} + {}^{n_v}k_{n_1}\cdot {}^{1}k_{\Sigma Q_f} + {}^{n_v}k_{n_2}\cdot {}^{2}k_{\Sigma Q_f}\right)^{-1} \longrightarrow \min. \qquad / \ 4.7 \ / $$

$$ \tau_v = {}^{\alpha_v}k_{n_v}^{-1} \longrightarrow \min. \qquad / \ 4.8 \ / $$

where

$$ {}^{1}k_{\Sigma Q_f} = {}^{1}k_{Q_f} + {}^{n_1}k_{n_2}\cdot {}^{n_2}k_{Q_f} \qquad / \ 4.9 \ / $$

$$ {}^{2}k_{\Sigma Q_f} = {}^{n_2}k_{Q_f} + {}^{n_2}k_{n_1}\cdot {}^{n_1}k_{Q_f}\left(1 - {}^{n_1}k_{n_2}\cdot {}^{n_2}k_{n_1}\right)^{-1} \qquad / \ 4.10 \ / $$

$$ {}^{2}\tau_{dQ_f} = {}^{n_2}k_{Q_f}\,\tau_1\left({}^{n_2}k_{Q_f} + {}^{n_1}k_{n_2}\cdot {}^{n_1}k_{Q_f}\right)^{-1} \qquad / \ 4.11 \ / $$

It is self evident to apply reverse, the other way round procedure and to derive the advisable trends and values of:

- ${}^{3R}M$ matrix coefficients $\Longrightarrow$ the important plant parameters, including control
- form of the K1 and K2 compressor characteristic /analogically it is possible to extend for turbine/

Compendiously and concisely f. e. :
For the urgent need to reduce the degenerative action n_2 on n_1 excepting

$$ {}^{\alpha_1}k_{n_2} \longrightarrow \text{min. and also } {}^{\alpha_1}k_{n_1} \longrightarrow \text{max.} \qquad / \ 4.12 \ / $$

$$ \Longrightarrow \ \uparrow K_{n_1},\ \downarrow K_{24},\ \downarrow K_5,\ \downarrow A_2, \qquad / \ 4.13' / $$

$$ \downarrow I_1,\ \uparrow [\pi_{K2c}]_0,\ \uparrow [\tau_{K2c}]_0, \ \ldots\ldots \qquad / \ 4.14 \ / $$

but also

$$ \downarrow K_{24} = \frac{\bar{n}_{1r}}{\bar{\tau}_{1r}} \Longrightarrow \quad \text{sheerer course} \quad \bar{\pi}_{K2c} = \bar{\pi}_{K2c}(\bar{n}_2) \qquad / \ 4.15 \ / $$

$\downarrow K_5 \Longrightarrow [T_{4c}]_0$ - higher temperature of gases in front of nozzle guide vanes T1

Full analysises of the values and trends bear many antagonismes.

5.0 Conclusion

This paper considers only trivial form of problems. In full extent using of the reverse procedure - analyse of the values and trends, which bear many antagonismes just onder the given conditions, quantitative calculations afford predominant work states for advantageous and reasonable compromises.

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